Lattice Gravity, Diffeomorphisms and Curvature

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Preview

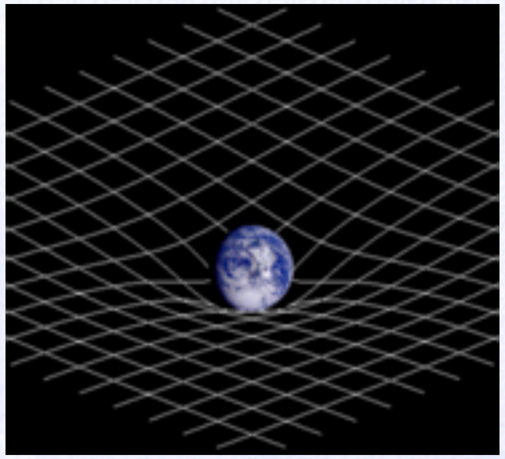
The context of my talk is the search for a ***theory of quantum gravity beyond perturbation theory*** and the ongoing research program of Causal Dynamical Triangulations (CDT) addressing the problem. Since time is much too short for a comprehensive overview, I will merely summarize the approach and then describe some new insights and structural aspects it has brought into focus.

My presentation will be about

- motivation and context
- lattice gravity and CDT in a nutshell
- the role of diffeomorphisms
- making sense of Ricci curvature in a quantum context

Life in the *Century of Gravity*

- **urgent**: complete our quantum gravity theories to make reliable predictions, minimizing free parameters and ad hoc assumptions
- **my route**: tackle quantum gravity and geometry *directly* in a non-perturbative, Planckian regime (no appeal to duality/dictionaries)
- the beauty of classical GR: "theory ***of*** spacetime", captured by its curvature properties
- given the central role of **curvature** classically, is it also true that



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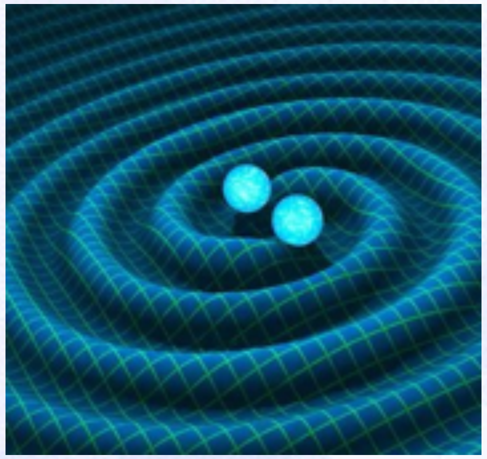

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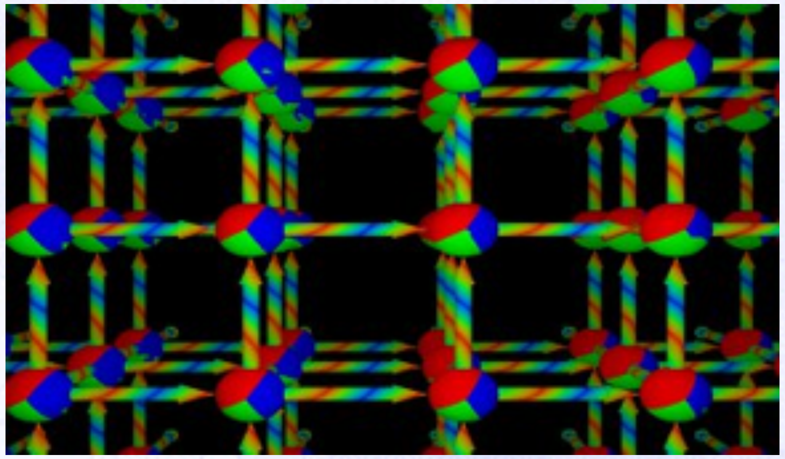
> nonperturb. quantum gravity = theory of quantum curvature?

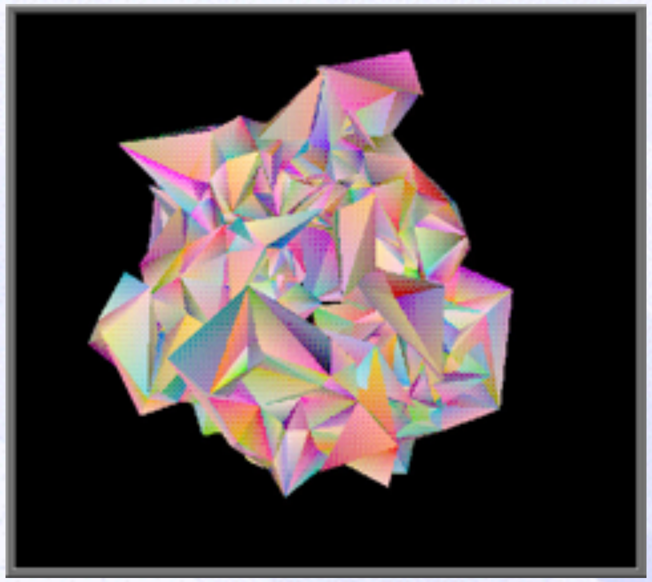
- So far, no-one has been able to make much sense of such a proposition. We have recently initiated a line of research into how to define and measure *quantum Ricci curvature* in quantum gravity.

