The Effective Field Theory approach to torsional geometrical modified gravities

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CANTATA
Cosmology and Astrophysics Network for Theoretical Advances and Training Actions

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Goal

- We construct and apply the EFT approach to torsional modified gravity.

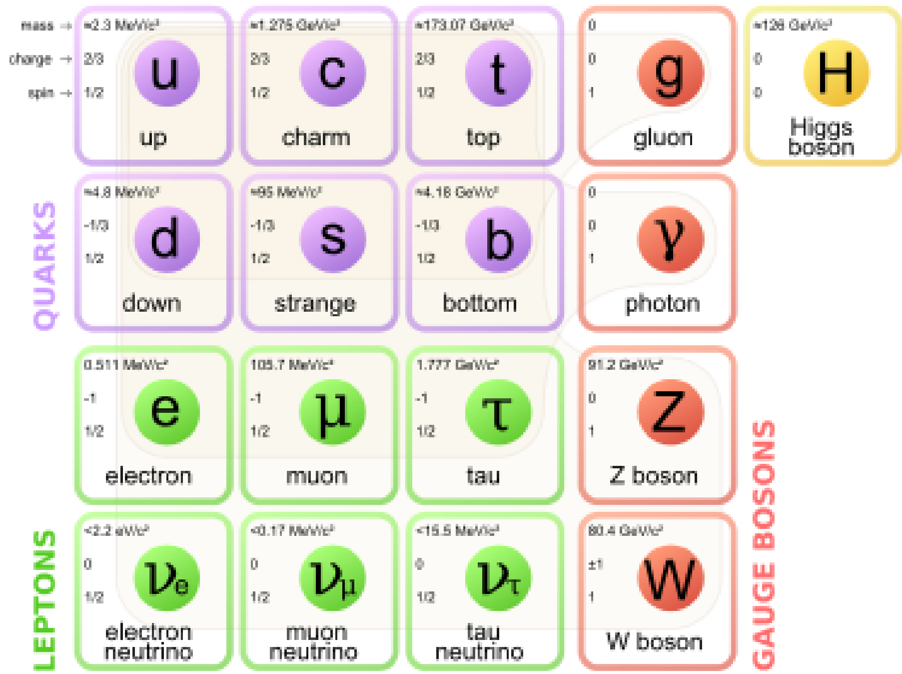
- High accuracy advancing Gravitational Waves observations offers a new window in testing Modified Gravity

Talk Plan

- 1) Introduction: Why Modified Gravity
- 2) Teleparallel Equivalent of General Relativity and f(T) modification
- 3) Non-minimal scalar-torsion theories
- 4) Teleparallel Equivalent of Gauss-Bonnet and f(T,T_G) modification
- 5) The EFT approach to torsional gravity
- 6) Background solutions
- 7) Gravitational Waves and observational signatures
- 8) Conclusions-Prospects

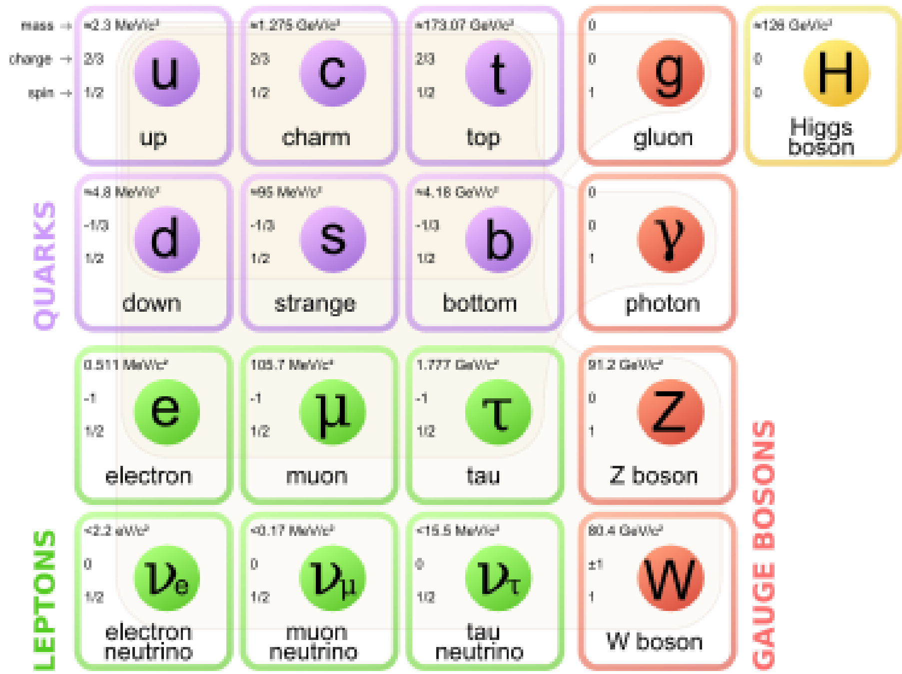
Why Modified Gravity?

