Renormalized Volume in AdS gravity

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- **Chang, Qing & Yang formula for 6D** [J. Math. Sci. 149, 1755 (2008)]

$$Vol_{ren}\left[M_6\right]=-\frac{8\pi^3}{15}\chi\left[M_6\right]+\frac{1}{240}\int\limits_{M_6}d^6x\sqrt{\hat{g}}\,J\left[W_{(E)}\right],$$

where

$$\begin{aligned} J\left[W\right] &= -\left|\nabla W\right|^2+8\left|W\right|^2+ \\ &+\frac{7}{3}W_{\mu\nu}{}^{\alpha\beta}W_{\alpha\beta}{}^{\lambda\rho}W_{\lambda\rho}{}^{\mu\nu}+\frac{4}{3}W_{\mu\nu\rho\lambda}W^{\mu\alpha\rho\beta}W^{\nu}{}_{\alpha}{}^{\lambda}{}_{\beta}, \end{aligned}$$

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- **Gubser, Klevanov, Polyakov; Witten (AdS/CFT)**

$$Z_{CFT}[\phi_0] \approx \exp(iI_{\text{grav}}[\phi])|_{\phi\to\phi_0}$$

- **For Asymptotically AdS (AAdS) spacetimes, Fefferman-Graham (FG) form of the metric**

$$ds^2 = \frac{\ell^2}{z^2}\, dz^2 + \frac{1}{z^2}\, g_{ij}(x,z)\, dx^i dx^j$$

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$$g_{ij}(x,\rho) = g_{(0)ij}(x) + zg_{(2)ij}(x) + z^2 g_{(4)ij}(x) + \cdots\,.$$

- $g_{(0)ij}$ **is the boundary data for the holographic reconstruction of the spacetime, i.e., solving** $g_{(k)}$ **as a covariant funtional of** $g_{(0)}$

Counterterm method

- **Renormalized AdS gravity action** (Holographic Renormalization)
  [Henningson, Skenderis JHEP 9807:023(1998)]

$$I_{ren} = \frac{1}{16\pi G}\int_M d^{d+1}x\,\sqrt{-\hat{g}}\,(R-2\Lambda) - \frac{1}{8\pi G}\int_{\partial M} d^d x\,\sqrt{-h}\,K + \\ + \int_{\partial M} d^d x\, L_{ct}(h,\mathcal{R},\nabla\mathcal{R})$$

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- **Renormalized quasi-local stress tensor**: $T^{ij}_{ren}[h] = \frac{2}{\sqrt{-h}}\frac{\delta I_{ren}}{\delta h_{ij}}$.

- **Holographic stress tensor** $T^{ij}[g_{(0)}] = \lim_{z\to 0}\left(\frac{1}{z^{d-1}}T^{ij}[h]\right)$.
  Contains the holographic information of the theory (e.g., Weyl anomaly)

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- $$L_{ct} = \frac{d-1}{\ell}\sqrt{-h} + \frac{\ell\sqrt{-h}}{2(d-2)}\mathcal{R} + \frac{\ell^3\sqrt{-h}}{2(d-2)^2(d-4)}\left(\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{d}{4(d-1)}\mathcal{R}^2\right)$$
$$+\frac{\ell^5\sqrt{-h}}{(d-2)^3(d-4)(d-6)}\left(\frac{3d-2}{4(d-1)}\mathcal{R}\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{d(d+2)}{16(d-1)^2}\mathcal{R}^3\right.$$
$$\left.-2\mathcal{R}^{ij}\mathcal{R}^{kl}\mathcal{R}_{ijkl} - \frac{d}{4(d-1)}\nabla_i\mathcal{R}\nabla^i\mathcal{R} + \nabla^k\mathcal{R}^{ij}\nabla_k\mathcal{R}_{ij}\right) + \ldots$$

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Topological Invariants in AdS Gravity

- **EH+GB in** $D = 4$

$$I = \frac{1}{16\pi G}\int\limits_{M} d^4x\sqrt{\hat{g}}\left[(R-2\Lambda)+\alpha(R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}-4R_{\mu\nu}R^{\mu\nu}+R^2)\right]$$

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$$G = \beta^{-1} I^E = \frac{M}{2} \left(1 + \frac{4}{\ell^2} \alpha \right) - TS + \lim_{r \to \infty} \frac{\pi r^3}{4 G \ell^2} \left(1 - \frac{4}{\ell^2} \alpha \right)$$

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- **Entropy** $S = \frac{Area}{4} + S_0$

