Effective Description of Critical QCD

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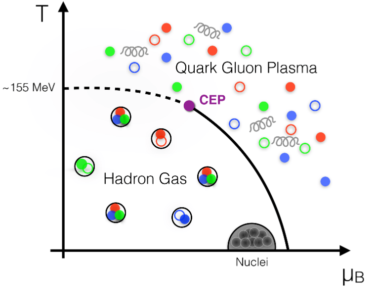
The phase diagram of QCD



from H-T. Ding et al,
Int. J. Mod. Phys. E 24, 1530007 (2015)

Standard description:

- at $\mu_b = 0$: crossover transition (exact result in Lattice QCD)
- at $\mu_b = \mu_c$: critical endpoint (CEP) 2nd order phase transition
- $\mu_b > \mu_c$: 1st order phase transition

Experimental territories

- LHC, RHIC: crossover
- SPS (NA49, NA61), RHIC-BES: critical endpoint
- FAIR, NICA: 1st order transition (baryon-rich region)

Observables (basic ingredient: order parameter)

- power-law fluctuations of factorial moments (intermittency) at the critical point (SPS).
- higher order cumulants: non-monotonic behaviour near the critical point (RHIC-BES).
- GW (from neutron stars) sensitive to EoS and phase transition in the baryon-rich regime.
(**New field:** Astronuclear Physics and Gravity)

The order parameter of critical QCD

- **Order parameter:** thermodynamics near a critical point!
- For QCD critical point $\Rightarrow$ chiral condensate: $\langle \bar{\Psi}\Psi \rangle = \bar{u}u + \bar{d}d$

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scalar-isoscalar (σ)

- σ is remnant from chiral phase transition:

$$(\sigma, \vec{\pi}), \quad O(4) \text{ univ. class} \longrightarrow \sigma, \quad Z_2 \text{ univ. class } (3d - \text{Ising})$$

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tricritical point (Wilczek) **critical endpoint**

- In a finite-density medium (T. Hatsuda, Quark Matter 1991):

$$\sigma \approx \langle \bar{\Psi}\Psi \rangle_0 - \frac{T^2}{8f_\pi^2} \langle \bar{\Psi}\Psi \rangle_0 - \frac{\Sigma_{\pi N}}{f_\pi^2 m_\pi^2} \langle \bar{\Psi}\Psi \rangle_0 n_b$$

$$\delta\sigma \approx -\frac{\Sigma_{\pi N}}{f_\pi^2 m_\pi^2} \langle \bar{\Psi}\Psi \rangle_0 \delta n_b \quad ; \quad \delta\sigma \sim \delta n_b$$

Same singular behaviour of $\delta\sigma$, δn_b for $T \to T_c$
(isothermal susceptibility)

- Net-baryon density $n_b \longrightarrow$ order parameter for QCD critical system!

- Dynamics: n_b is conserved $\Rightarrow$ slow variable

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dominant in long-wavelength limit ($k \to 0$): $\frac{\partial n_b}{\partial t} + i\vec{k} \cdot \vec{J}_b(\vec{k}, t) = 0$

Ising-QCD partition function

- **Critical QCD:** 3d Ising universality class $\Rightarrow$

  Critical exponents: $\alpha \approx 0,\ \beta \approx \frac{1}{3},\ \gamma \approx \frac{4}{3},\ \delta \approx 5,\ \nu \approx \frac{2}{3},\ \eta \approx 0$

- 3d-Ising effective action (M. M. Tsypin, Phys. Rev. Lett. 73, 2015 (1994)):

$$S_{eff} = \int_V d^3\mathbf{x} \left[\frac{1}{2}|\nabla\phi|^2 + U(\phi) - h\phi \right]$$

$$U(\phi) = \frac{1}{2}m^2\phi^2 + mg_4\phi^4 + g_6\phi^6 \quad ; \quad m = \beta_c \xi^{-1}\ (g_4 \approx 1,\ g_6 \approx 2)$$

$$\phi = \beta_c^3 \lim_{\delta V \to 0} \left(\frac{n_\uparrow - n_\downarrow}{\delta V} \right) \quad ; \quad \beta_c = \frac{1}{k_B T_c}$$