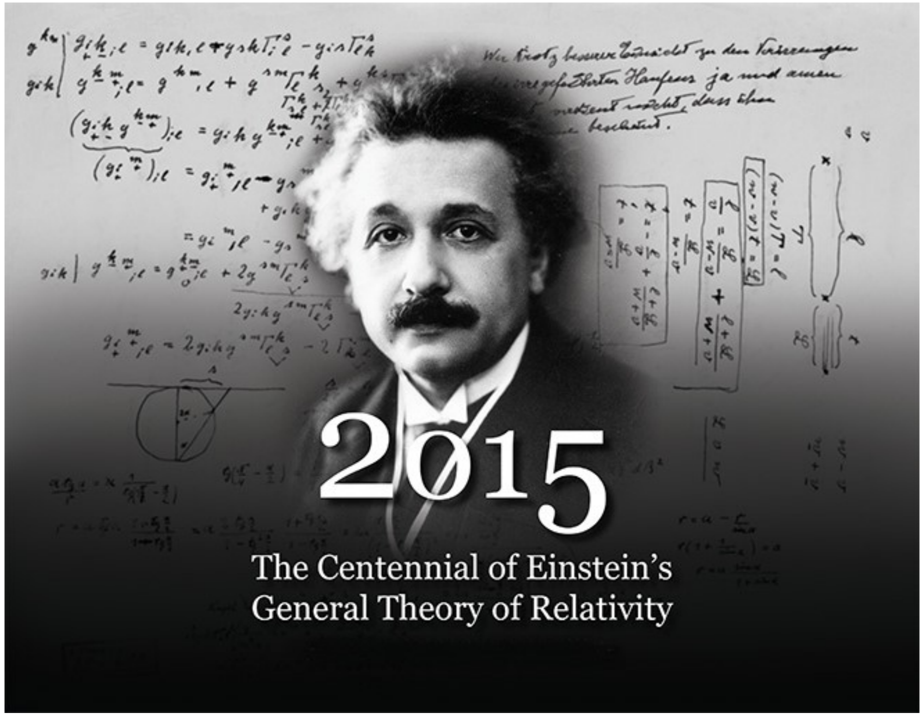
Knowledge of Physics: Standard Model



Why Modified Gravity?

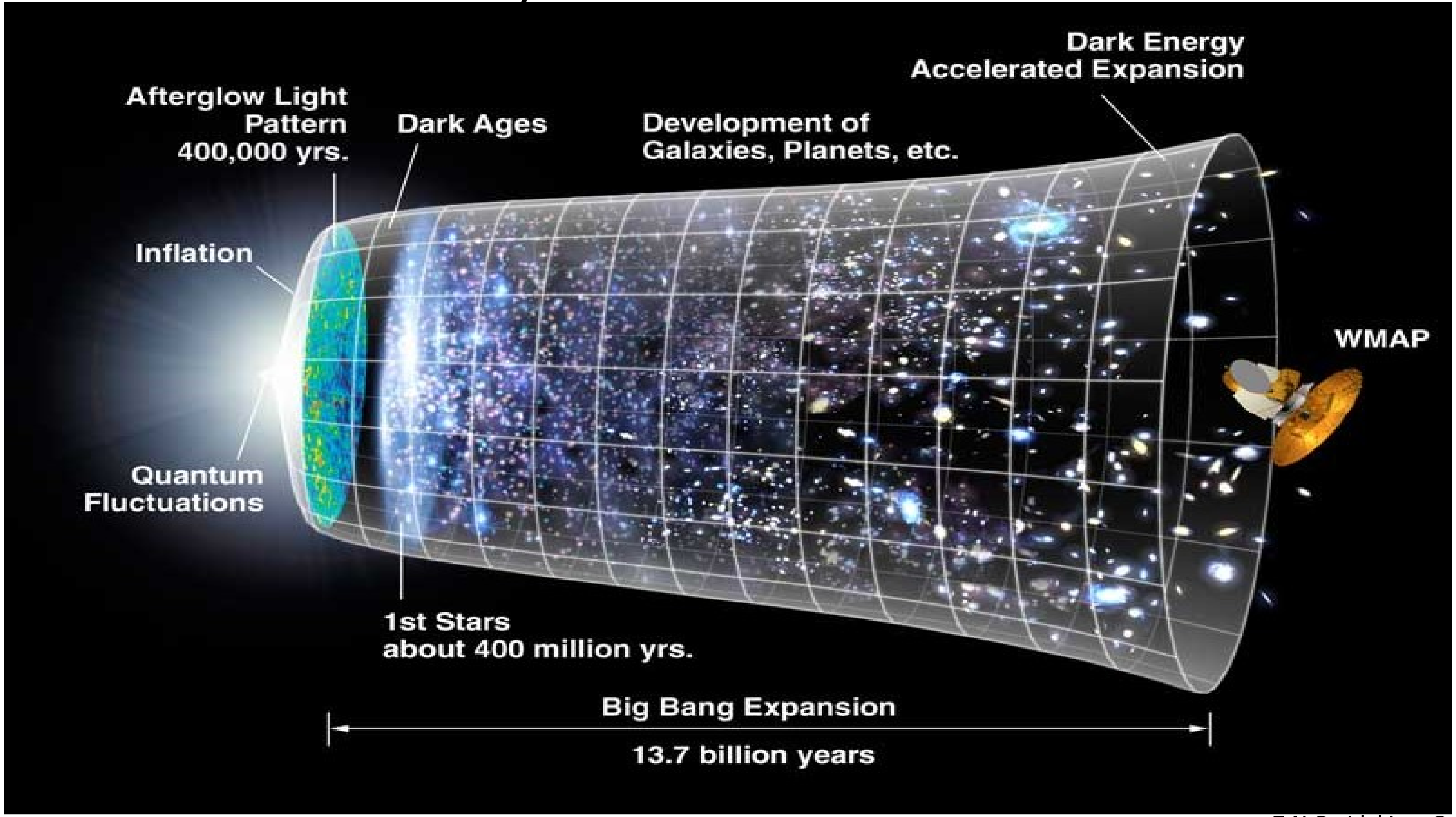
Knowledge of Physics: Standard Model + General Relativity





