Complex Langevin analysis

of the spontaneous symmetry breaking in the Euclidean type IIB matrix model

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Acknowledgments

work with

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Takehiro Azuma[b]
Yuta Ito[c]
Jun Nishimura[d]
Toshiyuki Okubo[e]

previous work on $D = 6$ IKKT CLM

[arXiv:1712.07562]

Outline

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The Type IIB string theory

[arXiv:9612115] N. Ishibashi, H. Kawai, Y. Kitazawa, A. Tsuchiya

Type IIB $D = 10$ Green-Schwarz action in the Schild gauge

$$S_{\text{Schild}} = \int d^2\sigma \left(\sqrt{h} g^{-2} \left(\frac{1}{4}\{X^\mu, X^\nu\}^2 - \frac{1}{2}\imath\bar{\psi}\Gamma^\mu\{X^\mu, \psi\} \right) \right)$$

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Matrix $D = 10$ IKKT (Eucleidean) model with fixed N

$$S = S_{\text{bozon}} + S_{\text{fermion}} = -N \ \text{tr} \left(\frac{1}{4} [A_\mu, A_\nu]^2 + \frac{1}{2} (\bar{\psi}_\alpha (\Gamma_\mu)_{\alpha\beta} [A_\mu, \psi_\beta]) \right)$$

$$\{\ldots, \ldots\} \to \imath[\ldots, \ldots], \int d^2\sigma \sqrt{h} \ldots \to \ \text{tr} \ldots \text{ and } g^{-2} = N$$

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$$\{\ldots,\ldots\} \to \imath[\ldots,\ldots], \int d^2\sigma \sqrt{h} \ldots \to \text{ tr} \ldots \text{ and } g^{-2} = N$$

Candidate for non-perturbative definition of Type IIB strings!

The IKKT model

Wick rotation

$A_D = -\imath A_0$ and $\Gamma_D = \imath \Gamma^0$

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Complex action

Integrating out the fermion action in partition function

$$Z = \int \mathcal{D}A \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp(-S) = \int \mathcal{D}A \, \mu(\mathcal{M}) \exp(-S_{\text{boson}})$$

contains a phase, $\mu(\mathcal{M}) \propto \exp(-\imath\Gamma)$, responsible for the SO(D) SSB.

Possible manifestations of the IKKT model for

$D = 4$, no phase exists and thus no SSB
$D = 6$, phase exists with SSB ($\mu \equiv$ determinant)
$D = 10$, phase exists with SSB ($\mu \equiv$ pfaffian)

Conjecture

[arXiv:1108.1293] Jun Nishimura, Toshiyuki Okubo, Fumihiko Sugino

Dynamical compactification of extra dimensions

SSB of original SO(10) to SO(4) by distinction of 4 dimensional extends from the other 6, indicates the compatification of extra dimensions is emergent from the theory itself!

Gaussian Expansion Method on $D = 10$ IKKT

Suggests that SO(10) breaks down to SO(3).

Outline

Monte Carlo

Calculating expectation value integrals,

$$\langle \mathcal{O} \rangle = \frac{\displaystyle\int \mathcal{D}\phi \mathcal{O}[\phi] e^{-S[\phi]}}{\displaystyle\int \mathcal{D}\phi e^{-S[\phi]}}$$

by sampling the x configuration space via markovian chains.

Markovian chains

Driven by a selection probability best defined on the Boltzmann factor e^{-S} containing the action $S : (X \longrightarrow \mathbb{R}) \longrightarrow \mathbb{R}$

Complex Action Problem

If the action $S : (X \longrightarrow \mathbb{R}) \longrightarrow \mathbb{C}$ is complex, e^{-S} is no longer a viable sampling probability!

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Regular Monte Carlo techniques relying on the action S fail

Re-weighting

Using the phase-quenched action S_0 ($S = S_0 - \imath\Gamma$)

A partial solution comes with re-weighting,

$$\langle\mathcal{O}\rangle = \frac{\dfrac{\displaystyle\int \mathcal{D}\phi\mathcal{O}[\phi]e^{\imath\Gamma}e^{-S_0[\phi]}}{\displaystyle\int \mathcal{D}\phi e^{-S_0[\phi]}}}{\dfrac{\displaystyle\int \mathcal{D}\phi e^{\imath\Gamma}e^{-S_0[\phi]}}{\displaystyle\int \mathcal{D}\phi e^{-S_0[\phi]}}} = \frac{\langle\mathcal{O}e^{\imath\Gamma}\rangle_0}{\langle e^{\imath\Gamma}\rangle_0}$$

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The problem is still NP hard!