- ξ: correlation length of **infinite** system:

$$\xi \sim |1 - \frac{T}{T_c}|^{-\nu}$$

- Ising-QCD correspondence: $(n_\uparrow, n_\downarrow) \longrightarrow (N_B, N_{\bar{B}})$

  $\phi \longrightarrow \beta_c^3 n_b$ (net-baryon density) ; $h \longrightarrow (\mu - \mu_c)\beta_c$

- The partition function: $\mathcal{Z} = \sum_{\{\phi\}} \exp\left(-S_{eff}\right)$

- n_b is **slow** mode $\Rightarrow$ long wavelength configurations dominate

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  $\{\phi\}$ contains mainly constant configurations $(\nabla\phi \approx 0)$

In the long-wavelength limit the grand-canonical description leads to (N. G. Antoniou, F. K. Diakonos, X. Maintas and C. E. Tsagkarakis, Phys. Rev. D97, 034015 (2018)):

$$\mathcal{Z} = \sum_{N_b=0}^{\Lambda} \zeta^{N_b} \exp\left[-\frac{1}{2} m^2 \frac{N_b^2}{\Lambda} - m g_4 \frac{N_b^4}{\Lambda^3} - g_6 \frac{N_b^6}{\Lambda^5}\right]$$

$$\Lambda = \beta_c^{-3} V \quad ; \quad m \sim |t|^{\nu} \ \left(t = \frac{T - T_c}{T_c}\right) \ ; \ \zeta = \exp\left(\frac{\mu - \mu_c}{T_c}\right)$$

Claim:

The Ising-QCD partition function $\mathcal{Z}$ describes correctly a critical system of protons, very close to the critical point $(\zeta \approx 1,\ t \approx 0)$.

Scaling laws for the infinite system

- General class of systems:

$$\mathcal{Z}(\zeta, \Lambda, T) = \sum_{N=0}^{\Lambda} \zeta^N \exp\left[-\beta F(N, \Lambda, T)\right]$$

where:

$$F(N, \Lambda, T_c) = F_c\left(\frac{N}{\Lambda^q}\right) \quad ; \quad q < 1\ ,\ \Lambda \gg 1$$

Ising-QCD partition function $\Rightarrow q = \frac{5}{6}$.

- **Lee-Yang theory** of phase transitions:

  Zeroes of $\mathcal{Z}$ in the complex ζ-plane: $\{\rho_i(\Lambda, T)\}$

  At $T = T_c$: accumulation of ρ_i at $\zeta = 1$ (critical point) for $\Lambda \to \infty$.

- Representation of $\mathcal{Z}$ with Lee-Yang zeroes:

$$\mathcal{Z} = \prod_{i=1}^{\Lambda} [\zeta - \rho_i(\Lambda, T)]$$

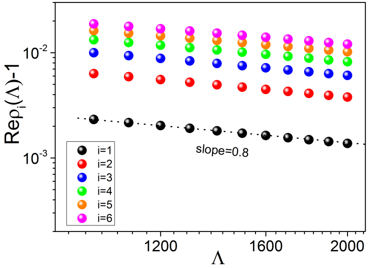
$$\Downarrow \quad \text{moments}$$

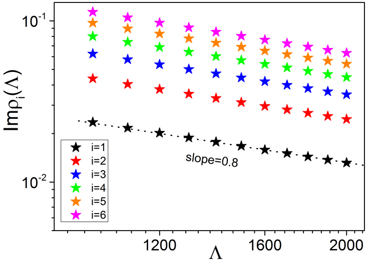
$$\langle N \rangle = \zeta \frac{\partial \ln \mathcal{Z}}{\partial \zeta} \quad ; \quad \langle N \rangle = \sum_{i=1}^{\Lambda} \frac{\zeta}{\zeta - \rho_i(\Lambda, T)}$$

- The q-class partition function $\mathcal{Z}$ satisfies the requirements of **scaling theory** for $\Lambda \gg 1$ if:

$$\rho_i(\Lambda, T_c) = 1 + \gamma_i \Lambda^{-q}$$

- This is verified for Ising-QCD partition function ($q = \frac{5}{6}$):





