Heavy-quark form factors

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Outline

1. Introduction
2. Calculation
3. Results

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Introduction

Considering form factors at three-loop order for the process

$$X \to Q + \overline{Q}$$

coupling through one of the vertices

$$\{1, \gamma_5, \gamma^\mu, \gamma^\mu \gamma_5\}$$

here:

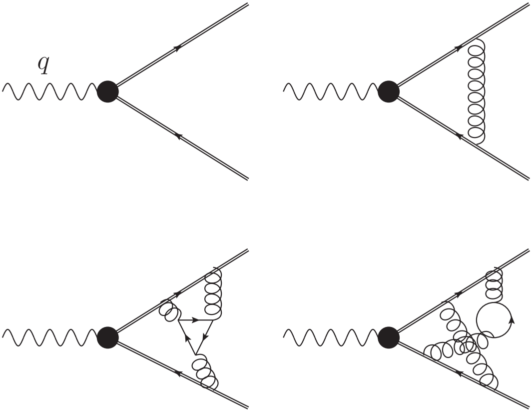
- only non-singlet contributions, i.e. the heavy-quark pair couples directly to the external current.
- at least one heavy-quark loop

Motivation

- heavy quark production
  - continuum production $e^+e^- \to t\bar{t}$
- particle decays
  - $H \to b\bar{b}$
  - $Z \to b\bar{b}$
  - $A \to t\bar{t}$
- technology development

Example: Vector case

$$\bar{\Psi}\Gamma^{\mu}_{V}\Psi = -i\bar{\Psi}\left(\gamma^{\mu} F_{V,1} + \frac{i}{2m}\sigma^{\mu\nu} q_{\nu} F_{V,2}\right)\Psi$$



History / Previous works

- two loop

  [Bernreuther,Bonciani,Gehrmann,Heinesch,Leineweber,Mastrolia,Remiddi '05]

  [Gluza,Mitov,Moch,Riemann '09]

  [Ablinger,Behring,Blümlein,Falcioni,De Freitas,PM,Rana,Schneider '18]

- three loop
  - light-fermionic contributions (HPLs)

    [Lee,Smirnov,Smirnov,Steinhauser'18]

    [Ablinger,Blümlein,PM,Rana,Schneider'18]
  - color-planar contributions (HPLs + cyclotomic HPLs)

    [Henn,Smirnov,Smirnov,Steinhauser '17]

    [Ablinger,Blümlein,PM,Rana,Schneider'18]
  - **NEW** heavy-fermionic contributions
- general infrared and high-energy structure [Ahmed,Henn,Steinhauser '17]

  [Blümlein, PM, Rana '18]

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Calculation

For the calculation of the form factors use the well-established multi-loop toolbox

- ✓ QGRAF for the generation of the diagrams
- ✓ use projectors to obtain scalar integrals
- ✓ FORM for the algebra
- ✓ use integration-by-parts identities [Chetyrkin,Tkachov]
  to reduce to an integral basis using Crusher [Seidel,PM]
  14 families, 104 master integral
- ??? calculate the required master integrals
- ✓ put everything together and renormalize
- ✓ final result still IR divergent – compare with predictions

Calculation / Strategy

Calculation of master integrals problematic since the heavy-fermionic and non-planar contributions contain structures beyond harmonic polylogarithms

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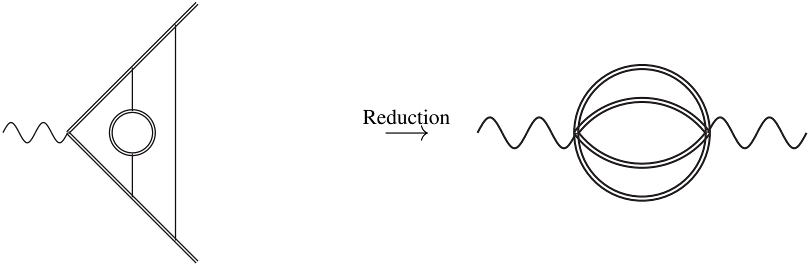
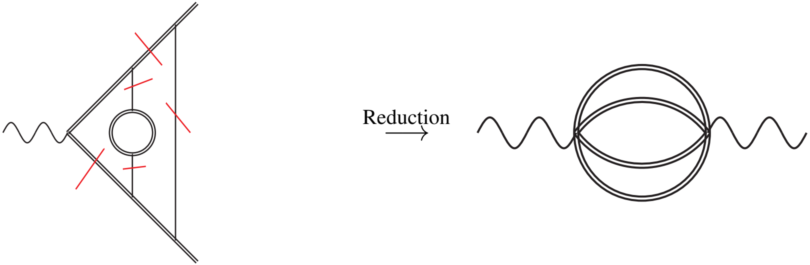