Outline

Stochastic Quantization

[DOI: 10.1016/0370-1573(87)90144-X] Poul Damgaard, Helmuth Huffel.

Stochastic evolution (simplified)

$$\frac{\partial}{\partial t}\phi(t) = v(\phi(t)) + \eta(t), \qquad \phi(t_0) = \phi_0 : X \longrightarrow \mathbb{R}$$

Drift $v : (X \longrightarrow \mathbb{R}) \longrightarrow \mathbb{R}$

$$v(\phi) = -\frac{\delta S}{\delta \phi}[\phi]$$

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Noise $\eta : X \longrightarrow \mathbb{R}$ obeying gaussian distribution

$$\langle \eta(t)\eta(t')\rangle_{\text{noise}} = 2\delta(t-t') \qquad \varrho_{\text{noise}}(\eta) = \exp\left(-\frac{1}{4}\eta^2\right)$$

Stochastic behaviour

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Properties:

- The solution ϕ is random and depends on noise η,
- Even the initial condition can be random as in $\phi_0 = \eta_0$,
- ϕ obeys a probability distribution $\varrho : (X \to \mathbb{R}) \times \mathbb{R}^+ \to \mathbb{R}$

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Fokker-Planck equation for solution probability

$$\frac{\partial}{\partial t}\varrho(\phi;t) = \mathcal{L}^*(\phi)\varrho(\phi;t) \qquad \varrho(\phi;t_0) = \delta(\phi - \phi_0)$$

EV evolution equation

$$\frac{\partial}{\partial t}\langle \mathcal{O}(t)\rangle_{\text{noise}} = \langle \mathcal{L}\mathcal{O}(t)\rangle_{\text{noise}}$$

Outline

Complexification

[arXiv:1802.01876] K. Nagata, J. Nishimura and S. Shimasaki.

Drift term becomes complex

$$v_a(\phi(x)) = -\frac{\delta S[\phi]}{\delta \phi_a(x)}$$

ϕ is extended to complex for consistency

$$\frac{\partial \phi_a}{\partial t}(t) = v_a(\phi(t)) + \eta_a(t),\ \phi : \mathbb{C}$$

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Noise chosen real (compatibility condition)

$$\langle \eta_a(t)\eta_{a'}(t')\rangle_{\text{noise}} = 2\alpha_{aa'}\delta(t-t') \qquad \alpha = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Validity of the Complex Langevin Method

[arXiv:1802.01876] K. Nagata, J. Nishimura and S. Shimasaki.

EV leading contributions

$$\langle \mathcal{O}(t) \rangle_{\text{noise}} \sim \int_0^\infty e^{(t-t_0)u} p(u,t) du$$

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$$p(u,\tau) \lesssim e^{-\kappa u} \qquad \kappa > 0$$

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Stochastic quantization assertion

$$\lim_{t\to\infty} \langle \mathcal{O}(t) \rangle_{\text{noise}} = \langle \mathcal{O} \rangle_w$$

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Stochastic quantization assertion in CLM

$$\int dA \mathcal{O}[A] \rho(A;t) = \int d\Re A d\Im A \mathcal{O}[\Re A + \imath \Im A] P(\Re A, \Im A; t)$$

Stochastic quantization assertion

$$\lim_{t\to\infty} \langle \mathcal{O}(t) \rangle_{\text{noise}} = \langle \mathcal{O} \rangle_w$$

Discretized Langevin equation

[arXiv:1802.01876] K. Nagata, J. Nishimura and S. Shimasaki.

Discretized time t with n time-steps fixed or variable

$$t = n\Delta t$$

Euler-like discretized Langevin process

$$\phi_a(t+\Delta t) = \phi_a(t) + \Delta t \upsilon_a(\phi(t)) + \sqrt{\Delta t}\eta_a(t)$$

Noise $\eta : \mathbb{R}$ with gaussian distribution noise : $\mathbb{R} \to \mathbb{R}_+$

$$\langle \eta_n \eta_{n'} \rangle_{\text{noise}} = 2\delta_{nn'}$$

Variable stepsize Δt used to contain the drift term.