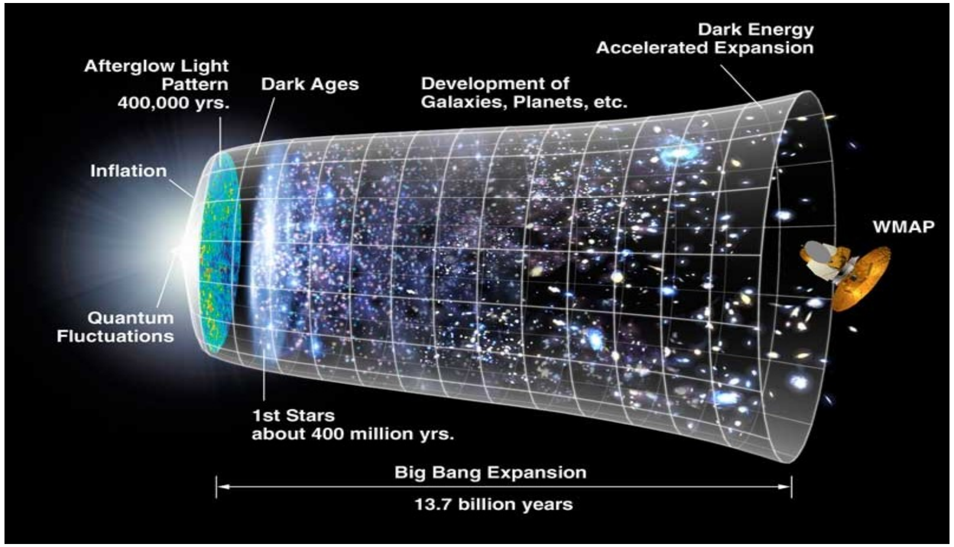
Why Modified Gravity?

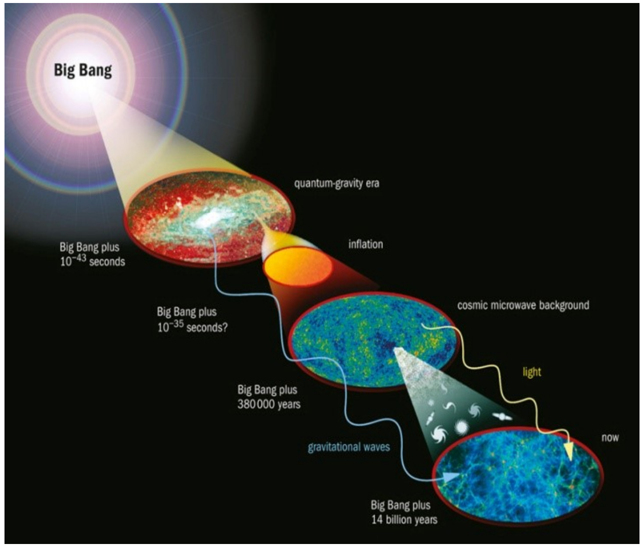
Universe History:



Why Modified Gravity?

So can our knowledge of Physics describes all these?





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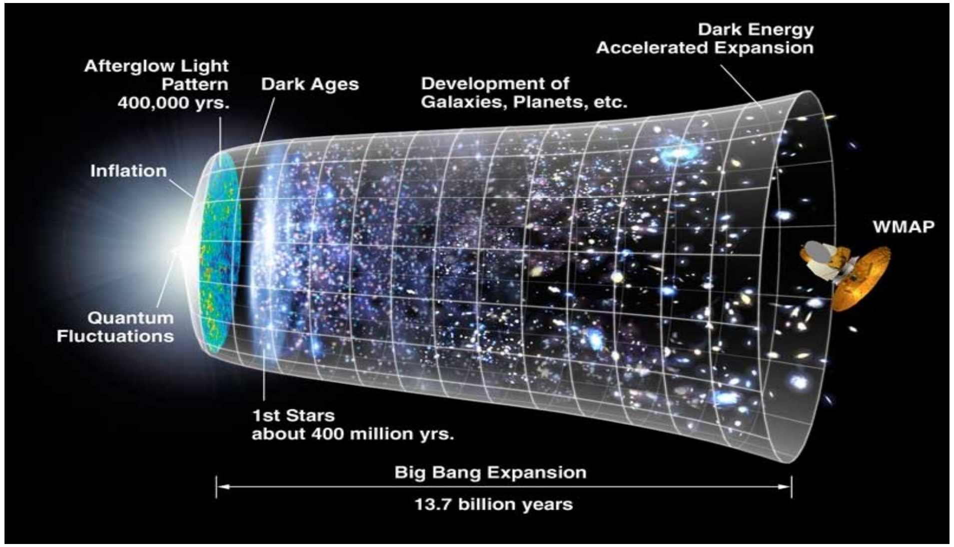
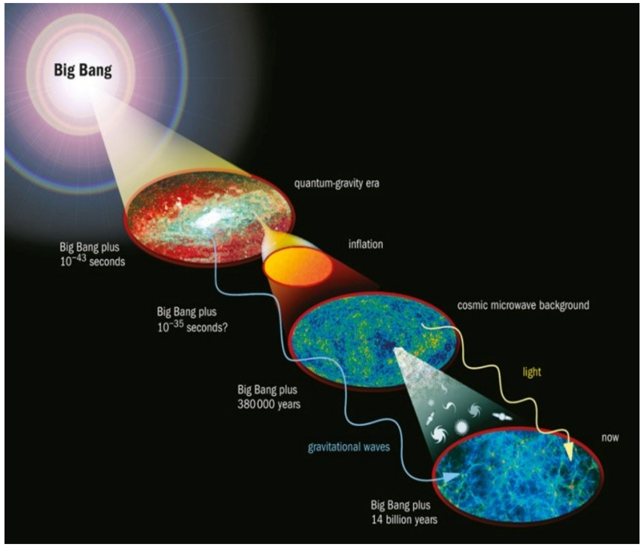




NO!