Calculation / Strategy

Calculation of master integrals problematic since the heavy-fermionic and non-planar contributions contain structures beyond harmonic polylogarithms



Calculation / Strategy

- Strategy: Sum simpler then the individual parts!
- turn everything into recurrences by considering the expansion around $q^2 = 0$
- try to derive a recurrence for the whole form factor and find a analytic solution for that [Blümlein,Schneider '17]

Method

- choose a more appropriate variable

$$\frac{q^2}{m^2} = -\frac{(1-x)^2}{x}$$

$q^2 \to \pm\infty \equiv x \to 0_{\mp}$
$q^2 \to 0 \equiv x \to 1$

- around $q^2 = 0$, i.e. $x = 1$ the non-singlet form factors can be expanded in a simple power series

$$\mathcal{F} = \sum_{n=0} C_n \left(\frac{q^2}{m^2}\right)^n \quad \Leftrightarrow \quad \mathcal{F} = \sum_{n=0} D_n (1-x)^n = \sum_{n=0} D_n y^n$$

Method

- start from the coupled system of diff. eqn. for the master integrals
- insert the power series ansatz

$$\mathcal{M}_i = \sum_{j=0} M_j^{(i)} y^j$$

  and obtain recurrences for the coefficients $M_j^{(i)}$
- calculate 2,000 - 8,000 terms in the expansion for the master integrals
- use these to obtain 2,000 - 8,000 terms in the expansions of the full form factors
- as initial condition we need the values at $x = 1$, i.e. on-shell propagators [Melnikov,v.Ritbergen]

Method

- the final expansion for the form factors has the form

$$\mathcal{F} = 1(\dots) + \zeta_2(\dots) + \zeta_3(\dots) + \ln(2)(\dots) + \mathrm{Li}_4(\tfrac{1}{2})(\dots) + \cdots$$

  where $(\dots)$ denote power series in y with rational coefficients
- this representation is unique
- can we do better?
  - Guess a recurrence [Kauers,Jaroschek,Johansson '15]
  - and try to solve it using `Sigma` [Schneider '07]
- if recurrence can be solved, i.e. first-order factorizing, one obtains (generalized) harmonic sums, which can be resumed using `HarmonicSums` [Ablinger '13]

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Results

We could find analytic results for all terms but for n_h $n_h\zeta_2$ $n_h\zeta_3$

		degree	order	remaining order
F_V	$g_1 n_h$	1288	54	15
	$g_1 n_h\zeta_3$	409	29	10
	$g_1 n_h\zeta_2$	295	24	6
	$g_2 n_h$	1324	55	15
	$g_2 n_h\zeta_3$	430	30	10
	$g_2 n_h\zeta_2$	273	23	6
F_S	n_h	1114	50	15
	$n_h\zeta_3$	350	27	10
	$n_h\zeta_2$	230	22	6

For leading color we could also solve the term $\propto N_c^2 n_h\zeta_2$

Results – Scalar form factor

$$
\begin{aligned}
F_S \;=\; & -\frac{1}{\varepsilon^3}\frac{1}{2(1+x)^2}\Bigg\{ n_h^2\left[-\frac{64}{27}(1+x)^2+\frac{64(1+x)\left(1+x^2\right)}{27(1-x)}H_0\right] \\
& +n_h\left[\frac{4}{27}\left(997+1418x+997x^2\right)-\frac{32H_0P_8^{(5)}}{27(1-x^2)}\right. \\
& \left.-n_l\left[\frac{32}{9}(1+x)^2-\frac{64(1+x)\left(1+x^2\right)}{27(1-x)}H_0\right]+\frac{256\left(1+x^2\right)^2}{27(1-x)^2}H_0^2\right]\Bigg\}
\end{aligned}
$$

