Exact results in AdS4/CFT3

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Bremsstrahlung functions

BPS Wilson Loops

BPS Wilson Loops in supersymmetric gauge theories: gauge invariant non–local operators that preserve some supercharges

The prototype example in 4D $\mathcal{N} = 4$ SYM

$$W = \mathrm{Tr} P e^{-i \int_\Gamma d\tau \mathcal{L}(\tau)}$$

$$\mathcal{L} = \dot{x}^\mu A_\mu + i|\dot{x}|\theta_I \Phi^I$$

It includes couplings to the six scalars

Maldacena, PRL80 (1998) 4859

Drukker, Gross, Ooguri, PRD60 (1999) 125006; Zarembo, NPB 643

The number of preserved supercharges depends on Γ and θ_I

⋆ They are in general non–protected operators and their expectation values

$$\langle W \rangle \sim \int D[A, \hat{A}, C, \bar{C}, \psi, \bar{\psi}]\, e^{-S}\, \mathrm{Tr} P \exp\left[-i \int_\Gamma d\tau \mathcal{L}(\tau)\right]$$

depend non-trivially on the coupling constant.

- Weak couplings $\Longrightarrow$ Ordinary perturbation theory
- Strong couplings $\Longrightarrow$ Holographic methods: **Dual description** in terms of fundamental strings or M2–branes. The expectation value at strong coupling is given by $\langle W \rangle = Z_{\mathrm{string}}$ for a minimal area worldsheet ending on the WL contour.
- Finite couplings $\Longrightarrow$ Localization techniques (**Matrix Model**)

The matrix model provides an **exact interpolating function** to check the AdS/CFT correspondence

Localization

Suppose we have a supersymmetric theory whose action $S[\Phi]$ is invariant under an odd generator Q, with $Q^2 = \delta$. We want to compute

$$Z = \int \mathcal{D}\Phi \, e^{-S[\Phi]} \qquad \text{and} \qquad \langle \mathcal{O} \rangle = \int \mathcal{D}\Phi \, \mathcal{O} \, e^{-S[\Phi]}$$

Consider

$$Z(t) = \int \mathcal{D}\Phi \, e^{-S[\Phi] - tQV[\Phi]} \qquad \text{with} \qquad \delta V[\Phi] = 0$$

Supersymmetry invariance then implies

$$\frac{dZ(t)}{dt} = 0 \quad \Longrightarrow \quad Z = Z(0) = Z(\infty)$$

At $t = \infty$ the path–integral localizes at loci Φ_0 where $QV[\Phi_0] = 0$.

Writing $\Phi = \Phi_0 + \frac{1}{\sqrt{t}} \Phi'$

$$Z = \int \mathcal{D}\Phi_0 e^{-S[\Phi_0]} \, Z_{1loop}[\Phi_0] \qquad \qquad \langle \mathcal{O} \rangle = \int \mathcal{D}\Phi_0 e^{-S[\Phi_0]} \, Z_{1loop}[\Phi_0] \, \mathcal{O}[\Phi_0]$$

If the space of Φ_0 solutions is a finite submanifold, the path–integral reduces to a finite dimensional integral, typically a Matrix Model

⋆ They are related to physical quantities like the **Bremsstrahlung function** and the **Cusp anomalous dimension**. Therefore, they are ultimately related to

$$\Downarrow$$

INTEGRABILITY IN AdS/CFT

⋆ Parametric WL (*latitude* WL) are related to correlation functions of the **1D defect CFT** defined on the WL contour.

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INTEGRABILITY IN AdS/CFT

⋆ Parametric WL (*latitude* WL) are related to correlation functions of the **1D defect CFT** defined on the WL contour.

