Phase transition(s) of the bosonic BMN model

Samuel Kováčik, Dublin Institute for Advanced Studies
with Denjoe O' Connor and Yuhma Asano (and Veselin Filev)

Corfu 2019

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Related to talks by: Asano, Filev, Hanada, Nishimura, Papadoudis, Bergner, Steinacker …

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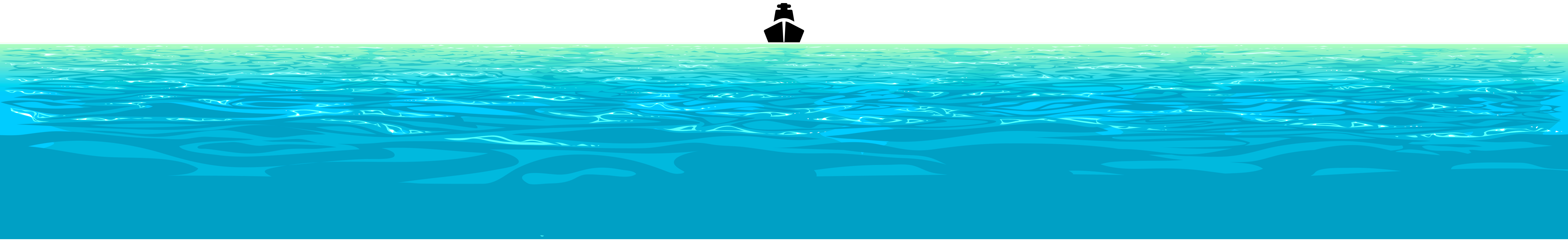
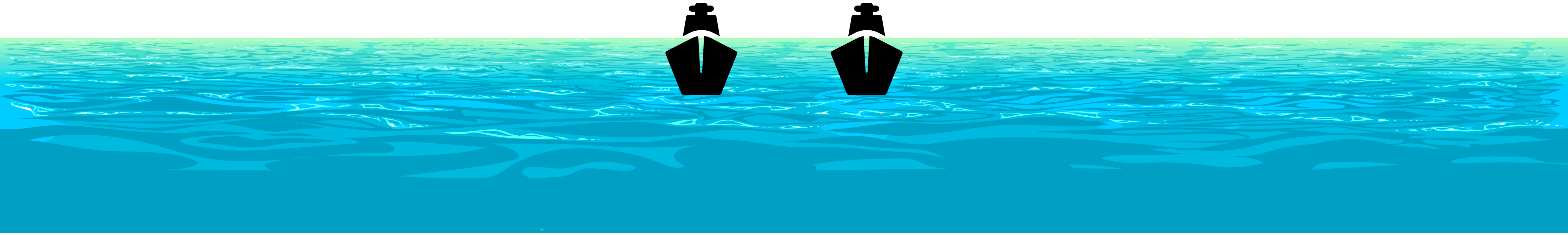
I promise!

Phase transition or phase transitions?





Matrix models from M-theory

- BFSS matrix model (Banks-Fischler-Shenker-Susskind)

$$S_E = \frac{1}{g^2} \int d\tau \, \mathrm{Tr} \left(\frac{1}{2} (D_\tau X^i)^2 - \frac{1}{4} [X^i, X^j]^2 + \frac{1}{2} \psi^T C_9 \, D_\tau \psi - \frac{1}{2} \psi^T C_9 \, \gamma^i [X^i, \psi] \right)$$

Matrix models from M-theory

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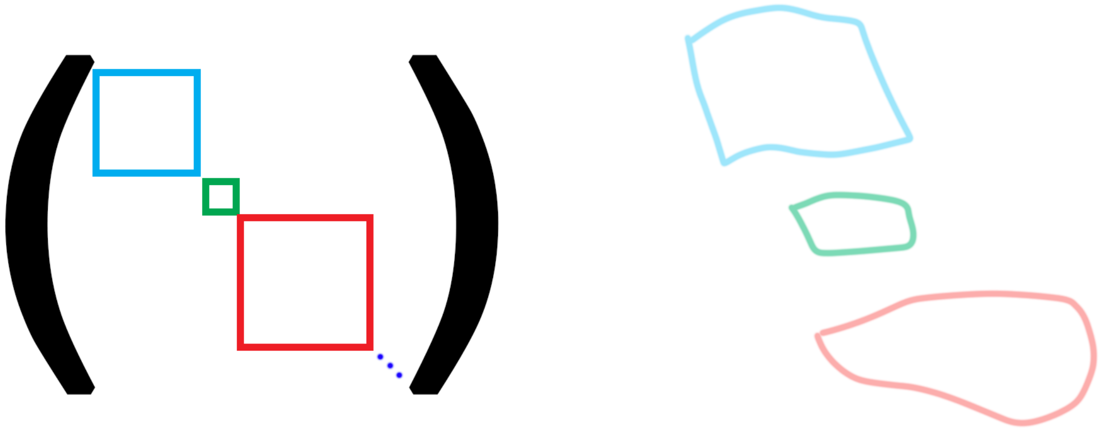

Matrix models from M-theory

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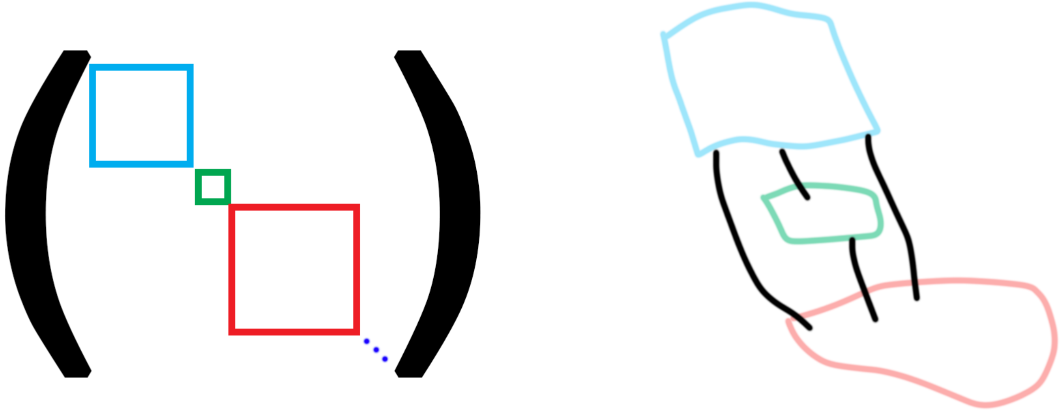

BD model

BD model

$$S_E = N\int_0^\beta d\tau \Bigg[\mathrm{Tr}\ \left(\frac{1}{2}D_\tau X^a D_\tau X^a + \frac{1}{2}D_\tau \bar{X}^{\rho\dot\rho} D_\tau X_{\rho\dot\rho} + \frac{1}{2}\lambda^{\dagger\rho} D_\tau \lambda_\rho + \frac{1}{2}\theta^{\dagger\dot\rho} D_\tau \theta_{\dot\rho}\right)$$

$$+ \mathrm{tr}\ \left(D_\tau \bar{\Phi}^\rho D_\tau \Phi_\rho + \chi^\dagger D_\tau \chi\right)$$

$$- \mathrm{Tr}\ \left(\frac{1}{4}[X^a, X^b]^2 + \frac{1}{2}[X^a, \bar{X}^{\rho\dot\rho}][X^a, X_{\rho\dot\rho}]\right)$$

$$+ \frac{1}{2}\mathrm{Tr}\sum_{A=1}^{3} \mathcal{D}^A \mathcal{D}^A + \mathrm{tr}\ \left(\bar{\Phi}^\rho (X^a - m^a)^2 \Phi_\rho\right)$$

$$- \mathrm{Tr}\ \left(-\frac{1}{2}\lambda^{\dagger\rho}\gamma^a[X^a, \lambda_\rho] + \frac{1}{2}\theta^{\dagger\dot\rho}\gamma^a[X^a, \theta_{\dot\rho}] - \sqrt{2}i\varepsilon^{\rho\sigma}\theta^{\dagger\dot\rho}[X_{\sigma\dot\rho}, \lambda_\rho]\right)$$

$$- \mathrm{tr}\ \left(\chi^\dagger \gamma^a (X^a - m^a)\chi + \sqrt{2}i\varepsilon^{\rho\sigma}\chi^\dagger \lambda_\rho \Phi_\sigma + \sqrt{2}i\varepsilon_{\rho\sigma}\bar{\Phi}^\rho \lambda^{\dagger\sigma}\chi\right) \Bigg]$$

BD model

$$S_E = N\int_0^\beta d\tau \Bigg[\mathrm{Tr}\ \left(\frac{1}{2}D_\tau X^a D_\tau X^a + \frac{1}{2}D_\tau \bar{X}^{\rho\dot\rho} D_\tau X_{\rho\dot\rho} + \frac{1}{2}\lambda^{\dagger\rho} D_\tau \lambda_\rho + \frac{1}{2}\theta^{\dagger\dot\rho} D_\tau \theta_{\dot\rho}\right)$$

$$+ \mathrm{tr}\ \left(D_\tau \bar{\Phi}^\rho D_\tau \Phi_\rho + \chi^\dagger D_\tau \chi\right)$$

$$-\mathrm{Tr}\ \left(\frac{1}{4}[X^a, X^b]^2 + \frac{1}{2}[X^a, \bar{X}^{\rho\dot\rho}][X^a, X_{\rho\dot\rho}]\right)$$

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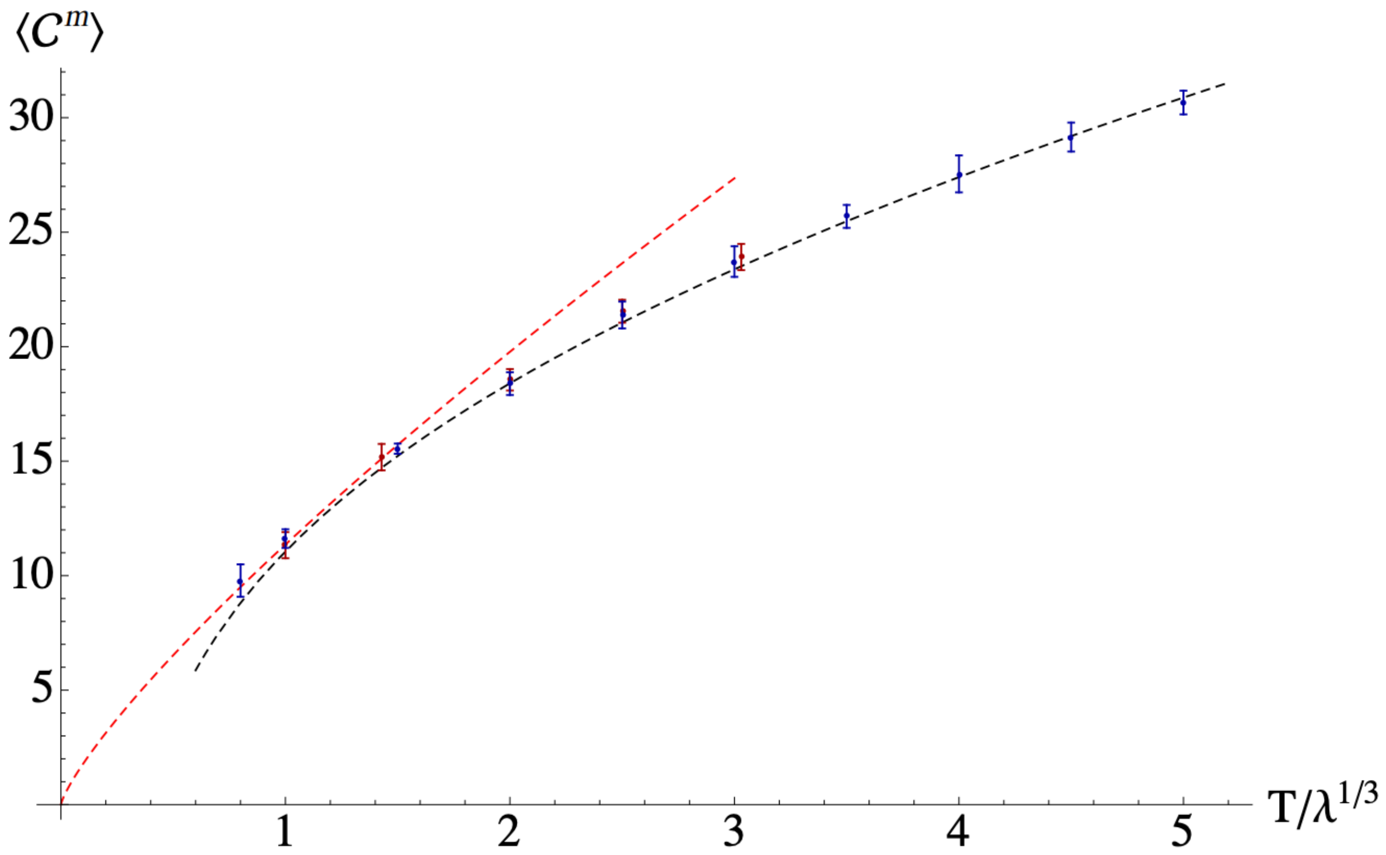
$$- \mathrm{tr}\ \left(\chi^\dagger \gamma^a (X^a - m^a)\chi + \sqrt{2}i\varepsilon^{\rho\sigma}\chi^\dagger \lambda_\rho \Phi_\sigma + \sqrt{2}i\varepsilon_{\rho\sigma}\bar{\Phi}^\rho \lambda^{\dagger\sigma}\chi\right)\Bigg]$$

BD model



A computer test of holographic flavour dynamics. Part II, Y. Asano, V. G. Filev, SK, Denjoe O' Connor
JHEP 1803 (2018) 055, [hep-th/1612.09281]

BMN model

$$S[X,\psi] = N\int_0^\beta d\tau\,\mathrm{Tr}\,\left[\frac{1}{2}D_\tau X^i D_\tau X^i - \frac{1}{4}\left([X^r,X^s] + \frac{i\mu}{3}\varepsilon^{rst}X_t\right)^2 \right.$$
$$- \frac{1}{2}[X^r,X^m]^2 - \frac{1}{4}[X^m,X^n]^2 + \frac{1}{2}\left(\frac{\mu}{6}\right)^2 X_m^2$$
$$\left. + \frac{1}{2}\psi^T\mathcal{C}\left(D_\tau - \frac{i\mu}{4}\gamma^{567}\right)\psi - \frac{1}{2}\psi^T\mathcal{C}\gamma^i[X^i,\psi]\right]$$

BMN model

$$S[X,\psi] = N\int_0^\beta d\tau\,\mathrm{Tr}\left[\frac{1}{2}D_\tau X^i D_\tau X^i - \frac{1}{4}\left([X^r,X^s] + \frac{i\mu}{3}\varepsilon^{rst}X_t\right)^2 \right.$$
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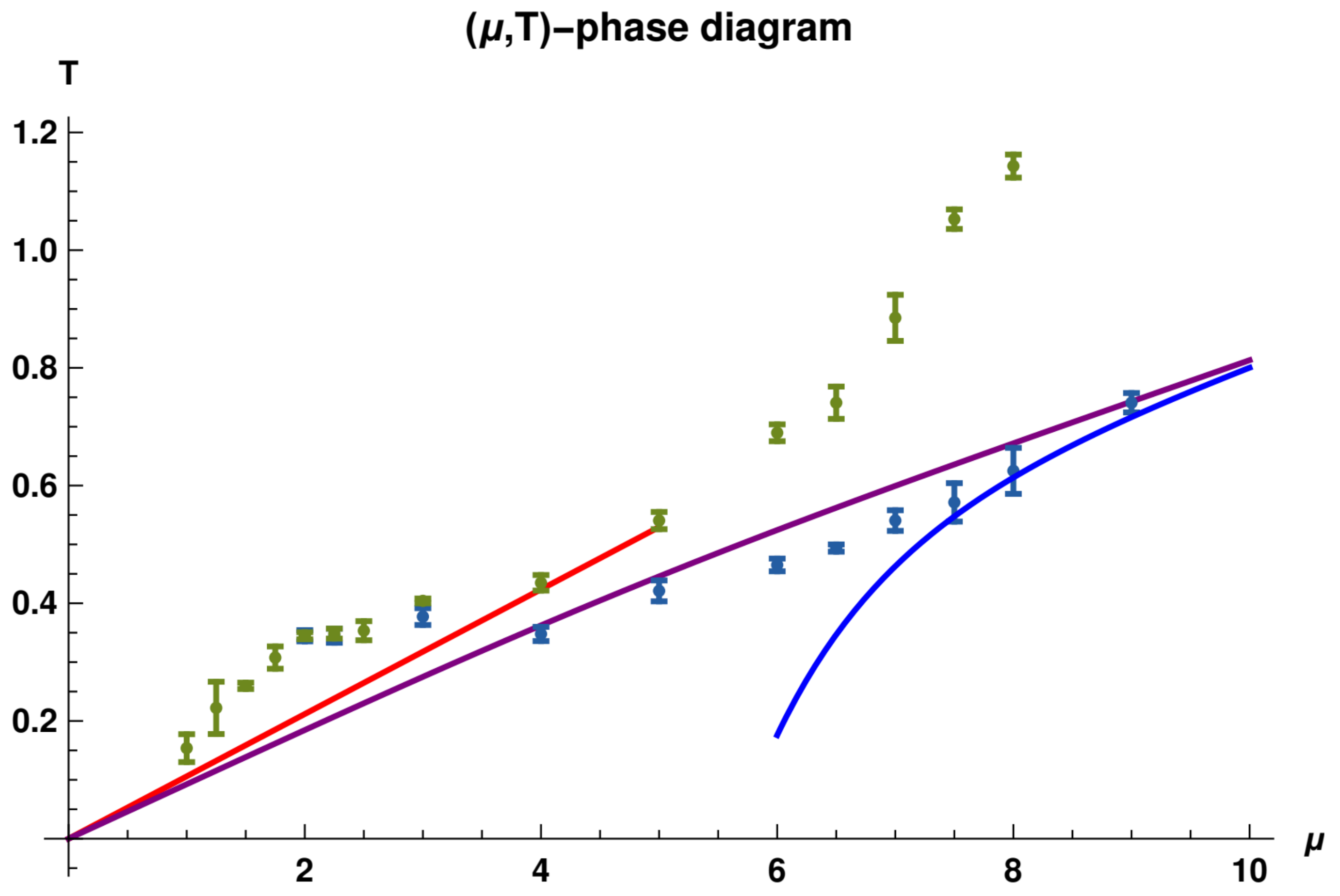
BMN model



The non-perturbative phase diagram of the BMN matrix model, Y. Asano, V. G. Filev, SK, Denjoe O’ Connor JHEP 1807 (2012) 154, [hep-th/1805.05314]

Bosonic BMN model

$$S[X,A] = N\int_0^\beta d\tau\,\mathrm{Tr}\,\left[\frac{1}{2}D_\tau X^i D_\tau X^i - \frac{1}{4}\left([X^r,X^s] + \frac{i\mu}{3}\varepsilon^{rst}X_t\right)^2 - \frac{1}{2}[X^r,X^m]^2 - \frac{1}{4}[X^m,X^n]^2 + \frac{1}{2}\left(\frac{\mu}{6}\right)^2 X_m^2\right]$$

Bosonic BMN model

$$S[X,A] = N\int_0^\beta d\tau\,\mathrm{Tr}\,\Bigg[\frac{1}{2}D_\tau X^i D_\tau X^i - \frac{1}{4}\left([X^r,X^s] + \frac{i\mu}{3}\varepsilon^{rst}X_t\right)^2 - \frac{1}{2}[X^r,X^m]^2 - \frac{1}{4}[X^m,X^n]^2 + \frac{1}{2}\left(\frac{\mu}{6}\right)^2 X_m^2\Bigg]$$

Bosonic BMN model

$$S[X,A] = N\int_0^\beta d\tau\,\mathrm{Tr}\left[\frac{1}{2}D_\tau X^i D_\tau X^i - \frac{1}{4}\left([X^r,X^s] + \frac{i\mu}{3}\varepsilon^{rst}X_t\right)^2 - \frac{1}{2}[X^r,X^m]^2 - \frac{1}{4}[X^m,X^n]^2 + \frac{1}{2}\left(\frac{\mu}{6}\right)^2 X_m^2\right]$$

- i = 1, …, 9
- X are N x N Hermitian matrices
- β = 1 / T, inversed temperature
- D = ∂ - i [A , .]
- A is the SU(N) gauge field (fixed not to depend on time and be)
- Temporal direction discretised
- Parameters: β, μ, N, Λ

Measuring (mean values of) observables

$$\langle \mathcal{O} \rangle = \frac{\int d[X]d[A]\mathcal{O}e^{-S[X,A]}}{Z}, \; Z = \int d[X]d[A]e^{-S[X,A]}$$

The setup

$$S[X,A] = N\int_0^\beta d\tau\,\mathrm{Tr}\,\left[\frac{1}{2}D_\tau X^i D_\tau X^i - \frac{1}{4}\left([X^r,X^s] + \frac{i\mu}{3}\varepsilon^{rst}X_t\right)^2 - \frac{1}{2}[X^r,X^m]^2 - \frac{1}{4}[X^m,X^n]^2 + \frac{1}{2}\left(\frac{\mu}{6}\right)^2 X_m^2\right]$$

$$\langle\mathcal{O}\rangle = \frac{\int d[X]d[A]\mathcal{O}e^{-S[X,A]}}{Z}$$

Observables

$$N^2 E = \langle H \rangle = -\partial_\beta \ln Z$$

$$C = \langle (H - \langle H \rangle)^2 \rangle = \partial_\beta^2 \ln Z$$

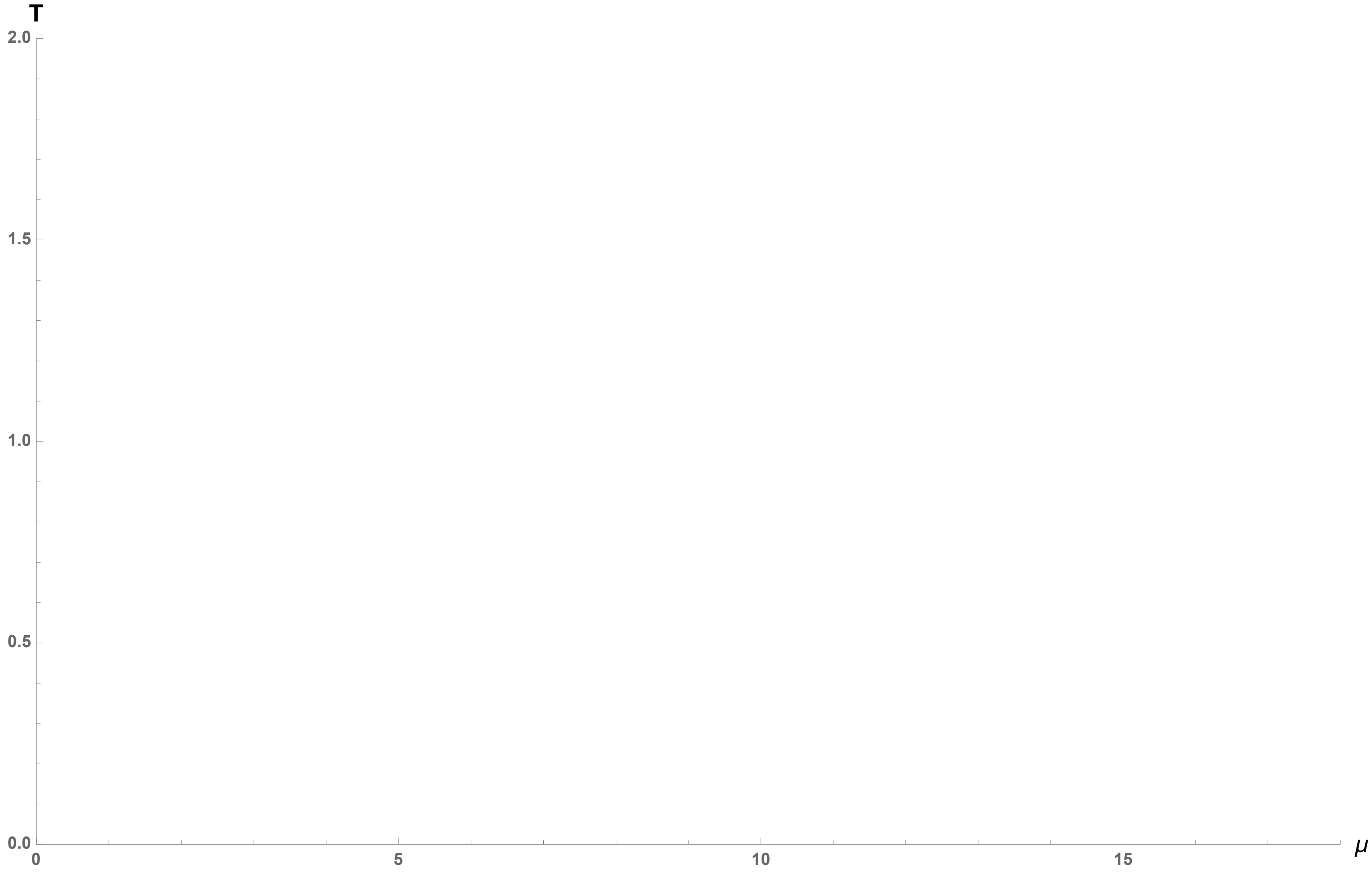
$$\langle |P| \rangle = \left\langle \frac{1}{N} |\text{Tr } (\exp [i\beta A])\,| \right\rangle$$

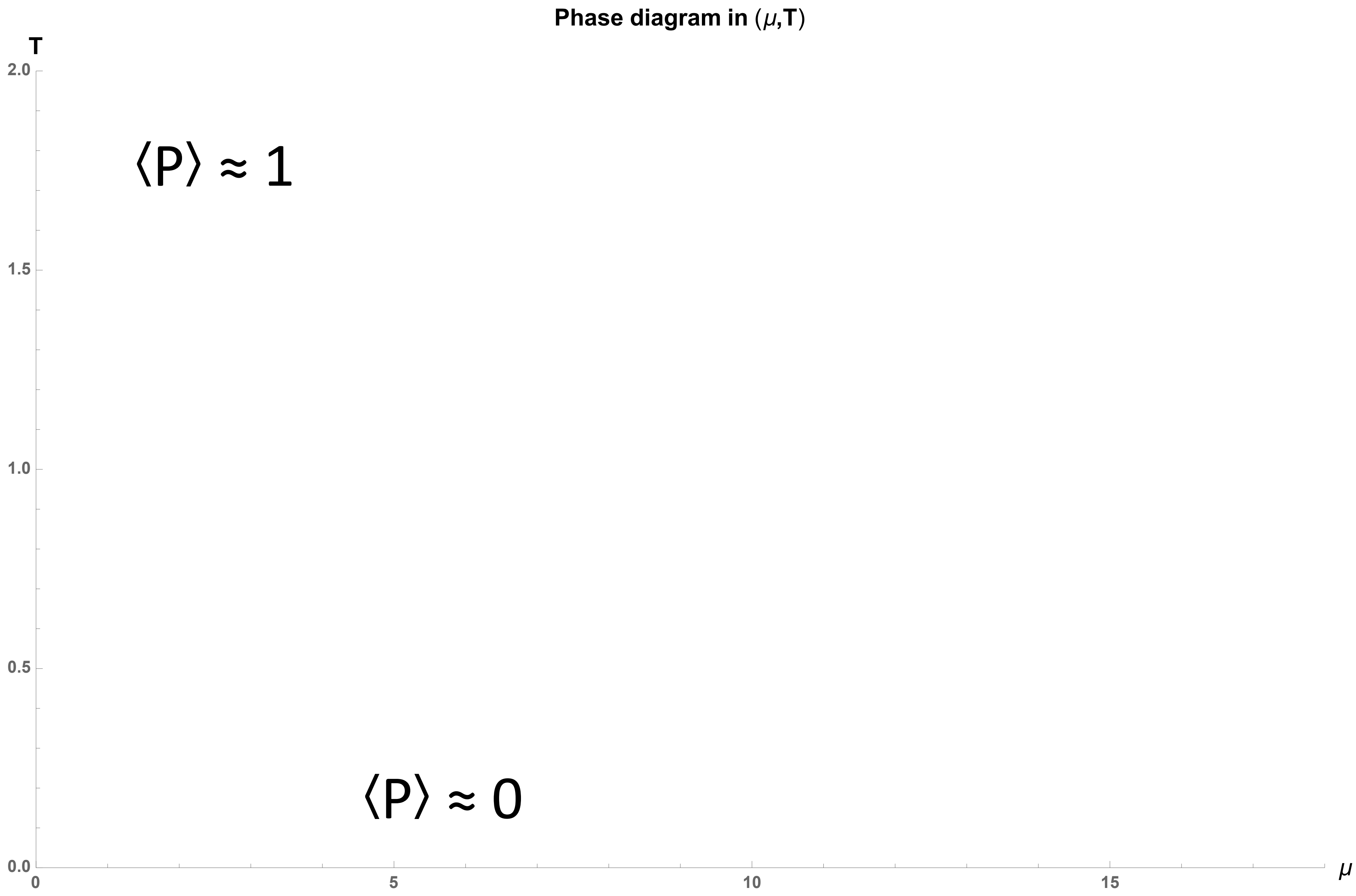
$$R_{ii}^2 = \left\langle \frac{1}{N\beta} \int_0^\beta d\tau \text{Tr } (X^i X^i) \right\rangle \quad \text{(no sum on } i\text{)}$$

