EXPONENTIALLY SUPPRESSED COSMOLOGICAL CONSTANT WITH ENHANCED GAUGE SYMMETRY IN HETEROTIC INTERPOLATING MODELS

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Introduction

When top-down approach from string theory is considered, there are two choices depending on where SUSY breaking scale is ;

1. **SUSY is broken at low energy in supersymmetric EFT ;**
2. **SUSY is already broken at high energy like string/Planck scale.**

In this talk, the second one is focused on, and non-supersymmetric string models are considered.

In particular, **the $SO(16)\times SO(16)$ model** is a unique **tachyon-free** non-supersymmetric string model in ten-dimensions.

[Dixon, Hervey, (1986)]

Introduction

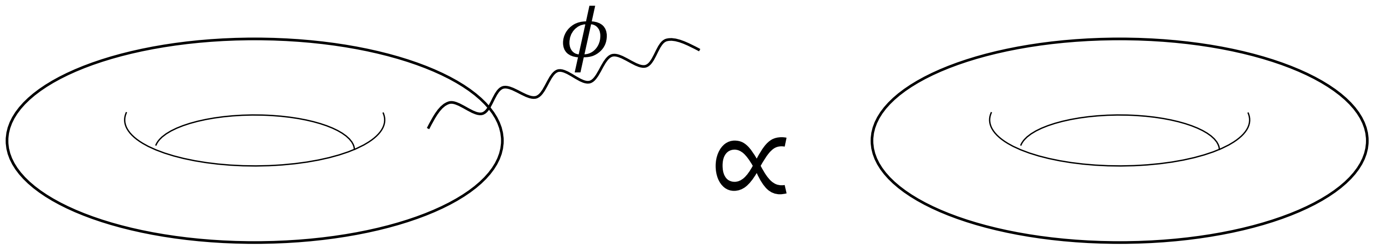
Considering non-supersymmetric string models, however, we face with the problem of vacuum instability arising from nonzero dilaton tadpoles;

$V(\phi)$: dilaton tadpole

$$V(\phi) \propto \Lambda$$

Λ : cosmological constant
(vacuum energy)

At 1-loop level,



The desired model is a non-supersymmetric one whose **cosmolosical constant is vanishing or very small.**

Interpolating models have the possibility of such properties.

Outline

1. Introduction
2. 9D Interpolating models
3. 9D Interpolating models with Wilson line
4. Summary

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2. **9D Interpolating models**
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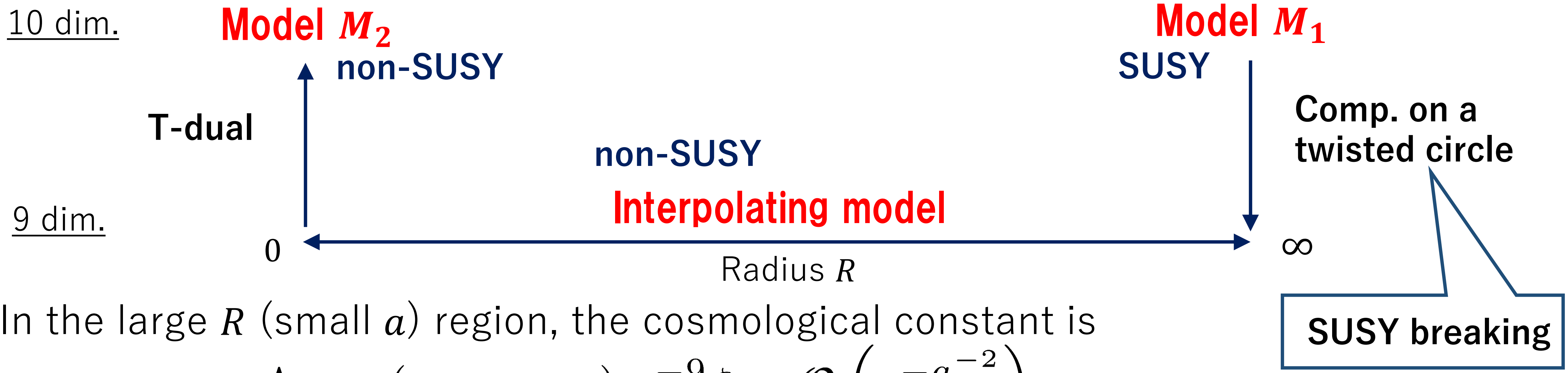
Interpolation between SUSY and non-SUSY

An interpolating model is a lower dimensional string model relating two different higher dimensional string models continuously.



In the large R (small a) region, the cosmological constant is

$$\Lambda_9 \simeq (n_F - n_B)\, a^{-9}\xi + \mathcal{O}\left(e^{-a^{-2}}\right)$$

$\left(a = \sqrt{\alpha'}/R, \quad \xi > 0, \quad n_F, \; n_B \right.$: **# of massless fermions, bosons** $\left. \right)$

If $n_F = n_B$, the cosmological constant is exponentially suppressed.

[Itoyama, Taylor, (1987)]

Interpolation between $SO(32)$ and $SO(16)\times SO(16)$

- **The one-loop partition function**

$$
\begin{aligned}
Z_{\rm int}^{(9)} = Z_B^{(7)} \{ &\Lambda_{0,0}\left[\bar{V}_8\left(O_{16}O_{16}+S_{16}S_{16}\right)-\bar{S}_8\left(V_{16}V_{16}+C_{16}C_{16}\right)\right] \\
&+\Lambda_{1/2,0}\left[\bar{V}_8\left(V_{16}V_{16}+C_{16}C_{16}\right)-\bar{S}_8\left(O_{16}O_{16}+S_{16}S_{16}\right)\right] \\
&+\Lambda_{0,1/2}\left[\bar{O}_8\left(V_{16}C_{16}+C_{16}V_{16}\right)-\bar{C}_8\left(O_{16}S_{16}+S_{16}O_{16}\right)\right] \\
&+\Lambda_{1/2,1/2}\left[\bar{O}_8\left(O_{16}S_{16}+S_{16}O_{16}\right)-\bar{C}_8\left(V_{16}C_{16}+C_{16}V_{16}\right)\right] \}
\end{aligned}
$$

$$
\Lambda_{\alpha,\beta} = (\bar{\eta}\eta)^{-1}\sum_{n,w}\bar{q}^{\alpha' p_R^2/2} q^{\alpha' p_L^2/2} = (\bar{\eta}\eta)^{-1}\sum_{n,w}\exp\left[2\pi i n w \tau_1 - \pi\tau_2\left(n^2a^2 + w^2/a^2\right)\right]
$$

where the sum is taken over $n \in 2(\boldsymbol{Z}+\alpha),\quad w \in \boldsymbol{Z}+\beta$

- $R \to \infty$: contribution from the zero winding # only $\quad\Rightarrow\quad$ $\Lambda_{\alpha,0} \to (2a)^{-1} Z_B^{(1)},\quad \Lambda_{\alpha,1/2} \to 0$

- $R \to 0$: contribution from the zero momentum only $\quad\Rightarrow\quad$ $\Lambda_{0,\beta} \to a Z_B^{(1)},\quad \Lambda_{1/2,\beta} \to 0$