Power-laws of Ising-QCD theory ($\Lambda \to \infty$)

- Order parameter ($T \to T_c$, $\zeta \to 1$):

$$\langle n_b \rangle \sim t^{\nu d(1-q)} \longrightarrow \beta = \nu d(1-q)$$

- Order parameter, isothermal ($T = T_c$, $\zeta \to 1$):

$$\langle n_b \rangle \sim (\zeta - 1)^{\frac{1-q}{q}} \longrightarrow \delta = \frac{1-q}{q}$$

- Susceptibility ($T \to T_c$, $\zeta = 1$):

$$\chi \sim t^{-\nu d(2q-1)} \longrightarrow \gamma = \nu d(2q-1)$$

- Specific heat ($T \to T_c$, $\zeta = 1$):

$$c \sim t^{\nu d - 2} \longrightarrow \alpha = \nu d - 2$$

- Correlation function for large r ($T \to T_c$, $\zeta \to 1$):

$$\Gamma(r) \sim r^{-2d(1-q)} \longrightarrow \eta = 2 + d - 2dq$$

Conclusion

- The Ising-QCD partition function $\mathcal{Z}$ containing the two fundamental critical indices ($q = \frac{D_h}{d}$, $\nu = \frac{1}{D_t}$) provides the **standard** solution of scaling theory for **critical exponents**:

$$\alpha = 2 - \nu d \quad ; \quad \beta = \nu d(1-q) \quad ; \quad \gamma = \nu d(2q-1)$$

$$\delta = \frac{q}{q-1} \quad ; \quad \eta = 2 + d - 2qd$$

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- **The infinite system's description with Ising-QCD is fully compatible with scaling theory laws!**

Finite-size scaling (FSS)

- The Ising-QCD partition function describes **equally well** the scaling laws for a system of finite volume Λ (finite-size scaling) **at the critical point** ($\zeta = 1$, $t = 0$).
- For $\xi_\infty \gg \Lambda^{\frac{1}{d}}$ (Λ large):

$$n_b \sim \Lambda^{-\frac{\beta}{\nu d}} \quad ; \quad c \sim \Lambda^{\frac{\alpha}{\nu d}} \quad ; \quad \chi \sim \Lambda^{\frac{\gamma}{\nu d}}$$

with (α, β, γ) for the **infinite system**:

$$n_b \sim t^{\beta} \quad ; \quad c \sim t^{-\alpha} \quad ; \quad \chi \sim t^{-\gamma}$$

Finite-size power law: $n_b \sim \Lambda^{-\frac{\beta}{\nu d}} \Rightarrow$ fractal structure: $\langle N_b \rangle \sim \Lambda^q$

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Fractal dimension of critical system $d_F = qd \Rightarrow$ for 3d-Ising univ. class ($q = \frac{5}{6}$, $d = 3$): $d_F = \frac{5}{2}$

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Measure of critical fluctuations (**observable** through intermittency)

The proposed Ising-QCD partition function satisfies **all the requirements of scaling and universality** near the QCD critical point $\Rightarrow$ effective description of critical QCD

Higher cumulants versus measurements at BNL-RHIC

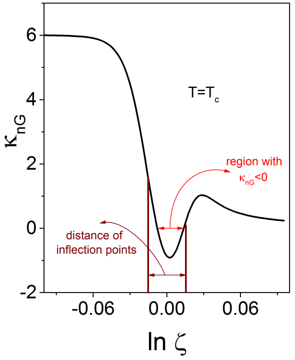
- Non-Gaussian kurtosis (κ_{nG}) of net protons is used in the search for the critical point (X. Luo and N. Xu, Nucl. Sci. Tech. 28, 112 (2017)):

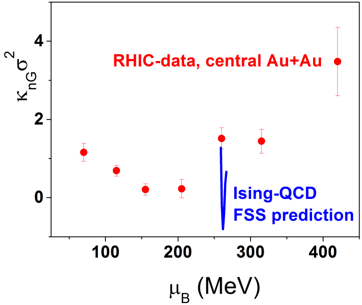
$$\kappa_{nG} = \frac{C_4 - 3C_2^2}{C_2^2} \quad ; \quad C_k = \langle (N - \langle N \rangle)^k \rangle \quad , \quad k = 2,\ 3, ..$$

- κ_{nG} possesses a non-monotonic behaviour, attaining a negative minimum when crossing the critical point (M. A. Stephanov, Phys. Rev. Lett. 107, 052301 (2011); N. G. Antoniou, F. K. Diakonos, N. Kalntis and A. Kanargias, Nucl. Phys. A 986, 167 (2019)).