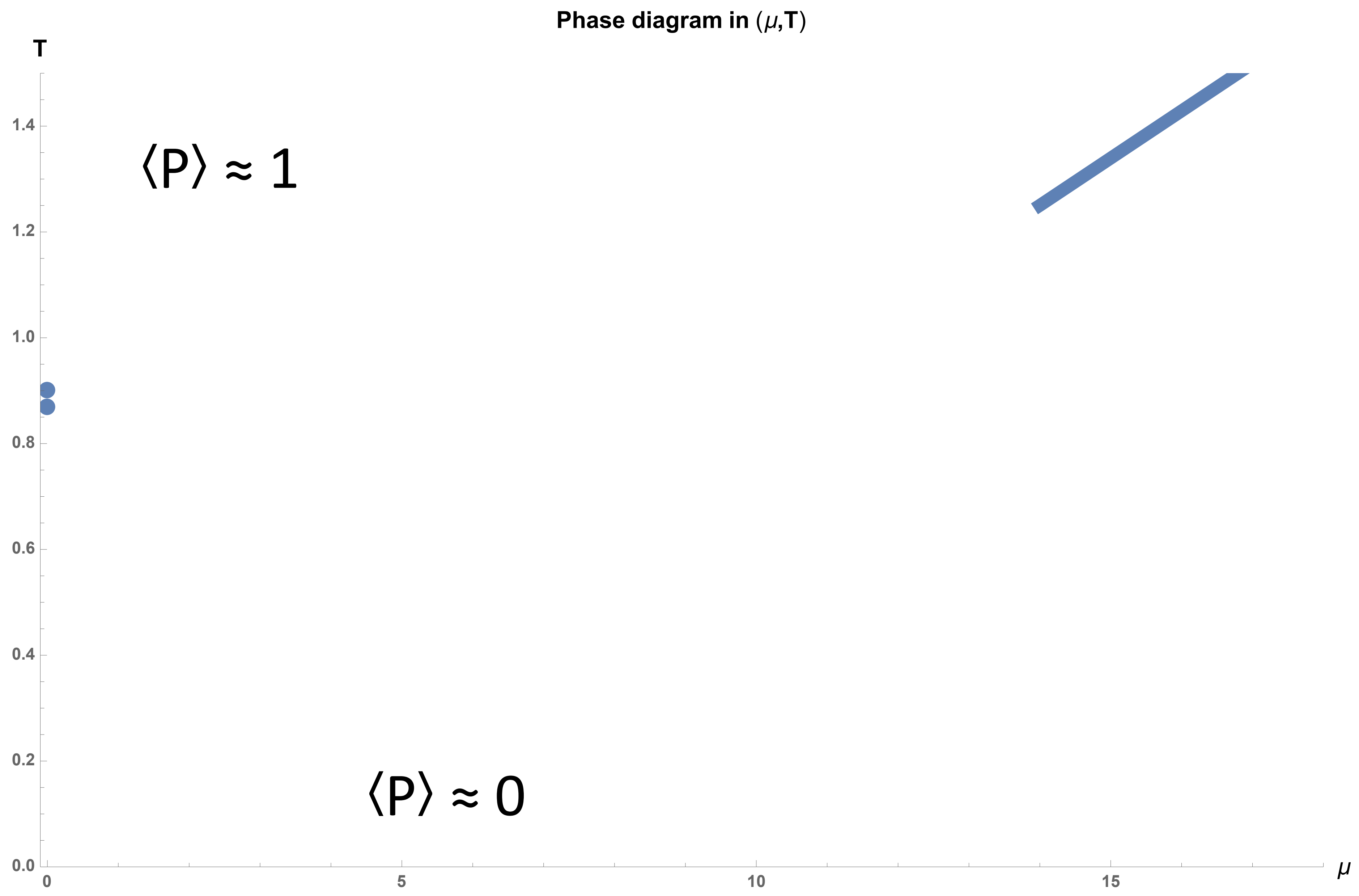
$$M = \left\langle \frac{i}{3N\beta} \int_0^\beta d\tau \epsilon_{rst} \text{Tr } (X^r X^s X^t) \right\rangle$$

What to expect?

Phase diagram in (μ,T)



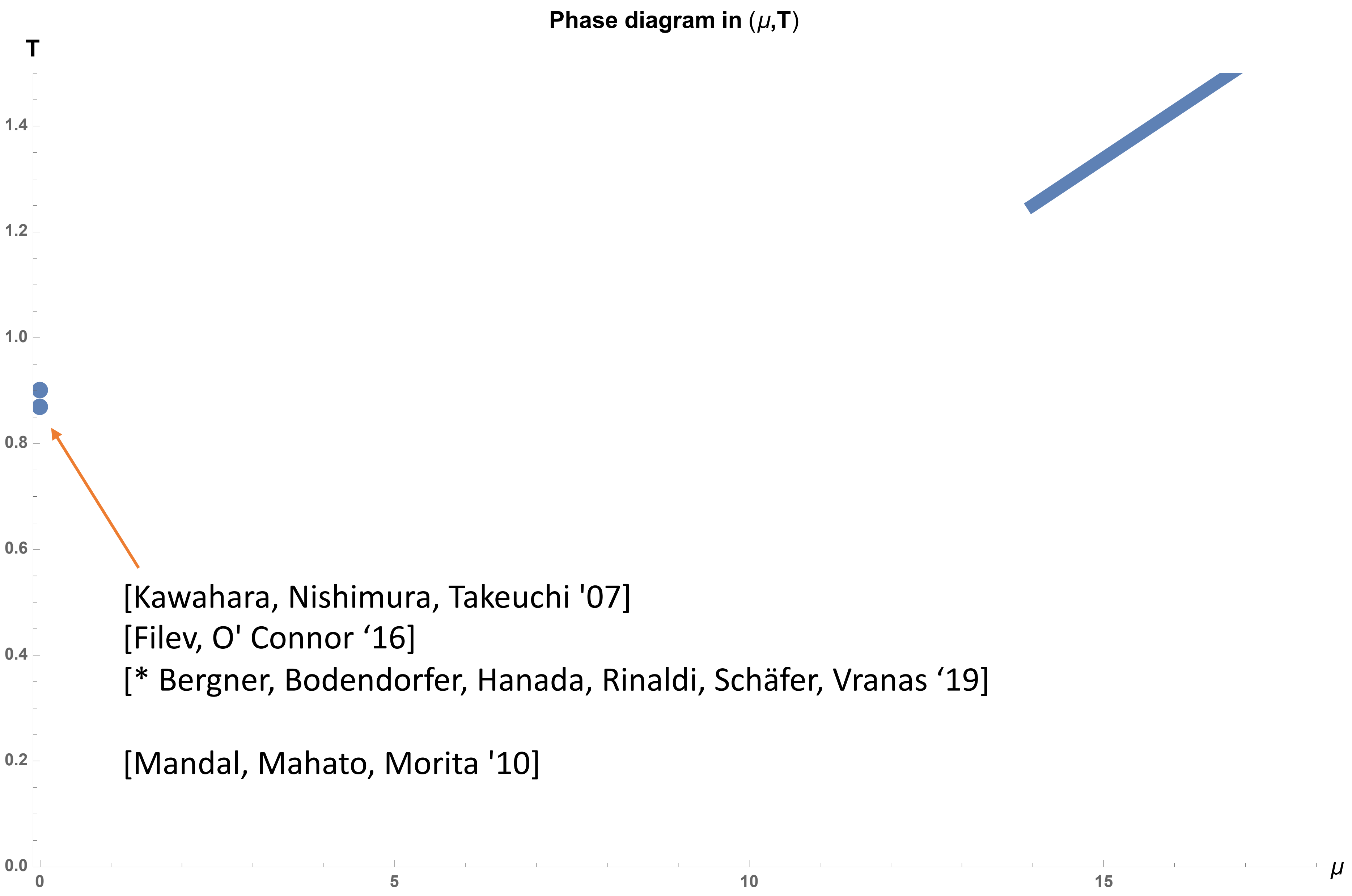

Phase diagram in (μ,T)
T
2.0
1.5
1.0
0.5
0.0
⟨P⟩ ≈ 1
⟨P⟩ ≈ 0
0
5
10
15
μ



Phase diagram in (μ,T)
T
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
μ
⟨P⟩ ≈ 1
⟨P⟩ ≈ 0

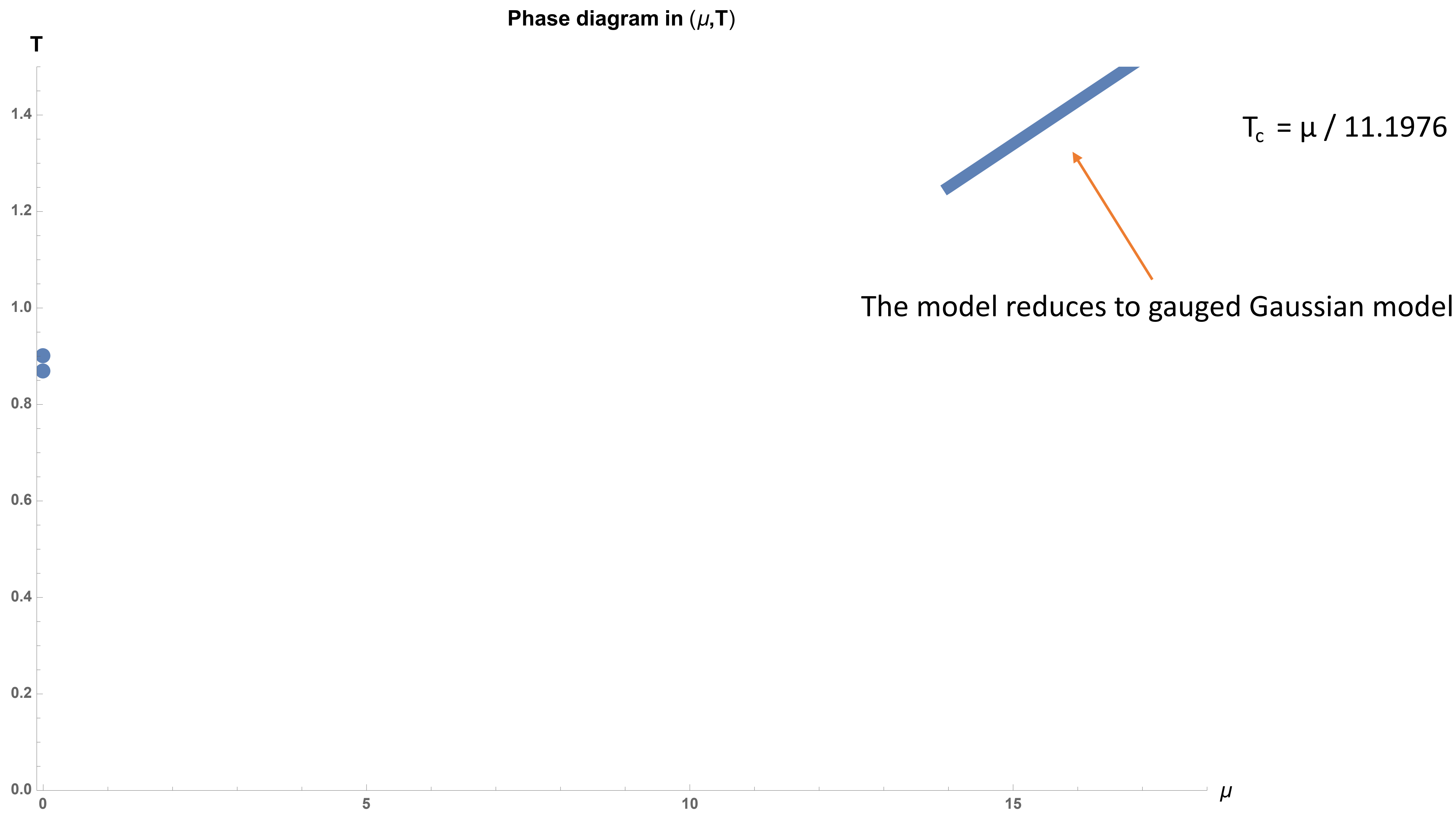
$T_{c2} \approx 0.9$

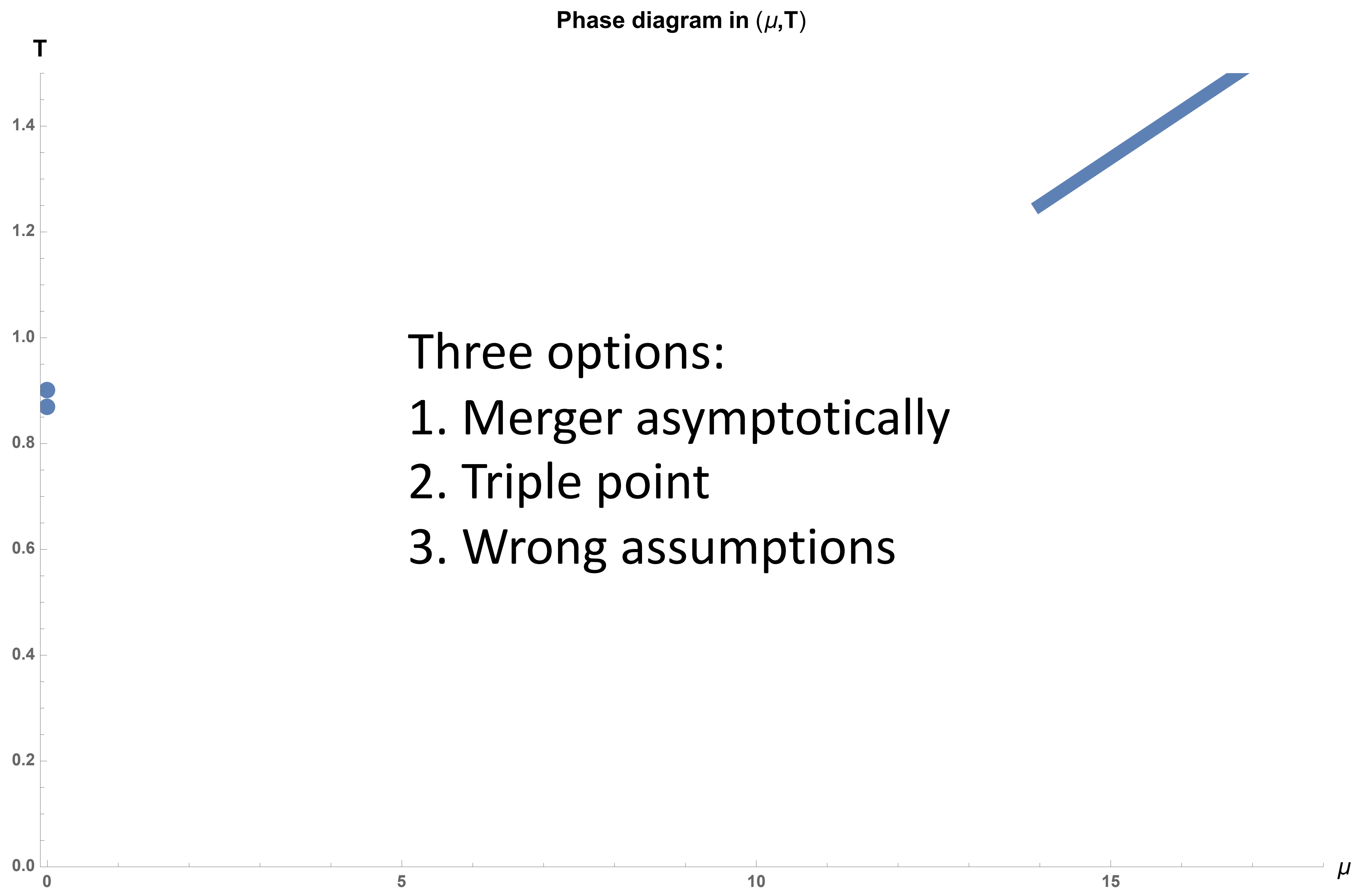
$T_{c1} \approx 0.87$



[Kawahara, Nishimura, Takeuchi '07]
[Filev, O' Connor ‘16]
[* Bergner, Bodendorfer, Hanada, Rinaldi, Schäfer, Vranas ‘19]

[Mandal, Mahato, Morita '10]


Phase diagram in (μ,T)
T
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
μ
$T_c = \mu / 11.1976$
The model reduces to gauged Gaussian model



Three options:

1. Merger asymptotically
2. Triple point
3. Wrong assumptions

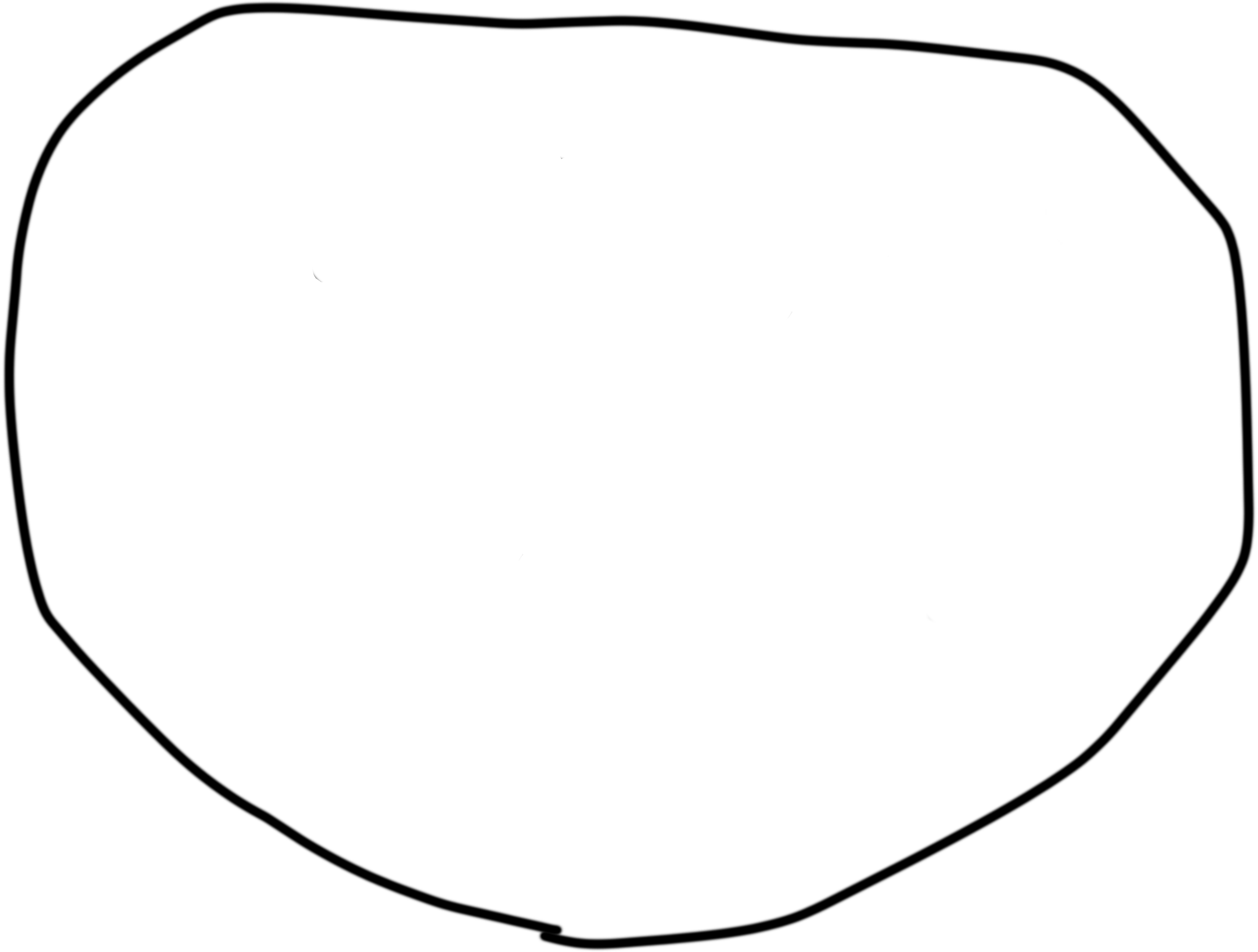
Hybrid Monte Carlo

Integration over **many** variables

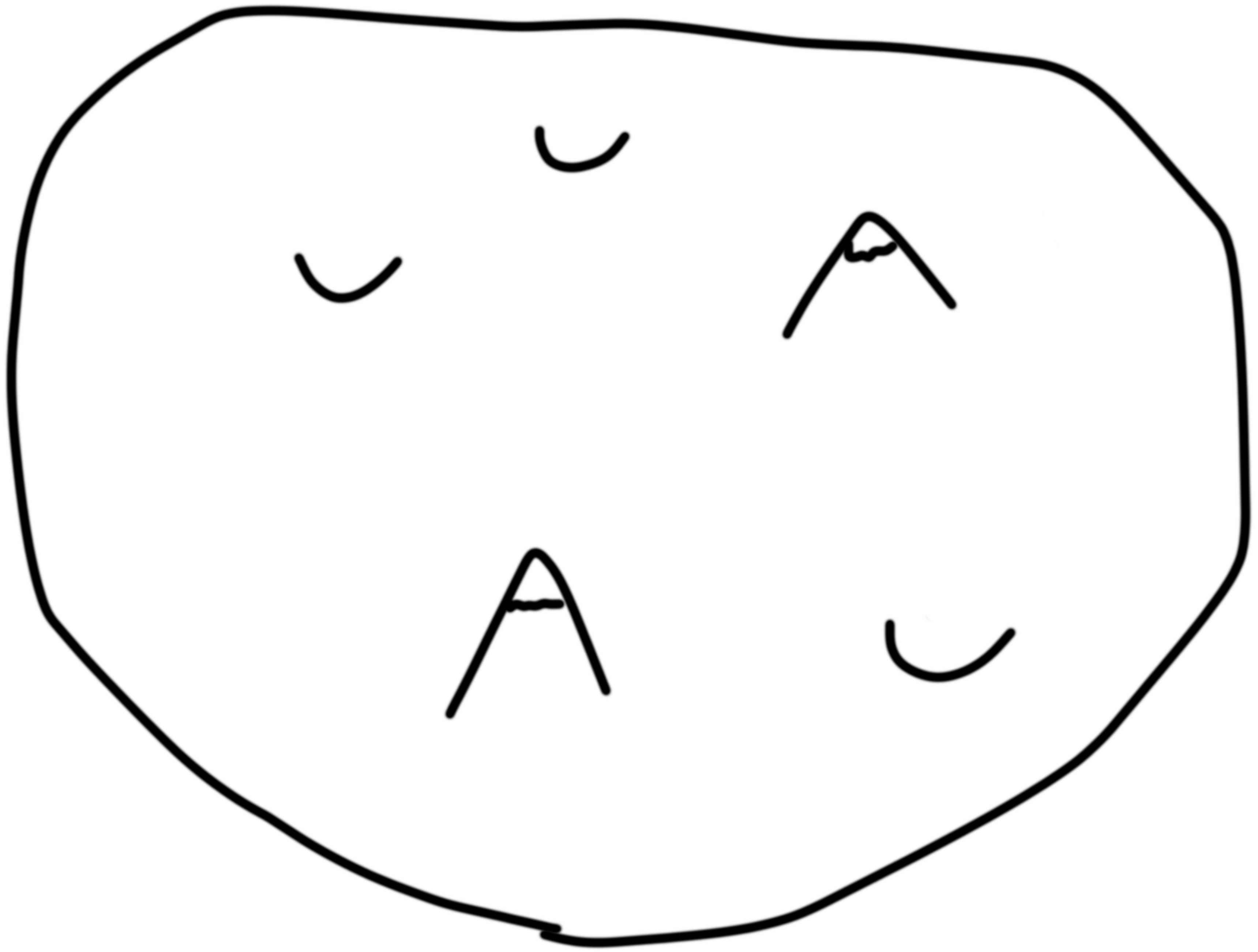
$$\langle \mathcal{O} \rangle = \frac{\int d[X] d[A] \mathcal{O} e^{-S[X,A]}}{Z}$$