Outline

IKKT Langevin dynamics

Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura, Toshiyuki Okubo and Stratos Kovalkov Papadoudis.

IKKT Langevin equation

$$\frac{d(A_\mu)_{ij}}{dt} = -\frac{\delta}{\delta(A_\mu)_{ji}} S_{\text{effective}} + (\eta_\mu)_{ij} \quad A_\mu \text{ hermitian and traceless}$$

IKKT effective action

$$S_{\text{effective}} = S_{\text{boson}} - \log \mu(\mathcal{M}) = \frac{1}{4} N [A_\mu, A_\nu]^2 - \log |\mu(\mathcal{M})| + \imath\Gamma$$

IKKT drift term

$$\frac{\delta}{\delta(A_\mu)_{ji}} S_{\text{effective}} = N[[A_\mu A_\nu]A_\nu]_{ij} - \frac{1}{2} \text{tr}\left(\left(\frac{\delta}{\delta(A_\mu)_{ji}} \mathcal{M}\right) \mathcal{M}^{-1}\right)$$

IKKT complexification

Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura, Toshiyuki Okubo and Stratos Kovalkov Papadoudis.

Complexification of A_μ:

- A_μ no longer hermitian, just traceless,
  - Still, the closer it is to hermitian the better,
  - SU(N) (internal) matrix gauge symmetry becomes SL(N).
- Noise η_μ remains hermitian and traceless.

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Observables $\mathcal{O}$ need to be holomorphic extensions of their real counter-part.

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IKKT Langevin problems

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Large excursions into the imaginary direction

Counter with gauge cooling: minimize the norm $\|A - A^{\dagger}\|$ under the SU(N) gauge freedom

Singular drift

Counter with shifting fermion matrix $\mathcal{M}$ off the origin:

$$\Delta S_{\text{fermion}} = \imath N m_{\text{fermion}} \text{tr}(\bar{\psi}_{\alpha}(\Gamma_8 \Gamma_9^{\dagger} \Gamma_{10})_{\alpha\beta}\psi_{\beta})$$

Manifestly breaks SO(10) $\longrightarrow$ SO(7)

m_{fermion} mass shift deformation (interpolation)

$m_{\text{fermion}} \to 0$ represents the true SYM (dimensionally reduced) theory.
$m_{\text{fermion}} \to \infty$ represents the pure bosonic (fermion-less) SYM theory.

SO(D) SSB probing

Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura, Toshiyuki Okubo and Stratos Kovalkov Papadoudis.

SSB order parameter and action term

$$\lambda_\mu = N^{-1}\ \mathrm{tr}(A_\mu)^2 \qquad\qquad \Delta S_{\text{boson}} = \frac{1}{2}N^2\epsilon\sum_\mu m_\mu\lambda_\mu$$

Emergent observable ordering:

- $\lambda_\mu : \mathbb{C}$ but $\langle\lambda_\mu\rangle : \mathbb{R}$ drift symmetry $A_\mu \longleftrightarrow \eta_{\mu\nu}(A_\nu)^\dagger$,
- For finite N, $\langle\lambda_\mu\rangle$ are ordered like m_μ: $\langle\lambda_1\rangle \geq \ldots \geq \langle\lambda_{10}\rangle$,
- For $N \to \infty$, $\langle\lambda_\mu\rangle$ are grouped together based on which subsymmetries survive.

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SO(2) minimum symmetry assumed

$$m_\mu = \left(\ \mathbf{2^{-1}}\quad \mathbf{2^{-1}}\quad 2^0\quad 2^1\quad 2^2\quad \mathbf{2^3}\quad \mathbf{2^3}\quad \mathbf{2^3}\quad \mathbf{2^3}\quad \mathbf{2^3}\ \right)$$

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Spacetime extend observables

Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura, Toshiyuki Okubo and Stratos Kovalkov Papadoudis.