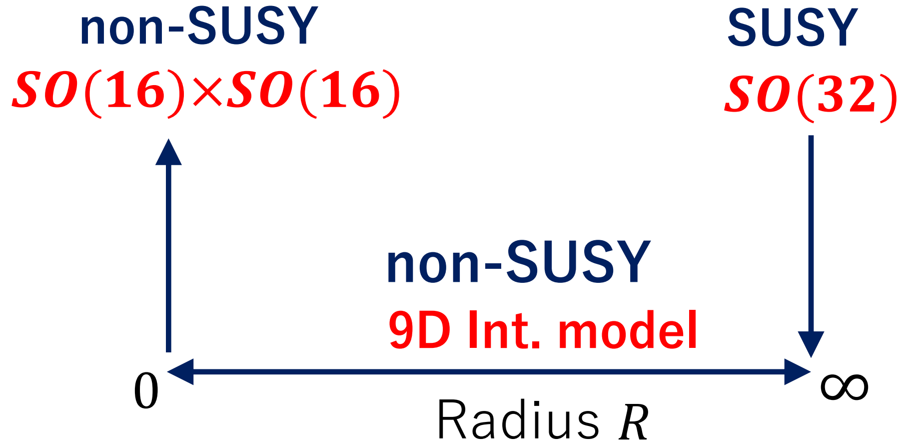
Interpolation between $SO(32)$ and $SO(16)\times SO(16)$

- **The one-loop partition function**

$$
\begin{aligned}
Z_{\mathrm{int}}^{(9)} = Z_B^{(7)} \Big\{ & \Lambda_{0,0}\left[\bar{V}_8\left(O_{16}O_{16}+S_{16}S_{16}\right)-\bar{S}_8\left(V_{16}V_{16}+C_{16}C_{16}\right)\right] \\
& +\Lambda_{1/2,0}\left[\bar{V}_8\left(V_{16}V_{16}+C_{16}C_{16}\right)-\bar{S}_8\left(O_{16}O_{16}+S_{16}S_{16}\right)\right] \\
& +\Lambda_{0,1/2}\left[\bar{O}_8\left(V_{16}C_{16}+C_{16}V_{16}\right)-\bar{C}_8\left(O_{16}S_{16}+S_{16}O_{16}\right)\right] \\
& +\Lambda_{1/2,1/2}\left[\bar{O}_8\left(O_{16}S_{16}+S_{16}O_{16}\right)-\bar{C}_8\left(V_{16}C_{16}+C_{16}V_{16}\right)\right] \Big\}
\end{aligned}
$$

- **The limiting cases**
  - $R\to\infty$: the 1st and 2nd lines survive
  - $R\to 0$: the 1st and 3rd lines survive

$$
Z_{\mathrm{int}}^{(9)} \ \text{realizes} \ \begin{cases} \text{SUSY } SO(32) \text{ model in } R\to\infty \\ SO(16)\times SO(16) \text{ model in } R\to 0 \end{cases}
$$



Interpolation between $SO(32)$ and $SO(16)\times SO(16)$

- **Massless spectrum**

at generic R, massless states come from $n=w=0$ part

$$
\begin{aligned}
Z_{\text{int}}^{(9)} = Z_B^{(7)} \Big\{ & \underline{\Lambda_{0,0}\left[\bar{V}_8\left(O_{16}O_{16}+S_{16}S_{16}\right)-\bar{S}_8\left(V_{16}V_{16}+C_{16}C_{16}\right)\right]} \\
&+\Lambda_{1/2,0}\left[\bar{V}_8\left(V_{16}V_{16}+C_{16}C_{16}\right)-\bar{S}_8\left(O_{16}O_{16}+S_{16}S_{16}\right)\right] \\
&+\Lambda_{0,1/2}\left[\bar{O}_8\left(V_{16}C_{16}+C_{16}V_{16}\right)-\bar{C}_8\left(O_{16}S_{16}+S_{16}O_{16}\right)\right] \\
&+\Lambda_{1/2,1/2}\left[\bar{O}_8\left(O_{16}S_{16}+S_{16}O_{16}\right)-\bar{C}_8\left(V_{16}C_{16}+C_{16}V_{16}\right)\right]\Big\}
\end{aligned}
$$

Massless bosons

- **9-dim. graviton, anti-symmetric tensor, dilaton:** $g_{\mu\nu},\quad B_{\mu\nu},\quad \phi$
- **Gauge bosons in adj rep of $SO(16)\times SO(16)\times U_{G,B}^2(1)$**

$g_{9\mu},\, B_{9\mu}$

Massless fermions

- $\mathbf{8}_S \otimes (\mathbf{16},\mathbf{16})$

$$n_F - n_B = 64$$

Interpolation between $E_8 \times E_8$ and $SO(16) \times SO(16)$

- **The one-loop partition function**

$$
\begin{aligned}
Z_{\text{int}}^{(9)} = Z_B^{(7)} \{ & \Lambda_{0,0} \left[\bar{V}_8 \left(O_{16} O_{16} + S_{16} S_{16} \right) - \bar{S}_8 \left(O_{16} S_{16} + S_{16} O_{16} \right) \right] \\
& + \Lambda_{1/2,0} \left[\bar{V}_8 \left(O_{16} S_{16} + S_{16} O_{16} \right) - \bar{S}_8 \left(O_{16} O_{16} + S_{16} S_{16} \right) \right] \\
& + \Lambda_{0,1/2} \left[\bar{O}_8 \left(V_{16} C_{16} + C_{16} V_{16} \right) - \bar{C}_8 \left(V_{16} V_{16} + C_{16} C_{16} \right) \right] \\
& + \Lambda_{1/2,1/2} \left[\bar{O}_8 \left(V_{16} V_{16} + C_{16} C_{16} \right) - \bar{C}_8 \left(V_{16} C_{16} + C_{16} V_{16} \right) \right] \}
\end{aligned}
$$

Interpolation between $E_8 \times E_8$ and $SO(16) \times SO(16)$

- **Massless spectrum**

at generic R, massless states come from $n=w=0$ part

$$Z_{\text{int}}^{(9)} = Z_B^{(7)} \left\{ \underline{\Lambda_{0,0} \left[\bar{V}_8 \left(O_{16}O_{16} + S_{16}S_{16} \right) - \bar{S}_8 \left(O_{16}S_{16} + S_{16}O_{16} \right) \right]} \right.$$
$$+ \Lambda_{1/2,0} \left[\bar{V}_8 \left(O_{16}S_{16} + S_{16}O_{16} \right) - \bar{S}_8 \left(O_{16}O_{16} + S_{16}S_{16} \right) \right]$$
$$+ \Lambda_{0,1/2} \left[\bar{O}_8 \left(V_{16}C_{16} + C_{16}V_{16} \right) - \bar{C}_8 \left(V_{16}V_{16} + C_{16}C_{16} \right) \right]$$
$$\left. + \Lambda_{1/2,1/2} \left[\bar{O}_8 \left(V_{16}V_{16} + C_{16}C_{16} \right) - \bar{C}_8 \left(V_{16}C_{16} + C_{16}V_{16} \right) \right] \right\}$$

Massless bosons

- 9-dim. graviton, anti-symmetric tensor, dilaton: $g_{\mu\nu}, \quad B_{\mu\nu}, \quad \phi$
- Gauge bosons in adj rep of $SO(16) \times SO(16) \times \underline{U_{G,B}^2(1)}$

$g_{9\mu}, B_{9\mu}$

Massless fermions

- $\mathbf{8}_S \otimes ((\mathbf{128}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{128}))$

$$n_F - n_B = 64$$

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Boost on momentum lattice

- Considering d-dimensional compactification, **the boost in the momentum lattice corresponds to putting massless constant backgrounds**, that is, adding the following term to the worldsheet action

$$C_{Aa}\int d^2z\,\partial X_L^A\bar{\partial}X_R^a \qquad \begin{pmatrix} a=10-d,\cdots,9 \\ A=(a,I)=10-d,\cdots,26 \end{pmatrix}$$

C_{ba}: metric and antisymmetric tensor, C_{Ia}: $U(1)^{16}$ gauge fields (WL)

[Narain, Sarmadi, Witten, (1986)]

- The d-dimensional compactifications are classified by the transformation

$$\frac{SO(16+d,d)}{SO(16+d)\times SO(d)},$$

whose DOF agree with that of C_{Aa}.