Results – Scalar form factor cont'd

$$
\begin{aligned}
&-\frac{1}{\varepsilon^2}\frac{1}{2(1+x)^2}\Bigg\{n_h^2\Bigg[-\frac{832}{81}(1+x)^2-\frac{256x(1+x)H_0}{27(1-x)}-\frac{128(1+x)(1+x^2)}{27(1-x)}H_{-1}H_0\\
&+\frac{32(1+x)(1+x^2)}{27(1-x)}H_0^2+\frac{128(1+x)(1+x^2)}{27(1-x)}H_{0,-1}-\frac{64(1+x)(1+x^2)}{27(1-x)}\zeta_2\Bigg]\\
&+n_h\Bigg[\frac{16}{27}(897+1786x+897x^2)+n_l\Bigg[-\frac{64}{3}(1+x)^2+\frac{64(1+x)(5-24x+5x^2)}{81(1-x)}H_0\\
&-\frac{256(1+x)(1+x^2)}{27(1-x)}H_{-1}H_0+\frac{64(1+x)(1+x^2)}{27(1-x)}H_0^2+\frac{256(1+x)(1+x^2)}{27(1-x)}H_{0,-1}\\
&-\frac{128(1+x)(1+x^2)}{27(1-x)}\zeta_2\Bigg]+\left(\frac{128H_{-1}P_7^{(5)}}{27(1-x^2)}-\frac{16P_{13}^{(5)}}{27(1-x^2)}\right)H_0+\left(\frac{64P_{26}^{(5)}}{27(1-x)^2(1+x)}\right.\\
&\left.-\frac{1024(1+x^2)^2}{27(1-x)^2}H_{-1}\right)H_0^2-\frac{128(-2+x^2)(1+x^2)}{27(1-x)^2}H_0^3-\frac{128(1+x)(1+x^2)}{3(1-x)}H_0H_1\\
&+\left(\frac{128(1+x)(1+x^2)}{3(1-x)}-\frac{128(1+x^2)^2}{3(1-x)^2}H_0\right)H_{0,1}-\left(\frac{128P_7^{(5)}}{27(1-x^2)}-\frac{2176(1+x^2)^2}{27(1-x)^2}H_0\right)\\
&\times H_{0,-1}+\frac{256(1+x^2)^2}{3(1-x)^2}H_{0,0,1}-\frac{256(1+x^2)^2}{3(1-x)^2}H_{0,0,-1}+\left(\frac{64P_5^{(5)}}{27(1-x^2)}\right.\\
&\left.-\frac{64(1+x^2)(-1+35x^2)}{27(1-x)^2}H_0\right)\zeta_2-\frac{64(1+x^2)^2}{3(1-x)^2}\zeta_3\Bigg]\Bigg\}
\end{aligned}
$$

Results – Scalar form factor – unsolved recurrences

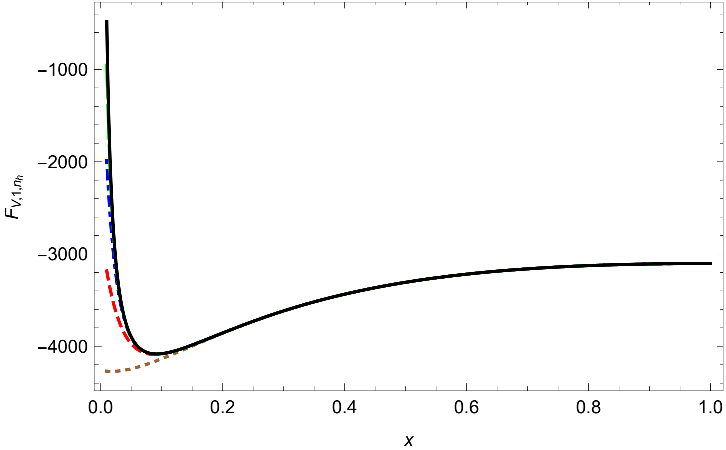
$$F_S = \ldots + n_h F^{(0)}_{S,1}(x) + n_h \zeta_2 F^{(0)}_{S,2}(x) + n_h \zeta_3 F^{(0)}_{S,3}(x)$$