The setting

- following the extremely successful example of QCD, we explore the nonperturbative regime quantitatively by **“lattice quantum gravity”**
- lattice gauge field configurations à la Wilson (PRD 10 (1974) 2445) are replaced by piecewise flat geometries (triangulations) à la Regge (Nuovo Cim. 19 (1961) 558)

(© G. Bergner, Jena)

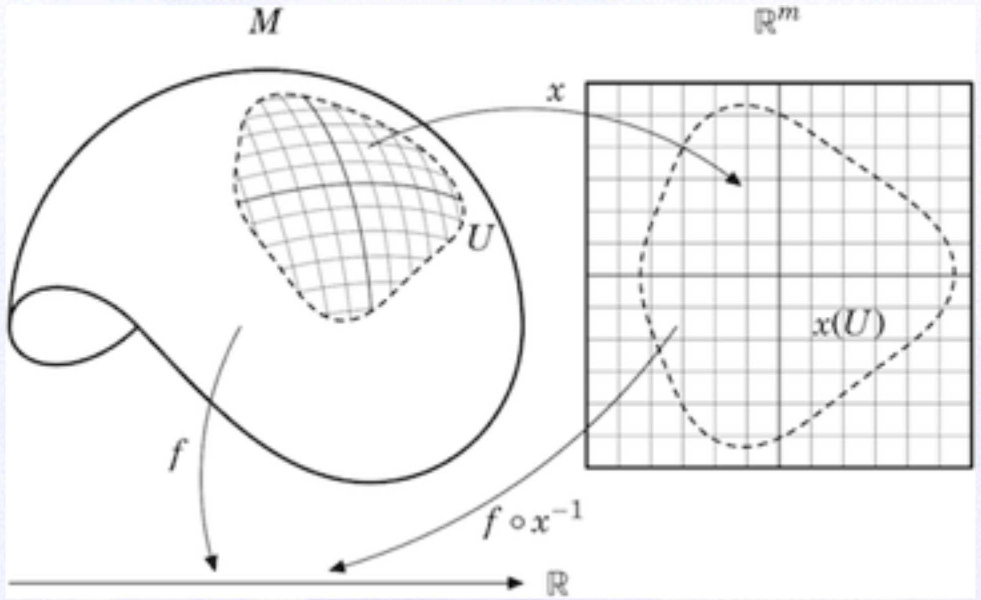




triangulated model of quantum space

- modern implementation: **Causal Dynamical Triangulations (CDT)**, a nonperturbative, background-independent, manifestly diffeo-morphism-invariant path integral, regularized on dynamical lattices
- N.B.: nontrivial scaling limit needed, no “fundamental discreteness”

Contrary to folklore, giving up smooth spacetimes and tensor calculus is not a crazy idea, but has been key to recent progress in quantum gravity (c.f. nonclassical 'discrete geometry' in maths).

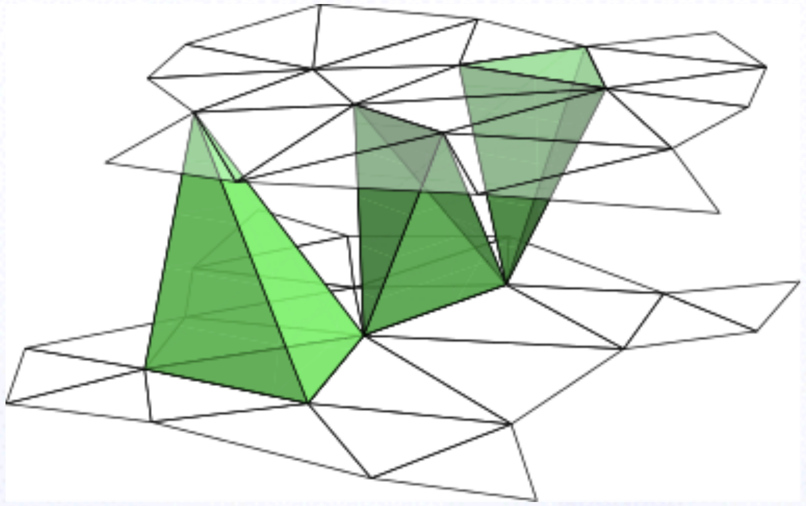


differentiable manifold *M* and a coordinate chart

- Classically, differentiable manifolds *M* provide powerful and extremely convenient models of spacetime.
- geometric properties encoded in the Riemann curvature tensor $R^{\kappa}{}_{\lambda\mu\nu}(x)$

- However, this description comes with an enormous redundancy, the "freedom to choose coordinates" without affecting the physics.
- The "gauge" group of GR is the infinite-dim. group of coordinate transformations (diffeomorphisms) on *M*. **The key challenges of quantum gravity are how to implement this symmetry and describe physics in terms of diffeomorphism-invariant *quantum observables*.**

CDT Quantum Gravity



part of a (piecewise flat) causal triangulation

The framework of **Causal Dynamical Triangulations (CDT)**, a nonperturbative candidate theory of quantum gravity, has proven both fruitful and well suited to studying the issue of observables.

Several quantum observables have been successfully defined and implemented, and their expectation values been measured.

CDT employs a *direct* quantization of classical spacetime geometry = *Metrics(M)/Diff(M)*, using a lattice regularization. As one would expect in standard QFT, the theory has divergences in the continuum limit as the UV regulator is removed, which must be renormalized appropriately.

CDT dynamics: nonperturbative, background-independent, unitary path integral; exactly soluble in *D=2*, Monte Carlo simulations in *D=4.*

CDT is a counterexample to the folklore that "putting gravity on the lattice breaks diffeomorphism invariance".

Quantum Gravity from CDT

The (formal, ill-defined) continuum gravitational path integral

$$Z(G_N,\Lambda) = \int_{\substack{\text{spacetimes}\\ g\in\mathcal{G}}} \mathcal{D}g\; \mathrm{e}^{iS^{\mathrm{EH}}_{\mathrm{G_N},\Lambda}[g]}$$

Newton's constant

cosmological constant

Einstein-Hilbert action

("sum over histories")

is turned into a finite regularized sum over triangulated spacetimes,

$$Z(G_N,\Lambda) := \lim_{\substack{a\to 0\\ N\to\infty}} \sum_{\substack{\text{inequiv.}\\ \text{triangul.s}\\ T\in\mathcal{G}_{a,N}}} \frac{1}{C(T)}\; \mathrm{e}^{iS^{\mathrm{Regge}}_{\mathrm{G_N},\Lambda}[T]}$$

UV cutoff

building blocks

|Aut(T)|

bare, discretized EH action

whose continuum limits are investigated after an analytic continuation.