SSB computed observables

$$\rho_\mu(m_{\text{fermion}}, \epsilon, N) = \frac{\langle \lambda_\mu \rangle}{\sum_\nu \langle \lambda_\nu \rangle}$$

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$$m_\mu = \begin{pmatrix} \mathbf{2^{-1}} & \mathbf{2^{-1}} & 2^0 & 2^1 & 2^2 & \mathbf{2^3} & \mathbf{2^3} & \mathbf{2^3} & \mathbf{2^3} & \mathbf{2^3} \end{pmatrix}$$

$$\frac{1}{2}\langle \lambda_1 + \lambda_2 \rangle, \langle \lambda_3 \rangle, \langle \lambda_4 \rangle, \langle \lambda_5 \rangle, \frac{1}{5}\langle \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} \rangle$$

$$\rho_1 = \rho_2 \geq \rho_3 \geq \rho_4 \geq \rho_5 \geq \rho_6 = \rho_7 = \rho_8 = \rho_9 = \rho_{10}$$

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Spacetime extend parameters

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Large N limit

$\lim_{N\to\infty} \rho_\mu(m_{\text{fermion}}, \epsilon, N) = \rho_\mu(m_{\text{fermion}}, \epsilon)$

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Order parameter expectation values

$$\rho_\mu(m_{\text{fermion}}, \epsilon) = \rho_\mu(m_{\text{fermion}}) + \sum_{k=1}^{n} b_k(m_{\text{fermion}})\epsilon^k$$

Large N limit

$\lim_{N\to\infty} \rho_\mu(m_{\text{fermion}}, \epsilon, N) = \rho_\mu(m_{\text{fermion}}, \epsilon)$

SSB limit

$\lim_{\varepsilon\to 0} \rho_\mu(m_{\text{fermion}}, \epsilon) = \rho_\mu(m_{\text{fermion}})$

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Original model limit

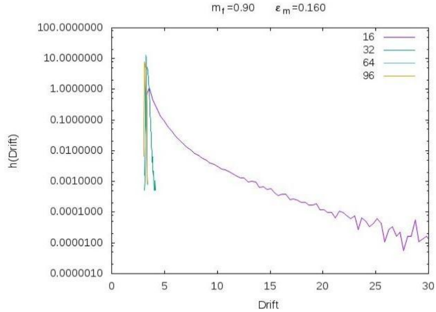
$\lim_{m_{\text{fermion}}\to 0} \rho_\mu(m_{\text{fermion}}) = \rho_\mu$

Outline

IKKT drift norm u

Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura, Toshiyuki Okubo and Stratos Kovalkov Papadoudis.

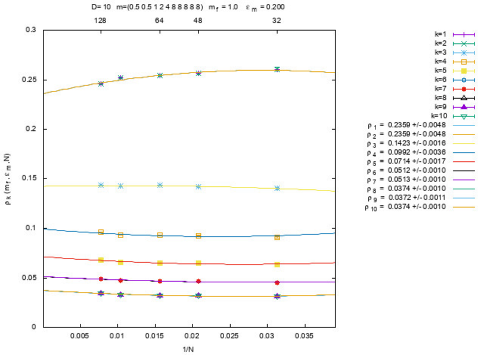
IKKT drift norm histogram: $u^2 = \frac{1}{DN^3} \sum_{\mu} \sum_{ij} \left| \frac{\partial S}{\partial (A_\mu)_{ji}} \right|^2$



IKKT order parameters $N \to \infty$ (example)

Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura, Toshiyuki Okubo and Stratos Kovalkov Papadoudis.

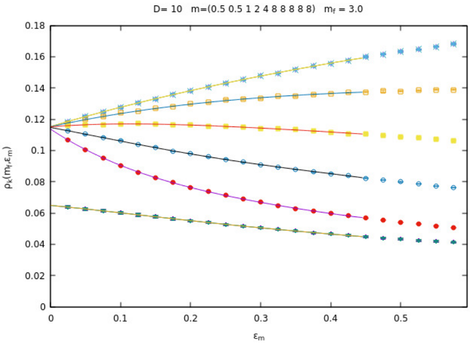
$$\rho_\mu(m_{\text{fermion}}, \epsilon, N) = \rho_\mu(m_{\text{fermion}}, \epsilon) + a_\mu^{(1)} N^{-1} + a_\mu^{(2)} N^{-2}$$



IKKT order parameters $\varepsilon \to 0$, $m_{\text{fermion}} = 3.0$

Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura, Toshiyuki Okubo and Stratos Kovalkov Papadoudis.