- In this work, we considered one-dimensional compactification and put a single WL background $A=C_{I=1,a=9}$ for simplicity.

Boost on momentum lattice

After turning on WL, the momenta of $X_L^{I=1}$, $X_L^{a=9}$ and $X_R^{a=9}$ are changed as

$$\begin{cases} l_L = \frac{1}{\sqrt{\alpha'}} m \\ p_L = \frac{1}{\sqrt{2\alpha'}} \left(an + \frac{w}{a} \right) \\ p_R = \frac{1}{\sqrt{2\alpha'}} \left(an - \frac{w}{a} \right) \end{cases} \xrightarrow{\text{boost and rotation}} \begin{cases} l'_L = \frac{1}{\sqrt{2\alpha'}} \left(\sqrt{2} m - 2 \frac{A}{\sqrt{1+A^2}} \frac{w}{a} \right) \\ p'_L = \frac{1}{\sqrt{2\alpha'}} \left(\sqrt{2} A m + \sqrt{1+A^2} an - \frac{1-A^2}{\sqrt{1+A^2}} \frac{w}{a} \right) \\ p'_R = \frac{1}{\sqrt{2\alpha'}} \left(\sqrt{2} A m + \sqrt{1+A^2} an - \sqrt{1+A^2} \frac{w}{a} \right) \end{cases}$$

l_L is the left-moving momentum of $X_L^{I=1}$

The effective change in the one-loop partition function is

$$\Lambda_{\alpha,\beta}(a) \left(\frac{\vartheta \begin{bmatrix} \gamma \\ \delta \end{bmatrix}}{\eta} \right)^8 \xrightarrow{\text{Turning on WL}} \Lambda^{(\alpha,\beta)}_{(\gamma,\delta)}(a, A) \left(\frac{\vartheta \begin{bmatrix} \gamma \\ \delta \end{bmatrix}}{\eta} \right)^7$$

$$\Lambda^{(\alpha,\beta)}_{(\gamma,\delta)} = (\bar{\eta}\eta)^{-1} \eta^{-1} \sum_{n,w,m} (-1)^{2\delta m} q^{\frac{\alpha'}{2}(l'^2_L + p'^2_L)} \bar{q}^{\frac{\alpha'}{2} p'^2_R} \qquad n \in 2(\boldsymbol{Z} + \alpha), \;\; w \in \boldsymbol{Z} + \beta, \;\; m \in \boldsymbol{Z} + \gamma$$

The fundamental region of moduli space

Do all the points in moduli space correspond to different models?

It is convenient to introduce a modular parameter $\tilde{\tau}$ as

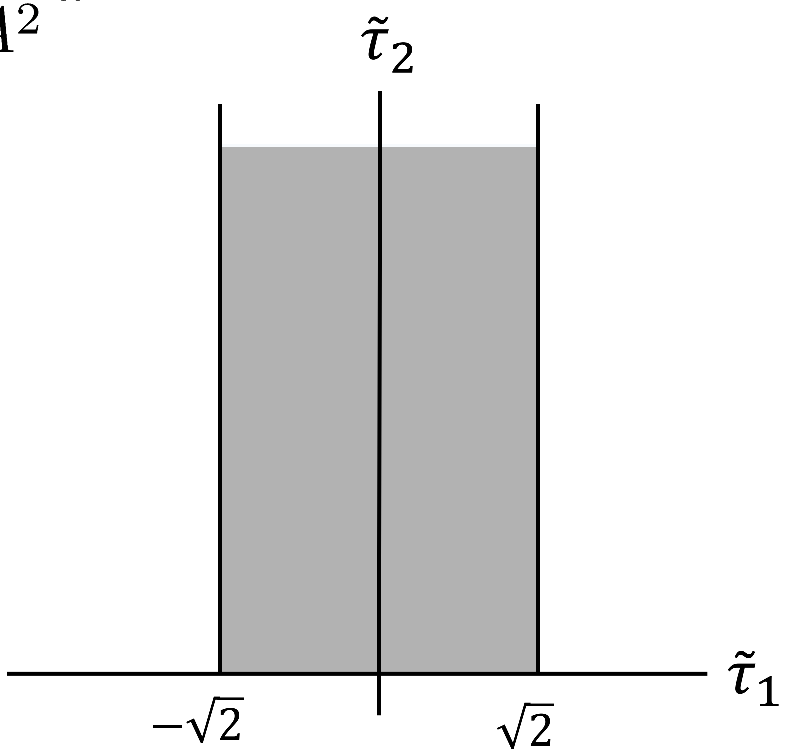
$$\tilde{\tau} = \tilde{\tau}_1 + i\tilde{\tau}_2 = \frac{A}{\sqrt{1+A^2}} a^{-1} + i \frac{1}{\sqrt{1+A^2}} a^{-1}$$

The momentum lattice $\Lambda^{(\alpha,\beta)}_{(\gamma,\delta)}$ is invariant under the shift

$$\tilde{\tau} \to \tilde{\tau} + 2\sqrt{2}$$

The fundamental region of moduli space is

$$-\sqrt{2} \leq \tilde{\tau}_1 \leq \sqrt{2}$$



Interpolation between $SO(32)$ and $SO(16)\times SO(16)$ with WL

- **The one-loop partition function**

$$
\begin{aligned}
Z_{\text{int}}^{(9)} = Z_B^{(7)} \Big\{ & \bar{V}_8 \left(O_{16}^{(0,0)} O_{16} + S_{16}^{(0,0)} S_{16}\right) - \bar{S}_8 \left(V_{16}^{(0,0)} V_{16} + C_{16}^{(0,0)} C_{16}\right) \\
& + \bar{V}_8 \left(V_{16}^{(1/2,0)} V_{16} + C_{16}^{(1/2,0)} C_{16}\right) - \bar{S}_8 \left(O_{16}^{(1/2,0)} O_{16} + S_{16}^{(1/2,0)} S_{16}\right) \\
& + \bar{O}_8 \left(V_{16}^{(0,1/2)} C_{16} + C_{16}^{(0,1/2)} V_{16}\right) - \bar{C}_8 \left(O_{16}^{(0,1/2)} S_{16} + S_{16}^{(0,1/2)} O_{16}\right) \\
& + \bar{O}_8 \left(O_{16}^{(1/2,1/2)} S_{16} + S_{16}^{(1/2,1/2)} O_{16}\right) - \bar{C}_8 \left(V_{16}^{(1/2,1/2)} C_{16} + C_{16}^{(1/2,1/2)} V_{16}\right) \Big\}
\end{aligned}
$$

$$
\begin{pmatrix} O_{16}^{(\alpha,\beta)} \\ V_{16}^{(\alpha,\beta)} \end{pmatrix} = \frac{1}{2\eta^7}\left(\Lambda_{(0,0)}^{(\alpha,\beta)} \vartheta \begin{bmatrix} 0 \\ 0 \end{bmatrix}^7 \pm \Lambda_{(0,1/2)}^{(\alpha,\beta)} \vartheta \begin{bmatrix} 0 \\ 1/2 \end{bmatrix}^7 \right) \qquad
\begin{pmatrix} S_{16}^{(\alpha,\beta)} \\ C_{16}^{(\alpha,\beta)} \end{pmatrix} = \frac{1}{2\eta^7}\left(\Lambda_{(1/2,0)}^{(\alpha,\beta)} \vartheta \begin{bmatrix} 1/2 \\ 0 \end{bmatrix}^7 \pm \Lambda_{(1/2,1/2)}^{(\alpha,\beta)} \vartheta \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}^7 \right)
$$