Why Modified Gravity?

- Einstein 1916: **General Relativity**:

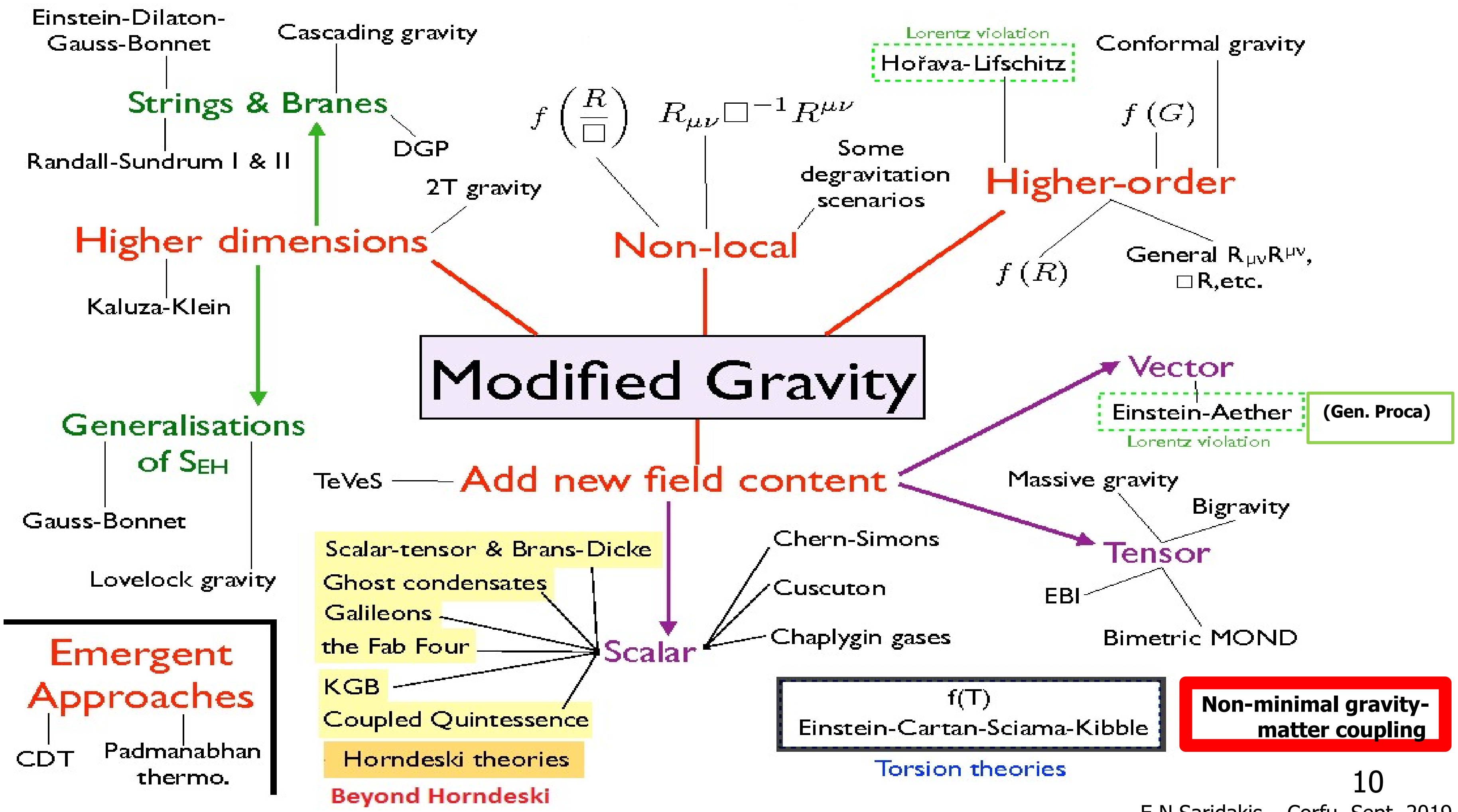
energy-momentum source of spacetime Curvature

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R-2\Lambda\right]+\int d^4x\ L_m\left(g_{\mu\nu},\psi\right)$$

$$\Rightarrow \quad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = 8\pi G\ T_{\mu\nu}$$

with $\quad T^{\mu\nu} \equiv \frac{2}{\sqrt{-g}}\frac{\delta L_m}{\delta g_{\mu\nu}}$

Modified Gravity



Introduction

- **Gauge Principle**: global symmetries replaced by local ones:

  The group generators give rise to the compensating fields

  It works perfect for the standard model of strong, weak and E/M interactions

  $$SU(3)\times SU(2)\times U(1)$$

- Can we apply this to gravity?

Introduction

- Formulating the gauge theory of gravity

  (mainly after 1960):
- Start from Special Relativity

$\Rightarrow$ Apply (Weyl-Yang-Mills) gauge principle to its Poincaré-group symmetries

$\Rightarrow$ Get Poinaré gauge theory:

Both curvature and torsion appear as field strengths

- Torsion is the field strength of the translational group

(Teleparallel and Einstein-Cartan theories are subcases of Poincaré theory)

[Blagojevic, Hehl, Imperial College Press, 2013]

Introduction

- One could extend the gravity gauge group (SUSY, conformal, scale, metric affine transformations)

  obtaining SUGRA, conformal, Weyl, metric affine

  gauge theories of gravity

- In all of them torsion is always related to the gauge structure.
- Thus, a possible way towards gravity quantization would need to bring torsion into gravity description.