BPS Wilson loops in $\mathcal{N} = 6$ ABJM theory

Plan of the talk

- Latitude “bosonic” and “fermionic” BPS WL in ABJM theory
- Matrix Model and exact results for latitude WL
- Bremsstrahlung functions
- Conclusions and Perspectives

$\mathcal{N} = 6$ ABJM theory

Aharony, Bergman, Jafferis, Maldacena, 0806.1218

$U(N)_k \times U(N)_{-k}$ CS-gauge vectors A_μ, $\hat{A}_\mu$ minimally coupled to

$SU(4)$ complex scalars C_I, $\bar{C}^I$ and fermions ψ_I, $\bar{\psi}^I$, $I = 1, \ldots, 4$

in the (anti)bifundamental representation of the gauge group with non-trivial potential.

$$S = S_{\rm CS} + S_{\rm mat} + S_{\rm pot}^{\rm bos} + S_{\rm pot}^{\rm ferm}$$

$$S_{\rm CS} = \frac{k}{4\pi i} \int d^3x \, \varepsilon^{\mu\nu\rho} \Big\{ {\rm Tr} \left(A_\mu \partial_\nu A_\rho + \frac{2}{3} i A_\mu A_\nu A_\rho \right) - {\rm Tr} \left(\hat{A}_\mu \partial_\nu \hat{A}_\rho + \frac{2}{3} i \hat{A}_\mu \hat{A}_\nu \hat{A}_\rho \right) \Big\}$$

$$S_{\rm mat} = \int d^3x \, {\rm Tr} \Big[D_\mu C_I D^\mu \bar{C}^I - i \bar{\Psi}^I \gamma^\mu D_\mu \Psi_I \Big] \qquad \lambda = N/k$$

$$D_\mu C_I = \partial_\mu C_I + i A_\mu C_I - i C_I \hat{A}_\mu$$

Dual to M–theory on $\mathrm{AdS}_4 \times S^7/Z_k$ or Type IIA on $\mathrm{AdS}_4 \times CP^3$

$N \gg k^5$ $\qquad$ $k \ll N \ll k^5$

Latitude Wilson loops M.Bianchi, L.Griguolo, M.Leoni, SP, D.Seminara (2014)

Bosonic BPS WL

$$W_B(\nu) = \mathrm{Tr} P \exp\left[-i\int_\Gamma d\tau(\dot{x}^\mu A_\mu - \frac{2\pi i}{k}|\dot{x}|M_J{}^I(\nu)\, C_I \bar{C}^J)\right]$$

$$M_J{}^I(\nu) = \begin{pmatrix} -\nu & e^{-i\tau}\sqrt{1-\nu^2} & 0 & 0 \\ e^{i\tau}\sqrt{1-\nu^2} & \nu & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \nu \in [0,1]$$

It preserves 1/12 of susy charges $\mathcal{Q}_1(\nu), \mathcal{Q}_2(\nu)$ $\Longrightarrow$ **1/12 BPS**

Enhancement of SUSY $W_B(\nu = 1) \equiv W_B$ $\Longrightarrow$ **1/6 BPS**

Drukker, Plefka, Young, JHEP 0811 (2008) 019

Similar construction for $\hat{W}_B$

Fermionic BPS WL $W_F(\nu) = \mathrm{Tr} P \exp\left[-i \int_\Gamma d\tau \mathcal{L}(\nu,\tau)\right]$ $U(N|N)$

$$\mathcal{L}(\nu,\tau) = \begin{pmatrix} \dot{x}^\mu A_\mu - \frac{2\pi i}{k}|\dot{x}| M_J^{\ I}(\nu) C_I \bar{C}^J & -i\sqrt{\frac{2\pi}{k}}|\dot{x}|\eta_I(\nu)\bar{\psi}^I \\ -i\sqrt{\frac{2\pi}{k}}|\dot{x}|\psi_I \bar{\eta}^I(\nu) & \dot{x}^\mu \hat{A}_\mu - \frac{2\pi i}{k}|\dot{x}| M_J^{\ I}(\nu) \bar{C}^J C_I \end{pmatrix}$$

$$M_I^{\ J} = \begin{pmatrix} -\nu & e^{-i\tau}\sqrt{1-\nu^2} & 0 & 0 \\ e^{i\tau}\sqrt{1-\nu^2} & \nu & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \eta_I^\alpha = \frac{e^{\frac{i\nu\tau}{2}}}{\sqrt{2}} \begin{pmatrix} \sqrt{1+\nu} \\ -\sqrt{1-\nu}\, e^{i\tau} \\ 0 \\ 0 \end{pmatrix}_I (1, -ie^{-i\tau})^\alpha$$