- $R \to \infty$: $\left(O_{16}^{(\alpha,\beta)}, V_{16}^{(\alpha,\beta)}, S_{16}^{(\alpha,\beta)}, C_{16}^{(\alpha,\beta)}\right) \longrightarrow \frac{\sqrt{\alpha'}}{2r_\infty} Z_B^{(1)} \left(O_{16}, V_{16}, S_{16}, C_{16}\right)\delta_{\beta,0}$

- $R \to 0$: $\left(O_{16}^{(\alpha,\beta)}, V_{16}^{(\alpha,\beta)}, S_{16}^{(\alpha,\beta)}, C_{16}^{(\alpha,\beta)}\right) \longrightarrow \sqrt{\alpha'} r_0 Z_B^{(1)} \left(O_{16}, V_{16}, S_{16}, C_{16}\right)\delta_{\alpha,0}$

$$
r_\infty = \frac{R}{\sqrt{1+A^2}} \qquad r_0 = \sqrt{1+A^2}\, R
$$

Interpolation between $SO(32)$ and $SO(16)\times SO(16)$ with WL

- **The one-loop partition function**

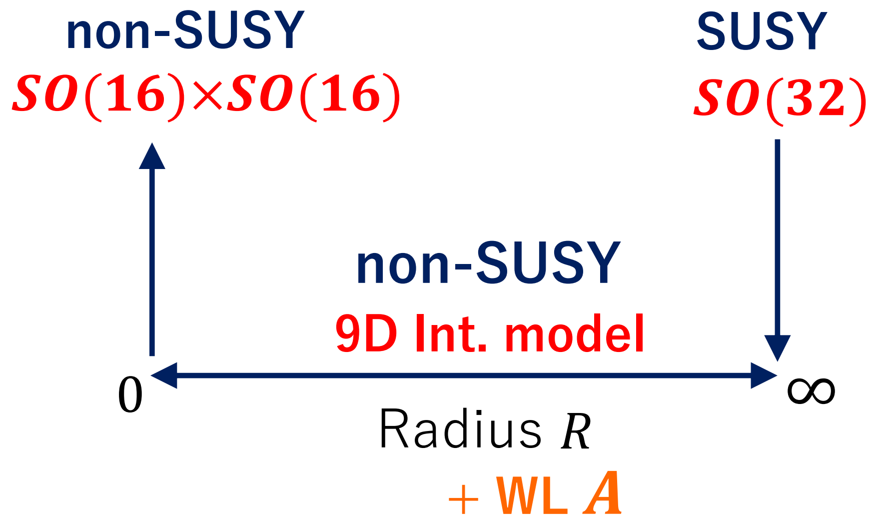
$$
\begin{aligned}
Z_{\text{int}}^{(9)} = Z_B^{(7)} \Big\{ & \bar{V}_8 \left(O_{16}^{(0,0)} O_{16} + S_{16}^{(0,0)} S_{16} \right) - \bar{S}_8 \left(V_{16}^{(0,0)} V_{16} + C_{16}^{(0,0)} C_{16} \right) \\
& + \bar{V}_8 \left(V_{16}^{(1/2,0)} V_{16} + C_{16}^{(1/2,0)} C_{16} \right) - \bar{S}_8 \left(O_{16}^{(1/2,0)} O_{16} + S_{16}^{(1/2,0)} S_{16} \right) \\
& + \bar{O}_8 \left(V_{16}^{(0,1/2)} C_{16} + C_{16}^{(0,1/2)} V_{16} \right) - \bar{C}_8 \left(O_{16}^{(0,1/2)} S_{16} + S_{16}^{(0,1/2)} O_{16} \right) \\
& + \bar{O}_8 \left(O_{16}^{(1/2,1/2)} S_{16} + S_{16}^{(1/2,1/2)} O_{16} \right) - \bar{C}_8 \left(V_{16}^{(1/2,1/2)} C_{16} + C_{16}^{(1/2,1/2)} V_{16} \right) \Big\}
\end{aligned}
$$

- **The limiting cases**
  - $R \to \infty$: the 1st and 2nd lines survive
  - $R \to 0$: the 1st and 3rd lines survive

For any WL A,

$Z_{\text{int}}^{(9)}$ realizes

$$
\begin{cases}
\text{SUSY } SO(32) \text{ model in } R \to \infty \\
SO(16)\times SO(16) \text{ model in } R \to 0
\end{cases}
$$



Interpolation between $SO(32)$ and $SO(16)\times SO(16)$ with WL

- **Massless spectrum**   **at generic *R*, massless states come from *n=w=0* part**

$$
\begin{aligned}
Z_{\text{int}}^{(9)} = Z_B^{(7)} \Big\{ & \underline{\bar{V}_8\left(O_{16}^{(0,0)}O_{16} + S_{16}^{(0,0)}S_{16}\right) - \bar{S}_8\left(V_{16}^{(0,0)}V_{16} + C_{16}^{(0,0)}C_{16}\right)} \\
& + \bar{V}_8\left(V_{16}^{(1/2,0)}V_{16} + C_{16}^{(1/2,0)}C_{16}\right) - \bar{S}_8\left(O_{16}^{(1/2,0)}O_{16} + S_{16}^{(1/2,0)}S_{16}\right) \\
& + \bar{O}_8\left(V_{16}^{(0,1/2)}C_{16} + C_{16}^{(0,1/2)}V_{16}\right) - \bar{C}_8\left(O_{16}^{(0,1/2)}S_{16} + S_{16}^{(0,1/2)}O_{16}\right) \\
& + \bar{O}_8\left(O_{16}^{(1/2,1/2)}S_{16} + S_{16}^{(1/2,1/2)}O_{16}\right) - \bar{C}_8\left(V_{16}^{(1/2,1/2)}C_{16} + C_{16}^{(1/2,1/2)}V_{16}\right)\Big\}
\end{aligned}
$$

Massless bosons

- **9-dim. graviton, anti-symmetric tensor, dilaton:** $g_{\mu\nu},\ B_{\mu\nu},\ \phi$
- **Gauge bosons in adj rep of $SO(16)\times SO(14)\times U(1)\times U_{G,B}^{2}(1)$**

Massless fermions

- $8_S \otimes (\mathbf{16}, \mathbf{14})$

$$n_F - n_B = 32$$

Interpolation between $SO(32)$ and $SO(16)\times SO(16)$ with WL

- **Massless spectrum** $\exists$ a few conditions under which the additional massless states appear

$$\begin{aligned}Z^{(9)}_{\text{int}} = Z^{(7)}_{B}\Big\{&\bar{V}_8\left(O^{(0,0)}_{16}O_{16}+S^{(0,0)}_{16}S_{16}\right)-\bar{S}_8\left(V^{(0,0)}_{16}V_{16}+C^{(0,0)}_{16}C_{16}\right)\\&+\bar{V}_8\left(V^{(1/2,0)}_{16}V_{16}+C^{(1/2,0)}_{16}C_{16}\right)-\bar{S}_8\left(O^{(1/2,0)}_{16}O_{16}+S^{(1/2,0)}_{16}S_{16}\right)\\&+\bar{O}_8\left(V^{(0,1/2)}_{16}C_{16}+C^{(0,1/2)}_{16}V_{16}\right)-\bar{C}_8\left(O^{(0,1/2)}_{16}S_{16}+S^{(0,1/2)}_{16}O_{16}\right)\\&+\bar{O}_8\left(O^{(1/2,1/2)}_{16}S_{16}+S^{(1/2,1/2)}_{16}O_{16}\right)-\bar{C}_8\left(V^{(1/2,1/2)}_{16}C_{16}+C^{(1/2,1/2)}_{16}V_{16}\right)\Big\}\end{aligned}$$