Introduction

- 1998: Universe acceleration

$\Rightarrow$ Thousands of work in Modified Gravity

(f(R), Gauss-Bonnet, Lovelock, nonminimal scalar coupling,
nonminimal derivative coupling, Galileons, Hordenski, massive etc)
[Copeland, Sami, Tsujikawa Int.J.Mod.Phys.D15], [Capozziello, De Laurentis, Phys. Rept. 509]

- Almost all in the curvature-based formulation of gravity

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- Almost all in the curvature-based formulation of gravity

- So question: Can we modify gravity starting from its torsion-based formulation?

  torsion $\Rightarrow$ gauge ? $\Rightarrow$ quantization

  modification $\Rightarrow$ full theory ? $\Rightarrow$ quantization

Introduction

- Einstein 1916: **General Relativity**:

  energy-momentum source of spacetime Curvature

  Levi-Civita connection: Zero Torsion

- Einstein 1928: **Teleparallel Equivalent of GR:**

  Weitzenbock connection: Zero Curvature

[Cai, Capozziello, De Laurentis, Saridakis, Rept.Prog.Phys. 79]

Curvature and Torsion

- Vierbeins e^{μ}_{A} : four linearly independent fields in the tangent space

  $$g_{\mu\nu}(x) = \eta_{AB}\ e^{A}_{\mu}(x)\ e^{B}_{\nu}(x)$$

- Connection: ω_{ABC}
- Curvature tensor: $R^{A}_{B\mu\nu} = \omega^{A}_{B\nu,\mu} - \omega^{A}_{B\mu,\nu} + \omega^{A}_{C\mu}\omega^{C}_{B\nu} - \omega^{A}_{C\nu}\omega^{C}_{B\mu}$
- Torsion tensor: $T^{A}_{\mu\nu} = e^{A}_{\nu,\mu} - e^{A}_{\mu,\nu} + \omega^{A}_{B\mu}e^{B}_{\nu} - \omega^{A}_{B\nu}e^{B}_{\mu}$
- Levi-Civita connection and Contorsion tensor: $\omega_{ABC} = \Gamma_{ABC} + K_{ABC}$

  $$K_{ABC} = \frac{1}{2}\left(T_{CAB} - T_{BCA} - T_{ABC}\right) = -K_{BAC}$$

- Curvature and Torsion Scalars: $R = \overline{R} + T - 2\left(T^{\nu\mu}_{\nu}\right)_{;\mu}$

$$R = g^{\mu\nu}R_{\mu\nu} = g^{\mu\nu}R^{\rho}_{\mu\rho\nu}$$

$$T = \frac{1}{4}T^{\rho\mu\nu}T_{\rho\mu\nu} + \frac{1}{2}T^{\rho\mu\nu}T_{\nu\mu\rho} - \mathrm{T}^{\rho}_{\rho\mu}\mathrm{T}^{\nu\mu}_{\nu}$$

Teleparallel Equivalent of General Relativity (TEGR)

- Let's start from the simplest tosion-based gravity formulation, namely TEGR:
- Vierbeins e_A^μ : four linearly independent fields in the tangent space

$$g_{\mu\nu}(x) = \eta_{AB}\ e_\mu^A(x)\ e_\nu^B(x)$$

- Use curvature-less Weitzenböck connection instead of torsion-less Levi-Civita one: $\Gamma_{\nu\mu}^{\lambda\{W\}} = e_A^\lambda \partial_\mu e_\nu^A$
- Torsion tensor:

$$T_{\mu\nu}^\lambda = \Gamma_{\nu\mu}^{\lambda\{W\}} - \Gamma_{\mu\nu}^{\lambda\{W\}} = e_A^\lambda\left(\partial_\mu e_\nu^A - \partial_\nu e_\mu^A\right)$$

[Einstein 1928], [Pereira: Introduction to TG]

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- Lagrangian (imposing coordinate, Lorentz, parity invariance, and up to 2nd order in torsion tensor)

$$L \equiv T = \frac{1}{4}T^{\rho\mu\nu}T_{\rho\mu\nu} + \frac{1}{2}T^{\rho\mu\nu}T_{\nu\mu\rho} - T_{\rho\mu}^{\rho}T_{\nu}^{\nu\mu}$$

- **Completely equivalent** with GR at the level of equations

[Einstein 1928], [Hayaski,Shirafuji PRD 19], [Pereira: Introduction to TG]

f(T) Gravity and f(T) Cosmology

- f(T) Gravity: Simplest torsion-based modified gravity
- Generalize T to f(T) (inspired by f(R))

$$S=\frac{1}{16\pi G}\int d^4x\, e\,\left[T+f(T)\right]+S_m$$

[Ferraro, Fiorini PRD 78], [Bengochea, Ferraro PRD 79]
[Linder PRD 82]

- Equations of motion:

$$e^{-1}\partial_\mu\left(e e_A^{\rho} S_\rho^{\ \mu\nu}\right)\left(1+f_T\right)-e_A^{\lambda}T^{\rho}_{\ \mu\lambda}S_\rho^{\ \nu\mu}+e_A^{\rho}S_\rho^{\ \mu\nu}\partial_\mu(T)f_{TT}-\frac{1}{4}e_A^{\nu}[T+f(T)]=4\pi G e_A^{\rho}\mathrm{T}_\rho^{\ \nu\{\mathrm{EM}\}}$$

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- f(T) Cosmology: Apply in FRW geometry:

$$e_\mu^A = diag\,(1, a, a, a) \;\Rightarrow\; ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j$$ (not unique choice)