Kounterterms

- Euler Theorem in $D = 4$ dimensions

$$\int_M d^4x\, GB = 32\pi^2 \chi(M) + \int_{\partial M} d^3x\, B_3$$

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- Kounterterms = counterterms of unusual sort (depend on K_{ij} and $\mathcal{R}^{kl}_{ij}(h)$)

$$\begin{aligned} B_3 &= 4\sqrt{-h}\,\delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]} K^{j_1}_{i_1}\left(\tfrac{1}{2}\,\mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \tfrac{1}{3}\,K^{j_2}_{i_2}K^{j_3}_{i_3}\right) \\ &= 4\sqrt{-h}\left[-2(\mathcal{R}^i_j - \tfrac{1}{2}\delta^i_j\mathcal{R})K^j_i - \tfrac{2}{3}K^i_jK^j_kK^k_i + K(K^i_jK^j_i - \tfrac{1}{3}K^2)\right] \\ c_3 &= \ell^2/64\pi G \end{aligned}$$

Kounterterms

- $D = 2n$ dimensions [Olea, JHEP 0506: 023 (2005)]

$$B_{2n-1} = 2n\sqrt{-h}\int\limits_0^1 dt\ \delta^{[i_1\cdots i_{2n-1}]}_{[j_1\cdots j_{2n-1}]} K^{j_1}_{i_1}\left(\frac{1}{2}\mathcal{R}^{j_2 j_3}_{i_2 i_3} - t^2 K^{j_2}_{i_2} K^{j_3}_{i_3}\right)\times\cdots$$

$$\cdots\times\left(\frac{1}{2}\mathcal{R}^{j_{2n-2} j_{2n-1}}_{i_{2n-2} i_{2n-1}} - t^2 K^{j_{2n-2}}_{i_{2n-2}} K^{j_{2n-1}}_{i_{2n-1}}\right)$$

$$c_{2n-1} = (-\ell^2)^{n-1}/(16\pi G n(2n-2)!)$$

From extrinsic to intrinsic regularization

- AdS gravity action + KTs

$$I = I_{EH} + \frac{\ell^2}{16\pi G} \int\limits_{\partial M} d^3x \sqrt{-h}\, \delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]} K^{j_1}_{i_1} \left(\frac{1}{2} \mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3} K^{j_2}_{i_2} K^{j_3}_{i_3} \right).$$

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- Adding zero...

$$I = I_{EH} - \frac{1}{8\pi G}\int\limits_{\partial M} d^3x\sqrt{-h}\,K + \int\limits_{\partial M} d^3x\, L_{ct}.$$

$$L_{ct} = \frac{\ell^2}{16\pi G}\sqrt{-h}\delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]} K^{j_1}_{i_1}\left(\frac{1}{2}\,\mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3}\,K^{j_2}_{i_2} K^{j_3}_{i_3} + \frac{1}{\ell^2}\,\delta^{j_2}_{i_2}\delta^{j_3}_{i_3}\right).$$

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- And expanding...

$$K^i_j = \frac{1}{\ell}\delta^i_j - \ell S^i_j(h) + \mathcal{O}(\mathcal{R}^2)$$

$$S^i_j(h) = \frac{1}{d-2}(\mathcal{R}^i_j(h) - \frac{1}{2(d-1)}\delta^i_j\mathcal{R}(h))$$

From extrinsic to intrinsic regularization

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$$\times\left(\frac{1}{2}\mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3}\left(\frac{\delta^{j_2}_{i_2}}{\ell} - \ell S^{i_2}_{j_2}\right)\left(\frac{\delta^{j_3}_{i_3}}{\ell} - \ell S^{i_3}_{j_3}\right) + \frac{1}{\ell^2}\delta^{j_2}_{i_2}\delta^{j_3}_{i_3}\right) + \ldots$$

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- *K*ounterterms turn into *c*ounterterms [O. Miskovic and R.O., arXiv:0902.2082]

$$L_{ct} = \frac{1}{8\pi G}\frac{\sqrt{-g}}{z^3}\left(\frac{2}{\ell} + \frac{\ell}{2}\mathcal{R}(h)\right) + \mathcal{O}(z)$$
$$= \frac{1}{8\pi G}\sqrt{-h}\left(\frac{2}{\ell} + \frac{\ell}{2}\mathcal{R}(h)\right)$$