condition ① $\quad \tilde{\tau}_1 = n_1/\sqrt{2} \qquad n_1 \in 2\boldsymbol{Z}$

new massless states： • two $\mathbf{8}_V \otimes (\mathbf{1},\mathbf{14})$ • two $\mathbf{8}_S \otimes (\mathbf{16},\mathbf{1})$

$$\Rightarrow \begin{cases} SO(16)\times SO(14)\times U(1) \longrightarrow SO(16)\times SO(16) \\ \mathbf{8}_S \otimes (\mathbf{16},\mathbf{14}) \longrightarrow \mathbf{8}_S \otimes (\mathbf{16},\mathbf{16}) \end{cases}$$

$$n_F - n_B = 64$$

Interpolation between $SO(32)$ and $SO(16)\times SO(16)$ with WL

- **Massless spectrum** ∃ a few conditions under which the additional massless states appear

$$Z_{\rm int}^{(9)} = Z_B^{(7)} \Big\{ \bar{V}_8 \left(O_{16}^{(0,0)} O_{16} + S_{16}^{(0,0)} S_{16} \right) - \bar{S}_8 \left(V_{16}^{(0,0)} V_{16} + C_{16}^{(0,0)} C_{16} \right)$$
$$+ \bar{V}_8 \left(V_{16}^{(1/2,0)} V_{16} + C_{16}^{(1/2,0)} C_{16} \right) - \bar{S}_8 \left(O_{16}^{(1/2,0)} O_{16} + S_{16}^{(1/2,0)} S_{16} \right)$$
$$+ \bar{O}_8 \left(V_{16}^{(0,1/2)} C_{16} + C_{16}^{(0,1/2)} V_{16} \right) - \bar{C}_8 \left(O_{16}^{(0,1/2)} S_{16} + S_{16}^{(0,1/2)} O_{16} \right)$$
$$+ \bar{O}_8 \left(O_{16}^{(1/2,1/2)} S_{16} + S_{16}^{(1/2,1/2)} O_{16} \right) - \bar{C}_8 \left(V_{16}^{(1/2,1/2)} C_{16} + C_{16}^{(1/2,1/2)} V_{16} \right) \Big\}$$

condition ② $\tilde{\tau}_1 = n_2/\sqrt{2} \quad n_2 \in 2\mathbf{Z}+1$

new massless states： • two $\mathbf{8}_V \otimes (\mathbf{16},\mathbf{1})$ • two $\mathbf{8}_S \otimes (\mathbf{1},\mathbf{14})$

$$\begin{cases} SO(16)\times SO(14)\times U(1) \longrightarrow SO(18)\times SO(14) \\ \mathbf{8}_S \otimes (\mathbf{16},\mathbf{14}) \longrightarrow \mathbf{8}_S \otimes (\mathbf{18},\mathbf{14}) \end{cases}$$

$$\boxed{n_F - n_B = 0}$$

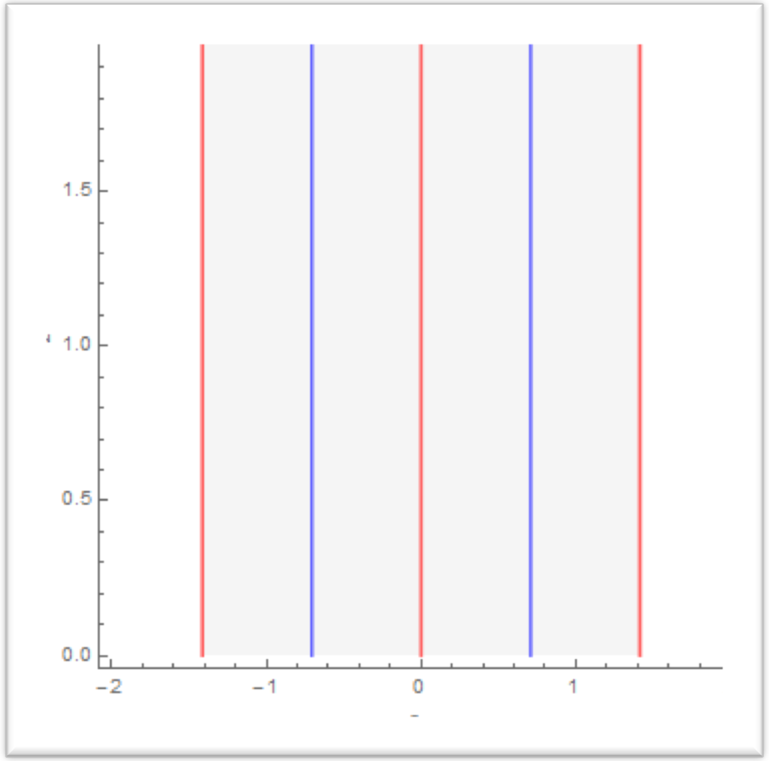
Interpolation between $SO(32)$ and $SO(16)\times SO(16)$ with WL

- **Summary of the conditions**

We have found the two conditions under which the additional massless states appear:

condition ① $\quad \tilde{\tau}_1 = n_1/\sqrt{2} \quad n_1 \in 2\boldsymbol{Z}$

condition ② $\quad \tilde{\tau}_1 = n_2/\sqrt{2} \quad n_2 \in 2\boldsymbol{Z}+1$



Actually, there are only four inequivalent orbits in the fundamental region:

Condition	$n_1 = 0$ and 2 (or -2)	$n_2 = -1$ and 1
Gauge gp	$SO(16)\times SO(16)$	$SO(18)\times SO(14)$
	$n_F > n_B$	$n_F = n_B$

Interpolation between $E_8 \times E_8$ and $SO(16)\times SO(16)$ with WL

- **The one-loop partition function**

$$
\begin{aligned}
Z_{\text{int}}^{(9)} = Z_B^{(7)} \Big\{ & \bar{V}_8 \left(O_{16}^{(0,0)} O_{16} + S_{16}^{(0,0)} S_{16}\right) - \bar{S}_8 \left(O_{16}^{(0,0)} S_{16} + S_{16}^{(0,0)} O_{16}\right) \\
& + \bar{V}_8 \left(O_{16}^{(1/2,0)} S_{16} + S_{16}^{(1/2,0)} O_{16}\right) - \bar{S}_8 \left(O_{16}^{(1/2,0)} O_{16} + S_{16}^{(1/2,0)} S_{16}\right) \\
& + \bar{O}_8 \left(V_{16}^{(0,1/2)} C_{16} + C_{16}^{(0,1/2)} V_{16}\right) - \bar{C}_8 \left(V_{16}^{(0,1/2)} V_{16} + C_{16}^{(0,1/2)} C_{16}\right) \\
& + \bar{O}_8 \left(V_{16}^{(1/2,1/2)} V_{16} + C_{16}^{(1/2,1/2)} C_{16}\right) - \bar{C}_8 \left(V_{16}^{(1/2,1/2)} C_{16} + C_{16}^{(1/2,1/2)} V_{16}\right) \Big\}
\end{aligned}
$$

$$
\left(\begin{pmatrix} O_{16}^{(\alpha,\beta)} \\ V_{16}^{(\alpha,\beta)} \end{pmatrix} = \frac{1}{2\eta^7} \left(\Lambda_{(0,0)}^{(\alpha,\beta)} \vartheta \begin{bmatrix} 0 \\ 0 \end{bmatrix}^7 \pm \Lambda_{(0,1/2)}^{(\alpha,\beta)} \vartheta \begin{bmatrix} 0 \\ 1/2 \end{bmatrix}^7 \right) \qquad \begin{pmatrix} S_{16}^{(\alpha,\beta)} \\ C_{16}^{(\alpha,\beta)} \end{pmatrix} = \frac{1}{2\eta^7} \left(\Lambda_{(1/2,0)}^{(\alpha,\beta)} \vartheta \begin{bmatrix} 1/2 \\ 0 \end{bmatrix}^7 \pm \Lambda_{(1/2,1/2)}^{(\alpha,\beta)} \vartheta \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}^7 \right) \right)
$$