- Friedmann equations:

$$H^2 = \frac{8\pi G}{3}\rho_m - \frac{f(T)}{6} - 2 f_T H^2$$

$$\dot{H} = -\frac{4\pi G(\rho_m + p_m)}{1 + f_T - 12H^2 f_{TT}}$$

- Find easily

$$T = -6H^2$$

f(T) Cosmology: Background

- Effective Dark Energy sector:

$$\rho_{DE} = \frac{3}{8\pi G}\left[-\frac{f}{6}+\frac{T}{3}f_T\right]$$

$$w_{DE} = -\frac{f - Tf_T + 2T^2 f_{TT}}{[1+f_T+2Tf_{TT}][f-2Tf_T]}$$

[Linder PRD 82]

- Interesting cosmological behavior: Acceleration, Inflation etc
- At the background level indistinguishable from other dynamical DE models

Non-minimally coupled scalar-torsion theory

- In curvature-based gravity, apart from $R+f(R)$ one can use $R+\xi R\varphi^2$
- Let's do the same in torsion-based gravity:

$$S=\int d^4x\, e\left[\frac{T}{2\kappa^2}+\frac{1}{2}\left(\partial_\mu\varphi\partial^\mu\varphi+\xi T\varphi^2\right)-V(\varphi)+L_m\right]$$

[Geng, Lee, Saridakis, Wu PLB 704]

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- Friedmann equations in FRW universe:

$$H^2=\frac{\kappa^2}{3}(\rho_m+\rho_{DE})$$

$$\dot{H}=-\frac{\kappa^2}{2}(\rho_m+p_m+\rho_{DE}+p_{DE})$$

with effective Dark Energy sector: $\rho_{DE}=\frac{\dot{\varphi}^2}{2}+V(\varphi)-3\xi H^2\varphi^2$

$$p_{DE}=\frac{\dot{\varphi}^2}{2}-V(\varphi)+4\xi H\varphi\dot{\varphi}+\xi\left(3H^2+2\dot{H}\right)\varphi^2$$

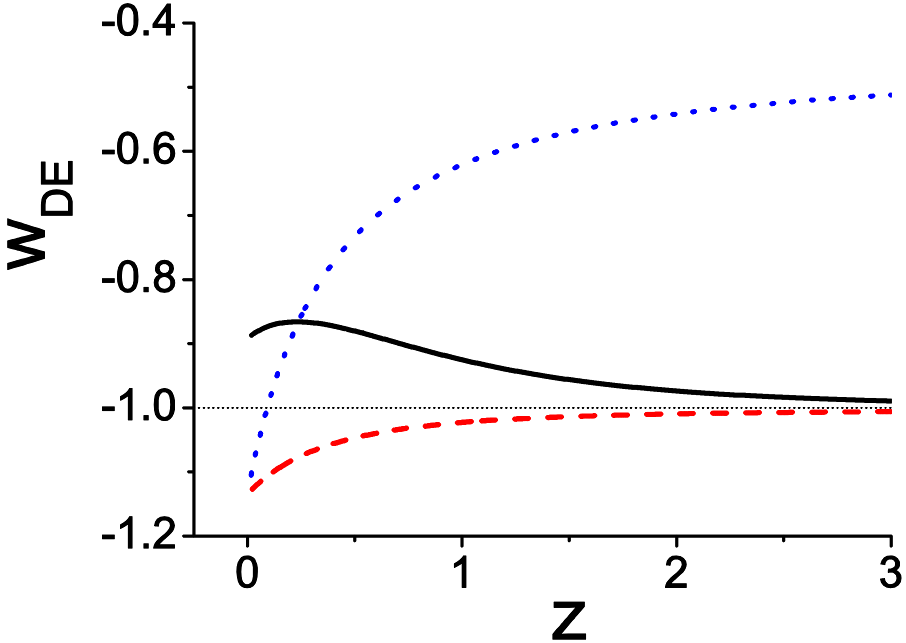
- **Different** than non-minimal quintessence!

  (no conformal transformation in the present case)

[Geng, Lee, Saridakis,Wu PLB 704]

Non-minimally coupled scalar-torsion theory

- Main advantage: Dark Energy may lie in the phantom regime or/and experience the phantom-divide crossing
- Teleparallel Dark Energy:



[Geng, Lee, Saridakis, Wu PLB 704]

Non-minimally matter-torsion coupled theory

- In curvature-based gravity, one can use $f(R)L_m$ coupling
- Let's do the same in torsion-based gravity:

$$S = \frac{1}{2\kappa^2}\int d^4x\, e\left\{T + f_1(T) + \left[1 + \lambda\, f_2(T)\right] L_m\right\}$$

[Harko, Lobo, Otalora, Saridakis, PRD 89]

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with effective Dark Energy sector: $\rho_{DE}=-\frac{1}{2\kappa^2}\left(f_1+12H^2f_1'\right)+\lambda\rho_m\left(f_2+12H^2f_2'\right)$