(N.B.: the inclusion of matter is straightforward)

gravity action: $S^{\mathrm{EH}} = \dfrac{1}{G_{\mathrm{N}}}\displaystyle\int d^4x\sqrt{-\det g}\,(R[g,\partial g,\partial^2 g] - 2\Lambda)$

What is the overall outlook of CDT QG?

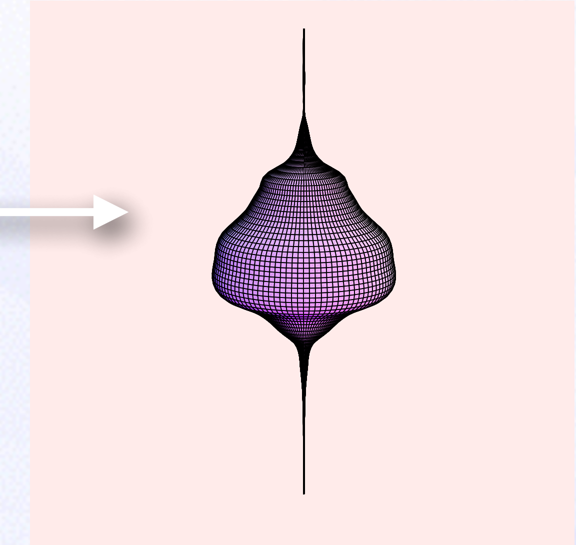
- CDT quantum gravity depends on a minimalist set of ingredients — metric d.o.f. and just two free parameters — and is conceptually simple.
- It builds on a significant body of analytical and numerical results on "dynamical triangulations" (a.k.a. "random geometry") since the 1980s, which give us a new view on geometry and the role of diffeomorphisms. (2D DT quantum gravity reproduces results of contin. Liouville gravity.)
- One has been able to extract *new and unique results* from evaluating a handful of **nonperturbative quantum observables**. These results are robust and quantitative, and potentially falsifiable (very rare in QG!).
- causal structure plays a crucial role (Euclidean QG 'not good enough')
- quantitative 4D results have so far been obtained in a highly quantum-fluctuating regime, far away from (semi-)classicality.

REVIEWS: J. Ambjørn, A. Görlich, J. Jurkiewicz & RL, Phys. Rep. 519 (2012) 127 [arXiv: 1203.3591]); **NEW**: RL, arXiv:1905.08669

The Emergence of Classicality from Causal Dynamical Triangulations (CDT)

From pure quantum excitations, CDT generates a spacetime with semiclassical properties *dynamically*, without using a background metric.

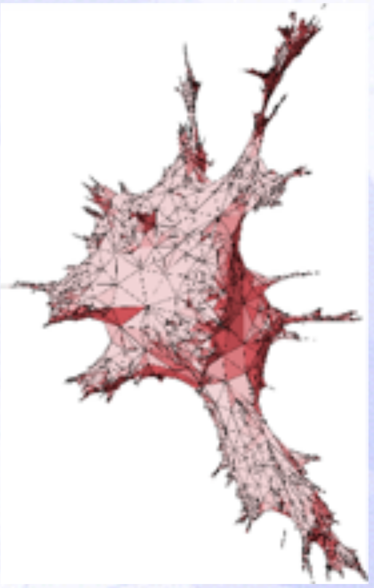
how to obtain a macroscopic universe with a ***de Sitter shape***:



Other key results/properties:

- crucial role of causal structure
- notion of discrete proper time (*not* coordinate time)
- existence of a well-defined "Wick rotation" (unique)
- amenable to computer simulations
- nontrivial phase structure, with "classical" phases
- second-order phase transitions (unique)
- scale-dependent spacetime dimension (2 → 4)
- applicability of renormalization group methods

from a superposition of "wild" path integral histories:



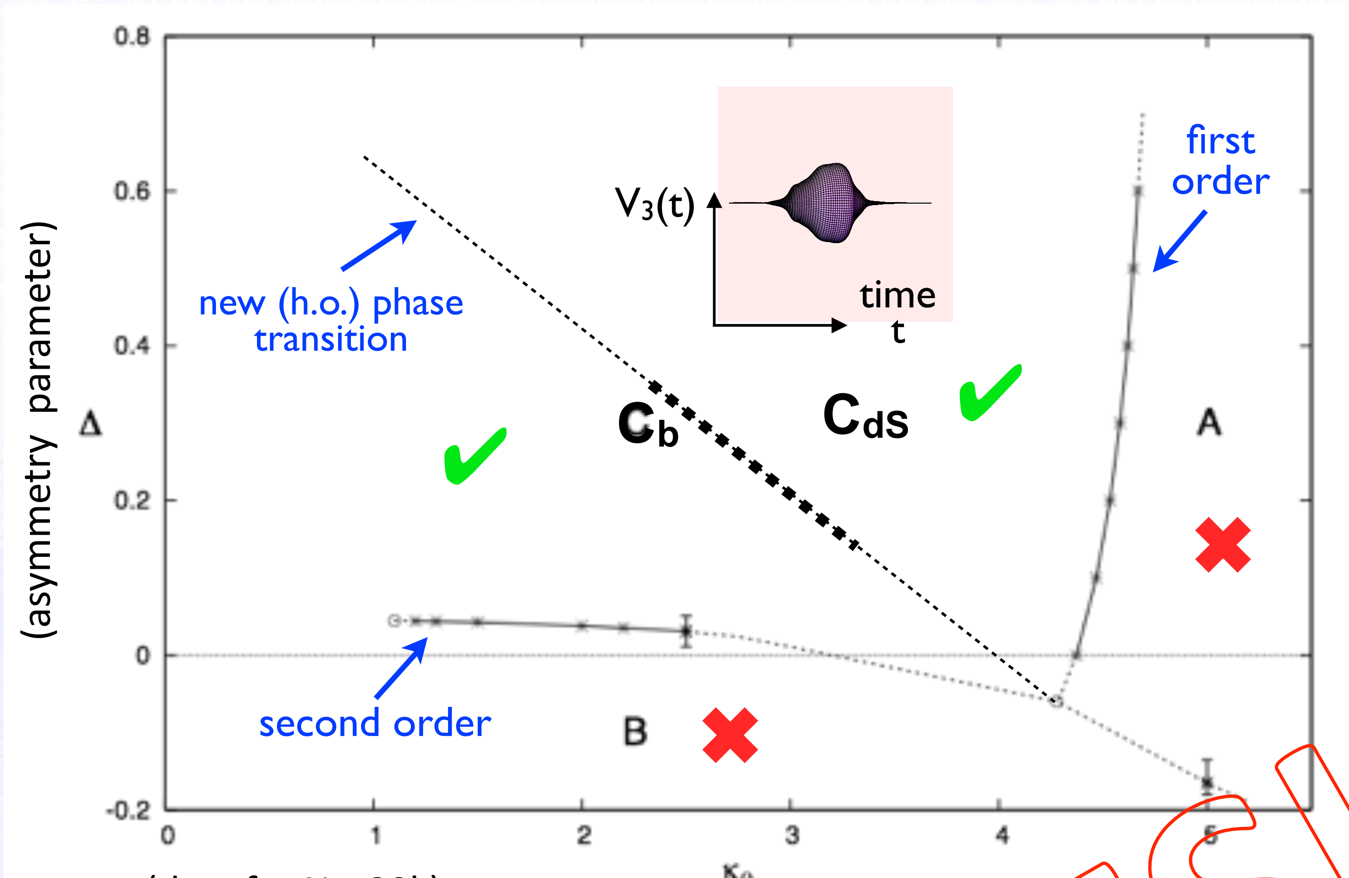
What we have learned so far in CDT quantum gravity about

(i) the phase structure and critical properties of the underlying statistical system of ‘random geometry’,
(ii) the system’s behaviour along RG trajectories, and
(iii) the properties of the dynamically generated “quantum spacetime”

comes from measuring a few <u>quantum observables</u>.