$$F^{(0)}_{S,1}(x) = -\frac{96756433y^5}{218700} - \frac{316061833y^4}{437400} - \frac{731018y^3}{729} - \frac{731018y^2}{729} - \frac{874750}{243} + O(y^6)$$

$$F^{(0)}_{S,2}(x) = \frac{3932123y^5}{18225} + \frac{16041283y^4}{36450} + \frac{2421832y^3}{3645} + \frac{2421832y^2}{3645} + \frac{343864}{81} + O(y^6)$$

$$F^{(0)}_{S,3}(x) = -\frac{7752703y^5}{48600} - \frac{21262303y^4}{97200} - \frac{22516y^3}{81} - \frac{22516y^2}{81} + \frac{62968}{27} + O(y^6).$$

Results – High-Energy region



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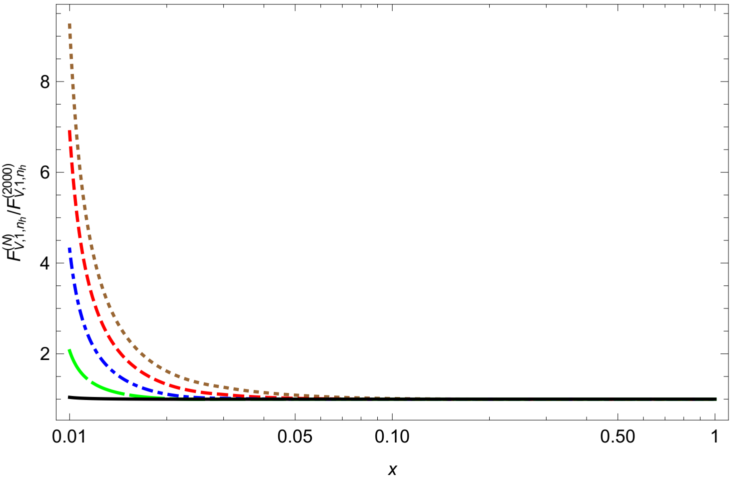
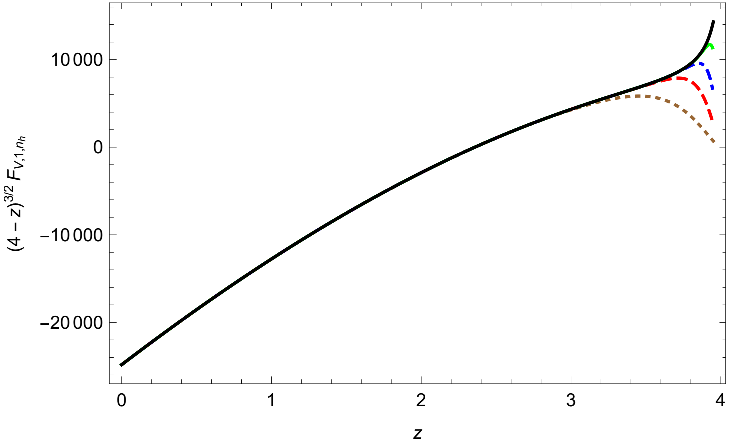


Results – Threshold region



Conclusions

- Calculated the heavy-fermionic corrections to the heavy-quark form factors in an expansion about $q^2 = 0$
- Many parts can be resumed and are available analytically
- For some parts only recurrences and thus deep expansions exist
- Results for pole terms agree with predictions
- ToDo: full color
- ToDo: singlet contributions
- ToDo: find a solution for the non-first-order factorizing recurrences