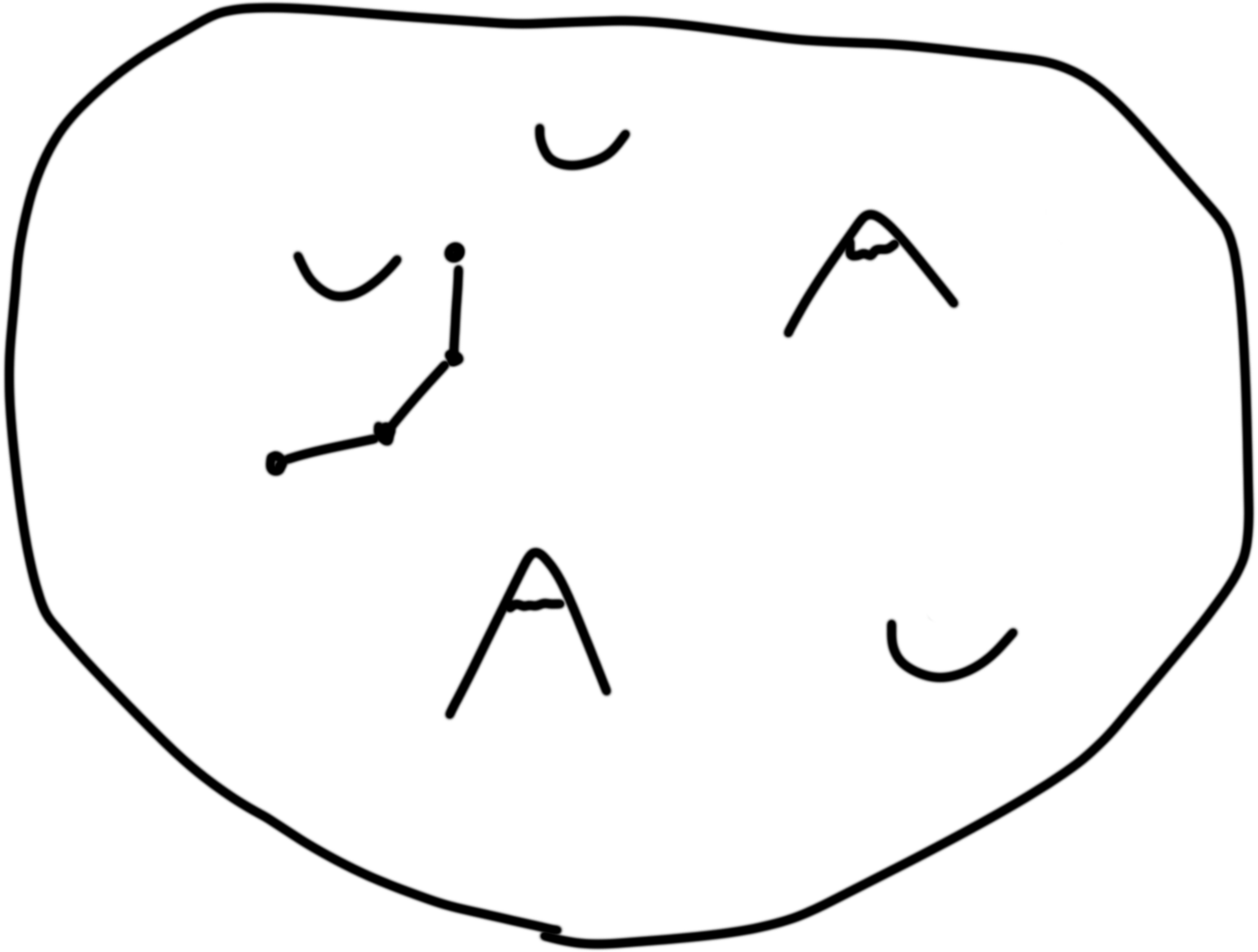
The idea is instead of computing this, we can produce a sample of configurations with the **same probability distribution** (probability sampling)

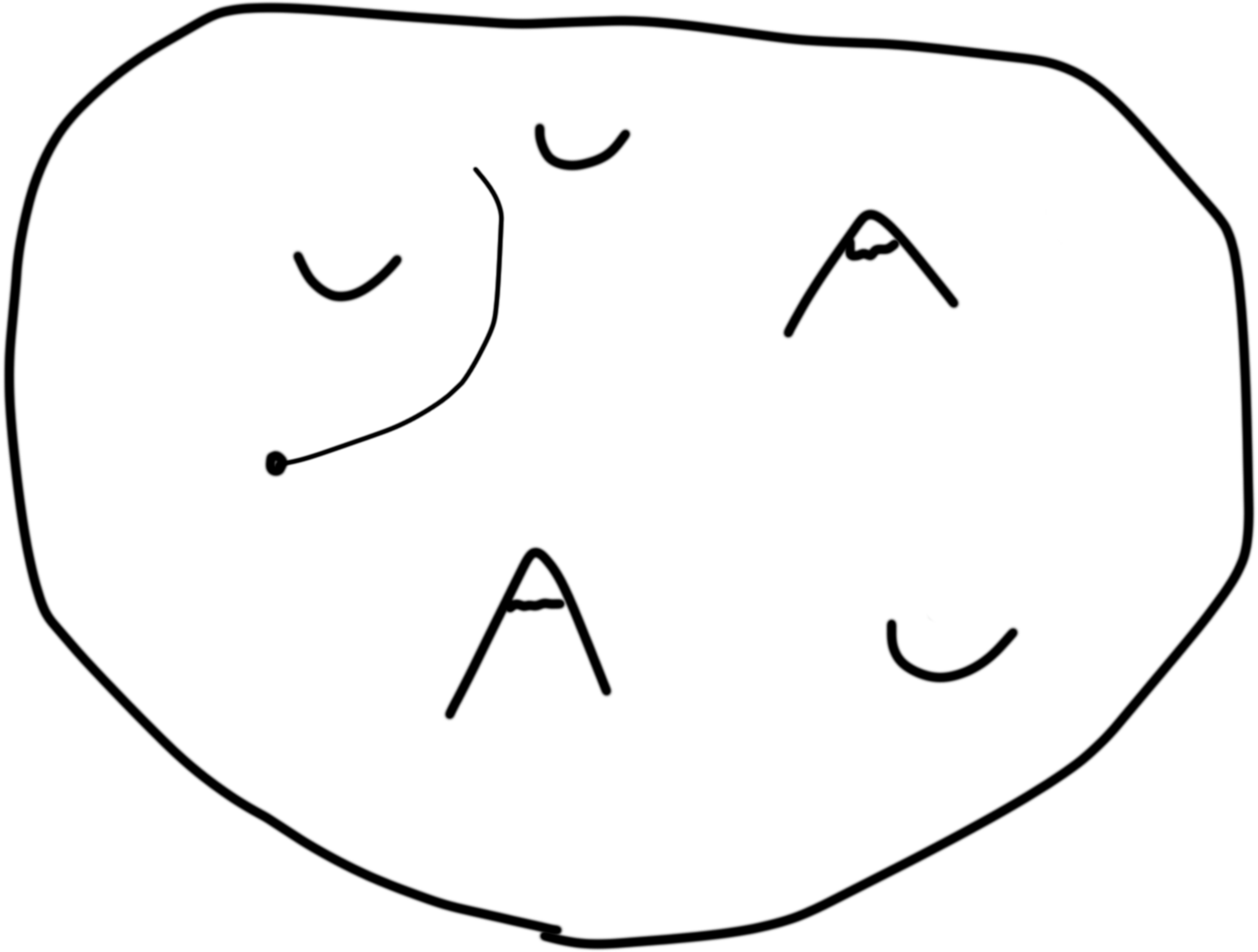
X

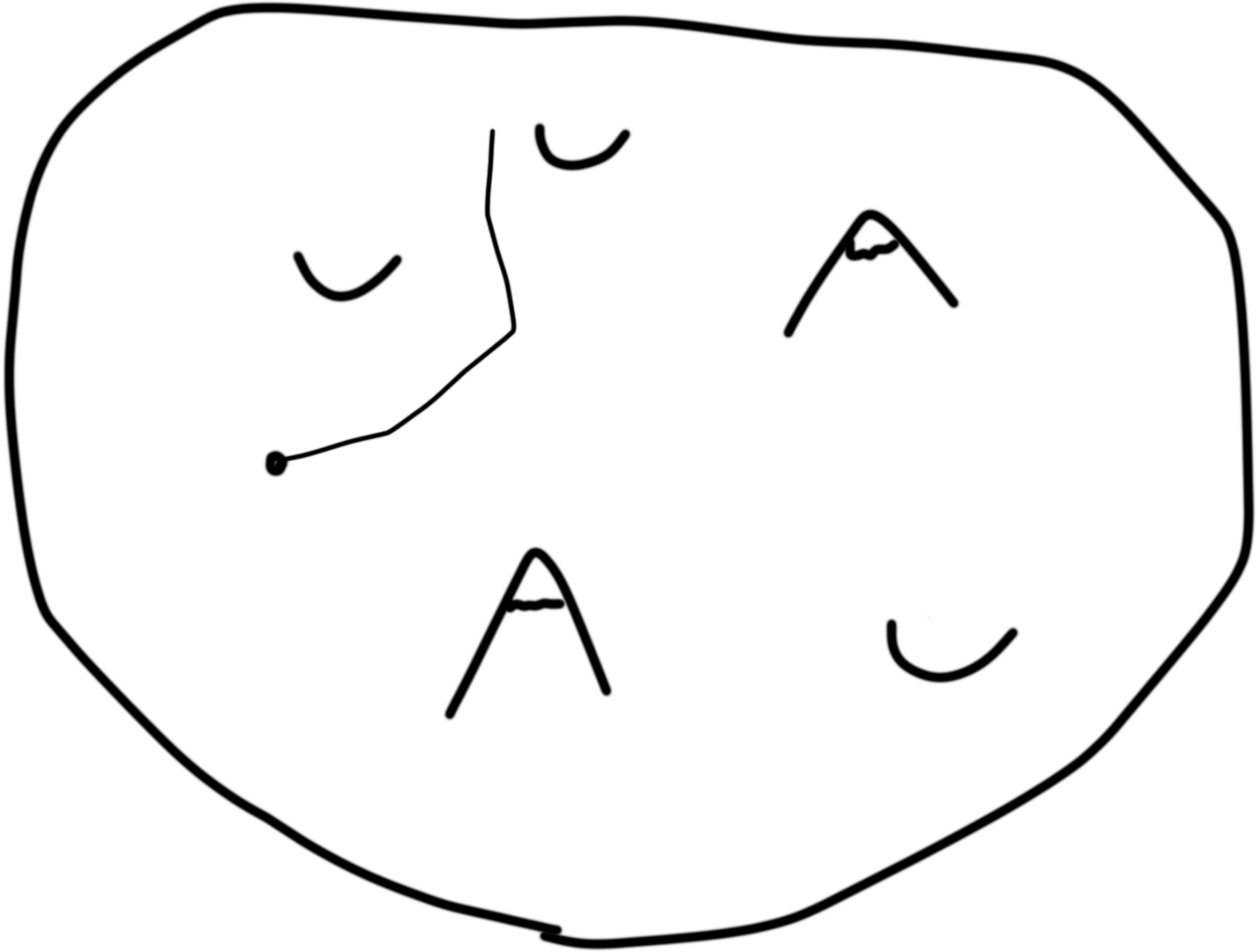


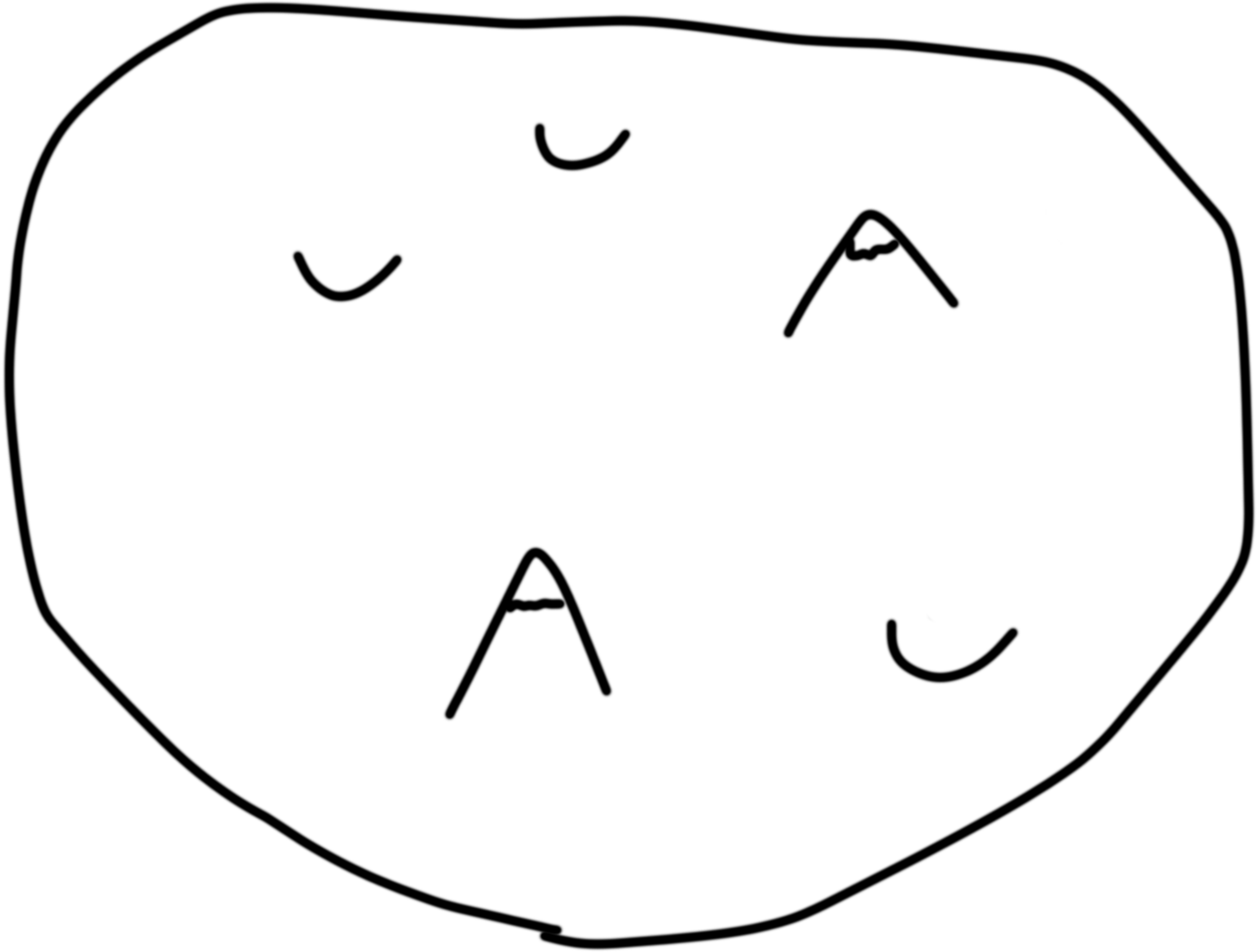
S(X)

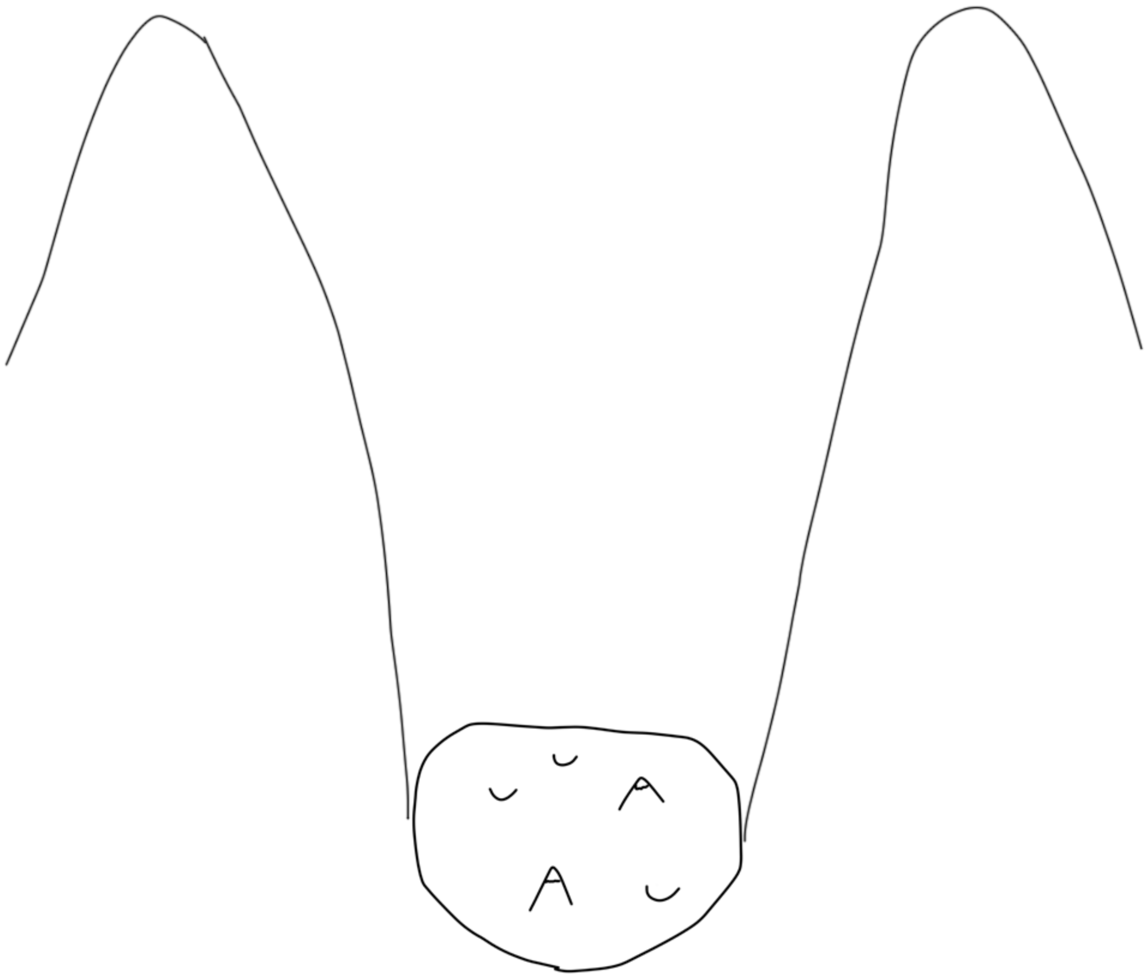


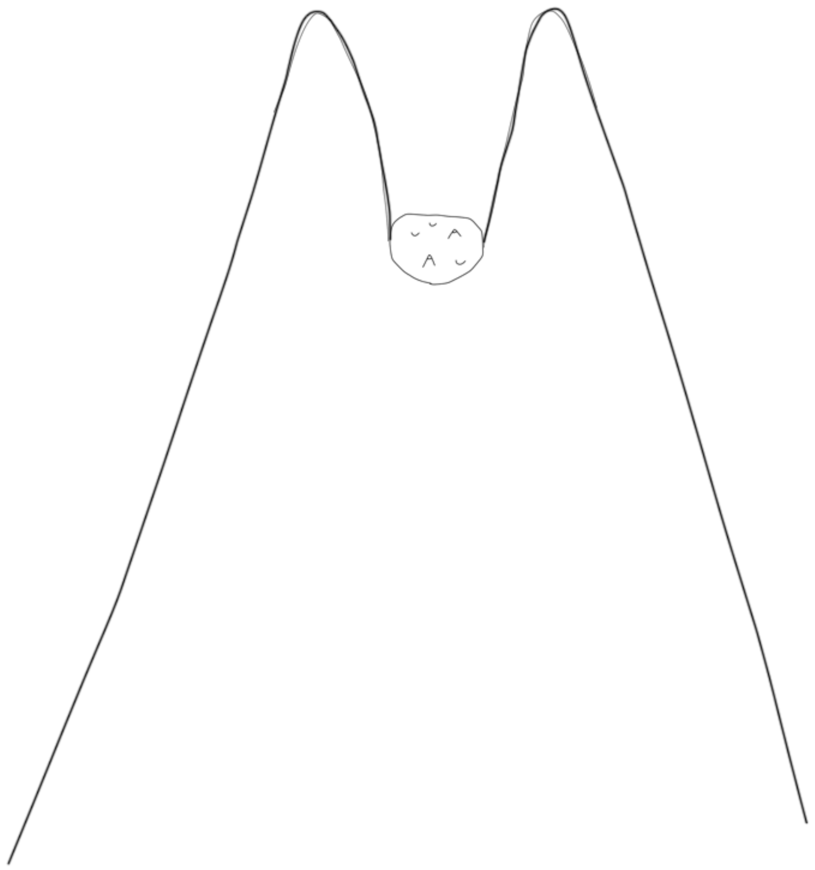








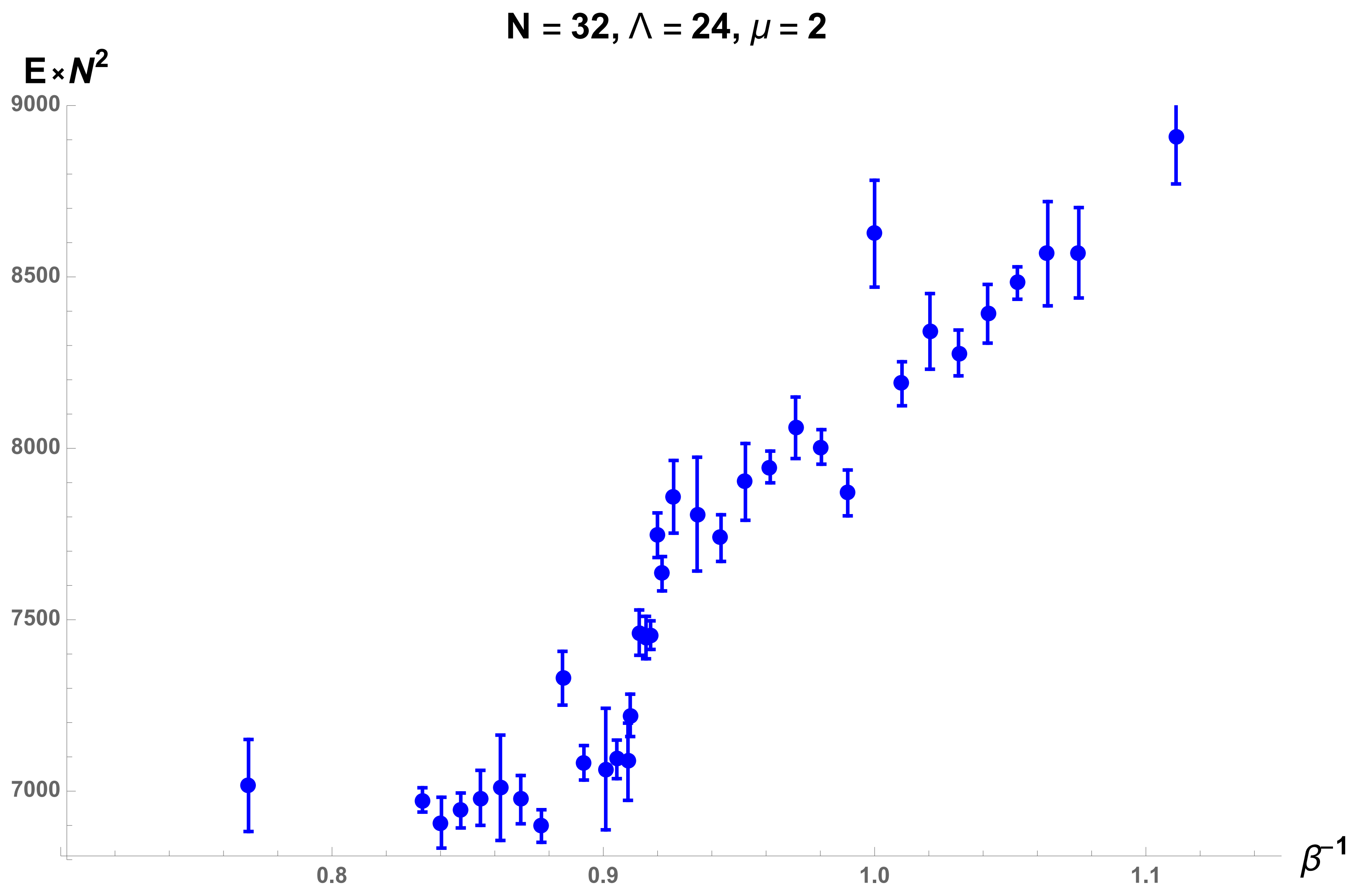




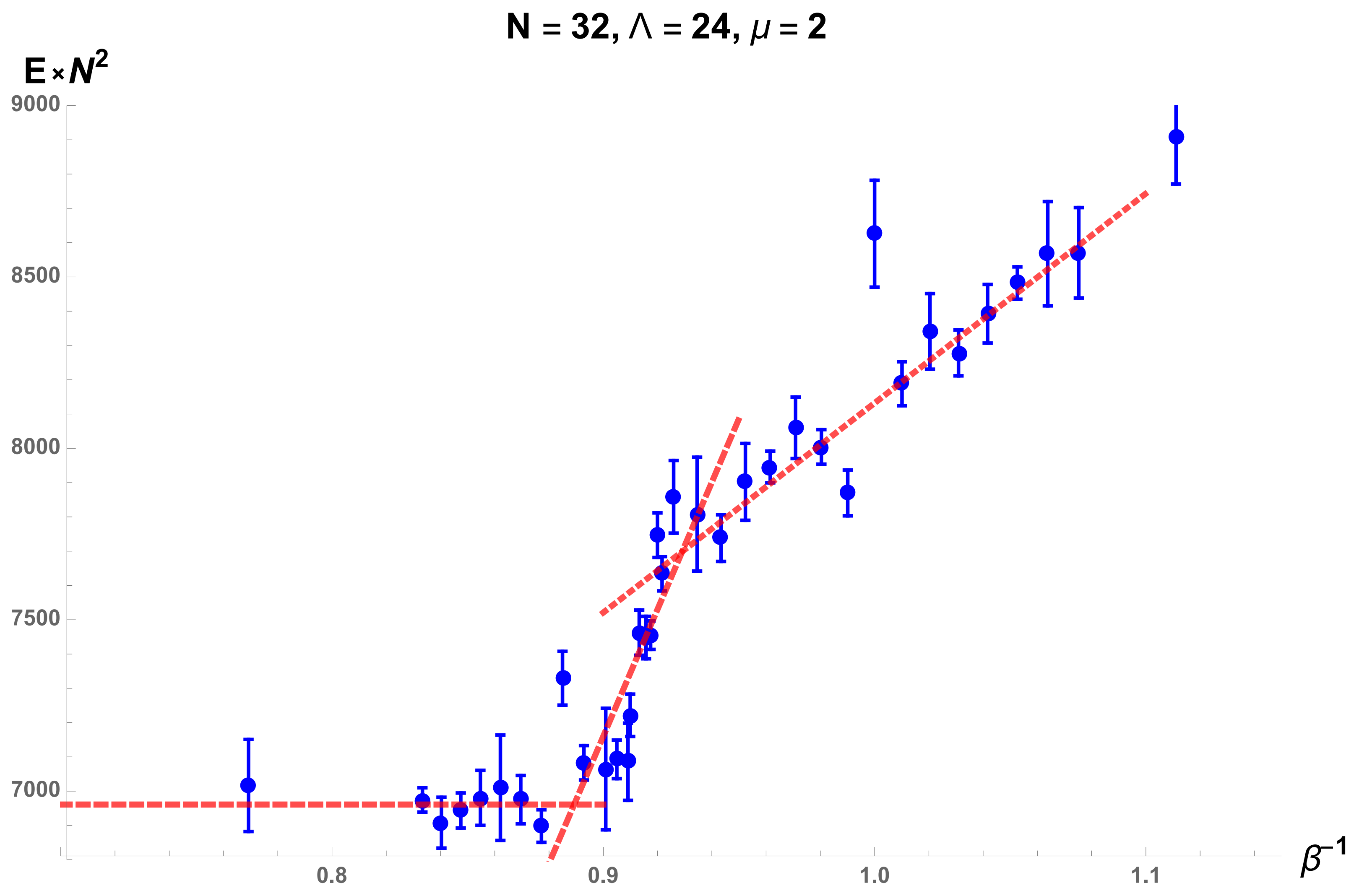
Simulations results

Simulations results

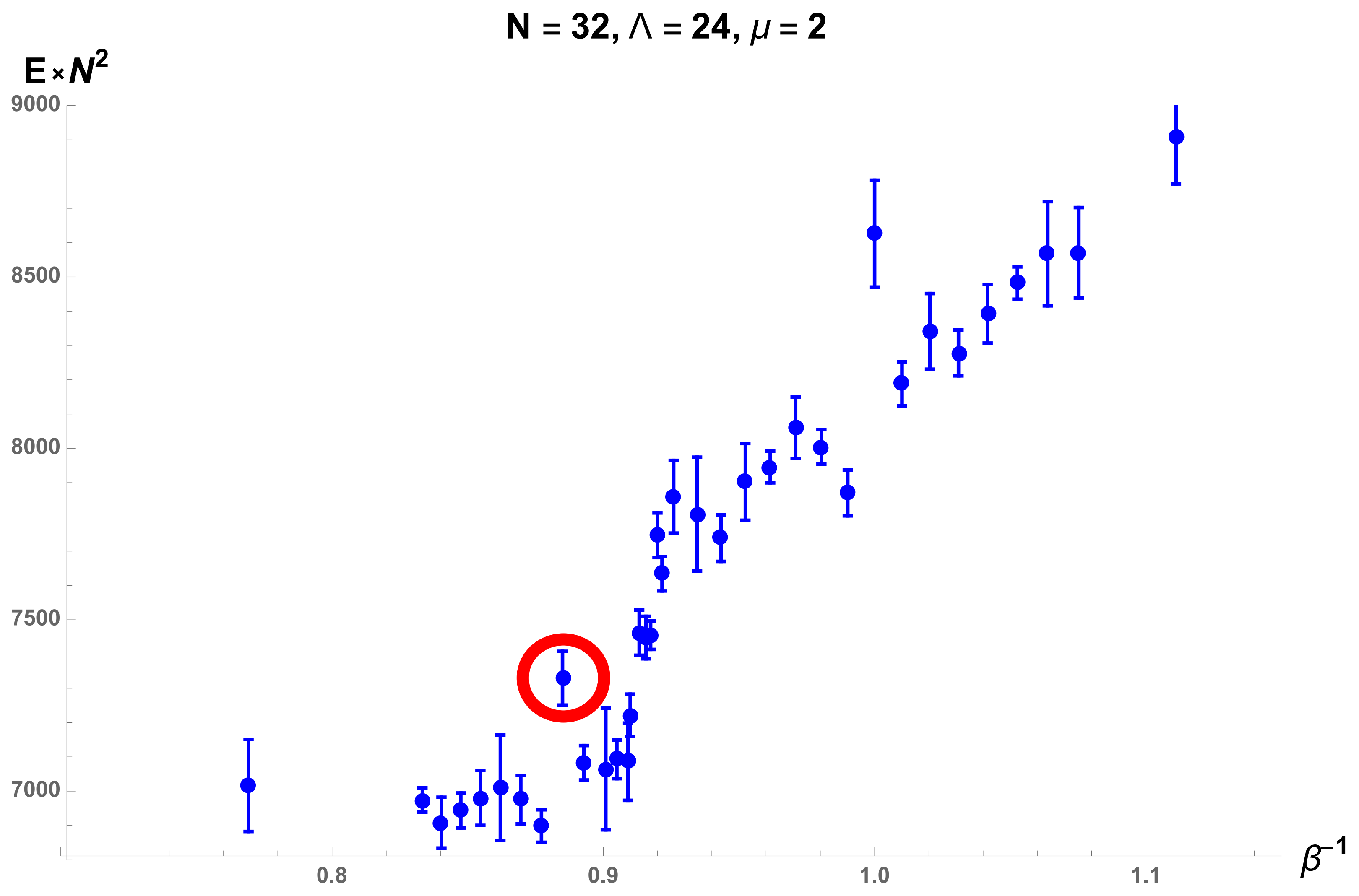
$$S[X,A] = N\int_0^\beta d\tau\,\mathrm{Tr}\,\left[\frac{1}{2}D_\tau X^i D_\tau X^i - \frac{1}{4}\left([X^r,X^s] + \frac{i\mu}{3}\varepsilon^{rst}X_t\right)^2 - \frac{1}{2}[X^r,X^m]^2 - \frac{1}{4}[X^m,X^n]^2 + \frac{1}{2}\left(\frac{\mu}{6}\right)^2 X_m^2\right]$$