- The non-Gaussian kurtosis can be calculated through the Ising-QCD partition function:

$$C_2 = \frac{\partial^2 \ln \mathcal{Z}}{\partial (\ln \zeta)^2} \quad ; \quad C_4 - 3C_2^2 = \frac{\partial^4 \ln \mathcal{Z}}{\partial (\ln \zeta)^4}$$





sharpness of minimum $\Rightarrow$ narrowness of critical region ($\Delta\mu \approx 5$ MeV)

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inaccessible by BES1 (RHIC-STAR)

sharp negative minimum for $\mu \approx \mu_c$

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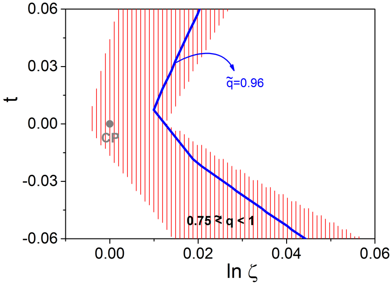
signature of critical point

Detailed study of the Ising-QCD partition function $\mathcal{Z}$



Scaling around the critical point: $\langle N_b \rangle \sim \Lambda^{\tilde{q}}$

$$\text{Critical region:} \quad \underbrace{\frac{3}{4}}_{\phi^4} < \tilde{q} < \underbrace{1}_{\text{conventional scaling}}$$

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Very narrow: $\Delta\mu = 5 - 7$ MeV for $T = T_c$

Critical fluctuations of proton density vs. measurements at CERN-SPS

- FSS property: $\langle N \rangle \sim \Lambda^{d_F/3}$

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fractal structure: $\langle n(\vec{x})n(\vec{x}')\rangle \sim |\vec{x}-\vec{x}'|^{d_F-3}$

at large scales $|\vec{x}-\vec{x}'| = O(\Lambda^{1/3})$

- singular density-density correlation in proton trans. momentum space: (N. G. Antoniou and F. K. Diakonos, J. Phys. G: Nucl. Part. Phys. 46, 035101 (2019))

$$\langle n(\vec{k})n(\vec{k}')\rangle \sim |\vec{k}-\vec{k}'|^{-\frac{2d_F}{3}} \quad (d_F = \frac{5}{2})$$

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detectable through intermittency in proton trans. momentum space

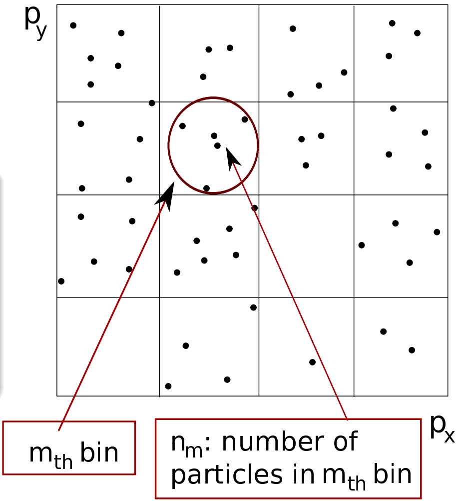
Intermittency in transverse momentum space:

Second factorial moment $F_2(M) \sim M^{2\phi_2}$ (critical index $\phi_2 = \frac{5}{6}$)

- Divide transverse momentum space in M^2 cells
- Calculate $F_2(M)$

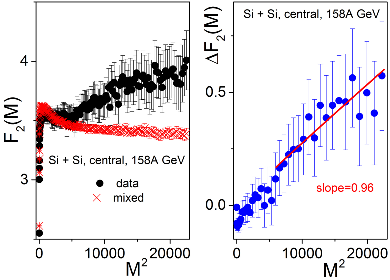
$$F_2(M) = \frac{\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i(n_i - 1) \rangle}{\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i \rangle^2}$$

$\langle .. \rangle \rightarrow$ event average



Measurements in A+A collisions at 158 GeV/c

NA49 experiment, CERN-SPS



T. Anticic *et al*,

EJP C 75, 587 (2015)

Intermittent behaviour for Si+Si system with $\tilde{q} = 0.96$ for $M^2 > 6000$!