- <u>$R \to \infty$:</u> $\left(O_{16}^{(\alpha,\beta)}, V_{16}^{(\alpha,\beta)}, S_{16}^{(\alpha,\beta)}, C_{16}^{(\alpha,\beta)}\right) \longrightarrow \dfrac{\sqrt{\alpha'}}{2r_\infty} Z_B^{(1)} \left(O_{16}, V_{16}, S_{16}, C_{16}\right) \delta_{\beta,0}$
- <u>$R \to 0$:</u> $\left(O_{16}^{(\alpha,\beta)}, V_{16}^{(\alpha,\beta)}, S_{16}^{(\alpha,\beta)}, C_{16}^{(\alpha,\beta)}\right) \longrightarrow \sqrt{\alpha'} r_0 Z_B^{(1)} \left(O_{16}, V_{16}, S_{16}, C_{16}\right) \delta_{\alpha,0}$

$$
\left(\begin{aligned} r_\infty &= \frac{R}{\sqrt{1+A^2}} \\ r_0 &= \sqrt{1+A^2} R \end{aligned} \right)
$$

Interpolation between $E_8 \times E_8$ and $SO(16)\times SO(16)$ with WL

- **Massless spectrum** — at generic R, massless states come from $n=w=0$ part

$$
\begin{aligned}
Z_{\rm int}^{(9)} = Z_B^{(7)} \Big\{ & \underline{\bar{V}_8 \left(O_{16}^{(0,0)} O_{16} + S_{16}^{(0,0)} S_{16} \right) - \bar{S}_8 \left(O_{16}^{(0,0)} S_{16} + S_{16}^{(0,0)} O_{16} \right)} \\
& + \bar{V}_8 \left(O_{16}^{(1/2,0)} S_{16} + S_{16}^{(1/2,0)} O_{16} \right) - \bar{S}_8 \left(O_{16}^{(1/2,0)} O_{16} + S_{16}^{(1/2,0)} S_{16} \right) \\
& + \bar{O}_8 \left(V_{16}^{(0,1/2)} C_{16} + C_{16}^{(0,1/2)} V_{16} \right) - \bar{C}_8 \left(V_{16}^{(0,1/2)} V_{16} + C_{16}^{(0,1/2)} C_{16} \right) \\
& + \bar{O}_8 \left(V_{16}^{(1/2,1/2)} V_{16} + C_{16}^{(1/2,1/2)} C_{16} \right) - \bar{C}_8 \left(V_{16}^{(1/2,1/2)} C_{16} + C_{16}^{(1/2,1/2)} V_{16} \right) \Big\}
\end{aligned}
$$

Massless bosons

- **9-dim. graviton, anti-symmetric tensor, dilaton:** $g_{\mu\nu}, \quad B_{\mu\nu}, \quad \phi$
- **Gauge bosons in adj rep of $SO(16)\times SO(14)\times U(1)\times U_{G,B}^2(1)$**

Massless fermions

- $\mathbf{8}_S \otimes (\mathbf{128}, \mathbf{1})$

$$n_F - n_B = -736$$

Interpolation between $E_8 \times E_8$ and $SO(16) \times SO(16)$ with WL

- Massless spectrum $\quad {}^{\exists}$ a few conditions under which the additional massless states appear

$$
\begin{aligned}
Z_{\text{int}}^{(9)} = Z_B^{(7)} \Big\{ & \bar{V}_8 \left(O_{16}^{(0,0)} O_{16} + S_{16}^{(0,0)} S_{16} \right) - \bar{S}_8 \left(O_{16}^{(0,0)} S_{16} + S_{16}^{(0,0)} O_{16} \right) \\
& + \bar{V}_8 \left(O_{16}^{(1/2,0)} S_{16} + S_{16}^{(1/2,0)} O_{16} \right) - \bar{S}_8 \left(O_{16}^{(1/2,0)} O_{16} + S_{16}^{(1/2,0)} S_{16} \right) \\
& + \bar{O}_8 \left(V_{16}^{(0,1/2)} C_{16} + C_{16}^{(0,1/2)} V_{16} \right) - \bar{C}_8 \left(V_{16}^{(0,1/2)} V_{16} + C_{16}^{(0,1/2)} C_{16} \right) \\
& + \bar{O}_8 \left(V_{16}^{(1/2,1/2)} V_{16} + C_{16}^{(1/2,1/2)} C_{16} \right) - \bar{C}_8 \left(V_{16}^{(1/2,1/2)} C_{16} + C_{16}^{(1/2,1/2)} V_{16} \right) \Big\}
\end{aligned}
$$

condition ① $\quad \tilde{\tau}_1 = n_1/\sqrt{2} \quad n_1 \in 2Z$

new massless states： • two $\quad 8_V \otimes (1, 14)$

$$SO(16) \times SO(14) \times U(1) \longrightarrow SO(16) \times SO(16)$$

Furthermore, the different additional massless states appear depending on whether $n_1/2$ is even or odd.

Interpolation between $E_8 \times E_8$ and $SO(16) \times SO(16)$ with WL

- **Massless spectrum** $\exists$ a few conditions under which the additional massless states appear

$$
\begin{aligned}
Z_{\text{int}}^{(9)} = Z_B^{(7)} \Big\{ & \bar{V}_8 \left(O_{16}^{(0,0)} O_{16} + S_{16}^{(0,0)} S_{16} \right) - \bar{S}_8 \left(O_{16}^{(0,0)} S_{16} + S_{16}^{(0,0)} O_{16} \right) \\
& + \bar{V}_8 \left(O_{16}^{(1/2,0)} S_{16} + S_{16}^{(1/2,0)} O_{16} \right) - \bar{S}_8 \left(O_{16}^{(1/2,0)} O_{16} + S_{16}^{(1/2,0)} S_{16} \right) \\
& + \bar{O}_8 \left(V_{16}^{(0,1/2)} C_{16} + C_{16}^{(0,1/2)} V_{16} \right) - \bar{C}_8 \left(V_{16}^{(0,1/2)} V_{16} + C_{16}^{(0,1/2)} C_{16} \right) \\
& + \bar{O}_8 \left(V_{16}^{(1/2,1/2)} V_{16} + C_{16}^{(1/2,1/2)} C_{16} \right) - \bar{C}_8 \left(V_{16}^{(1/2,1/2)} C_{16} + C_{16}^{(1/2,1/2)} V_{16} \right) \Big\}
\end{aligned}
$$

condition ①-1 $\quad \tilde{\tau}_1 = n_1/\sqrt{2} \qquad n_1/2 \in 2Z$

new massless states : • two $\mathbf{8}_V \otimes (\mathbf{1}, \mathbf{14})$ • two $\mathbf{8}_S \otimes (\mathbf{1}, \mathbf{64})$

$$\mathbf{8}_S \otimes (\mathbf{128}, \mathbf{1}) \longrightarrow \mathbf{8}_S \otimes ((\mathbf{128}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{128}))$$

In the fundamental region, this condition is $\tilde{\boldsymbol{\tau}}_1 = \mathbf{0}$, which corresponds to the no WL case.