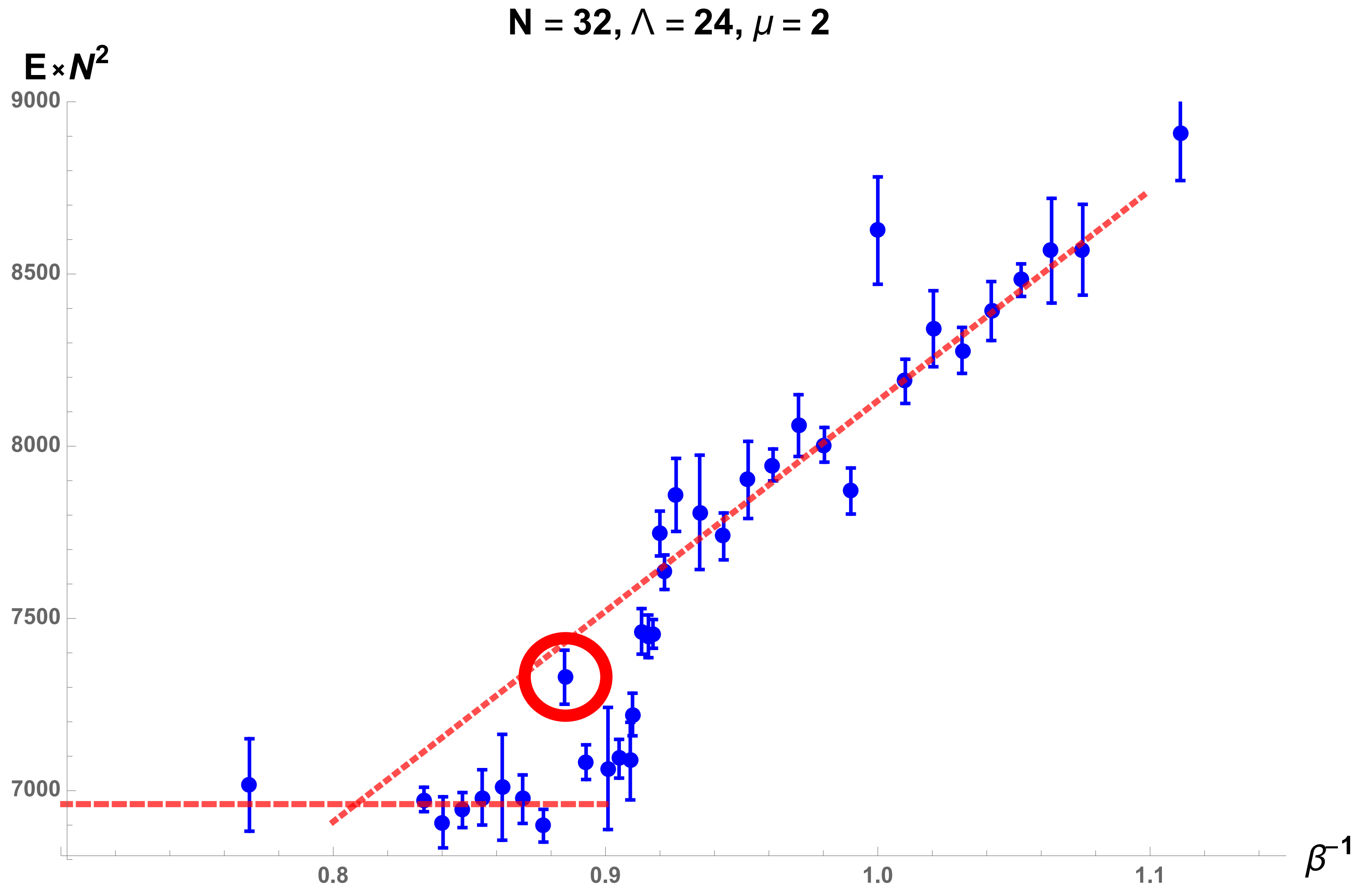


N = 32, Λ = 24, μ = 2
E × N²
9000
8500
8000
7500
7000
0.8
0.9
1.0
1.1
β⁻¹

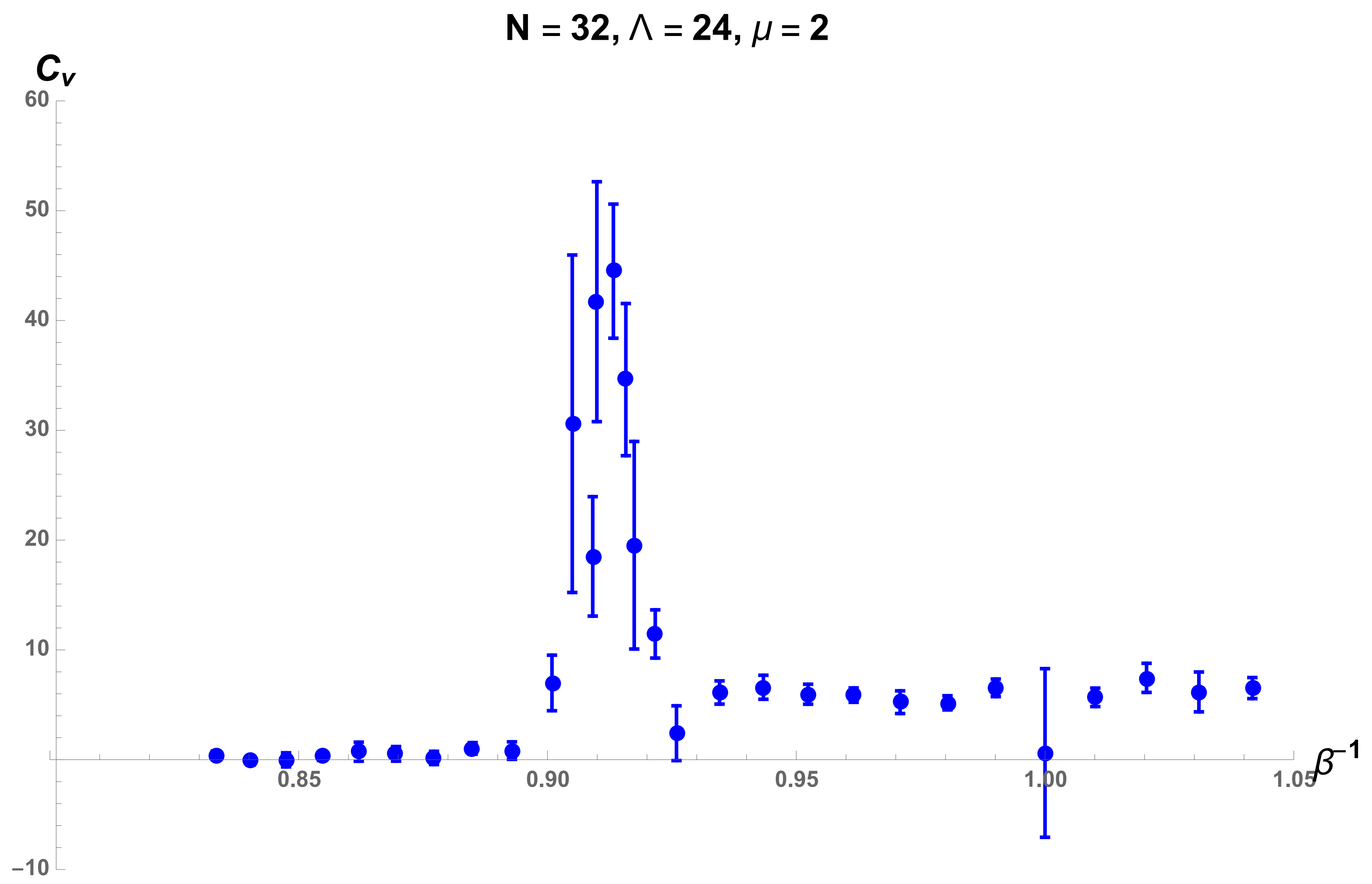


N = 32, Λ = 24, μ = 2
E × N²
9000
8500
8000
7500
7000
0.8
0.9
1.0
1.1
β⁻¹


N = 32, Λ = 24, μ = 2
E × N²
9000
8500
8000
7500
7000
0.8
0.9
1.0
1.1
β⁻¹

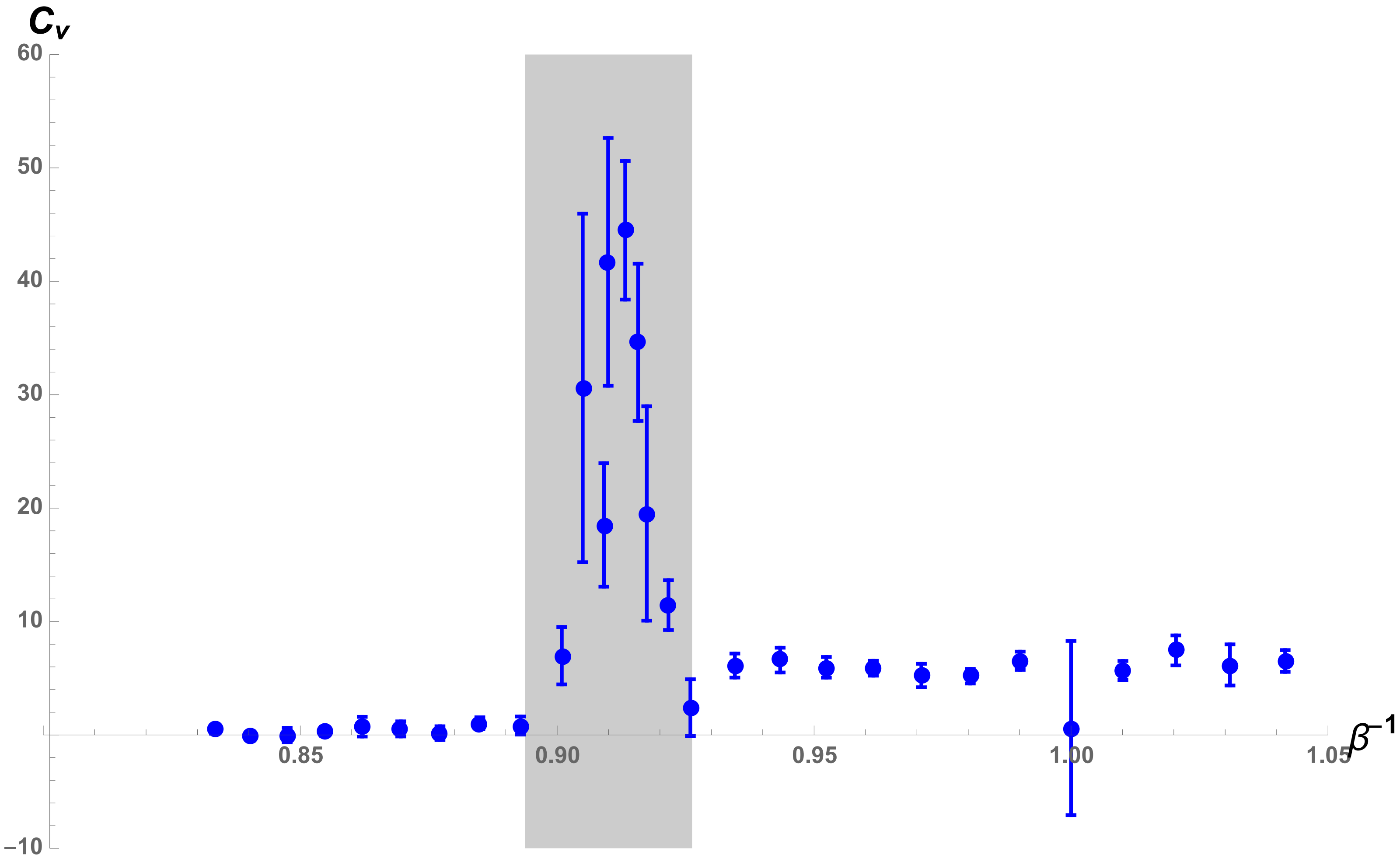


N = 32, Λ = 24, μ = 2
E × N2
9000
8500
8000
7500
7000
0.8
0.9
1.0
1.1
β−1

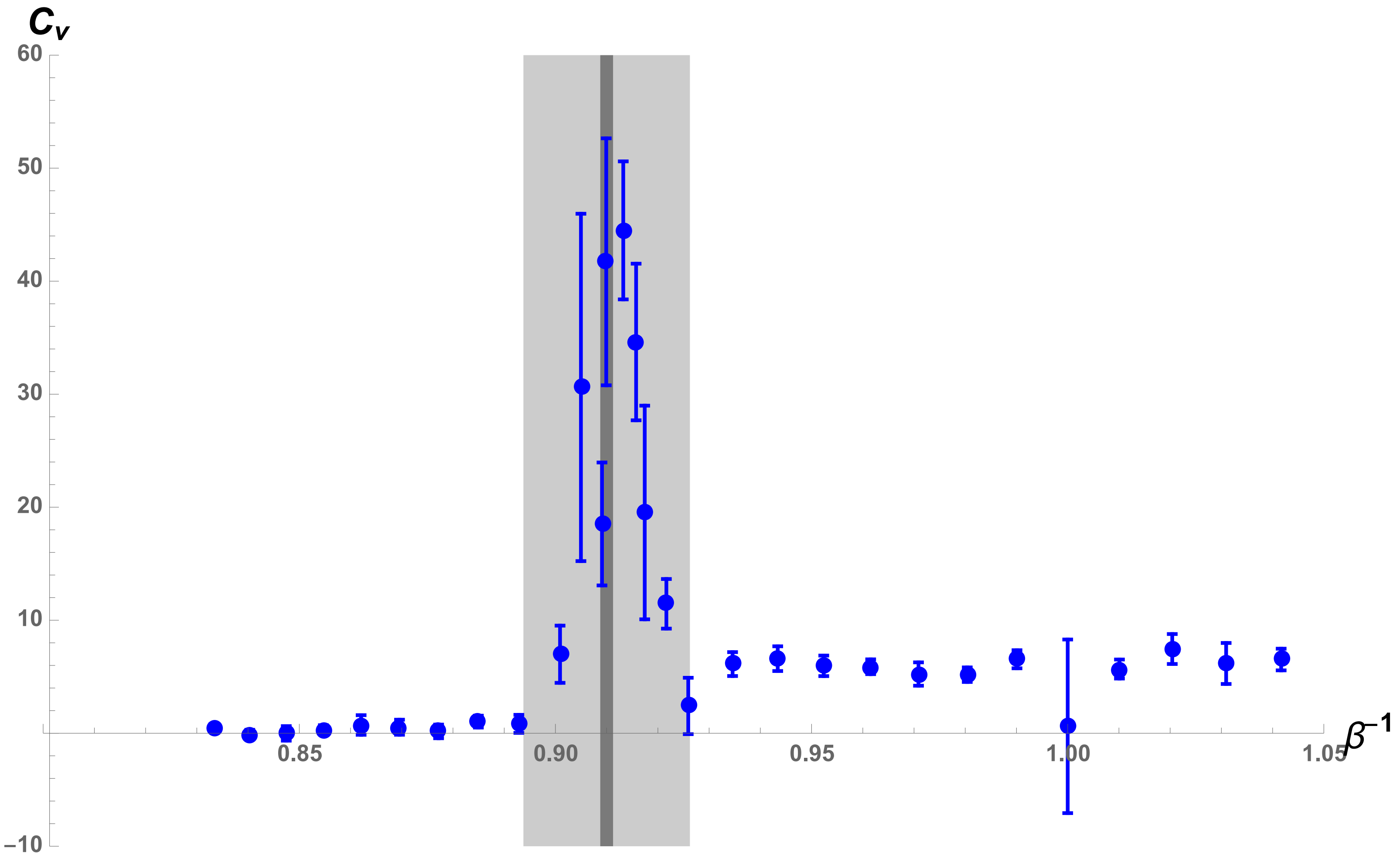


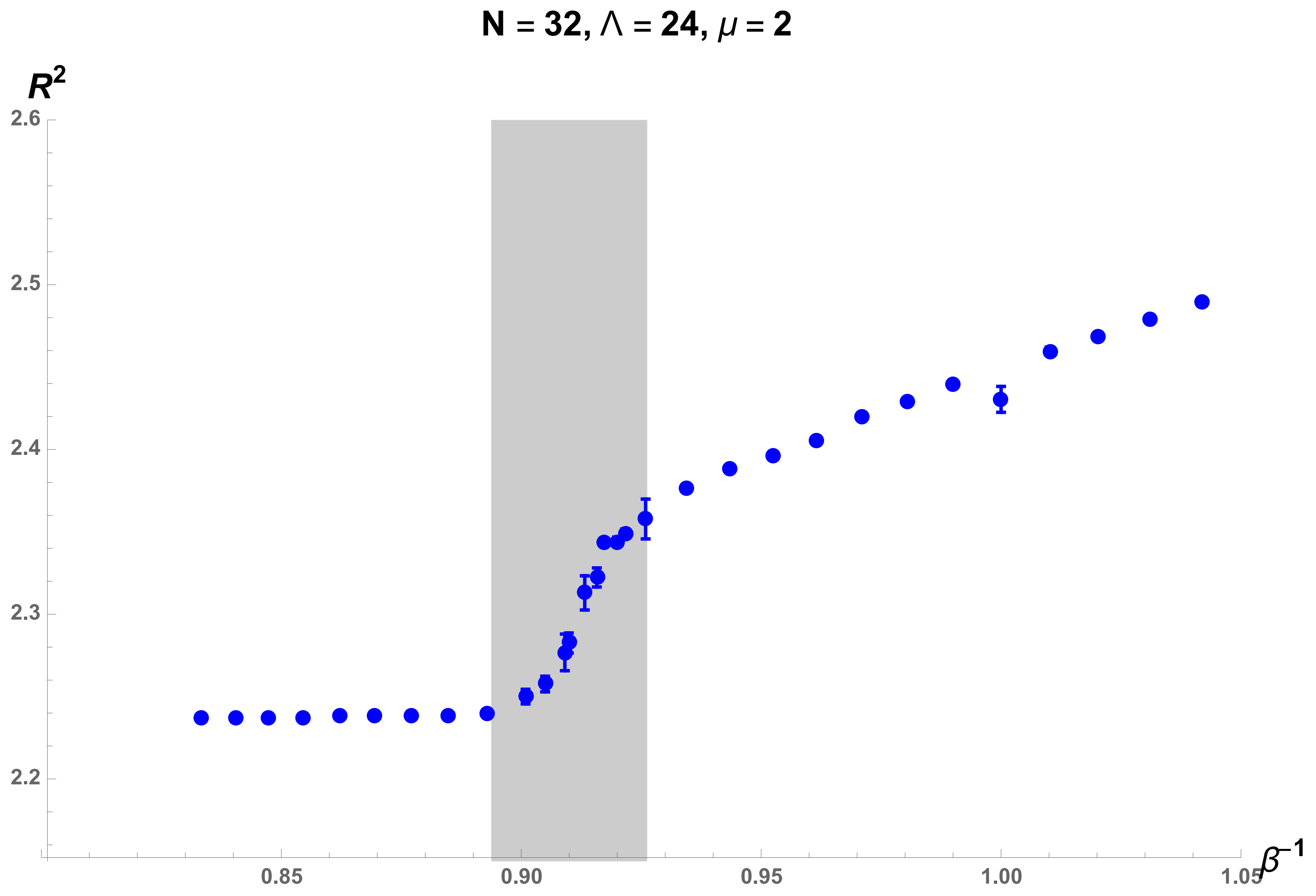
N = 32, Λ = 24, μ = 2
C_v
60
50
40
30
20
10
−10
0.85
0.90
0.95
1.00
1.05
β^−1