No signal for the other systems: C+C, Pb+Pb

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Si+Si fireball (central collisions) freezes out within the critical region, close to the boundary

Constraints imposed by:

(a) finite-size scaling (b) Ising universality

(c) Lattice QCD (T_c) (d) recent measurements of freeze-out states

(e) intermittency measurements in Si+Si, Pb+Pb, C+C collisions at CERN-SPS (NA49)

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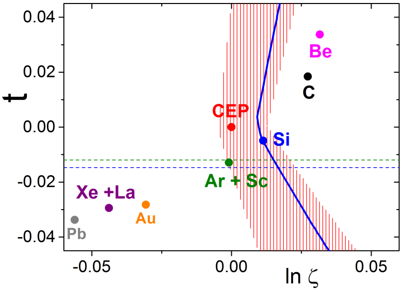
- locate various systems in the phase diagram
- find their distance from the critical region area
- only **Si+Si** and **Ar+Sc** freeze-out within (or close to the boundary of) the critical region



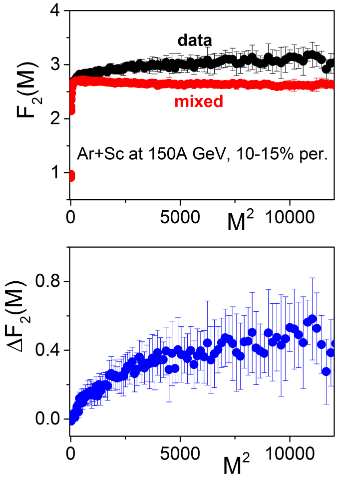
Predictions based on Ising-QCD partition function

- Estimate for the critical point: $T_c \approx 160$ MeV, $\mu_c \approx 260$ MeV.
- Intermittency effect in peripheral Ar+Sc collisions at 150 GeV/c.

Preliminary NA61 results show:

- a rather strong intermittency effect in Ar+Sc collision at 10-15% peripherality
- weak or no effect in more central collisions
- A slight temperature increase to enter into the critical region is needed!

Efforts to minimize statistical and systematic uncertainties!



N. Davis (NA61), CPOD 2018, Corfu

Summary

1. Ising-QCD universality $\Rightarrow$ Ising-QCD partition function

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$$\underbrace{\text{Description of QCD (net baryons) close to the critical point}}$$

satisfies all requirements of scaling and universality!

2. **Predictions** relevant to measurements in the search for the QCD critical point (not depending on details $\rightarrow$ **universality** and **FSS**):
   - Fractal structure at the critical point (FSS) with $d_F = \frac{5}{2}$ (measure of critical fluctuations) $\Rightarrow$ self-similar transverse momentum fluctuations (**intermittency**)
   - Critical region $\Rightarrow$ narrow along μ axis ($\Delta\mu \approx 5 - 7$ MeV)
   - Sharp minimum of kurtosis close to the critical point. Sharpness linked to the critical region narrowness!

3. BES1 measurement (RHIC-STAR) of net-proton kurtosis:

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broad minimum, not related to critical point!

Sharp minimum (critical) cannot be captured by BES1 measurements!

4. NA49 and NA61 measurements (CERN-SPS) of proton factorial moments in transverse momentum space:

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Indications of critical fluctuations (intermittency) in the freeze-out states of:

- Si+Si central collisions at 158A GeV/c (NA49)
- Ar+Sc collisions in peripherality 10-15% at 150A GeV/c (NA61)

5. Final news from NA61 by the end of the year (**Quark Matter 2019**)

High-precision measurements at RHIC expected early in 2020 (**BES2**)

THANK YOU!