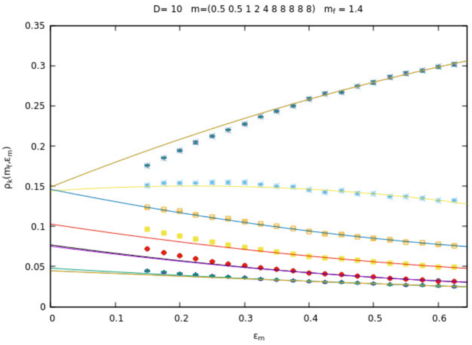
$$\rho_\mu(m_{\text{fermion}}, \epsilon) = \rho_\mu(m_{\text{fermion}}) + b_\mu^{(1)}\epsilon + b_\mu^{(2)}\epsilon^2 + b_\mu^{(3)}\epsilon^3 + b_\mu^{(4)}\epsilon^4$$



IKKT order parameters $\varepsilon \to 0$, $m_{\text{fermion}} = 1.4$

Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura, Toshiyuki Okubo and Stratos Kovalkov Papadoudis.

$$\rho_\mu(m_{\text{fermion}}, \epsilon) = \rho_\mu(m_{\text{fermion}}) + b_\mu^{(1)}\epsilon + b_\mu^{(2)}\epsilon^2$$



IKKT order parameters $\varepsilon \to 0$, $m_{\text{fermion}} = 1.0$

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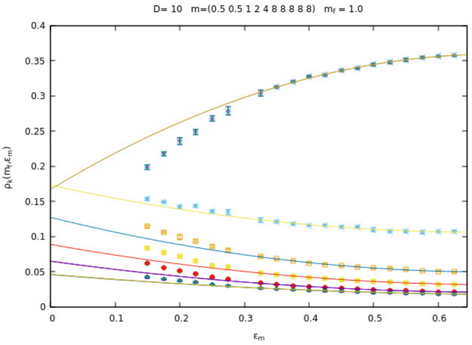
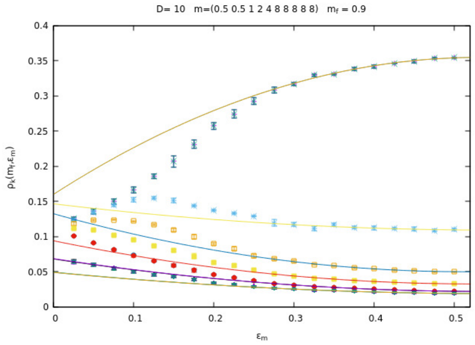


IKKT order parameters $\varepsilon \to 0$, $m_{\text{fermion}} = 0.9$

Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura, Toshiyuki Okubo and Stratos Kovalkov Papadoudis.

$$\rho_\mu(m_{\text{fermion}}, \epsilon) = \rho_\mu(m_{\text{fermion}}) + b_\mu^{(1)}\epsilon + b_\mu^{(2)}\epsilon^2$$



IKKT order parameters $\varepsilon \to 0$, $m_{\text{fermion}} = 0.7$

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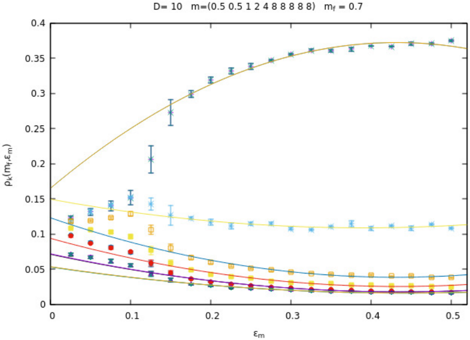
$$\rho_\mu(m_{\text{fermion}}, \epsilon) = \rho_\mu(m_{\text{fermion}}) + b_\mu^{(1)}\epsilon + b_\mu^{(2)}\epsilon^2$$



Summary

Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura, Toshiyuki Okubo and Stratos Kovalkov Papadoudis.

Conclusions:

Outlook:

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Spontaneous Symmetry Breaking of the rotational symmetry of the euclidean $D = 10$ IKKT model was observed.

At $N \to \infty$, $\varepsilon \to 0$, as m_{fermion} decreases, symmetry seems to breakdown to SO(3) as indicated also by the GEM result.

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Study of the Lorentzian signature IKKT model (true reduced Type IIB superstring model)

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Further exploration of model variations, as the SSB seems to be present.

∼ The End? ∼