It preserves 1/6 of susy charges $\mathcal{Q}_{1,2,3,4}(\nu)$ $\Longrightarrow$ **1/6 BPS**

Enhancement of SUSY $W_F(\nu=1) \equiv W_F$ $\Longrightarrow$ **1/2 BPS**

Drukker, Trancanelli, JHEP 02 (2010) 058

The $\nu = 1$ case

Exact Matrix Model Kapustin, Willett, Yaakov (2010) 089

$$\langle W_B \rangle_1 = e^{i\Phi_B} |\langle W_B \rangle| \qquad \langle \hat{W}_B \rangle_1 = e^{-i\Phi_B} |\langle \hat{W}_B \rangle|$$

Complex functions at framing one

- Framing function $\Phi_B = \pi \left(\frac{N}{k}\right) 1 - \frac{\pi^3}{2} \left(\frac{N}{k}\right)^3 + \mathcal{O}\left(\left(\frac{N}{k}\right)^5\right)$
- Framing independent $|\langle W_B \rangle|, |\langle \hat{W}_B \rangle|$, functions of N/k

Comohological equivalence

$$W_F = \frac{W_B + \hat{W}_B}{2} + \mathcal{Q}(\text{something}) \quad \Longrightarrow \quad \langle W_F \rangle_1 = \frac{\langle W_B \rangle_1 + \langle \hat{W}_B \rangle_1}{2}$$

Checked perturbatively

The general case, $\nu \neq 1$

Comohological equivalence

$$W_F(\nu) = \frac{e^{-\frac{i\pi\nu}{2}}\, W_B(\nu) - e^{\frac{i\pi\nu}{2}}\, \hat{W}_B(\nu)}{e^{-\frac{i\pi\nu}{2}} - e^{\frac{i\pi\nu}{2}}} + \mathcal{Q}(\nu)(\text{something})$$

Localizing the path integral with $\mathcal{Q}(\nu) \Longrightarrow$ cohomological equivalence implemented at quantum level in the same form.

Perturbatively it is true at framing ν

$$\langle W_F(\nu)\rangle_\nu = \frac{e^{-\frac{i\pi\nu}{2}}\, \langle W_B(\nu)\rangle_\nu - e^{\frac{i\pi\nu}{2}}\, \langle \hat{W}_B(\nu)\rangle_\nu}{e^{-\frac{i\pi\nu}{2}} - e^{\frac{i\pi\nu}{2}}}$$

where up to two loops we find $\quad \Phi_B = \pi\left(\frac{N}{k}\right)\nu + \mathcal{O}\left(\left(\frac{N}{k}\right)^3\right)$

M.Bianchi, L.Griguolo, M.Leoni, SP, D.Seminara (2014)

The Matrix Model

In order to respect cohomological equivalence at quantum level, we would like to perform localization with $\mathcal{Q}(\nu) \Longrightarrow$ Matrix Model will depend on ν

But we are not able yet to implement localization, since $\mathcal{Q}(\nu)$ is not chiral.

Alternatively, we guess the structure of the MM and perform consistency checks

$$\langle W_B(\nu) \rangle_\nu = \left\langle \frac{1}{N} \sum_{a=1}^{N} e^{2\pi \sqrt{\nu}\, \lambda_a} \right\rangle$$

$$Z = \int \prod_{a=1}^{N} d\lambda_a \; e^{i\pi k \lambda_a^2} \prod_{b=1}^{N} d\mu_b \; e^{-i\pi k \mu_b^2}$$
$$\times \frac{\prod_{a<b}^{N} \sinh \sqrt{\nu}\pi(\lambda_a - \lambda_b) \sinh \frac{\pi(\lambda_a - \lambda_b)}{\sqrt{\nu}} \prod_{a<b}^{N} \sinh \sqrt{\nu}\pi(\mu_a - \mu_b) \sinh \frac{\pi(\mu_a - \mu_b)}{\sqrt{\nu}}}{\prod_{a=1}^{N} \prod_{b=1}^{N} \cosh \sqrt{\nu}\pi(\lambda_a - \mu_b) \cosh \frac{\pi(\lambda_a - \mu_b)}{\sqrt{\nu}}}$$

Planar limit (large N)