Topological terms and Wald charges

- **Wald charges for an action** $I = \frac{1}{16\pi G}\int \sqrt{\hat{g}}\mathcal{L}$

$$Q^{\alpha}_{\text{Wald}}[\xi] = \frac{1}{8\pi G}\int\limits_{\Sigma} dS_{\beta}\,\nabla^{\mu}\xi^{\nu}\,E^{\alpha\beta}_{\mu\nu}$$

$$E^{\alpha\beta}_{\mu\nu} = \frac{\delta\mathcal{L}}{\delta R^{\mu\nu}_{\alpha\beta}}$$

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- **For EH+GB in 4D**

$$E^{\alpha\beta}_{\mu\nu} = \frac{\ell^2}{8}\delta^{[\alpha\beta\gamma\delta]}_{[\mu\nu\sigma\lambda]}\left(R^{\sigma\lambda}_{\gamma\delta} + \frac{1}{\ell^2}\,\delta^{[\sigma\lambda]}_{[\gamma\delta]}\right)$$

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- **Conformal (Ashtekar-Magnon-Das) Mass in 4D**
  Kofinas, Jatkar, Miskovic and RO, [1404.1411]

$$\text{Wald charges} + \text{Top. terms} \implies \text{AMD charges}$$

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  $$I_{CG}\sim\int\limits_M d^4x\,\sqrt{-\hat{g}}\,W_{\mu\nu\alpha\beta}W^{\mu\nu\alpha\beta}$$
- **Fefferman-Graham expansion for AAdS spaces in CG**
  $$ds^2=\frac{\ell^2}{z^2}\,dz^2+\frac{1}{z^2}\,g_{ij}(x,z)\,dx^i dx^j$$
  $$\begin{aligned} g_{ij}(x,\rho) &= g_{(0)ij}(x)+z^2 g_{(2)ij}(x)+\cdots \\ &\quad +zg_{(1)ij}(x)+\cdots \end{aligned}$$

AdS gravity in D=2n dim

- **Renormalized Action**

$$I_{ren}^{2n} = \frac{1}{16\pi G}\int_M d^{2n}x\sqrt{-\hat{g}}\Big[R-2\Lambda- \\ -\frac{(-\ell^2)^{n-1}}{2^n n(2n-2)!}\,\delta^{[\nu_1\cdots\nu_{2n}]}_{[\mu_1\cdots\mu_{2n}]}R^{\mu_1\mu_2}_{\nu_1\nu_2}\cdots R^{\mu_{2n-1}\mu_{2n}}_{\nu_{2n-1}\nu_{2n}}\Big]\,,$$

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- **In terms of fully-antisymmetric objects**

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AdS gravity in D=2n dim

- **Schematically represented by the polynomial**

$$\begin{aligned} I(x,y) &= \frac{\ell^{2n-2}}{2^{n+4}\pi G n(2n-2)!}\left((-1)^n x^n + n\,xy^{n-1} + (n-1)y^n\right) \\ &= (x+y)^2 Q(x,y) \end{aligned}$$

where $x = R^{\alpha\beta}_{\mu\nu}$ and $y = \delta^{[\alpha\beta]}_{[\mu\nu]}/\ell^2$.

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- `Miskovic, RO and Tsoukalas, [arXiv:1404.5993]`

$$I^{2n}_{ren} = \frac{\ell^{2n-2}(n-1)}{2^{n+4}\pi G(2n-2)!}\int\limits_M d^{2n}x\sqrt{-\hat{g}}\,\delta^{[\nu_1\cdots\nu_{2n}]}_{[\mu_1\cdots\mu_{2n}]} W^{\mu_1\mu_2}_{(E)\nu_1\nu_2} W^{\mu_3\mu_4}_{(E)\nu_3\nu_4}\times$$
$$\times\int\limits_0^1 dt\,t\,\Xi^{\mu_5\mu_6}_{\nu_5\nu_6}(t)\times\cdots\times\Xi^{\mu_{2n-1}\mu_{2n}}_{\nu_{2n-1}\nu_{2n}}(t)\,,$$

where

$$\Xi^{\alpha\beta}_{\mu\nu}(t) = \frac{1}{\ell^2}\delta^{[\alpha\beta]}_{[\mu\nu]} - (1-t)\,W^{\alpha\beta}_{(E)\mu\nu}\,,$$

Six-dim AdS gravity

- Renormalized AdS Action

$$I^{6}_{ren} = \frac{\ell^4}{16\pi G} \int\limits_{M_6} d^6x \sqrt{\hat{g}} P[W_{(E)}]$$