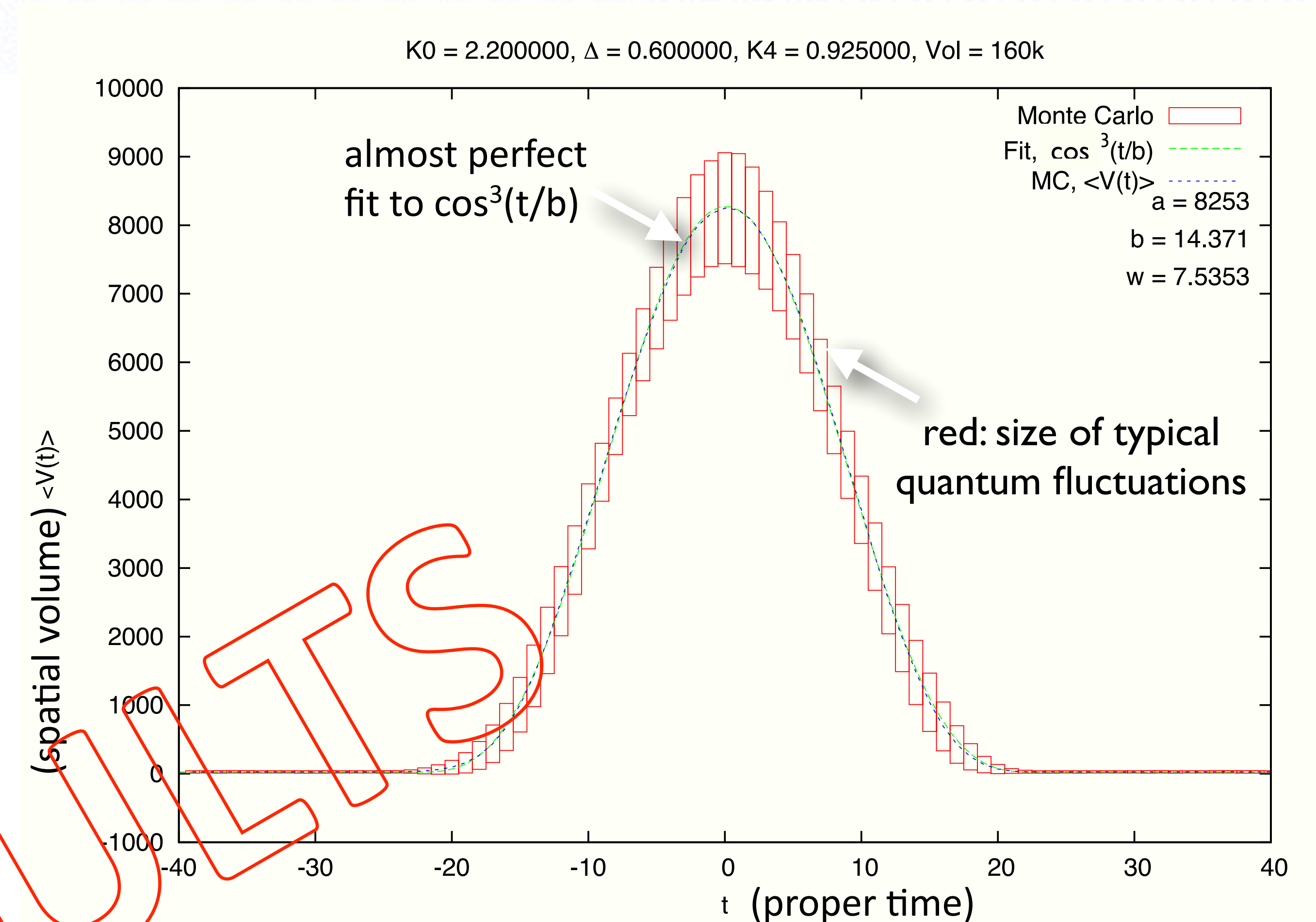
RESULTS

Phase diagram of CDT QG

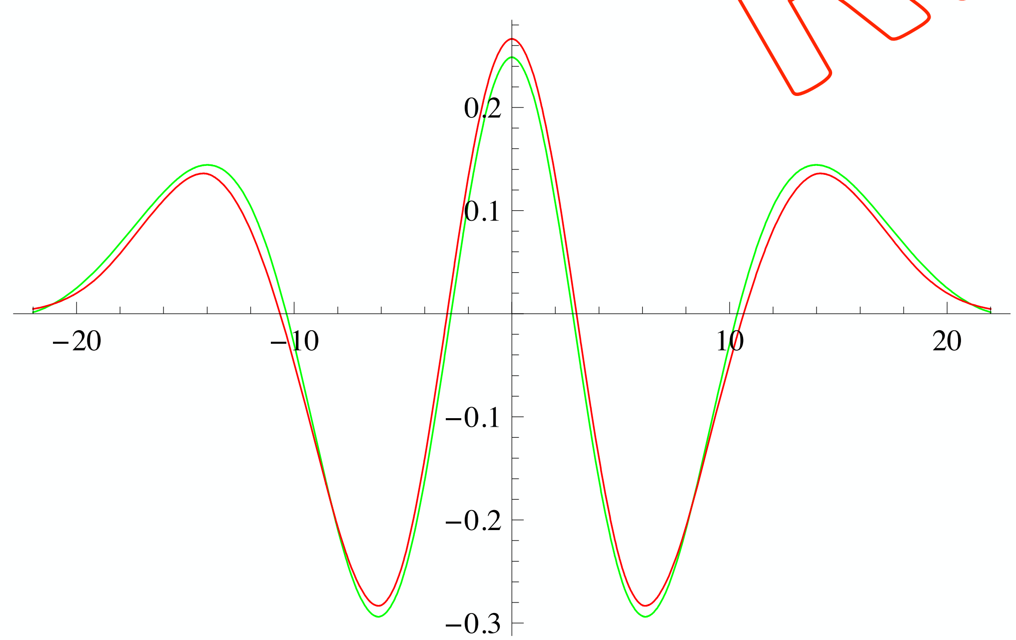


(data for N_4=80k) (bare inverse Newton constant)