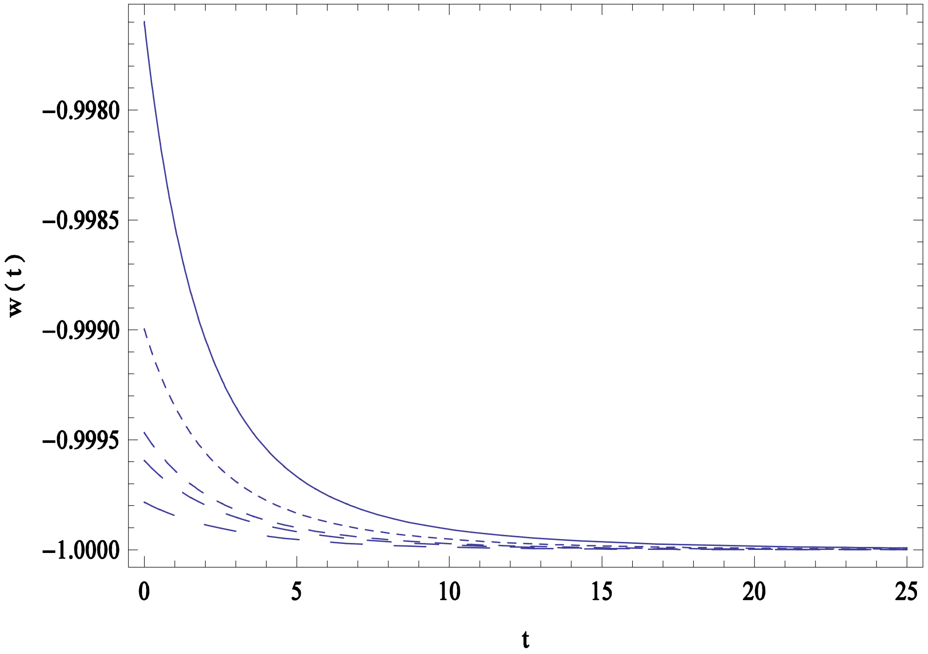
$$p_{DE}=(\rho_m+p_m)\left[\frac{1+\lambda\left(f_2+12H^2f_2'\right)}{1+f_1'-12H^2f_1''-2\kappa^2\lambda\rho_m\left(f_2'-12H^2f_2''\right)}\right]+\frac{\lambda\left(f_1+12H^2f_1'\right)}{2\kappa^2}-\lambda\rho_m\left(f_2+12H^2f_2'\right)$$

- **Different** than non-minimal matter-curvature coupled theory

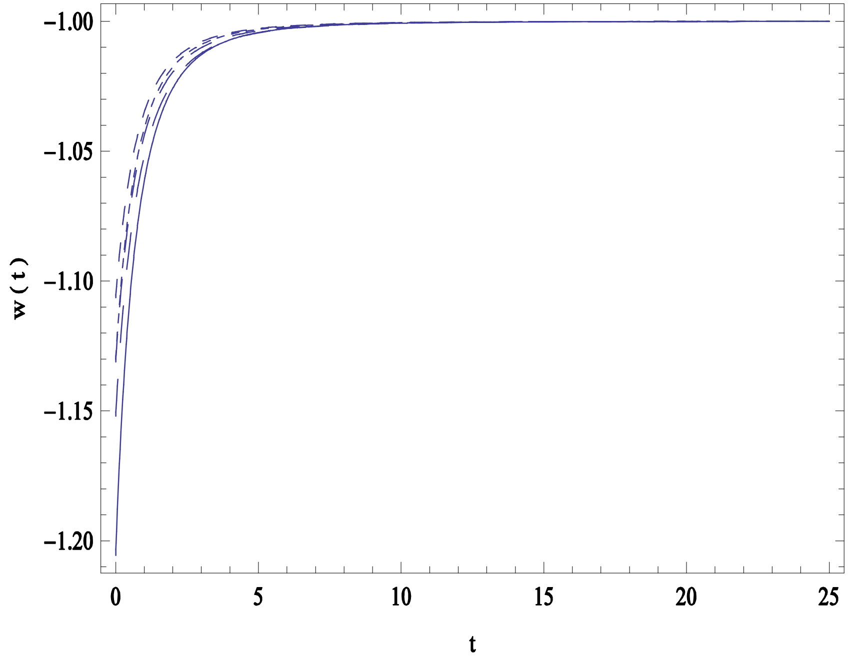
[Harko, Lobo, Otalora, Saridakis, PRD 89]

Non-minimally matter-torsion coupled theory

- Interesting phenomenology



$$f_1(T) = -\Lambda + \alpha_1 T^2, \quad f_2(T) = \beta_1 T^2$$



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Teleparallel Equivalent of Gauss-Bonnet and f(T,T_G) gravity

- In curvature-based gravity, one can use higher-order invariants like the Gauss-Bonnet one $G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\kappa\lambda}R^{\mu\nu\kappa\lambda}$
- Let's do the same in torsion-based gravity:
- Similar to $e\overline{R} = -eT + 2\left(eT_{\nu}^{\nu\mu}\right)_{,\mu}$ we construct $e\overline{G} = eT_G + tot.diverg$ with

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$$T_G = \left(K^{a_1}_{\ ea_2}K^{ea_2}_{\ \ b}K^{a_3}_{\ fc}K^{fa_4}_{\ \ d} - 2K^{a_1a_2}_{\ \ a}K^{a_3}_{\ eb}K^{e}_{\ fc}K^{fa_4}_{\ \ d} + 2K^{a_1a_2}_{\ \ a}K^{a_3}_{\ eb}K^{ea_4}_{\ \ f}K^{f}_{\ cd} + +2K^{a_1a_2}_{\ \ a}K^{a_3}_{\ eb}K^{ea_4}_{\ \ f}K^{f}_{\ c,d}\right)\delta^{abcd}_{a_1a_2a_3a_4}$$

- $f(\mathrm{T},T_G)$ gravity:

$$S = \frac{1}{2\kappa^2}\int d^4x\, e\, \{T + f(T,T_G)\} + S_m$$

[Kofinas, Saridakis, PRD 90a]
[Kofinas, Saridakis, PRD 90b]
[Kofinas, Leon, Saridakis, CQG 31]