Interpolation between $E_8 \times E_8$ and $SO(16) \times SO(16)$ with WL

- **Massless spectrum** $\quad$ ∃ a few conditions under which the additional massless states appear

$$
\begin{aligned}
Z_{\rm int}^{(9)} = Z_B^{(7)} \Big\{ & \bar{V}_8 \left(O_{16}^{(0,0)} O_{16} + S_{16}^{(0,0)} S_{16} \right) - \bar{S}_8 \left(O_{16}^{(0,0)} S_{16} + S_{16}^{(0,0)} O_{16} \right) \\
& + \bar{V}_8 \left(O_{16}^{(1/2,0)} S_{16} + S_{16}^{(1/2,0)} O_{16} \right) - \bar{S}_8 \left(O_{16}^{(1/2,0)} O_{16} + S_{16}^{(1/2,0)} S_{16} \right) \\
& + \bar{O}_8 \left(V_{16}^{(0,1/2)} C_{16} + C_{16}^{(0,1/2)} V_{16} \right) - \bar{C}_8 \left(V_{16}^{(0,1/2)} V_{16} + C_{16}^{(0,1/2)} C_{16} \right) \\
& + \bar{O}_8 \left(V_{16}^{(1/2,1/2)} V_{16} + C_{16}^{(1/2,1/2)} C_{16} \right) - \bar{C}_8 \left(V_{16}^{(1/2,1/2)} C_{16} + C_{16}^{(1/2,1/2)} V_{16} \right) \Big\}
\end{aligned}
$$

condition ①-2 $\quad \tilde{\tau}_1 = n_1/\sqrt{2} \qquad n_1/2 \in 2\boldsymbol{Z}+1$

new massless states : • **two** $\mathbf{8}_V \otimes (\mathbf{1}, \mathbf{14})$ $\quad$ • **two** $\mathbf{8}_V \otimes (\mathbf{1}, \mathbf{64})$

➡ $\quad SO(16) \times SO(14) \times U(1) \longrightarrow SO(16) \times E_8$

In the fundamental region, this condition is $\tilde{\tau}_1 = \sqrt{2}$ (or $\tilde{\tau}_1 = -\sqrt{2}$).

Interpolation between $E_8 \times E_8$ and $SO(16) \times SO(16)$ with WL

- **Massless spectrum** $\quad {}^{\exists}$ a few conditions under which the additional massless states appear

$$
\begin{aligned}
Z_{\rm int}^{(9)} = Z_B^{(7)} \Big\{ & \bar{V}_8 \left(O_{16}^{(0,0)} O_{16} + S_{16}^{(0,0)} S_{16} \right) - \bar{S}_8 \left(O_{16}^{(0,0)} S_{16} + S_{16}^{(0,0)} O_{16} \right) \\
& + \bar{V}_8 \left(O_{16}^{(1/2,0)} S_{16} + S_{16}^{(1/2,0)} O_{16} \right) - \bar{S}_8 \left(O_{16}^{(1/2,0)} O_{16} + S_{16}^{(1/2,0)} S_{16} \right) \\
& + \bar{O}_8 \left(V_{16}^{(0,1/2)} C_{16} + C_{16}^{(0,1/2)} V_{16} \right) - \bar{C}_8 \left(V_{16}^{(0,1/2)} V_{16} + C_{16}^{(0,1/2)} C_{16} \right) \\
& + \bar{O}_8 \left(V_{16}^{(1/2,1/2)} V_{16} + C_{16}^{(1/2,1/2)} C_{16} \right) - \bar{C}_8 \left(V_{16}^{(1/2,1/2)} C_{16} + C_{16}^{(1/2,1/2)} V_{16} \right) \Big\}
\end{aligned}
$$

condition ②　$\tilde{\tau}_1 = n_2/\sqrt{2} \quad n_2 \in 2Z+1$

new massless states： • two $8_S \otimes (1, 14)$

Gauge group is not enhanced

In the fundamental region, this condition is $\tilde{\tau}_1 = \sqrt{2}/2$ and $\tilde{\tau}_1 = -\sqrt{2}/2$.

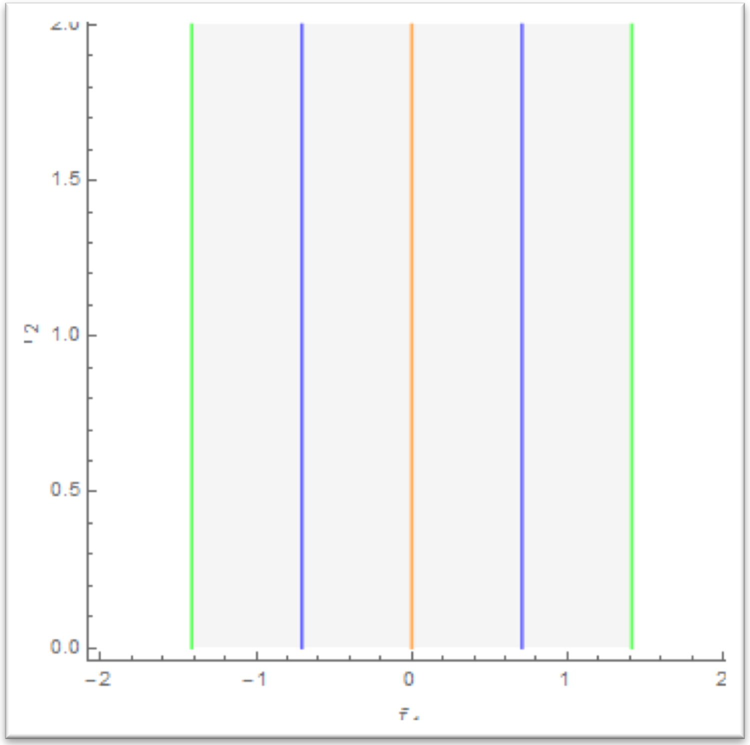
There is no condition such that $n_F = n_B$ in this example.

Interpolation between $E_8 \times E_8$ and $SO(16) \times SO(16)$ with WL

- **Summary of the conditions**

We have found the three conditions under which the additional massless states appear:

condition ①-1	$\tilde{\tau}_1 = n_1/\sqrt{2}$	$n_1/2 \in 2\boldsymbol{Z}$
condition ①-2	$\tilde{\tau}_1 = n_1/\sqrt{2}$	$n_1/2 \in 2\boldsymbol{Z}+1$
condition ②	$\tilde{\tau}_1 = n_2/\sqrt{2}$	$n_2 \in 2\boldsymbol{Z}+1$



Actually, there are only four inequivalent orbits in the fundamental region:

Condition	$n_1 = 0$	$n_1 = 2$ (or -2)	$n_2 = 1$ and -1
Gauge gp	$SO(16)\times SO(16)$	$SO(16)\times E_8$	$SO(16)\times SO(14)\times U(1)$
	$n_F > n_B$	$n_F < n_B$	$n_F < n_B$

The leading terms of the cosmological constant

The cosmological constant is written as

$$\Lambda_{int}^{(9)}(a,R) = -\frac{1}{2}\left(4\pi^2\alpha'\right)^{-9/2}\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2^2}Z_{int}^{(9)}(a,R;\tau)$$

Up to exponentially suppressed terms, the results are

- $SO(32)$ - $SO(16)\times SO(16)$ interpolation

$$\Lambda_{int}^{(9)}(a,R) \simeq 48\pi^{-14}\left(\frac{a_0}{\sqrt{\alpha'}}\right)^9 \times 8\left\{(224-220)+2(16-14)\cos\left(\sqrt{2}\pi\tilde{\tau}_1\right)\right\}$$