The universe is de Sitter-shaped

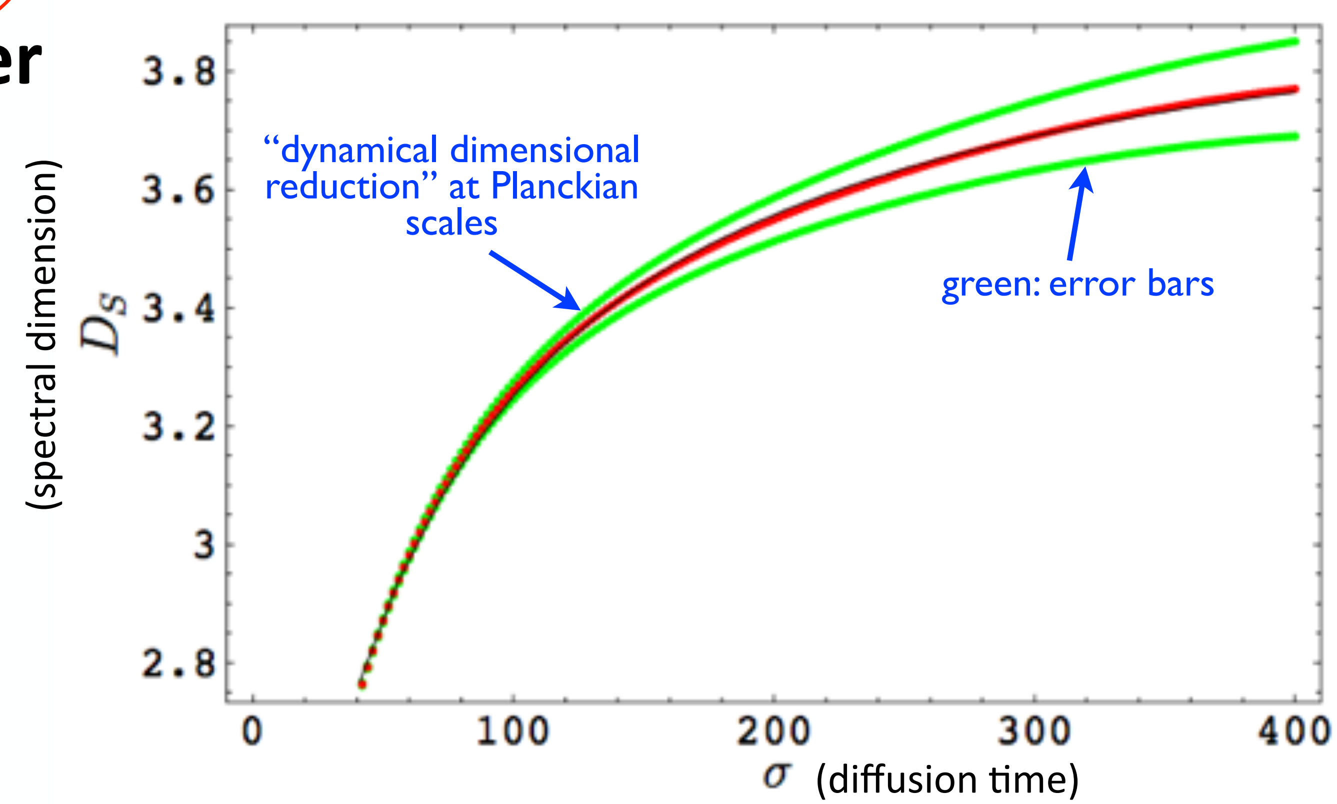


Volume fluctuations around de Sitter



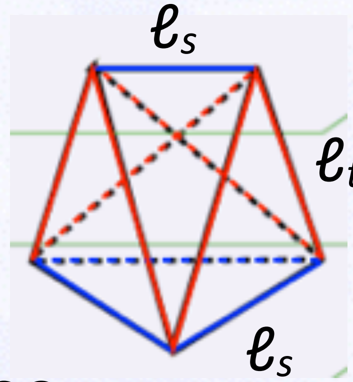
(low-lying eigenmode matches with semiclassical expectation)

Spectral dimension of the universe

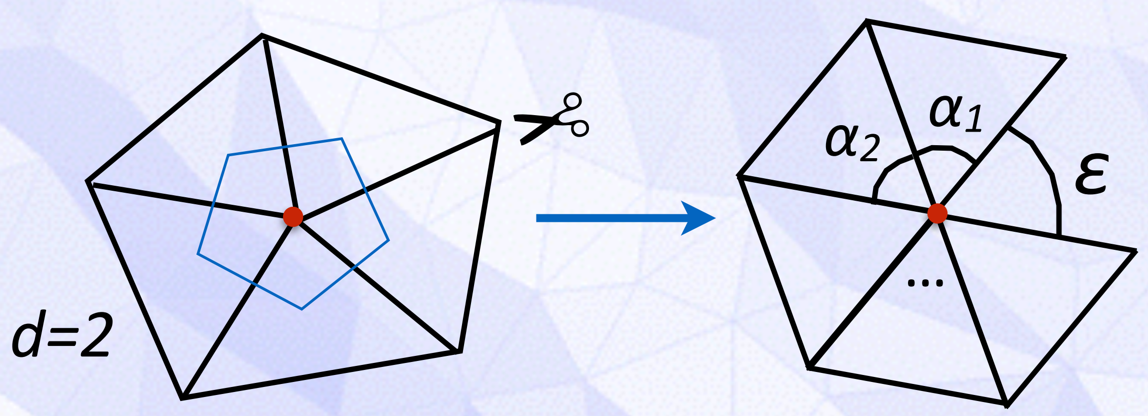


Crucial: QG <u>without</u> diffeomorphisms

Strategy: at a regularized level, represent curved spacetimes by simplicial manifolds, following the profound, but underappreciated idea of “General Relativity without Coordinates” (T. Regge, 1961).



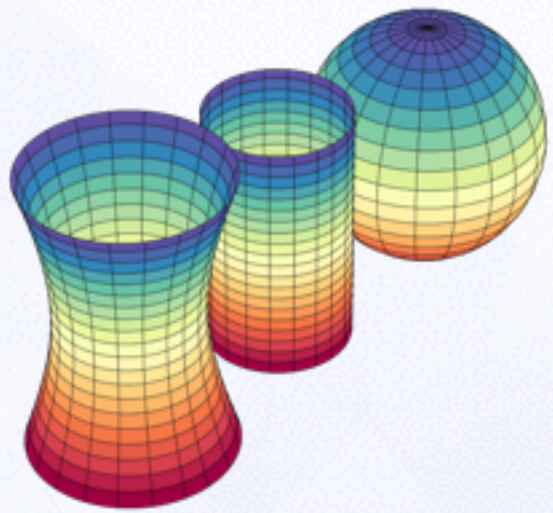
- Use ‘piecewise flat’ gluings of 4D triangular building blocks (four-simplices) to describe intrinsically curved spacetimes.
- Geometry is specified uniquely by the edge lengths ℓ of these simplices and how simplices are ‘glued’ together. **No coordinates are needed** and **the CDT path integral has no coordinate redundancies**.



Gluing five equilateral triangles around a vertex generates a surface with Gaussian curvature (deficit angle ε) at the vertex.

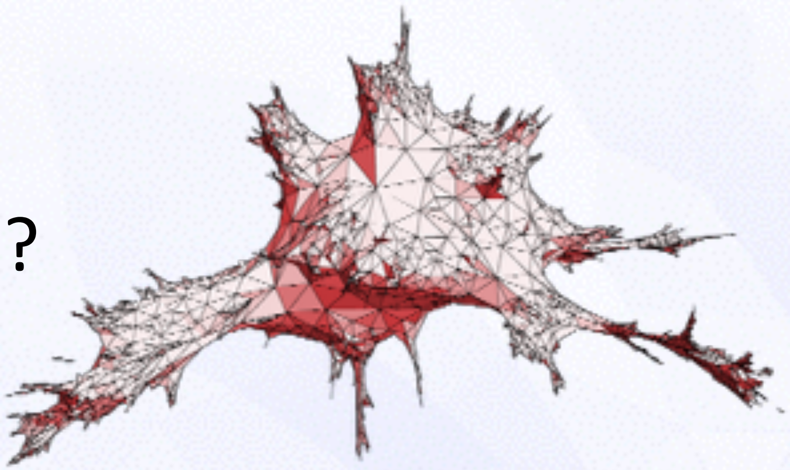
- distinct from Regge calculus: all edges have identical length $\ell = a$ (up to global time vs. space scaling)
- study superpositions of such geometries in continuum limit $a \rightarrow 0$ (removal of UV cut-off)

The challenge of “quantum curvature”



from classical

to quantum?