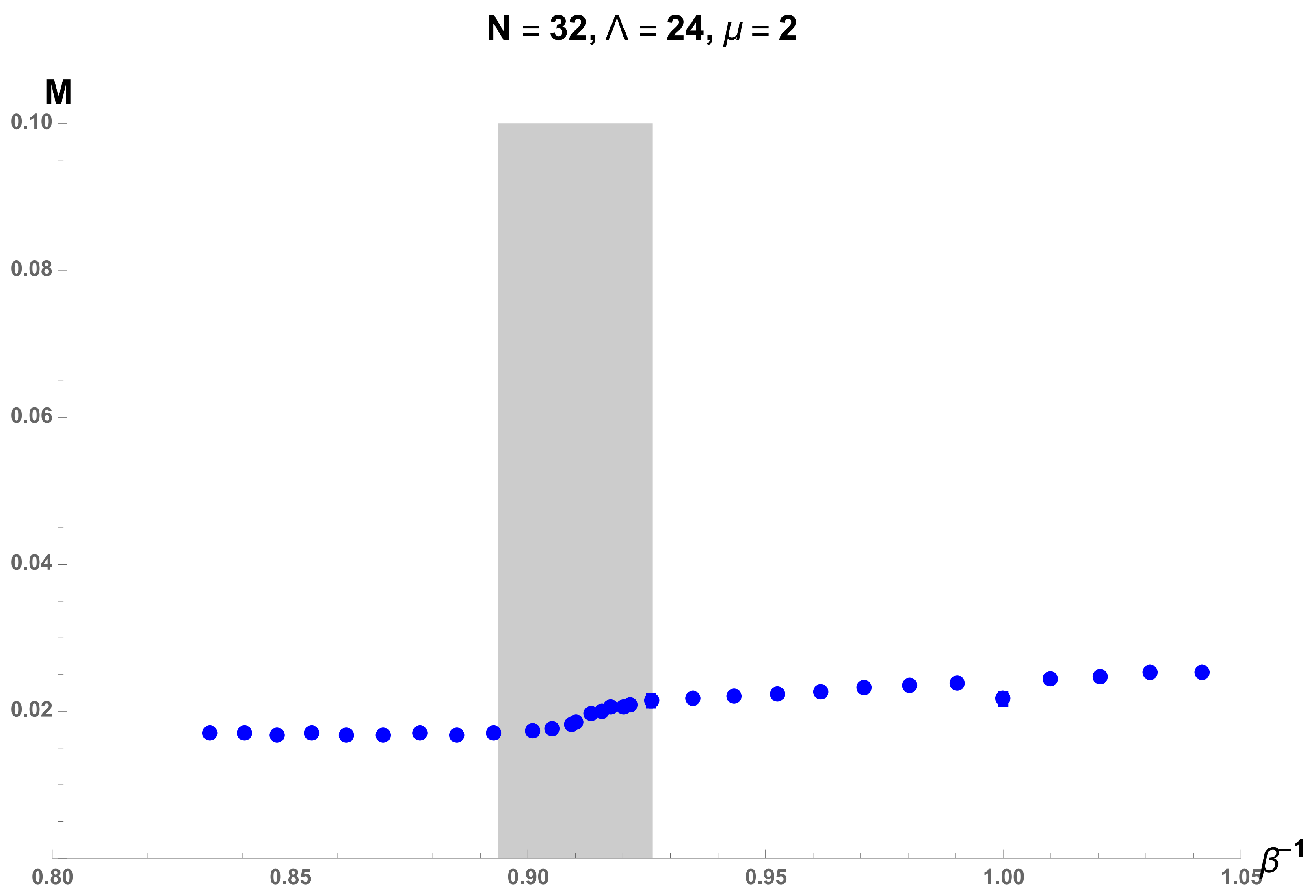
N = 32, Λ = 24, μ = 2



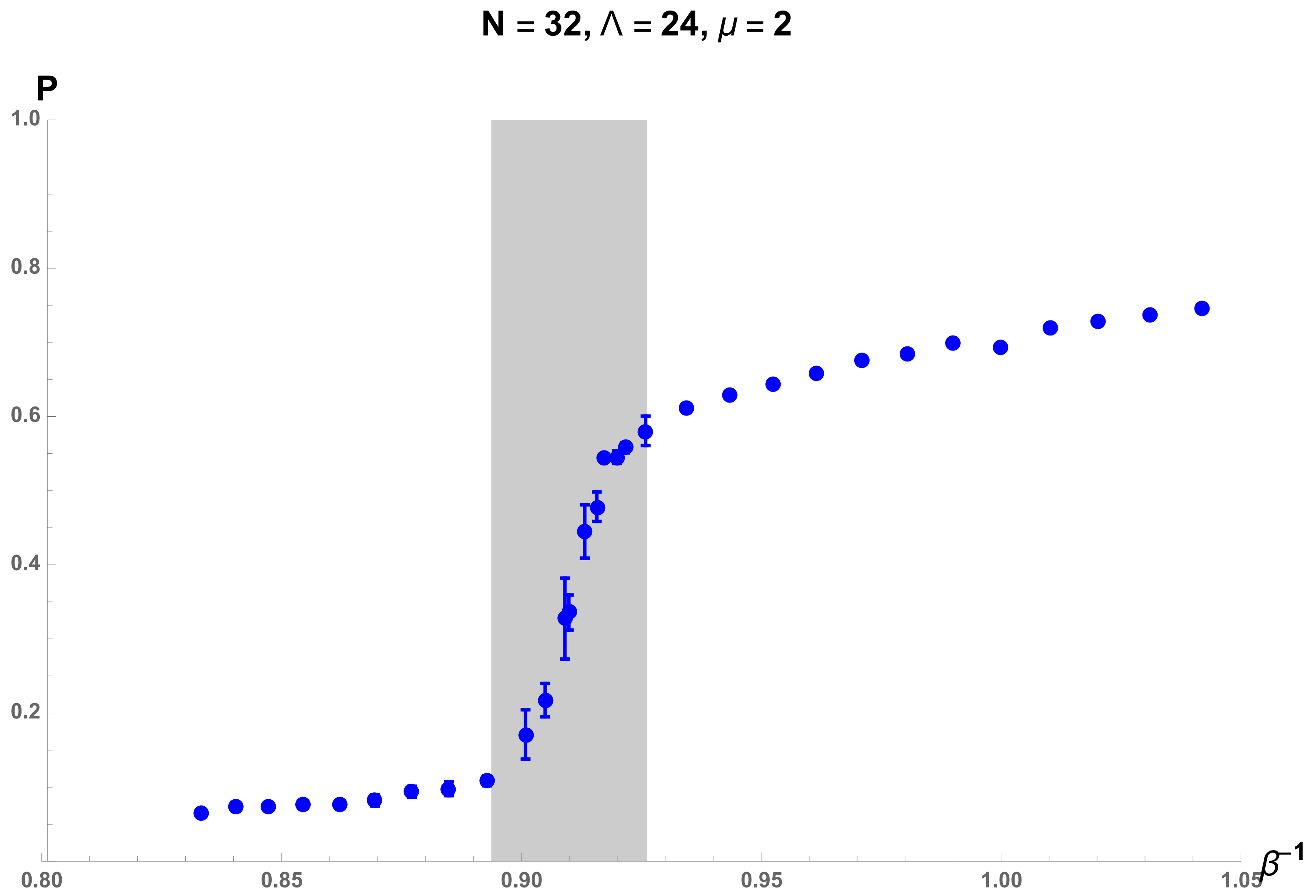
$N = 32, \Lambda = 24, \mu = 2$



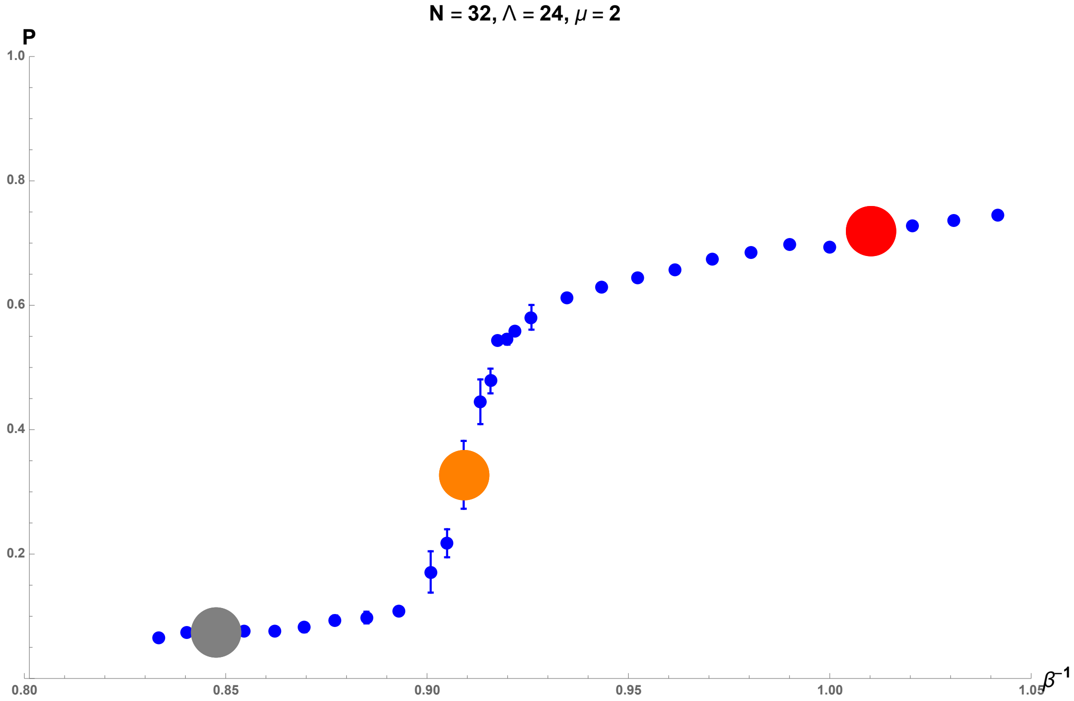

N = 32, Λ = 24, μ = 2
R2
2.6
2.5
2.4
2.3
2.2
0.85
0.90
0.95
1.00
1.05
β−1



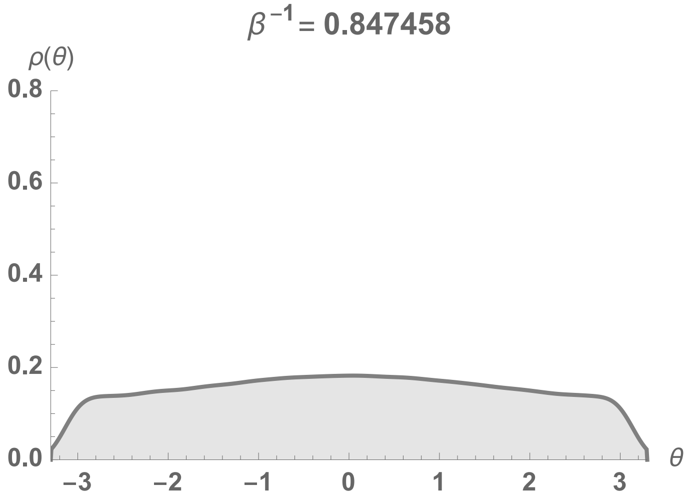
N = 32, Λ = 24, μ = 2
M
0.10
0.08
0.06
0.04
0.02
0.80
0.85
0.90
0.95
1.00
1.05
β−1

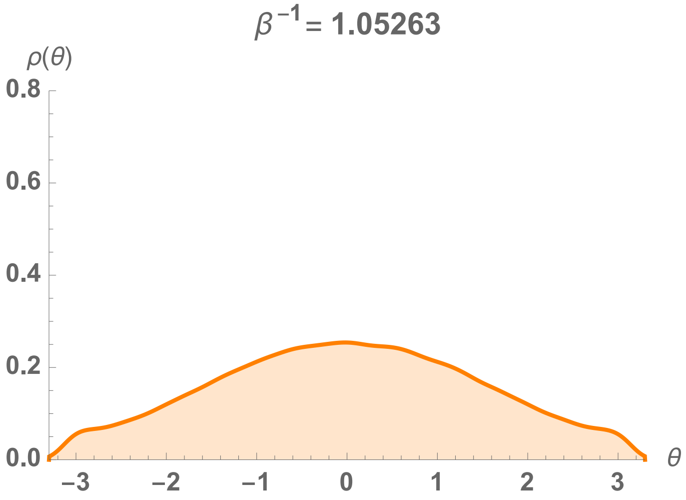


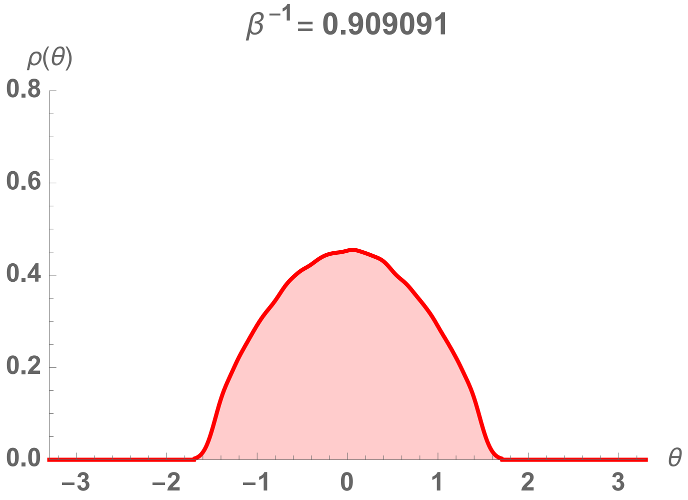
N = 32, Λ = 24, μ = 2
P
1.0
0.8
0.6
0.4
0.2
0.80
0.85
0.90
0.95
1.00
1.05
β⁻¹



Eigenvalue distribution of A







Moments of the e.v. distribution

Moments of the e.v. distribution

$$u_n = \int_{-\pi}^{\pi} \rho(\theta) e^{in\theta} d\theta$$

$$\rho(\theta) = \frac{1}{N} \sum_{i=1}^{N} \delta(\theta - \theta_i)$$

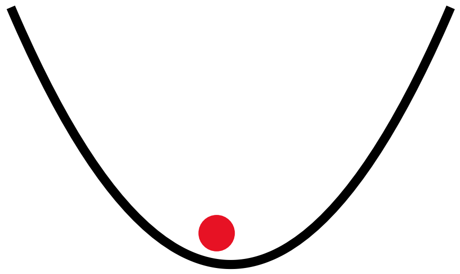
$$u_1 = \frac{1}{N} \sum_{i=1}^{N} e^{i\theta_i} = P$$

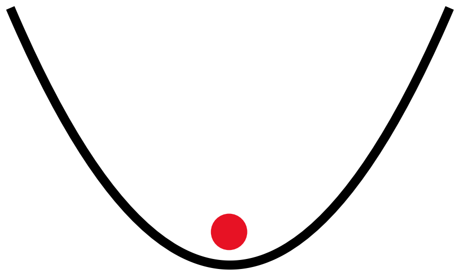
Effective action for the moments

$$S_e(\{u_n\}) = S_0 + \left(a_1|u_1|^2 + b_1|u_1|^4 + ...\right) + \left(a_2|u_2|^2 + b_2|u_2|^4 + ...\right)$$

Effective action for the moments

T ≈ 0

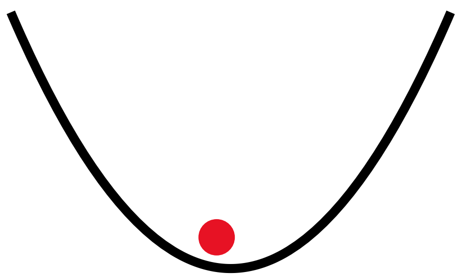


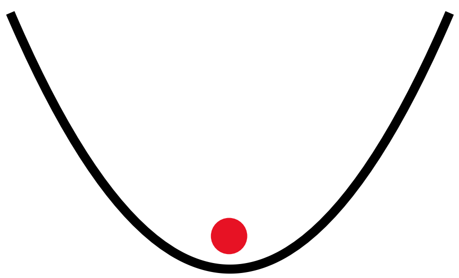


$$S_e(\{u_n\}) = S_0 + \left(a_1|u_1|^2 + b_1|u_1|^4 + ...\right) + \left(a_2|u_2|^2 + b_2|u_2|^4 + ...\right)$$

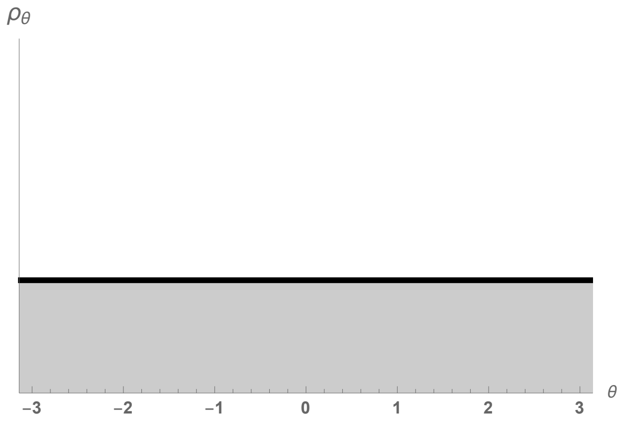
Effective action for the moments

T ≈ 0





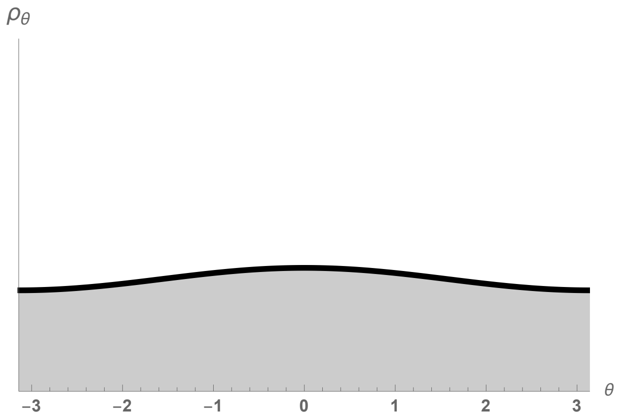
$$S_e(\{u_n\}) = S_0 + \left(a_1|u_1|^2 + b_1|u_1|^4 + \ldots\right) + \left(a_2|u_2|^2 + b_2|u_2|^4 + \ldots\right)$$



Effective action for the moments

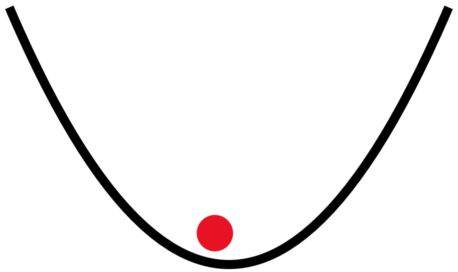
T ≈ 0

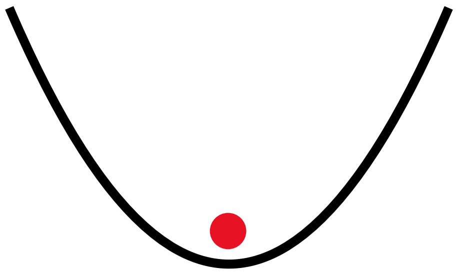
$$S_e(\{u_n\}) = S_0 + \left(a_1|u_1|^2 + b_1|u_1|^4 + \ldots\right) + \left(a_2|u_2|^2 + b_2|u_2|^4 + \ldots\right)$$



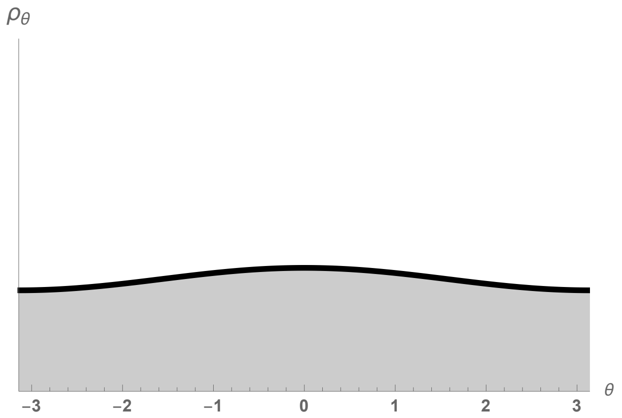
Effective action for the moments

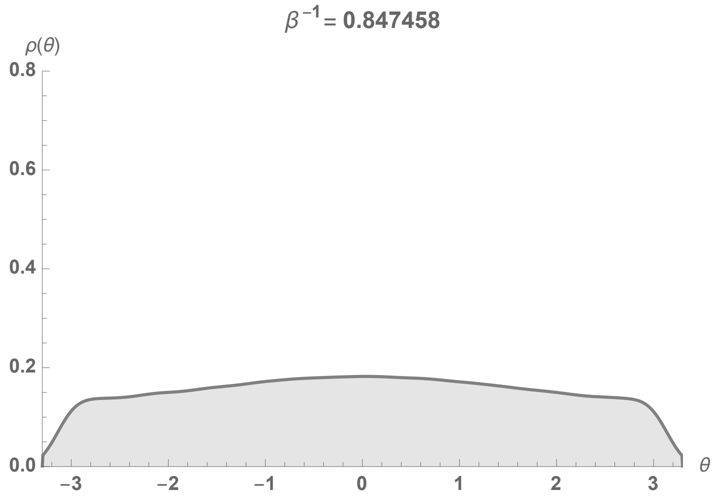
T ≈ 0





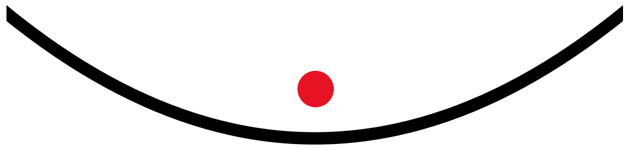
$$S_e(\{u_n\}) = S_0 + \left(a_1|u_1|^2 + b_1|u_1|^4 + \ldots\right) + \left(a_2|u_2|^2 + b_2|u_2|^4 + \ldots\right)$$

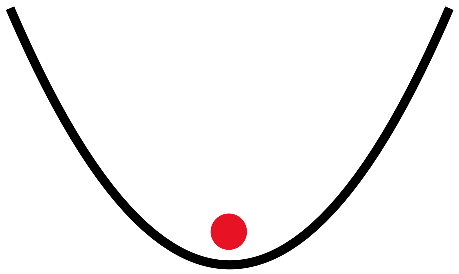




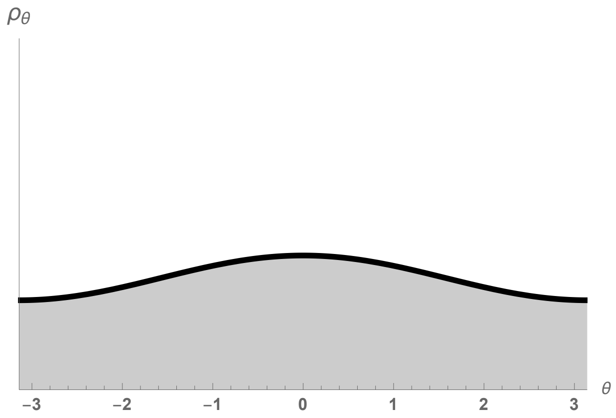
Effective action for the moments

T > 0



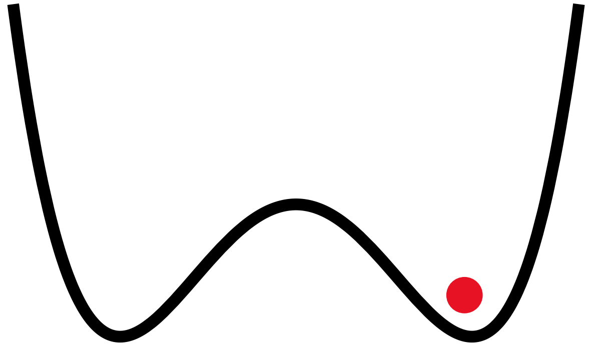


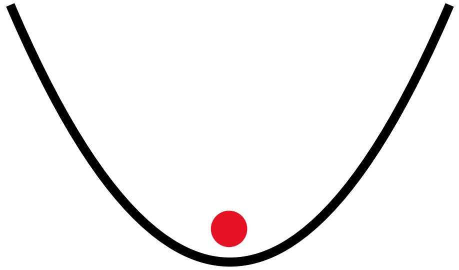
$$S_e(\{u_n\}) = S_0 + \left(a_1|u_1|^2 + b_1|u_1|^4 + \ldots\right) + \left(a_2|u_2|^2 + b_2|u_2|^4 + \ldots\right)$$



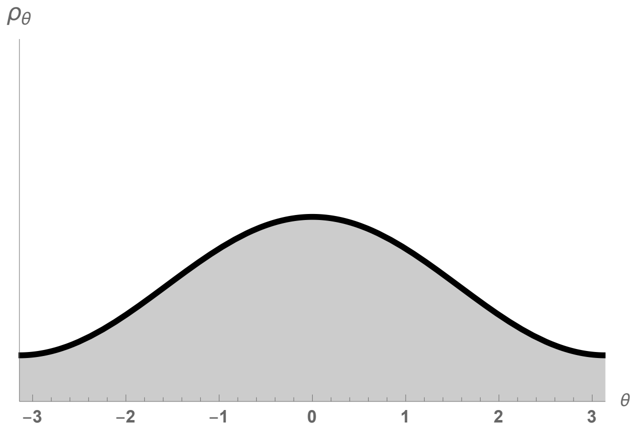
Effective action for the moments

T ≈ T_c



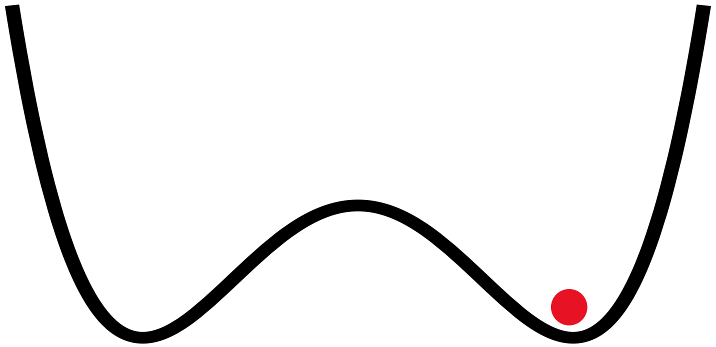


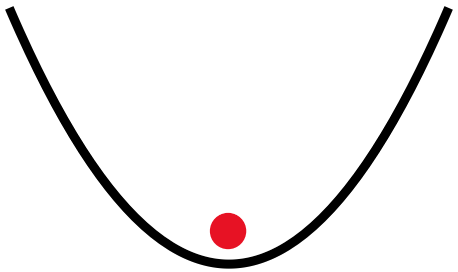
$$S_e(\{u_n\}) = S_0 + \left(a_1|u_1|^2 + b_1|u_1|^4 + \ldots\right) + \left(a_2|u_2|^2 + b_2|u_2|^4 + \ldots\right)$$



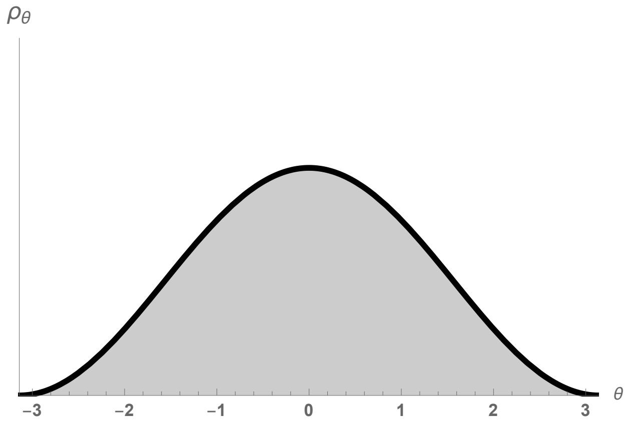
Effective action for the moments

T > Tc





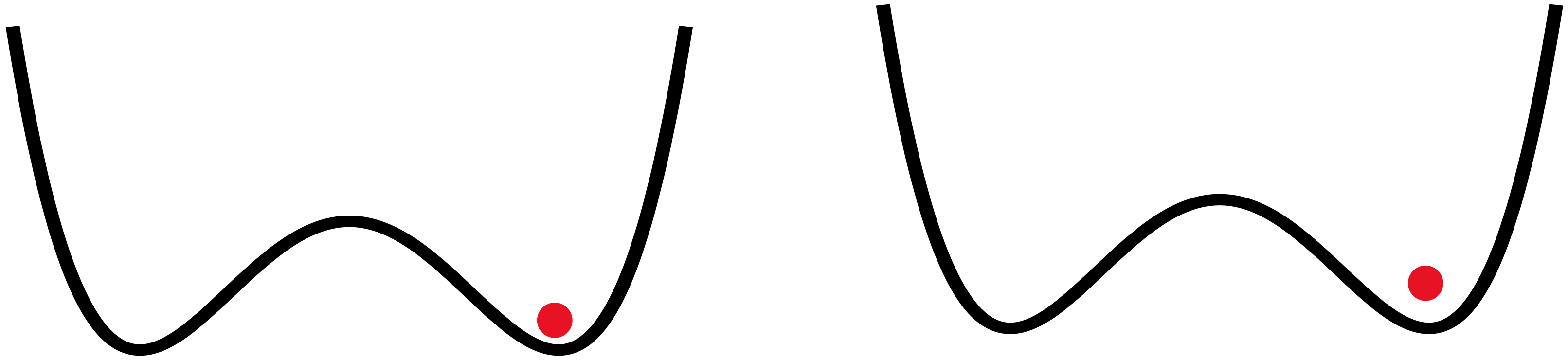
$$S_e(\{u_n\}) = S_0 + \left(a_1|u_1|^2 + b_1|u_1|^4 + \ldots\right) + \left(a_2|u_2|^2 + b_2|u_2|^4 + \ldots\right)$$



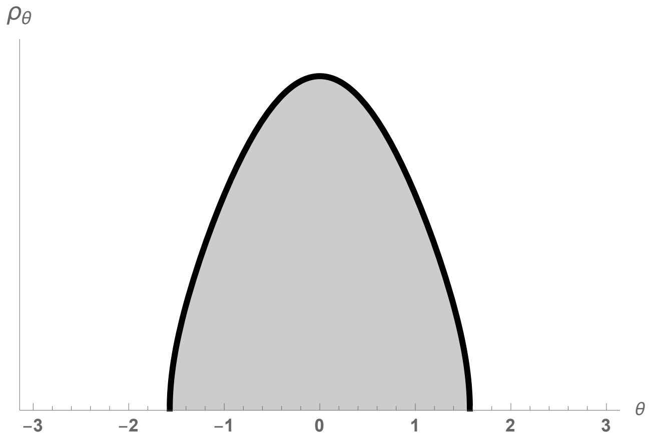
$\langle P \rangle \approx 1/2$

Effective action for the moments

$T > T_{c2}$



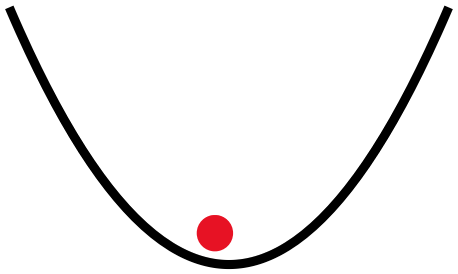
$$S_e(\{u_n\}) = S_0 + \left(a_1|u_1|^2 + b_1|u_1|^4 + \ldots\right) + \left(a_2|u_2|^2 + b_2|u_2|^4 + \ldots\right)$$