For N large, we have computed

$$\langle W_B(\nu)\rangle_\nu = \frac{1}{8\pi^2}\left(2\,\pi^2\,\csc\frac{2\pi}{k}\,\left(1+i\tan\frac{\pi\nu}{2}\right)\frac{\mathrm{Ai}\left(\left(\frac{k}{2\pi^2}\right)^{-1/3}\left(N-\frac{k}{24}-\frac{7}{3k}\right)\right)}{\mathrm{Ai}\left(\left(\frac{k}{2\pi^2}\right)^{-1/3}\left(N-\frac{k}{24}-\frac{1}{3k}\right)\right)}\right.$$
$$\left.+2\,i\,\pi\,\nu\,\frac{\Gamma\left(\frac{\nu-1}{2}\right)\,\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma(\nu+1)}\,\csc\frac{2\pi\nu}{k}\,\frac{\mathrm{Ai}\left(\left(\frac{k}{2\pi^2}\right)^{-1/3}\left(N-\frac{k}{24}-\frac{6\nu+1}{3k}\right)\right)}{\mathrm{Ai}\left(\left(\frac{k}{2\pi^2}\right)^{-1/3}\left(N-\frac{k}{24}-\frac{1}{3k}\right)\right)}\right)$$

$$\langle W_F(\nu)\rangle_\nu = -\frac{\nu\,\Gamma\left(-\frac{\nu}{2}\right)\,\csc\left(\frac{2\pi\nu}{k}\right)\,\mathrm{Ai}\left(\left(\frac{k}{2\pi^2}\right)^{-1/3}\left(N-\frac{k}{24}-\frac{6\nu+1}{3k}\right)\right)}{2^{\nu+2}\sqrt{\pi}\,\Gamma\left(\frac{3-\nu}{2}\right)\,\mathrm{Ai}\left(\left(\frac{k}{2\pi^2}\right)^{-1/3}\left(N-\frac{k}{24}-\frac{1}{3k}\right)\right)}$$

EXACT INTERPOLATING FUNCTIONS

Checks

- Strong consistency check: Partition function Z does not depend on ν
- Its weak coupling expansion matches a genuine three–loop calculation done at framing ν (1802.07742)
- Its leading behavior at strong coupling in the large N limit reproduces the holographic prediction

$$\langle W_F(\nu) \rangle \sim e^{\pi\nu\sqrt{2N/k}}$$

Correa, Aguilera-Damia, Silva, JHEP 06 (2014)

- Recent result for $\left.\frac{\langle W_F(1) \rangle}{\langle W_F(\nu) \rangle}\right|_{1-\text{loop}}$ at strong coupling matches our prediction

Medina-Rincon, 1907.02984

Bremsstrahlung function

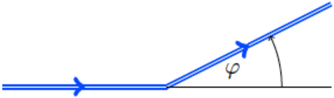
We now prove that latitude WLs have physical meaning, being related to the Bremsstrahlung functions. In particular, we disclose the physical meaning of the framing function.

Energy lost by a slowly moving massive quark

$$\Delta E = 2\pi B \int dt\, \dot{v}^2 \qquad |v| \ll 1$$

In CFT

$$\langle W^{\angle} \rangle_{\varphi} \sim e^{-\Gamma_{cusp}(\varphi) \log \frac{L}{\epsilon}} \qquad \Gamma_{cusp}(\varphi) \underset{\varphi \ll 1}{\sim} -B\, \varphi^2$$



Correa, Henn, Maldacena, Sever, JHEP 06 (2012)

Bremsstrahlung functions in ABJM

$$\langle W_F^{\angle} \rangle_\varphi \sim e^{-\Gamma_{cusp}^{1/2}(\varphi) \log \frac{L}{\epsilon}} \qquad \Gamma_{cusp}^{1/2}(\varphi) \underset{\varphi \ll 1}{\sim} -B_{1/2} \varphi^2$$

$$\langle W_B^{\angle} \rangle_\varphi \sim e^{-\Gamma_{cusp}^{1/6}(\varphi) \log \frac{L}{\epsilon}} \qquad \Gamma_{cusp}^{1/6}(\varphi) \underset{\varphi \ll 1}{\sim} -B_{1/6}^{\varphi} \varphi^2$$

They are functions of the coupling constant N/k

How to relate B to quantities exactly computable via localization?