Individual spacetime geometries (= path integral histories) in CDT are continuous, but *not* smooth, and far from (semi-)classical.

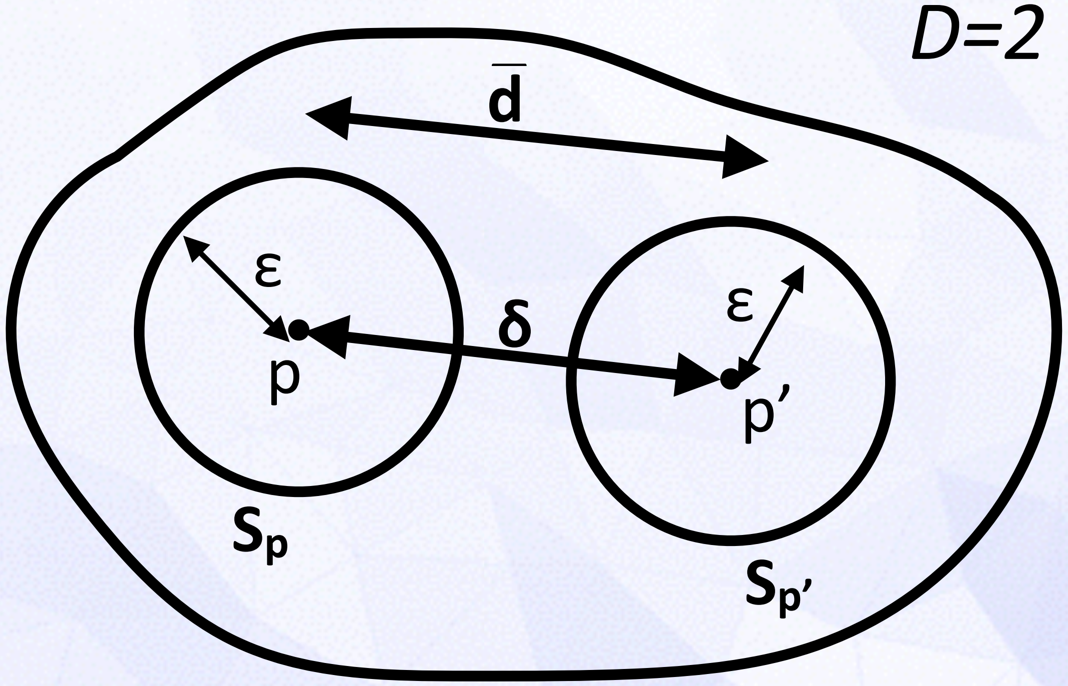
- Which properties continue to hold on such spaces?
- How can we make sense of curvature and curvature tensors?
- How can we average/coarse-grain them?

We have successfully defined and tested ***quantum Ricci curvature.***
(N. Klitgaard & RL, PRD 97 (2018) no.4, 0460008 and no.10, 106017, work in progress with J. Brunekreef and N. Klitgaard)

Introducing quantum Ricci curvature

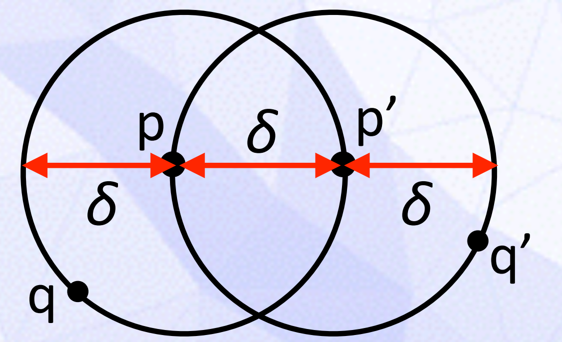
In D dimensions, the key idea is to compare the distance $\bar{d}$ between two $(D\text{-}1)$-spheres with the distance δ between their centres.

The sphere-distance criterion:
"On a metric space with positive (negative) Ricci curvature, the distance $\bar{d}$ of two nearby spheres S_p and $S_{p'}$ is smaller (bigger) than the distance δ of their centres."



(c.f. Y. Ollivier, J. Funct. Anal. 256 (2009) 810)

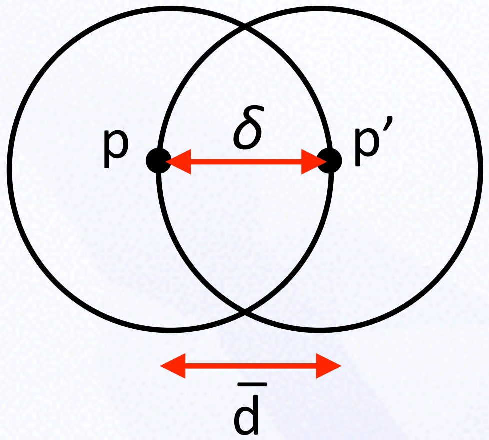
Our variant uses the ***average sphere distance*** $\bar{d}$ of two spheres of radius δ whose centres are a distance δ apart,



- involves only distance and volume measurements
- the directional/tensorial character is captured by the "double sphere"
- coarse-graining is captured by the variable scale δ