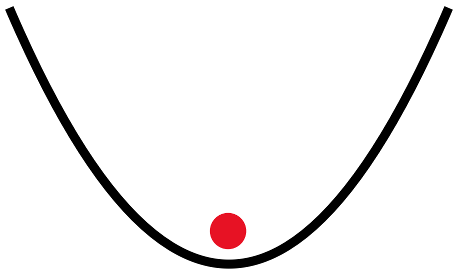


$1 > \langle P \rangle > 1/2$

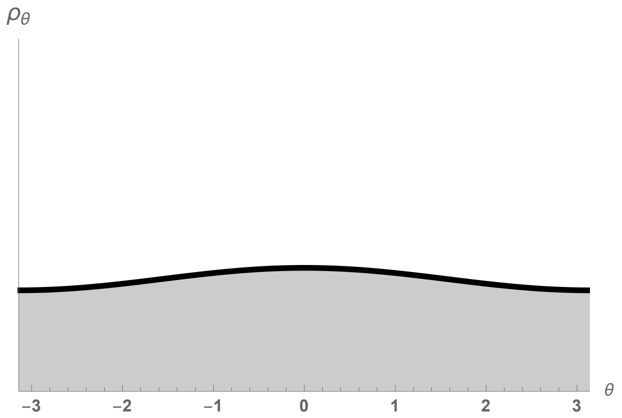
Effective action for the moments

T ≈ 0





$$S_e(\{u_n\}) = S_0 + \left(a_1|u_1|^2 + b_1|u_1|^4 + \ldots\right) + \left(a_2|u_2|^2 + b_2|u_2|^4 + \ldots\right)$$



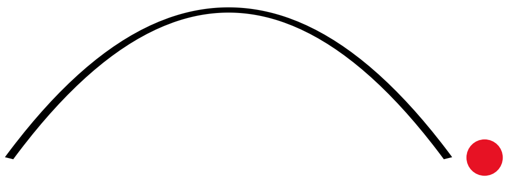
Effective action for the moments

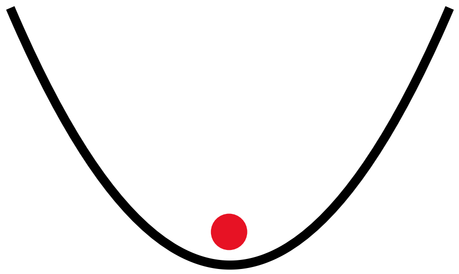
$T \approx T_c$

$$S_e(\{u_n\}) = S_0 + \left(a_1|u_1|^2 + b_1|u_1|^4 + ...\right) + \left(a_2|u_2|^2 + b_2|u_2|^4 + ...\right)$$

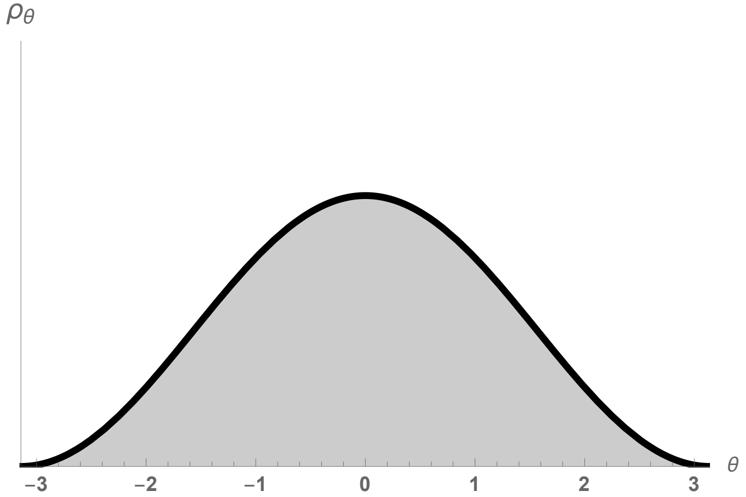
Effective action for the moments

$T \approx T_c$



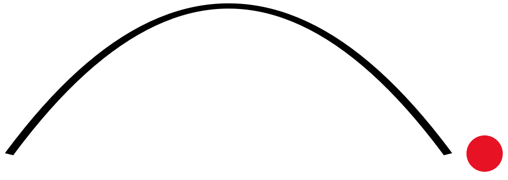


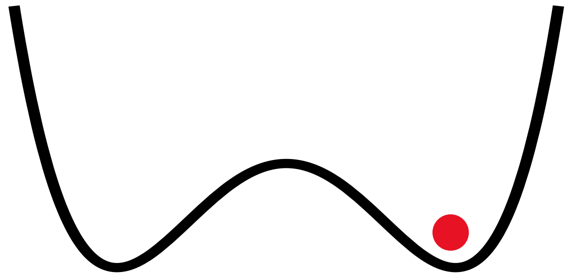
$$S_e(\{u_n\}) = S_0 + \left(a_1|u_1|^2 + b_1|u_1|^4 + \ldots\right) + \left(a_2|u_2|^2 + b_2|u_2|^4 + \ldots\right)$$



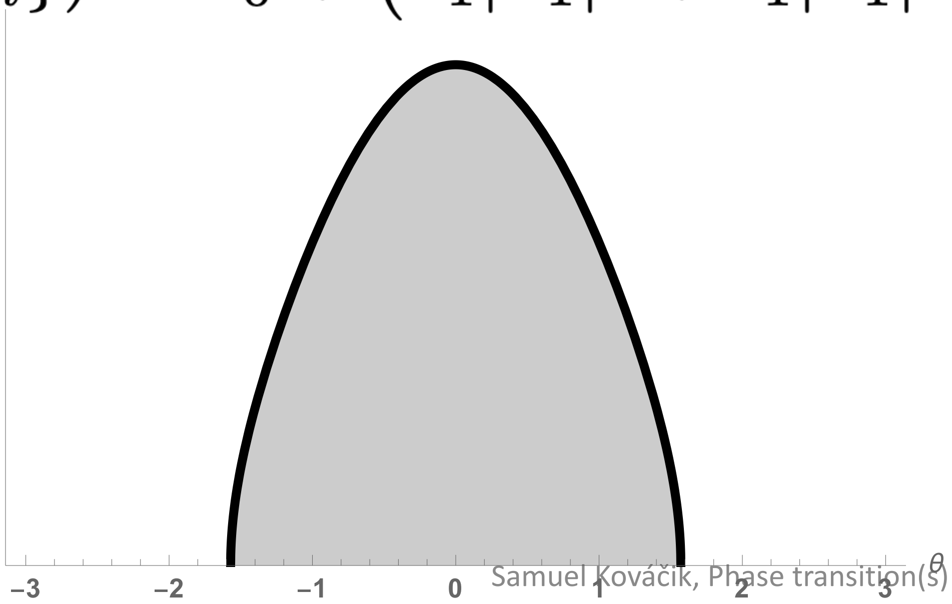
Effective action for the moments

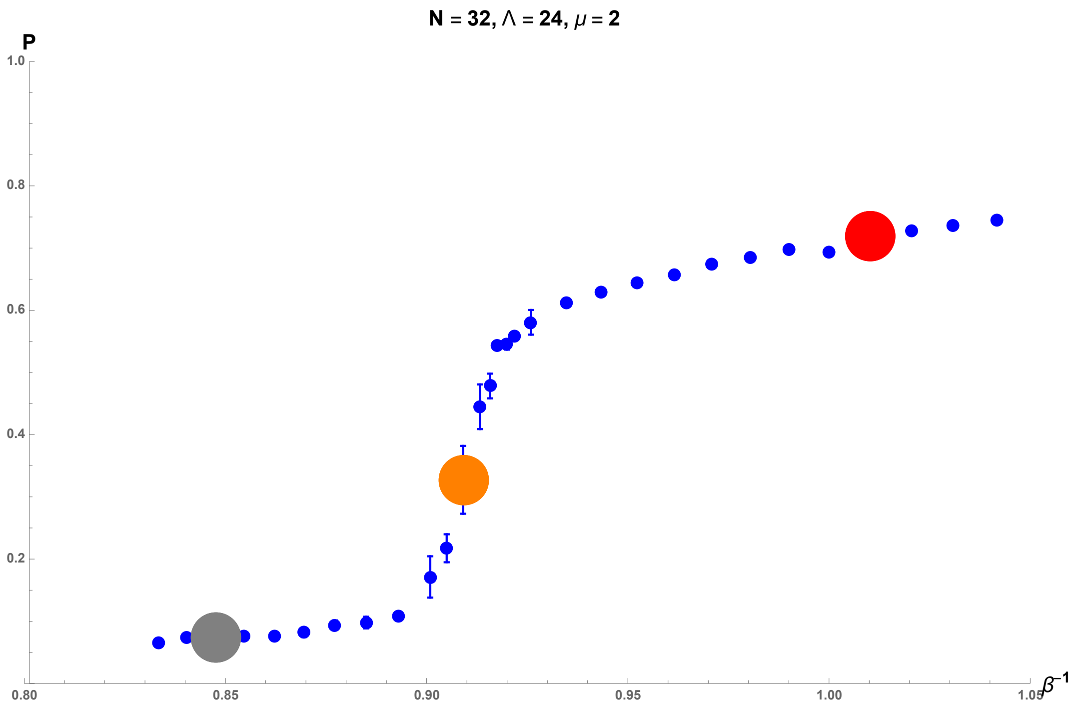
T > T_c





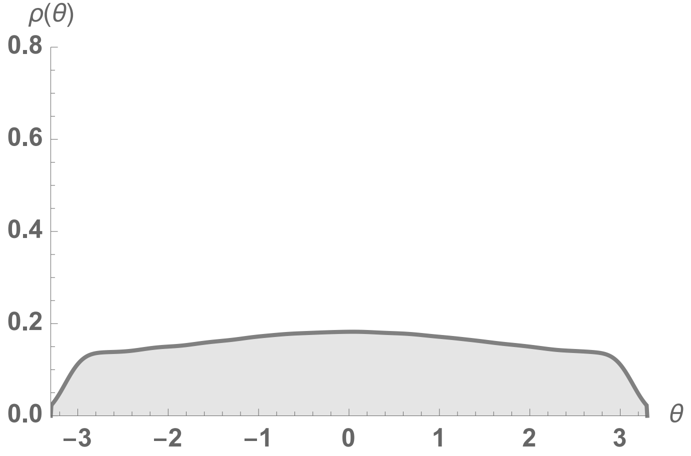
$$S_e(\{u_n\}) = S_0 + \left(a_1|u_1|^2 + b_1|u_1|^4 + \ldots\right) + \left(a_2|u_2|^2 + b_2|u_2|^4 + \ldots\right)$$



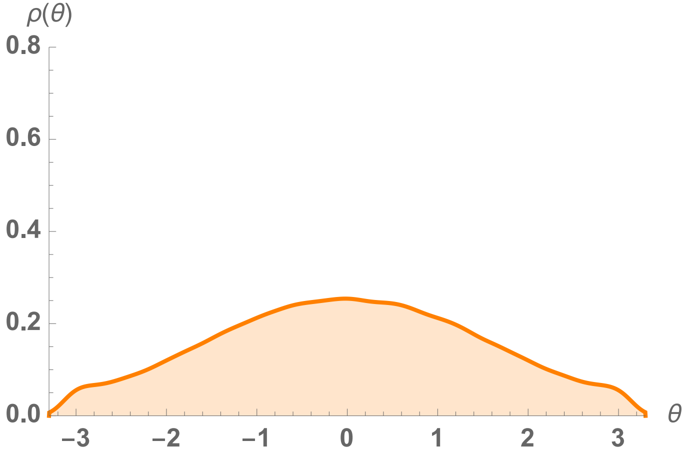


Eigenvalue distribution of A

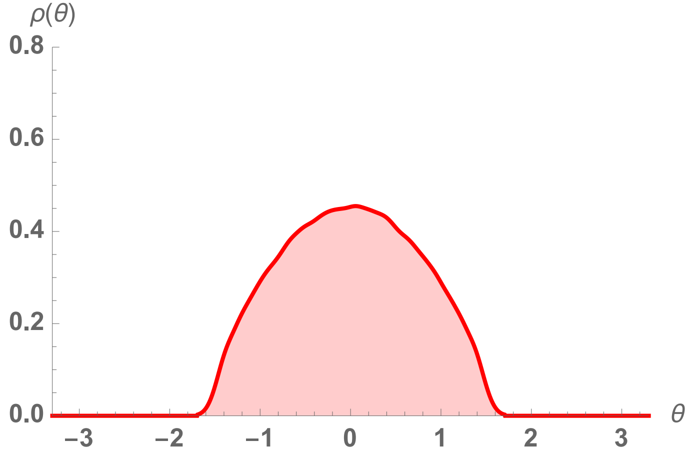
$\beta^{-1} = 0.847458$



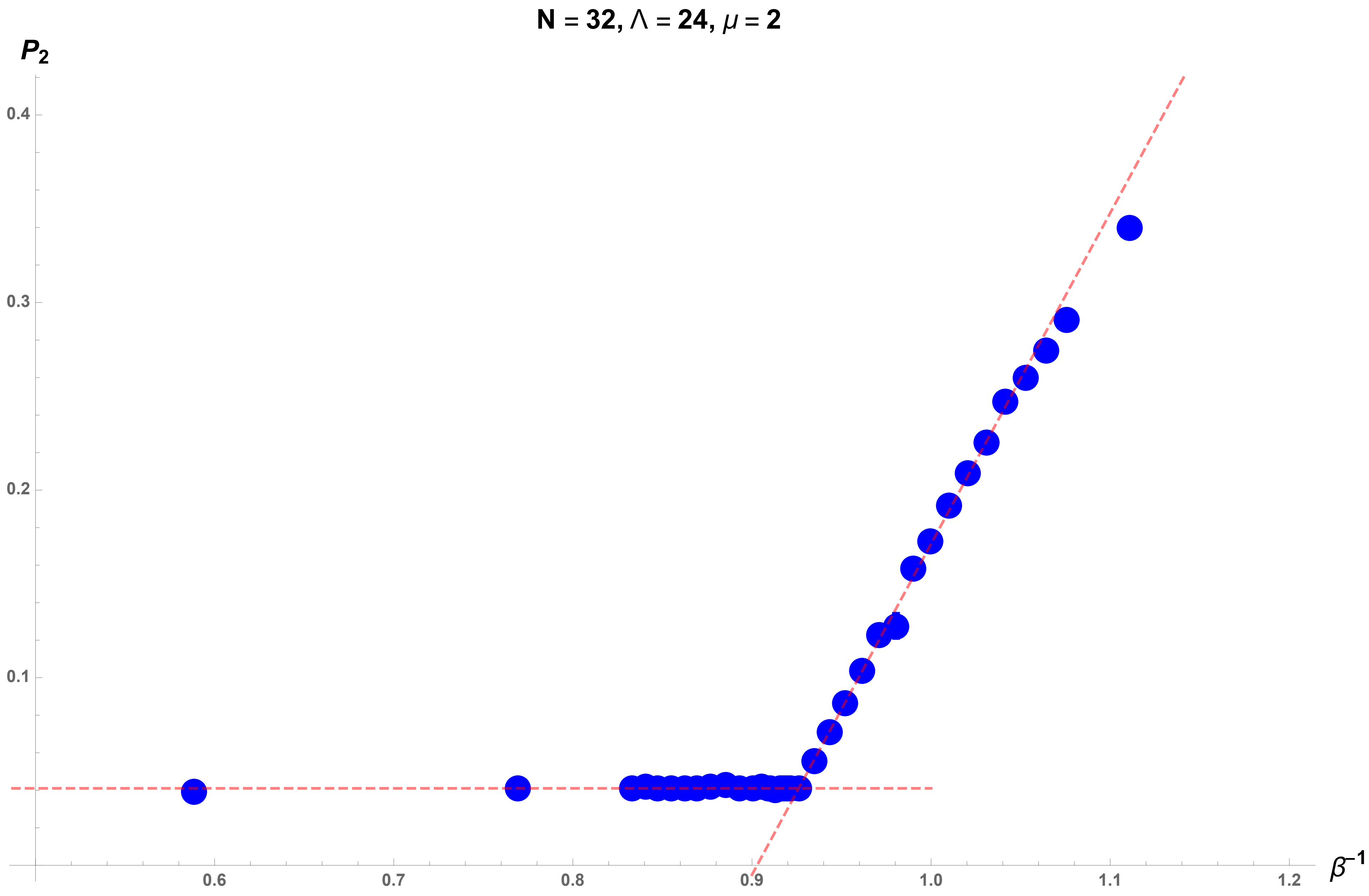
$\beta^{-1} = 1.05263$



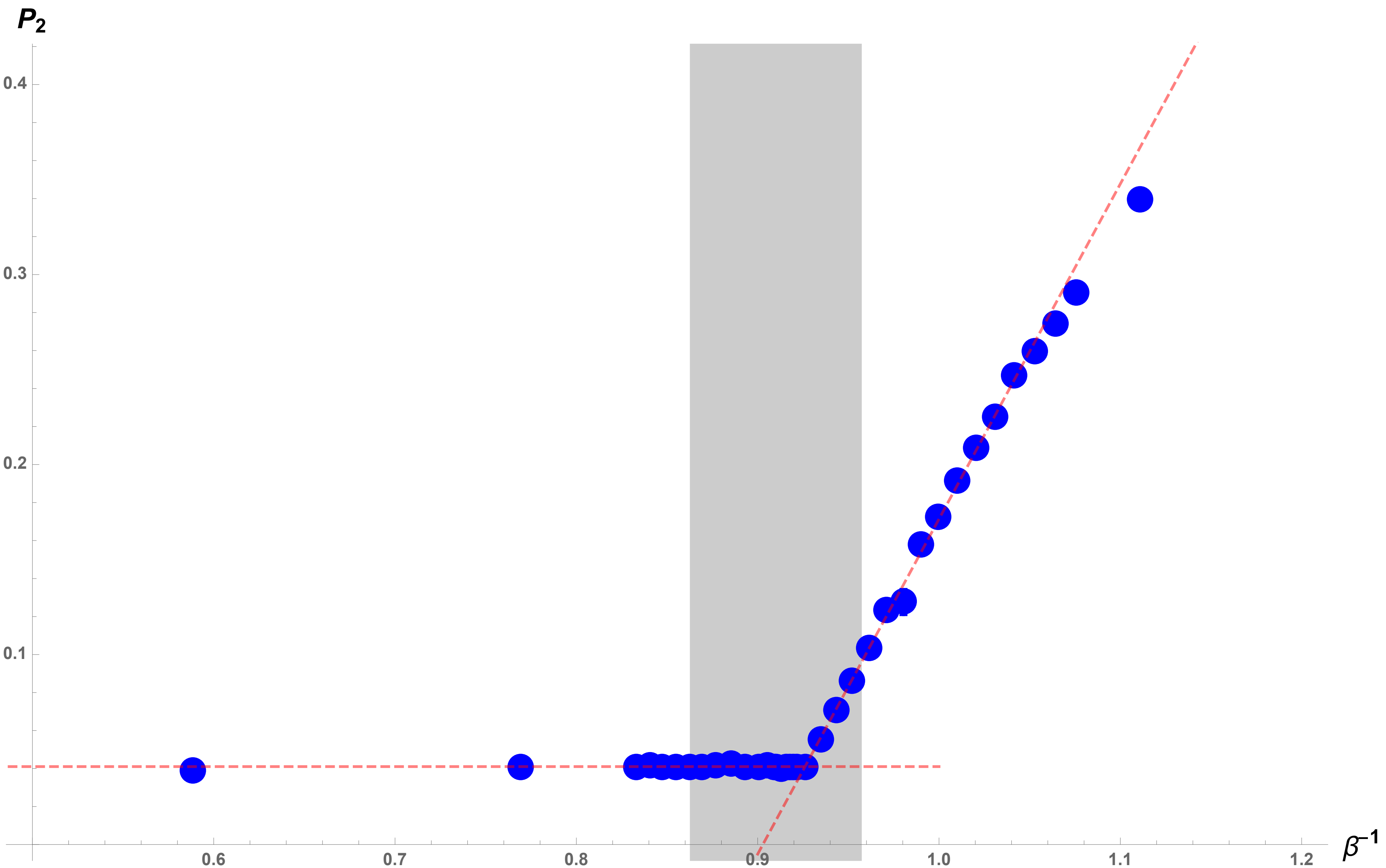
$\beta^{-1} = 0.909091$



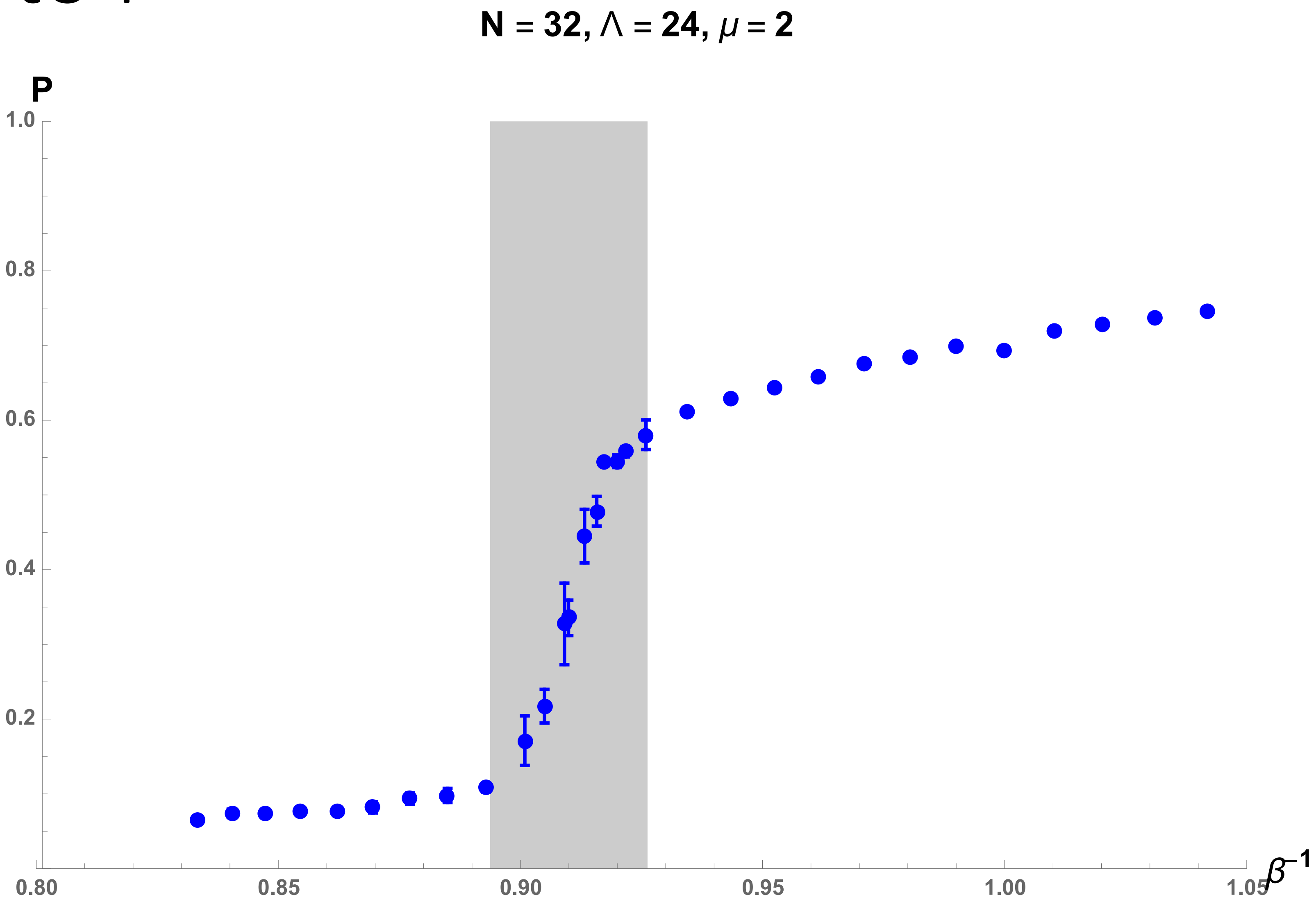
Plots for P_2

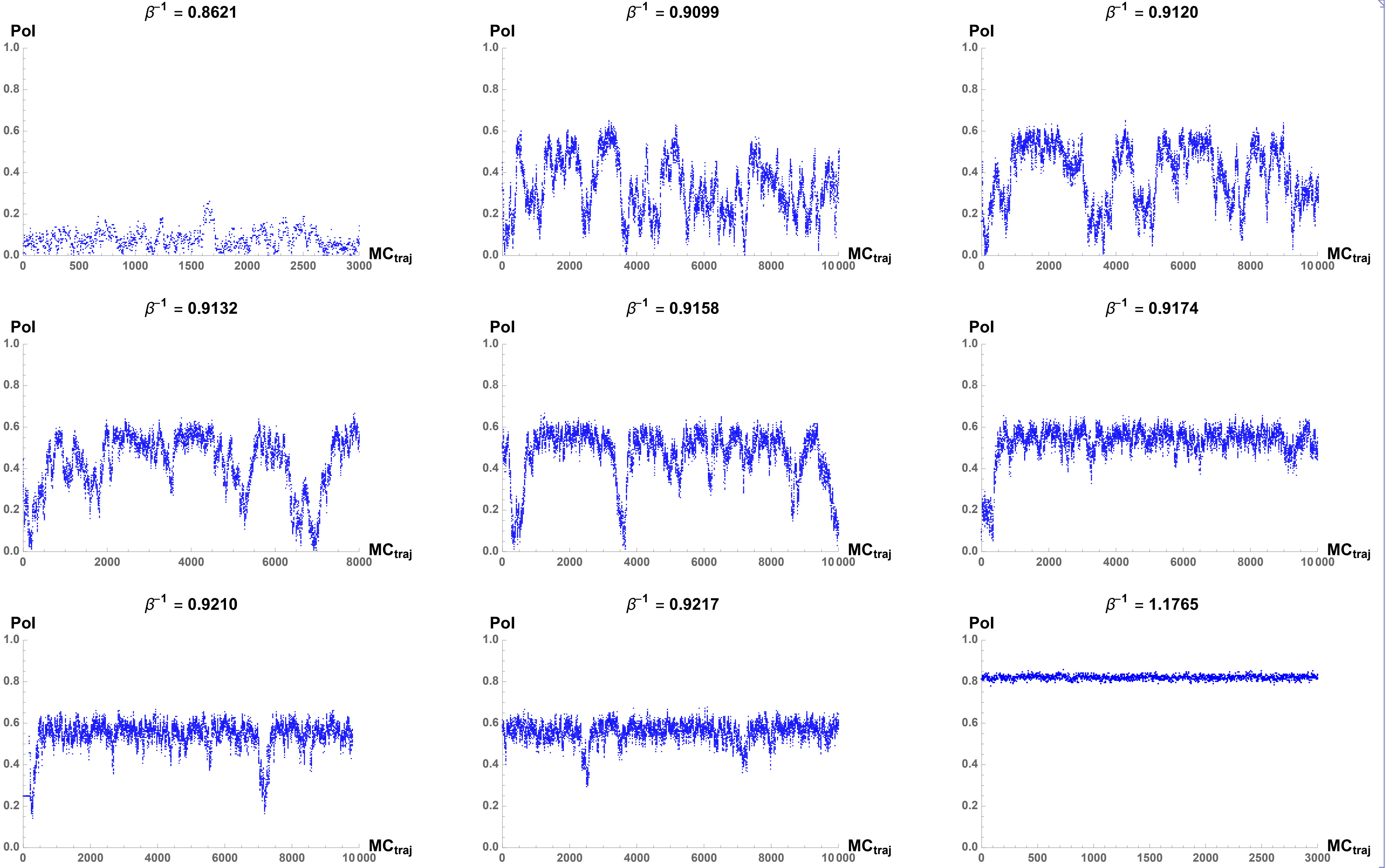

N = 32, Λ = 24, μ = 2
P_2
0.4
0.3
0.2
0.1
0.6
0.7
0.8
0.9
1.0
1.1
1.2
β^{-1}

N = 32, Λ = 24, μ = 2



Back to P



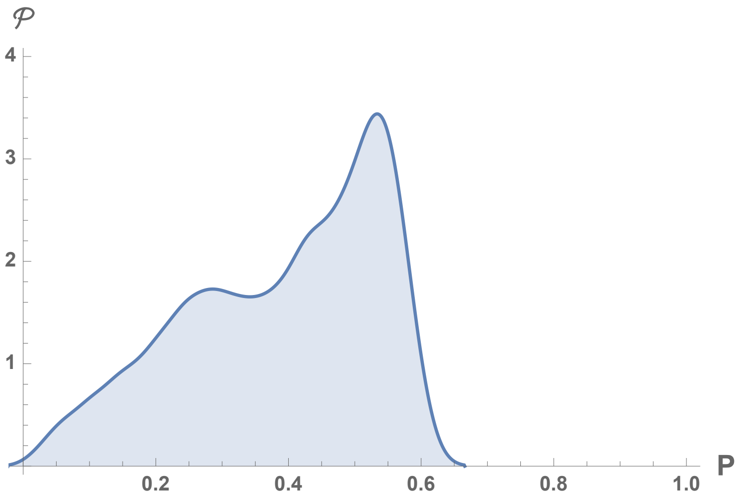


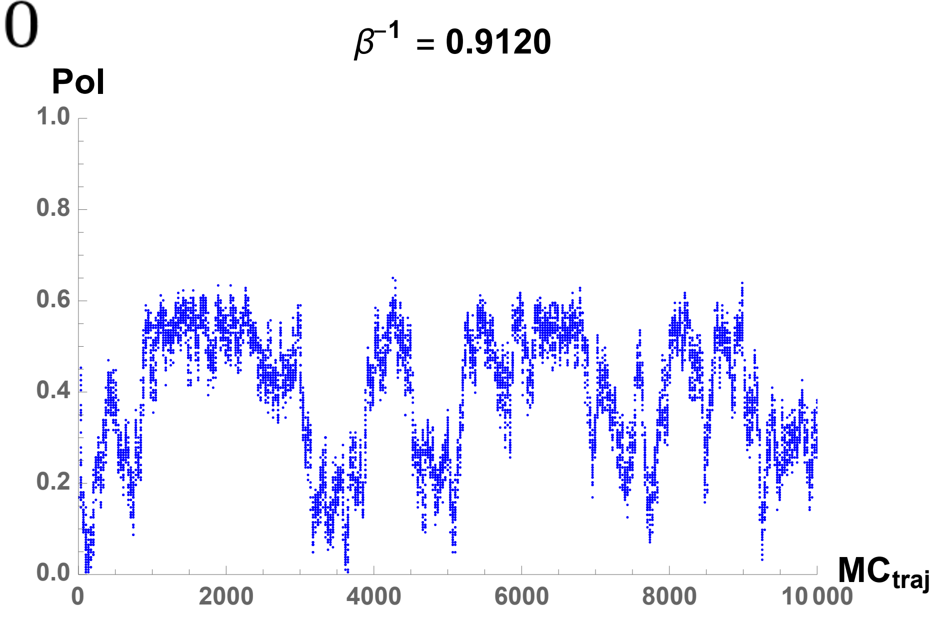
β⁻¹ = 0.8621
β⁻¹ = 0.9099
β⁻¹ = 0.9120
β⁻¹ = 0.9132
β⁻¹ = 0.9158
β⁻¹ = 0.9174
β⁻¹ = 0.9210
β⁻¹ = 0.9217
β⁻¹ = 1.1765
Pol
MCtraj