First proposal

$$B_{1/6}^{\varphi} = \frac{1}{4\pi^2} \partial_m \log |\langle W_B^m \rangle| \Big|_{m=1}$$

Lewkowycz, Maldacena, JHEP 05 (2014)

Our proposal

$$B_{1/2} = \frac{1}{4\pi^2}\,\partial_\nu \log\left|\,\langle W_F(\nu)\rangle\,\right|\,\Big|_{\nu=1}$$

- Perturbative checks up to three loops for $B_{1/2}$ by a direct computation of Γ_{cusp}

  M. Bianchi, Griguolo, Leoni, SP, Seminara (2014); Griguolo, Marmiroli, Martelloni, Seminara (2014); M. Bianchi, Mauri (2017)

- $B_{1/2}$ matches the string prediction at next-to-leading order

  Forini, Giangreco Puletti, Ohlsson Sax (2013); Correa, Aguilera-Damia, Silva 1412.4084

- Exact proof using two–point correlation functions in 1d defect CFT

  L. Bianchi, Griguolo, Preti, Seminara (2017)

Relation with framing

M. Bianchi, Griguolo, Leoni, SP, Seminara (2014)
L. Bianchi, Preti, Vescovi (2018)

We write

$$\langle W_B(\nu)\rangle_\nu = e^{i\Phi_B(\nu)} |\langle W_B(\nu)\rangle_\nu|$$

where the phase $\Phi_B(\nu)$ includes all (but not only) framing effects.

Using cohomological equivalence and $\partial_\nu \log\left(\langle W_B(\nu)\rangle_\nu + \langle \hat{W}_B(\nu)\rangle_\nu\right)\Big|_{\nu=1} = 0$ we find

$$B_{1/2} = \frac{1}{8\pi} \tan\Phi_B$$

The undeformed $\Phi_B \equiv \Phi_B(1)$ is the framing function of 1/6 BPS Wilson loop

New physical interpretation of framing for non–topological models

Conclusions & Perspectives

1) **Matrix Model**

- We have found the Matrix Model for latitude WLs in ABJM. This is a new exact, interpolating function that allows to test AdS_4/CFT_3 in the large N limit. Localization procedure?
- Generalization to the $U(N_1) \times U(N_2)$ ABJ theory Aharony, Bergman, Jafferis '08

2) **Bremsstrahlung functions**

- We have provided an exact prescription for computing the Bremsstrahlung functions in terms of latitude WL. Exploiting integrability the exact B could be also computed by a system of TBA equations, as done in $\mathcal{N} = 4$ SYM Drukker (2012); Correa-Maldacena-Sever (2012)

  The matching would be crucial for checking the famous

  interpolating function $h(\lambda)$ of ABJM Gromov, Sizov (2014)

3) Relations with the 1D defect CFT (Work in progress)

Derivatives of latitude WL respect to the parameter produce correlation functions of scalar local operators of the 1D defect CFT

$$\partial_\nu \, \log\langle W_B(\nu)\rangle \Big|_{\nu=1} = -\frac{8\pi^2}{k^2} \int_0^{2\pi} d\tau_1 \int_0^{\tau_1} d\tau_2 \, \langle\!\langle \chi(\tau_1)\chi(\tau_2)\rangle\!\rangle_{\mathrm{WL}}$$

where $\chi(\tau) = m_J^I(\tau) C_I(\tau) \bar{C}^J(\tau)$

and $$\langle\!\langle \chi(\tau_1)\chi(\tau_2)\rangle\!\rangle_{\mathrm{WL}} \equiv \frac{\langle \mathrm{Tr} P \chi(\tau_1)\chi(\tau_2) e^{-i\oint d\tau \mathcal{L}_B(\tau)} \rangle}{\langle W_B \rangle}$$

Important questions:

- What is the relation between the two 1D CFT defined on W_B and W_F?
- Exact results for correlation functions of 1D CFT from the MM. Implications for 1D bootstrap?
- More generally, what is the effect of framing on the 1D CFT? What is its meaning in 1D?