Implementing quantum Ricci curvature

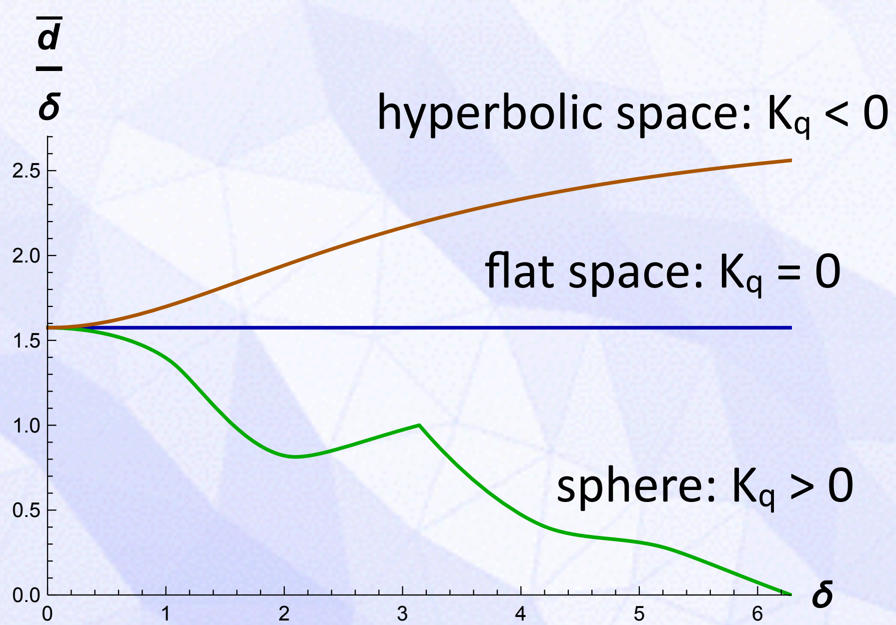
We measure the **"quantum Ricci curvature K_q at scale δ"**,



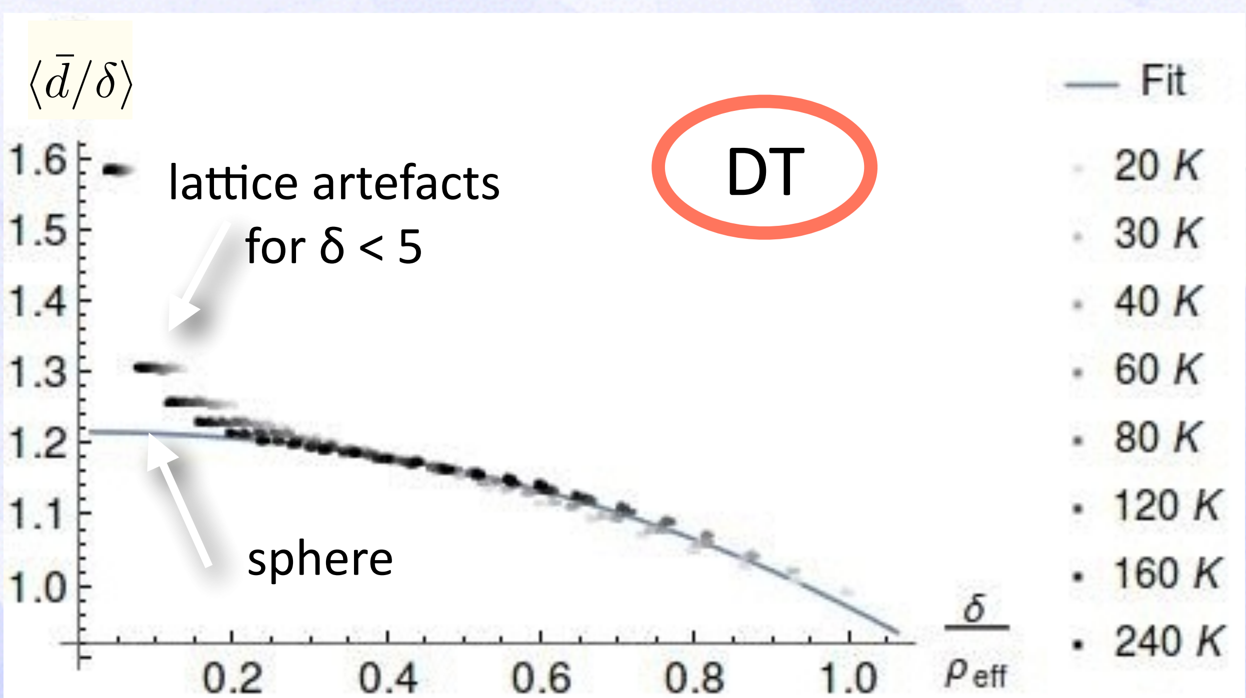
non-univ. constant

$$\frac{\bar{d}(S_p^\delta, S_{p'}^\delta)}{\delta} = c_q(1 - K_q(p,p')), \quad \delta = d(p,p'), \quad 0 < c_q < 3,$$

on the quantum ensemble and compare it with the behaviour on simple continuum "reference spaces" (constantly curved; ellipsoids; cones). Remarkably, for the highly fractal quantum geometry of 2D quantum gravity, quantum Ricci curvature displays a robust, sphere-like scaling behaviour:



K_q on classical, constantly curved spaces in D=2 (curvature radius 1)



triangles N∈[20k, 240k]; error bars too small to be shown

Summary

Nonperturbative quantum gravity can be studied in a **lattice** setting, in close analogy with lattice QCD, but taking into account the dynamical nature of geometry, as exemplified by CDT.

The **CDT** approach has been making significant strides towards a full-fledged quantum theory. Its well-defined computational lattice framework allows for quantitative evaluation and "reality checks".

The full power of Regge's idea of describing geometry without coordinates unfolds in nonperturbative QG in terms of CDT, yielding a manifestly **diffeomorphism-invariant** formulation.

Despite the absence of smoothness, one can define a notion of **curvature** that appears to be well-defined, including in a Planckian regime, and gives us a new tool to understand the properties of **quantum gravity** and the quantum geometry emergent from it.

Lattice Gravity, Diffeomorphisms and Curvature

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Thank you!