- Polynomial in the Weyl tensor

$$-4!P_6 \left[W_{(E)} \right] = -2 \left| W_{(E)} \right|^2 + \frac{7}{3} I_2 - \frac{4}{3} I_2 + (4I_1 - I_2)$$

$$\begin{aligned} I_1 &= W_{(E)\alpha\beta\mu\nu} W^{\alpha\rho\lambda\nu}_{(E)} W_{(E)\rho}{}^{\beta\mu}{}_{\lambda} \\ I_2 &= W^{\alpha\beta}_{(E)\mu\nu} W^{\mu\nu}_{(E)\sigma\lambda} W^{\sigma\lambda}_{(E)\alpha\beta} \end{aligned}$$

Six-dim AdS gravity

- Identity by Osborn and Stergiou

$$4I_1 - I_2 = W^{\alpha\beta\mu\nu}\Box W_{\alpha\beta\mu\nu} + 10\left|W_{(E)}\right|^2$$

- Integrating by parts

$$4I_1 - I_2 = -\left|\nabla W_{(E)}\right|^2 + 10\left|W_{(E)}\right|^2$$

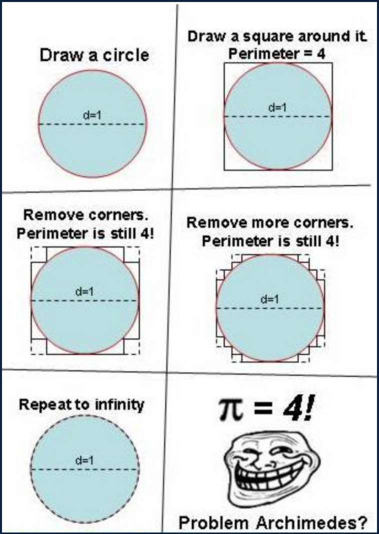
Six-dim AdS gravity

- Chang, Qing and Yang

$$
\begin{aligned}
J\left[W\right] &= -\left|\nabla W\right|^2 + 8\left|W\right|^2 + \\
&+ \frac{7}{3} W_{\mu\nu}{}^{\alpha\beta} W_{\alpha\beta}{}^{\lambda\rho} W_{\lambda\rho}{}^{\mu\nu} + \frac{4}{3} W_{\mu\nu\rho\lambda} W^{\mu\alpha\rho\beta} W^{\nu}{}_{\alpha}{}^{\lambda}{}_{\beta},
\end{aligned}
$$

$$
P_6\left[W_{(E)}\right] = -\frac{1}{4!} J\left[W_{(E)}\right].
$$

Renormalized Volume: à la physicist



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- Or using the GB theorem

$$Vol_{ren}\left[M_4\right] = \frac{4}{3}\ell^4\pi^2\chi\left[M_4\right] + \frac{\ell^4}{6}\int\limits_{M_4} d^4x\sqrt{\hat{g}}W_{(E)}^2 \qquad (1)$$

[M.T. Anderson, Math.Res. Lett. 8, 171 (2001)]

Renormalized volume

- 6D case
  [Anastasiou, Araya, Arias and RO, arXiv:1806.10708]

$$Vol_{ren}\left[M_6\right] = -\frac{8\pi^3 \ell^6}{15}\chi\left[M_6\right] + \frac{\ell^6}{240}\int\limits_{M_6} d^6x\sqrt{\hat{g}}J\left[W_{(E)}\right]$$

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- Polynomial in the Weyl tensor + 6-derivative term (Einstein part of 6D CG)

$$\begin{aligned} J\left[W\right] &= -\left|\nabla W\right|^2 + \frac{8}{\ell^2}\left|W\right|^2 + \\ &\quad + \frac{7}{3}W_{\mu\nu}{}^{\alpha\beta}W_{\alpha\beta}{}^{\lambda\rho}W_{\lambda\rho}{}^{\mu\nu} + \frac{4}{3}W_{\mu\nu\rho\lambda}W^{\mu\alpha\rho\beta}W^{\nu}{}_{\alpha}{}^{\lambda}{}_{\beta}, \end{aligned}$$

[Chang, Qing and Yang, J. Math. Sci. 149, 1755 (2008)]

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$$Vol_{ren}\left[M_{2n}^{(\alpha)}\right] = Vol_{ren}\left[M_{2n}^{(\alpha)}/\Sigma\right] - (1-\alpha)\,\frac{2\pi\ell^2}{(2n-1)}\,Vol_{ren}\left[\Sigma\right].$$

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- **For constant-curvature entangling surfaces, HEE is purely topological (Euler characteristic)**

$$S_{EE} \sim \chi(\Sigma).$$

Conclusions: Renormalization with Topological Terms

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- **Odd-dim case and HEE: Anastasiou, Araya, Guijosa & RO [1908.11447]**