Let us define a convenient observable

$$Q = \int_{1/2}^{1} \mathcal{P} dP$$

$$\int_{0}^{1} \mathcal{P} dP = 1, \ \mathcal{P} \geq 0$$





Let us define a convenient observable

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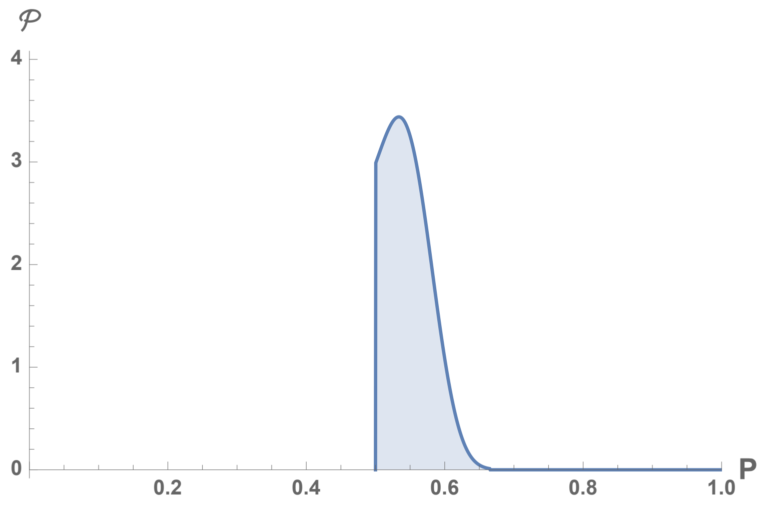
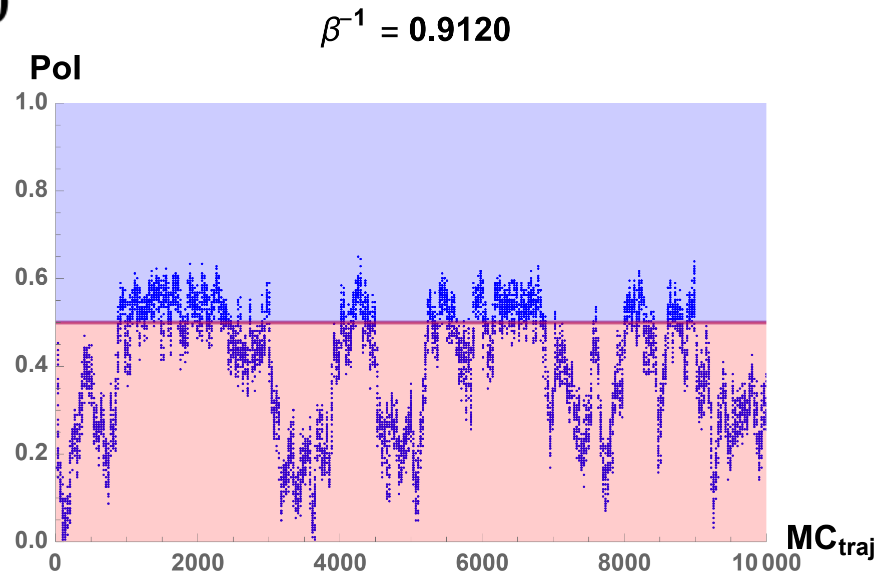
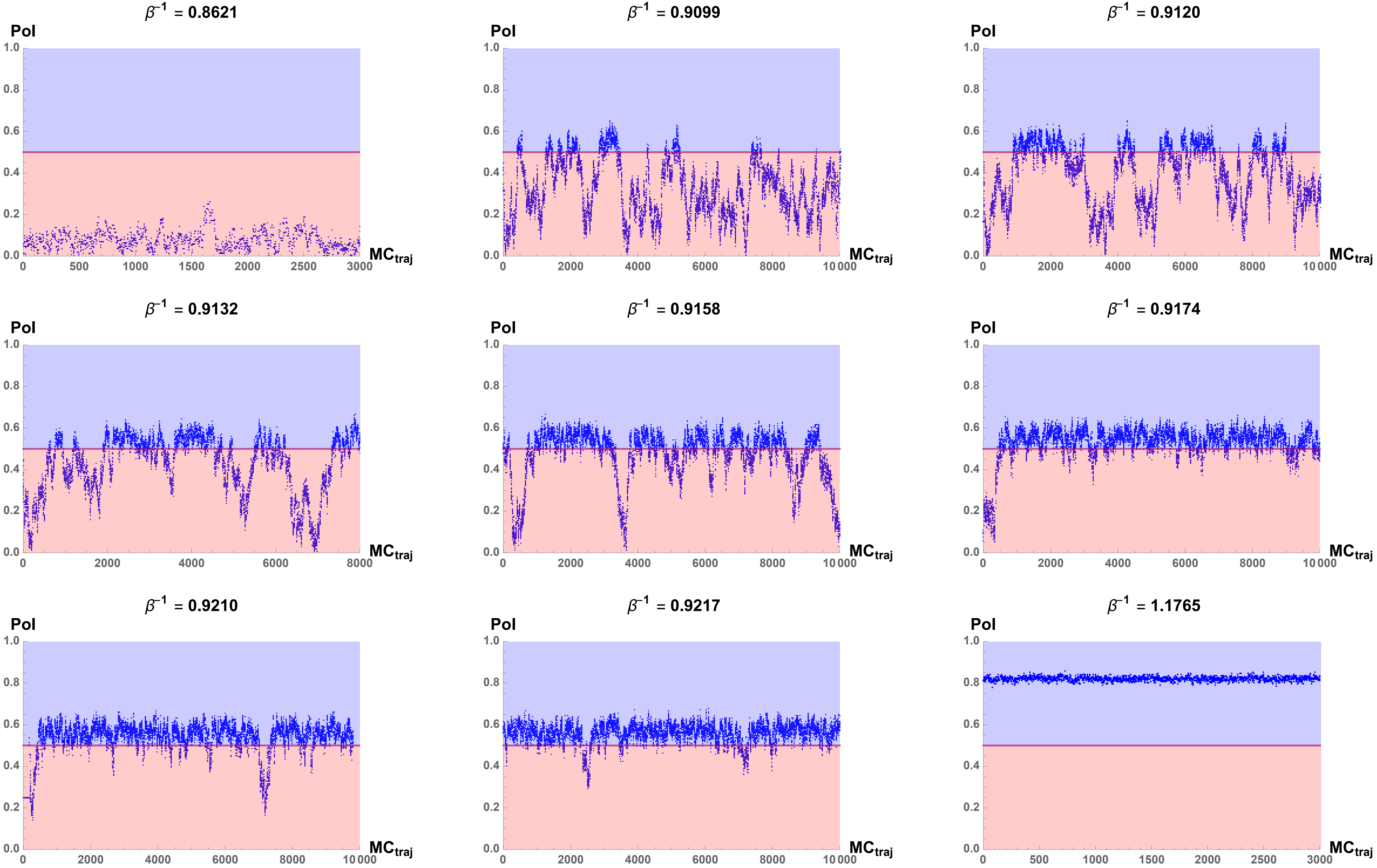


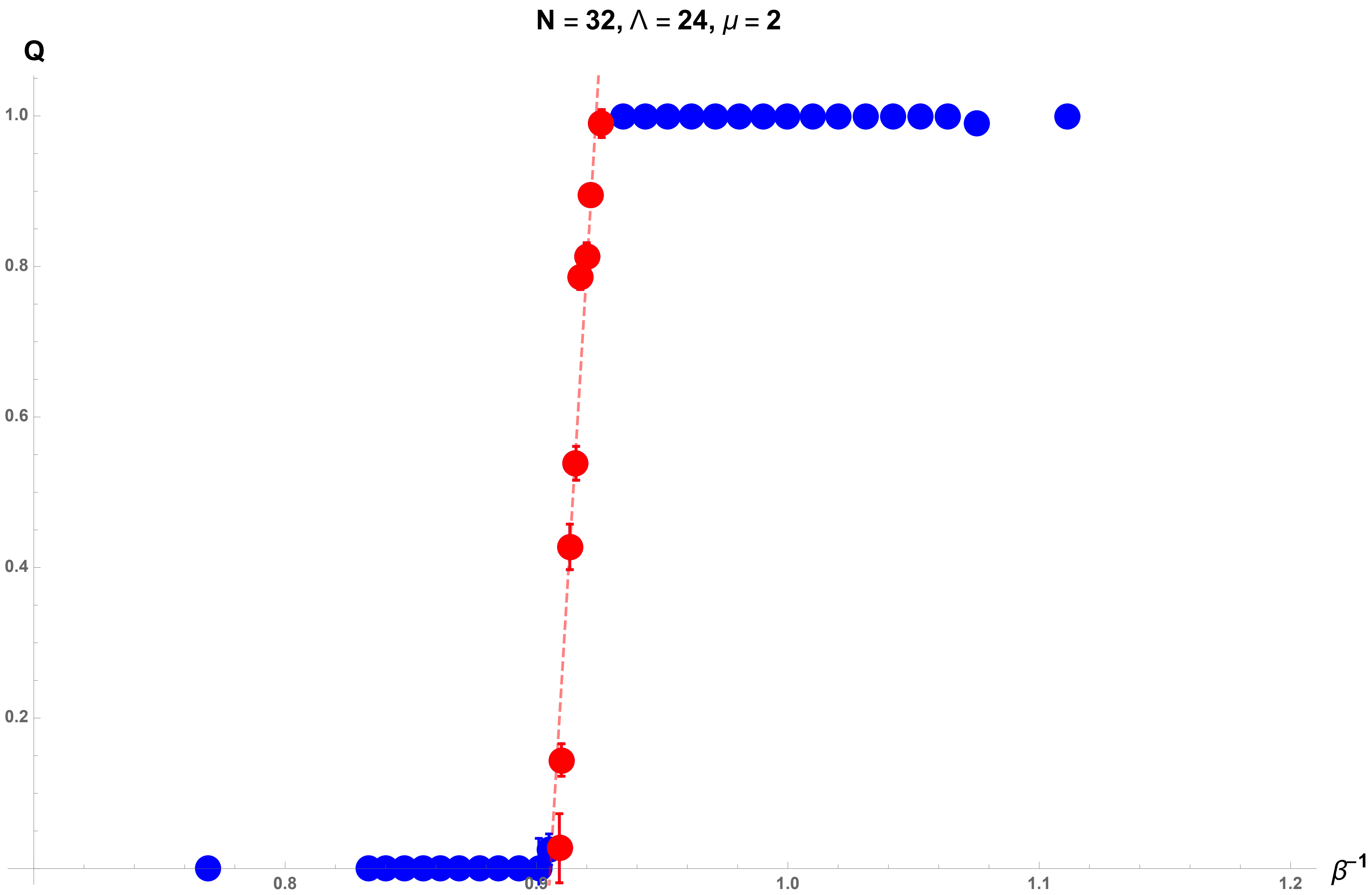


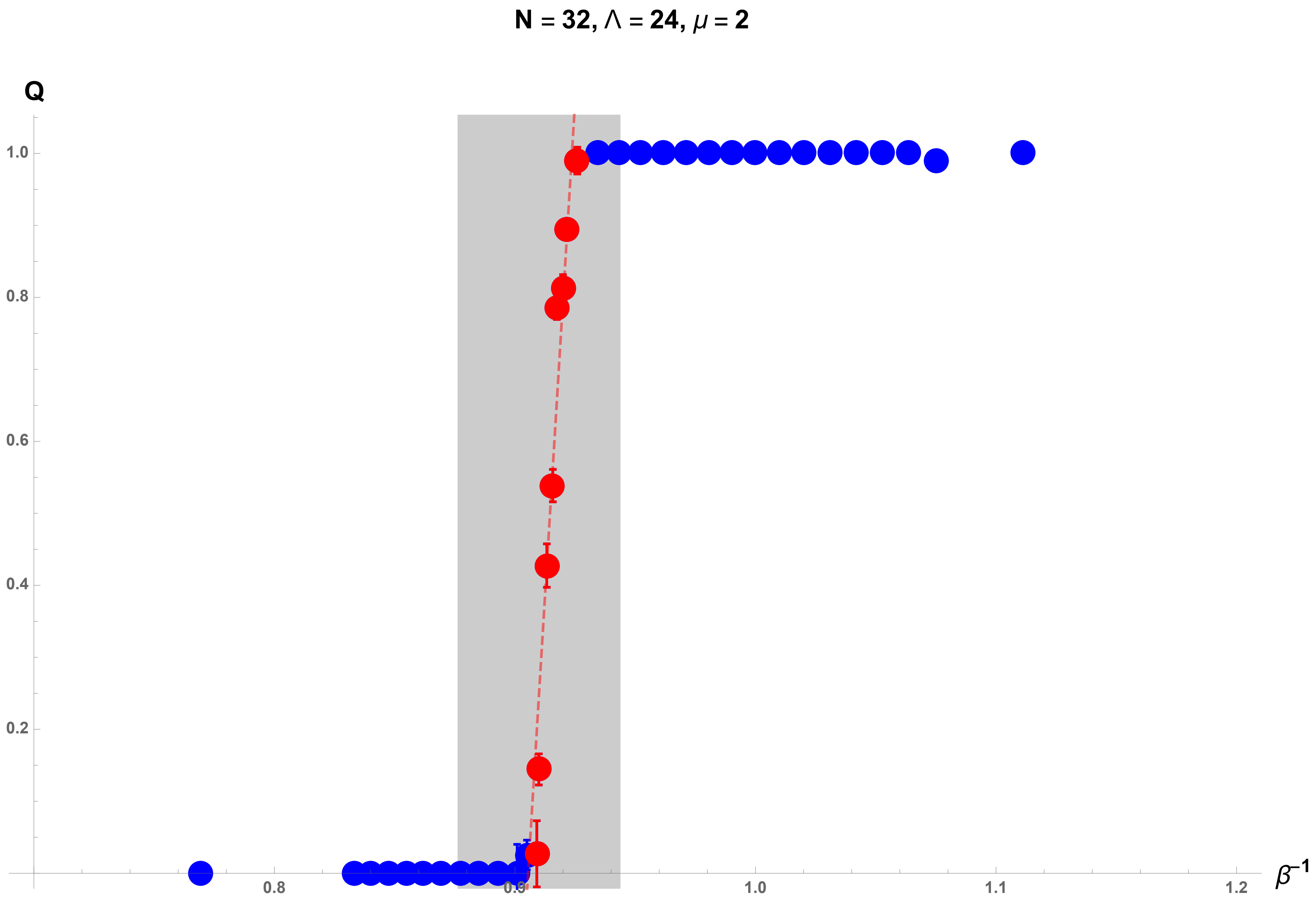
Q ≈ 0.3



β^{-1} = 0.8621
β^{-1} = 0.9099
β^{-1} = 0.9120
β^{-1} = 0.9132
β^{-1} = 0.9158
β^{-1} = 0.9174
β^{-1} = 0.9210
β^{-1} = 0.9217
β^{-1} = 1.1765
Pol
MC_{traj}

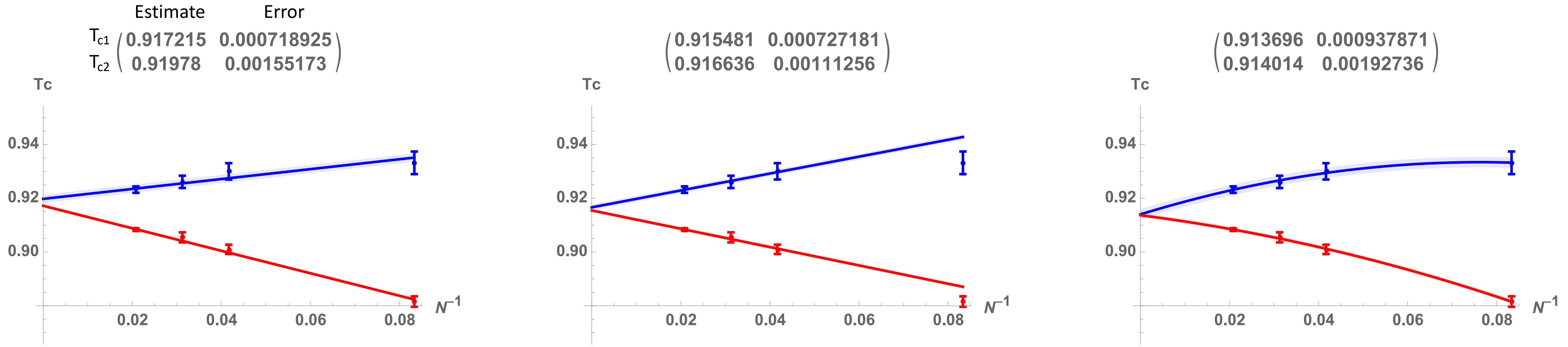
Plots of Q


N = 32, Λ = 24, μ = 2
Q
1.0
0.8
0.6
0.4
0.2
0.8
0.9
1.0
1.1
1.2
β⁻¹


N = 32, Λ = 24, μ = 2
Q
1.0
0.8
0.6
0.4
0.2
0.8
0.9
1.0
1.1
1.2
β⁻¹

Very accurate estimates for critical temperatures allows us to extrapolate to large-N limit …

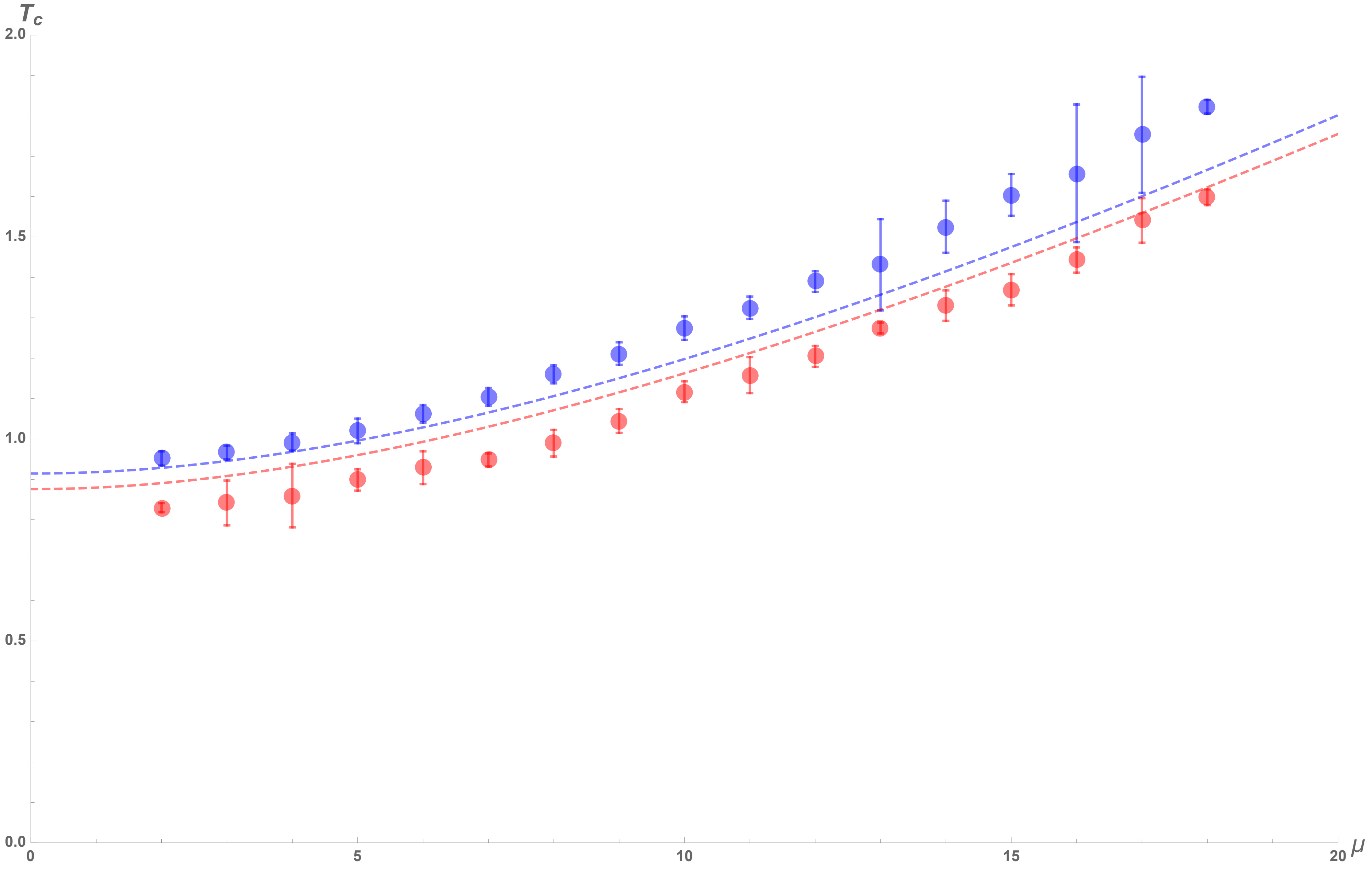
Very accurate estimates for critical temperatures allows us to extrapolate to large-N limit …



… to see that there is only a single one!