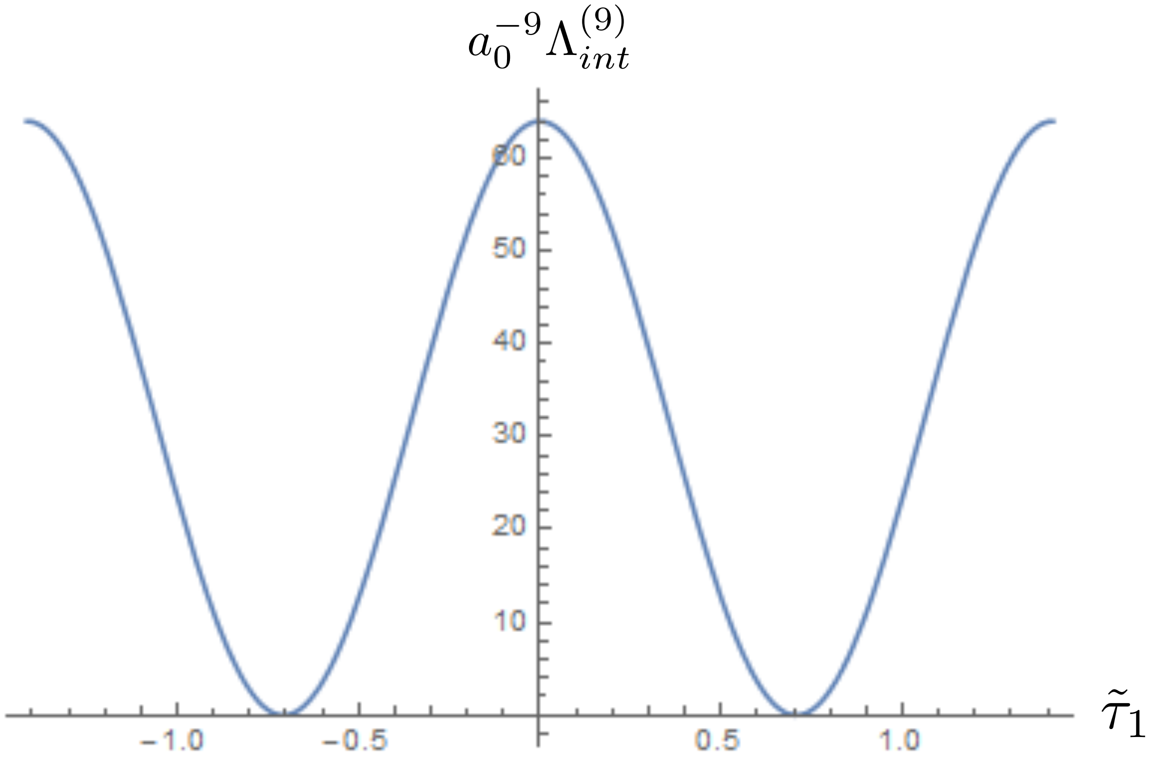
- $E_8\times E_8$ - $SO(16)\times SO(16)$ interpolation

$$\Lambda_{int}^{(9)}(a,R) \simeq 48\pi^{-14}\left(\frac{a_0}{\sqrt{\alpha'}}\right)^9 \times 8\left\{(2^7-220)-2\cdot 14\cos\left(\sqrt{2}\pi\tilde{\tau}_1\right)+2\cdot 2^6\cos\left(\frac{\sqrt{2}}{2}\pi\tilde{\tau}_1\right)\right\}$$

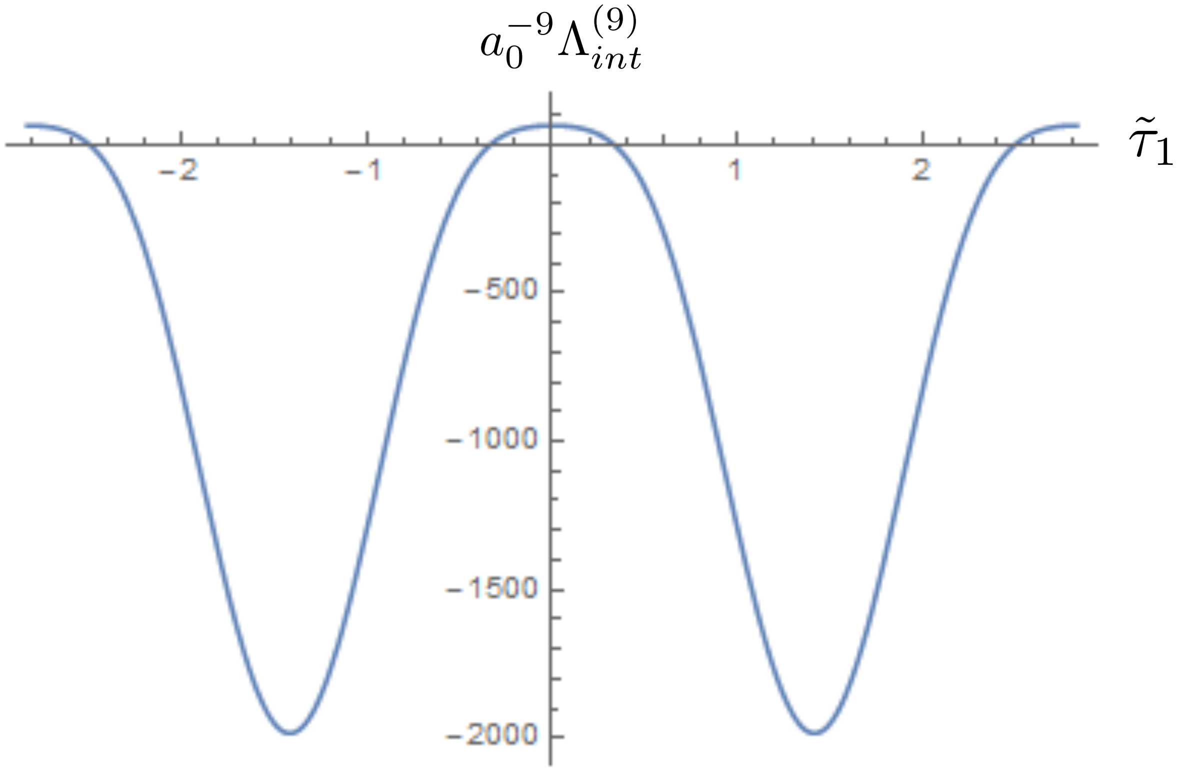
These results reflect the shift symmetry $\tilde{\tau}\to\tilde{\tau}+2\sqrt{2}$ and the conditions under which the additional massless states appear.

The leading terms of the cosmological constant

- $SO(32)$ - $SO(16)\times SO(16)$ interpolation



- $E_8\times E_8$ - $SO(16)\times SO(16)$ interpolation



Outline

1. Introduction
2. 9D Interpolating models
3. 9D Interpolating models with Wilson line
4. **Summary**

Conclusion

- We have constructed 9D interpolating models with two parameters, radius R and WL A, and studied the massless spectra.
- We have found some conditions for (R, A) under which the additional massless states appear.
- We have found that an example under which the cosmological const. is exponentially suppressed simultaneously with the gauge group enhancement to $SO(18)\times SO(14)$.

Outlook

- How are SM-like or GUT-like 4D models with $n_F = n_B$ constructed ?

- We can generalize interpolating models by putting more WL and the other backgrounds. In fact, compactifying d-dimensions, the compactifications are classified by $\frac{SO(16+d,d)}{SO(16+d)\times SO(d)}$, whose DOF is $d(16+d)$.

Thank you!