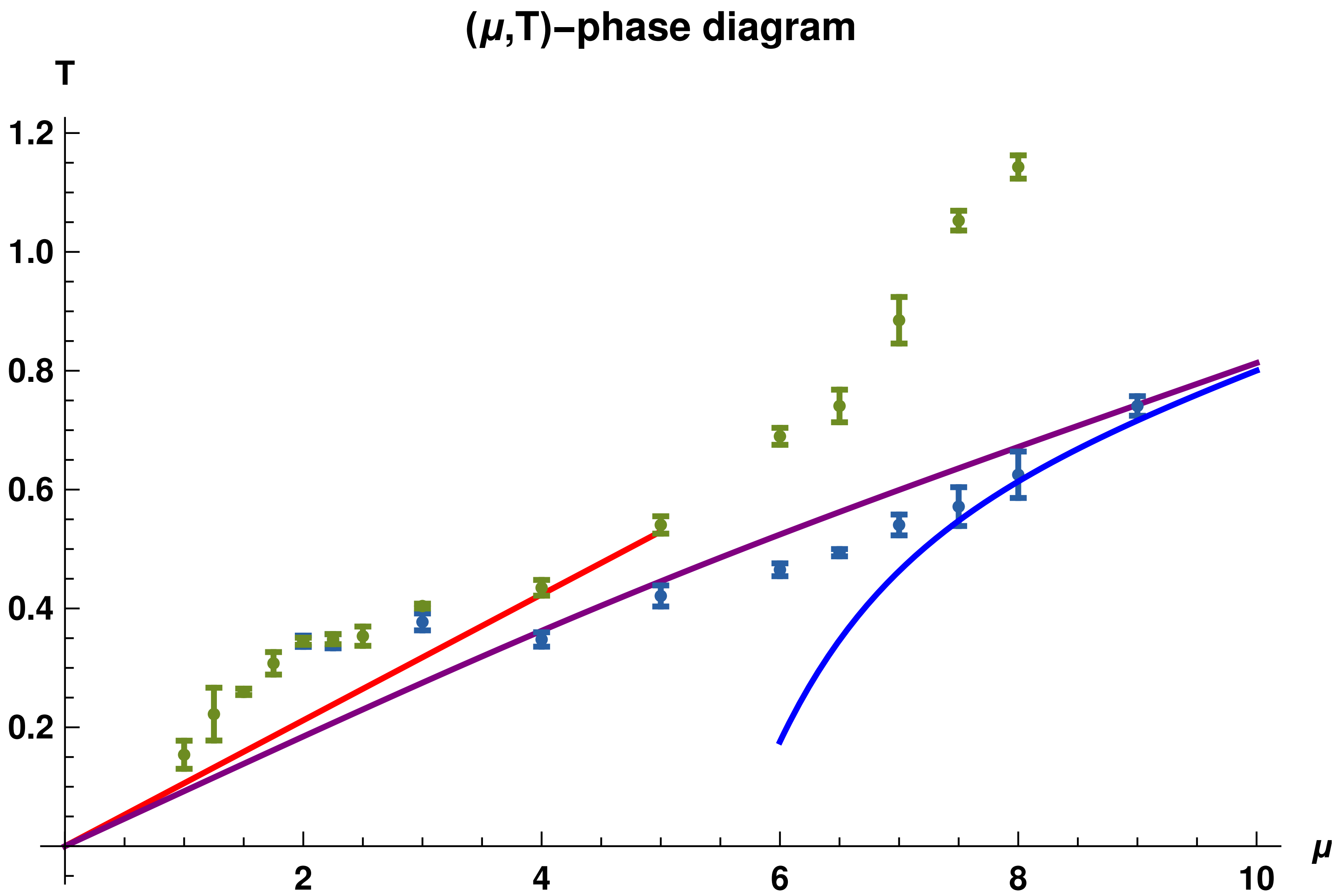
Phase diagram for N = 12

T_{c1}, T_{c2}, Pade approximation

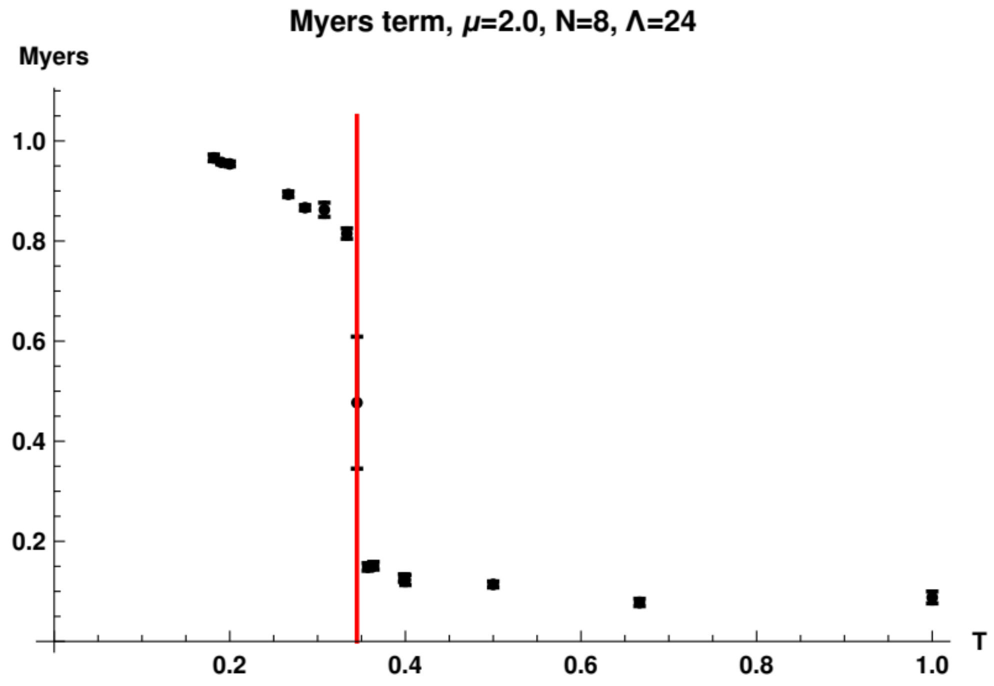


Just to mention: phase transitions of BMN

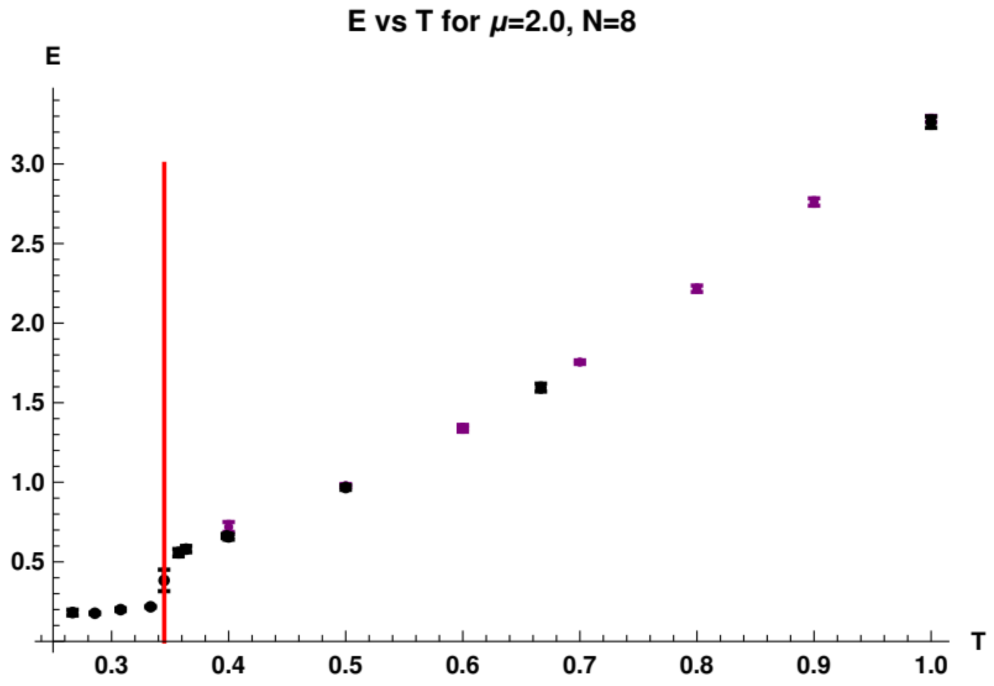
$$S[X,\psi] = N\int_0^\beta d\tau\,\mathrm{Tr}\,\Bigg[\frac{1}{2}D_\tau X^i D_\tau X^i - \frac{1}{4}\left([X^r,X^s] + \frac{i\mu}{3}\varepsilon^{rst}X_t\right)^2 - \frac{1}{2}[X^r,X^m]^2 - \frac{1}{4}[X^m,X^n]^2 + \frac{1}{2}\left(\frac{\mu}{6}\right)^2 X_m^2 + \frac{1}{2}\psi^T\mathcal{C}\left(D_\tau - \frac{i\mu}{4}\gamma^{567}\right)\psi - \frac{1}{2}\psi^T\mathcal{C}\gamma^i[X^i,\psi]\Bigg]$$



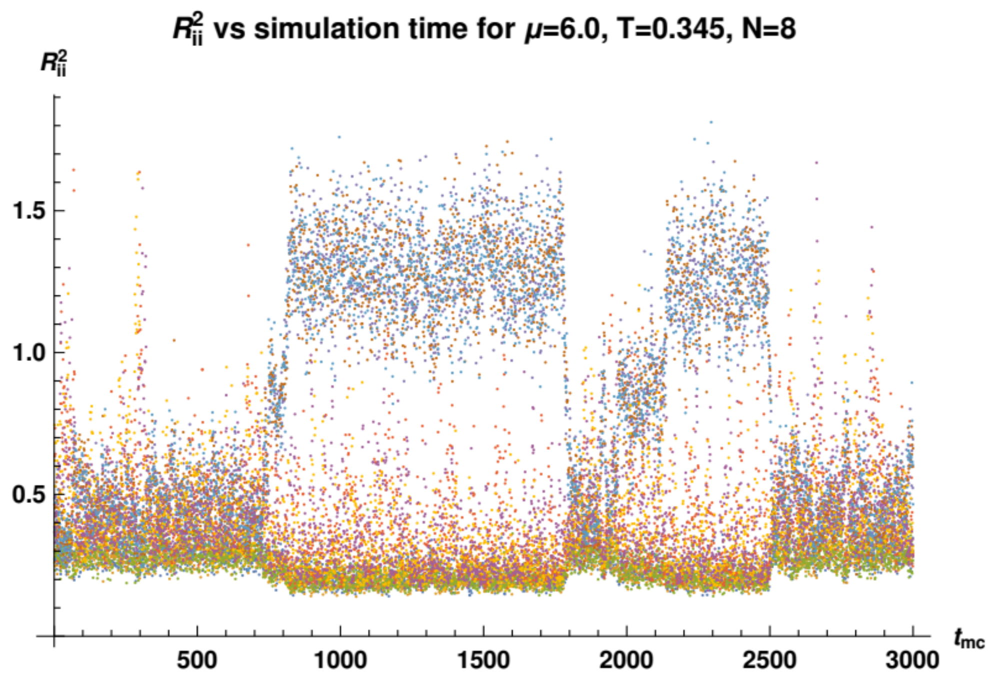
(μ,T)–phase diagram
T
1.2
1.0
0.8
0.6
0.4
0.2
2
4
6
8
10
μ



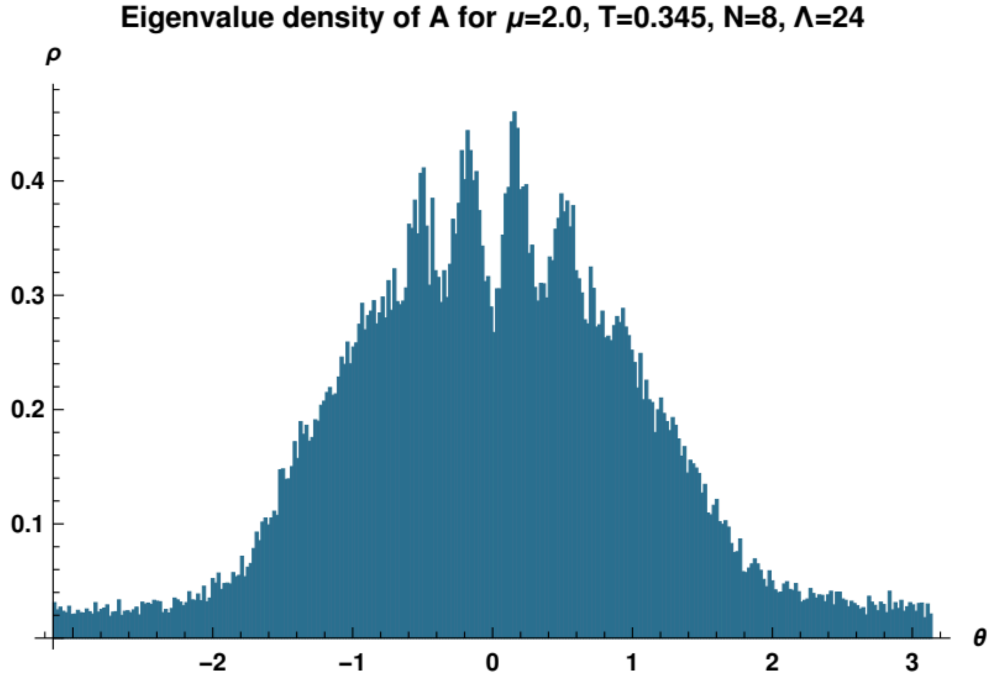
Myers term, μ=2.0, N=8, Λ=24
Myers
1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0
T



E vs T for μ=2.0, N=8
E
3.0
2.5
2.0
1.5
1.0
0.5
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
T



R^2_{ii} vs simulation time for μ=6.0, T=0.345, N=8
R^2_{ii}
1.5
1.0
0.5
500
1000
1500
2000
2500
3000
t_{mc}

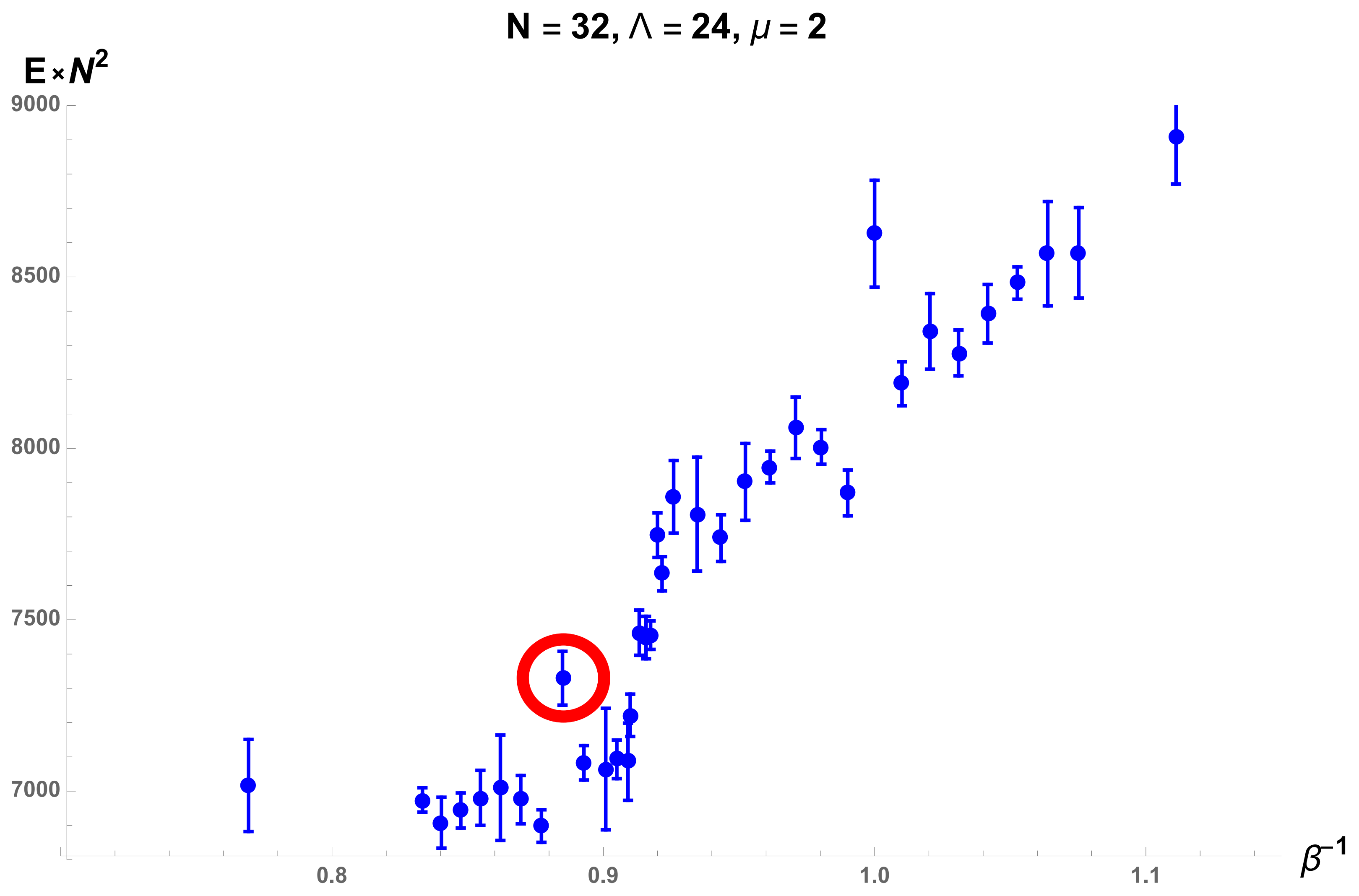


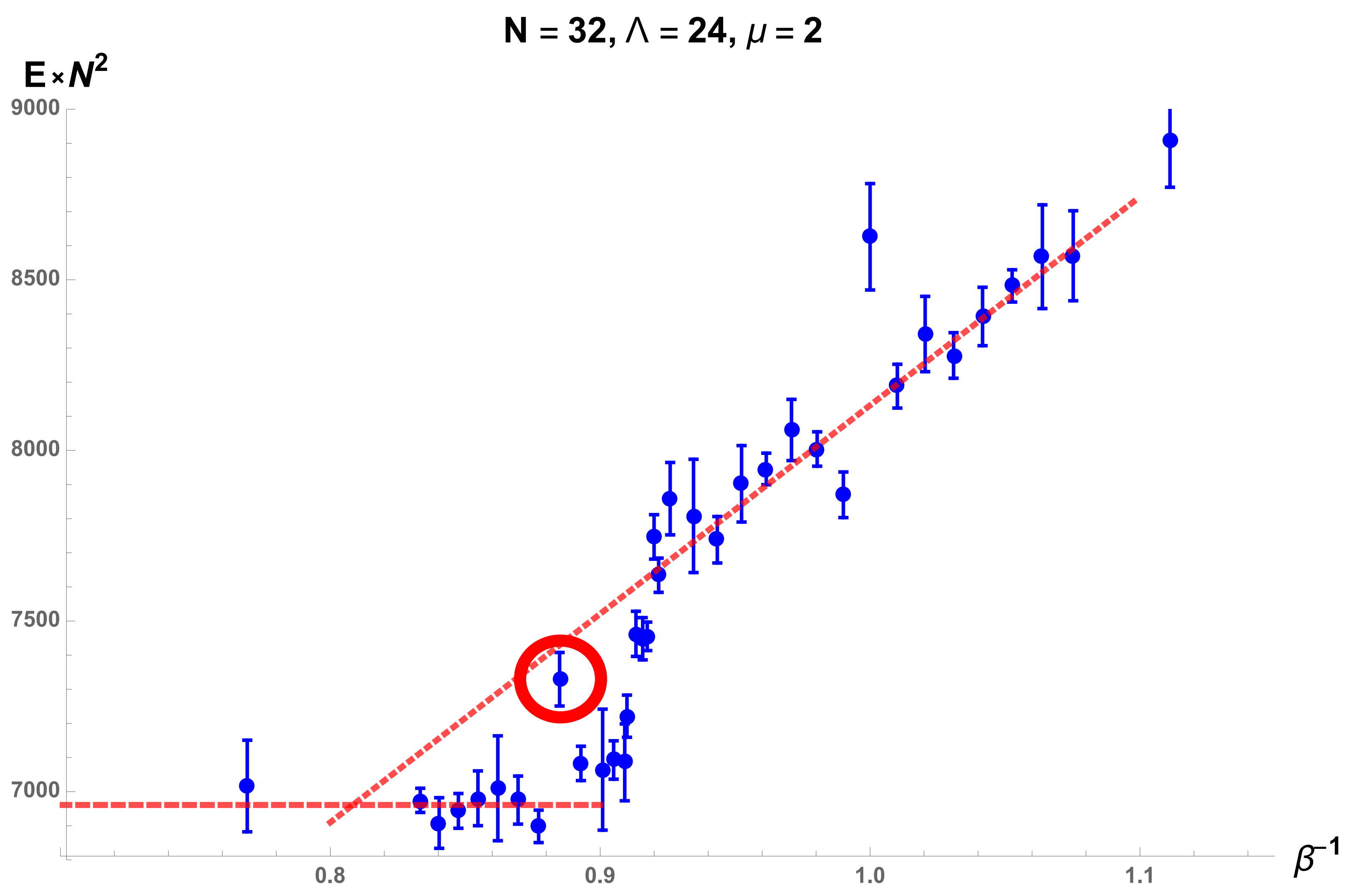
Eigenvalue density of A for μ=2.0, T=0.345, N=8, Λ=24
ρ
0.4
0.3
0.2
0.1
−2
−1
0
1
2
3
θ

Conclusions

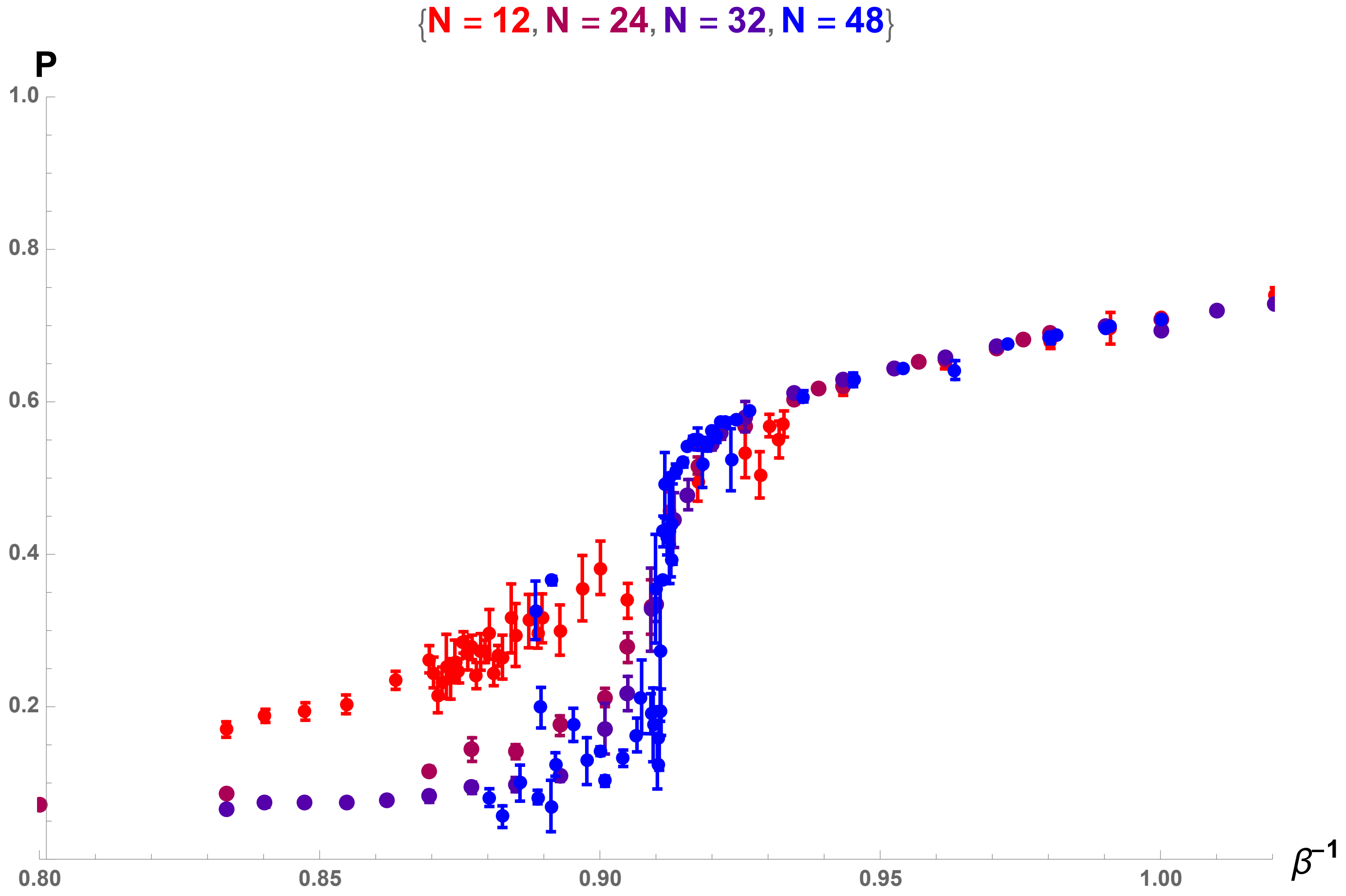
- There is a **single** (1^{st} order, confined/deconfined) phase transition for the bosonic BMN model
- There are **two different** phase transitions in the SUSY BMN model (confined/deconfined and transition into fuzzy sphere background)
- There is **yet another** phase transition in the BD model (which is hard to study numerically with satisfactory precision)
- Unsettling question: What N is large enough?

Thank you!

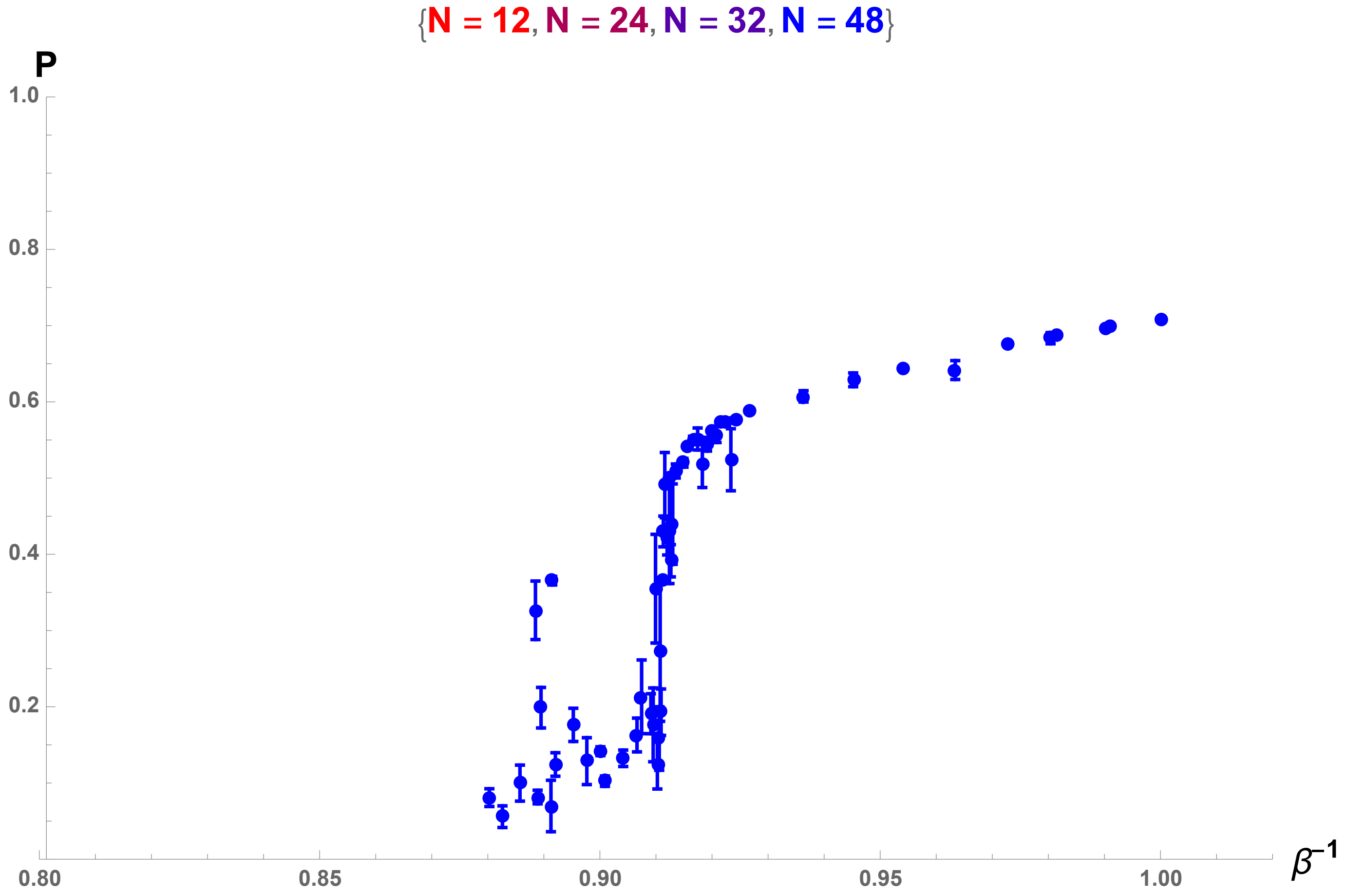

N = 32, Λ = 24, μ = 2
E × N²
9000
8500
8000
7500
7000
0.8
0.9
1.0
1.1
β⁻¹



N = 32, Λ = 24, μ = 2
E × N^2
9000
8500
8000
7500
7000
0.8
0.9
1.0
1.1
β^-1



{N = 12, N = 24, N = 32, N = 48}
P
1.0
0.8
0.6
0.4
0.2
0.80
0.85
0.90
0.95
1.00
β^{-1}


{N = 12, N = 24, N = 32, N = 48}
P
1.0
0.8
0.6
0.4
0.2
0.80
0.85
0.90
0.95
1.00
β⁻¹