- **Different** from $f(R,G)$ and $f(T)$ gravities

Teleparallel Equivalent of Gauss-Bonnet and f(T,T_G) gravity

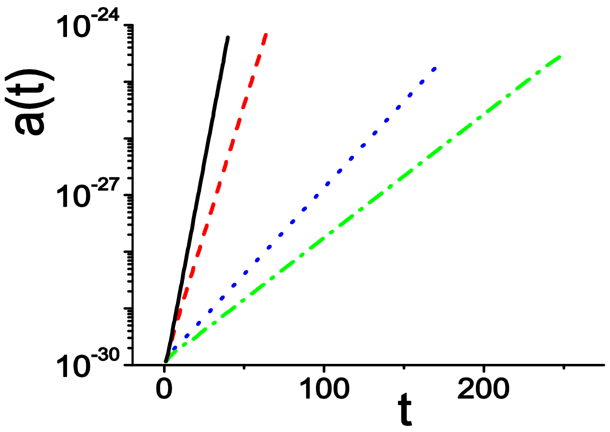
- Cosmological application:

$$\rho_{DE} = -\frac{1}{2\kappa^2}\left[f - 12H^2 f_T - T_G f_{T_G} + 24H^3 \dot{f}_{T_G}\right]$$

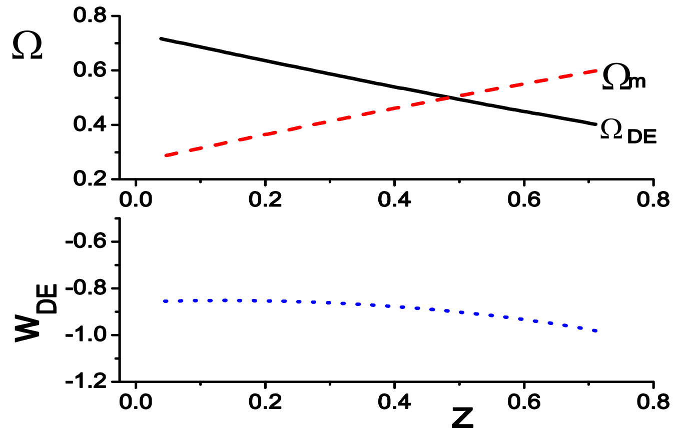
$$p_{DE} = \frac{1}{2\kappa^2}\left[f - 4\left(\dot{H} + 3H^2\right)f_T - 4H\dot{f}_T - T_G f_{T_G} + \frac{2}{3H}T_G \dot{f}_{T_G} + 8H^2 \ddot{f}_{T_G}\right]$$

$$T = 6H^2$$

$$T_G = 24H^2\left(\dot{H} + H^2\right)$$



$$f(T,T_G) = \alpha_1 T^2 + \alpha_2 T\sqrt{|T_G|}$$



$$f(T,T_G) = \beta_1\sqrt{T^2 + \beta_2 T_G}$$

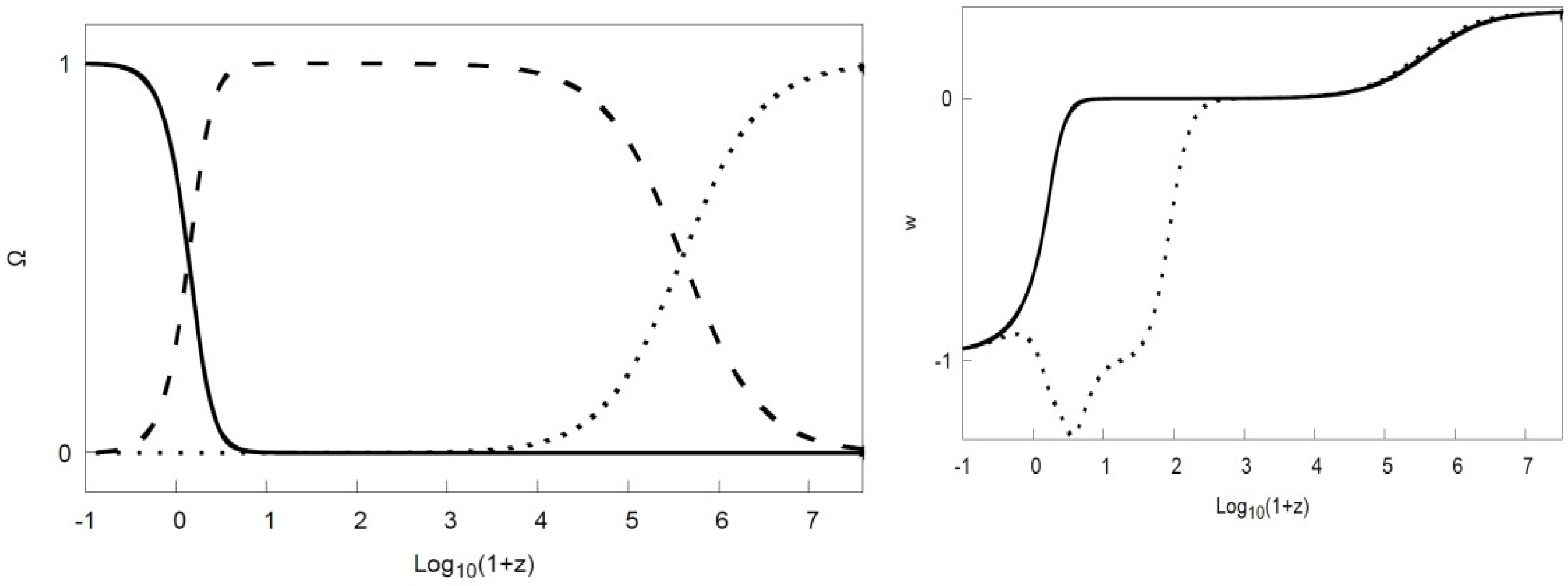
[Kofinas, Saridakis, PRD 90a]

[Kofinas, Saridakis, PRD 90b]

[Kofinas, Leon, Saridakis, CQG 31]

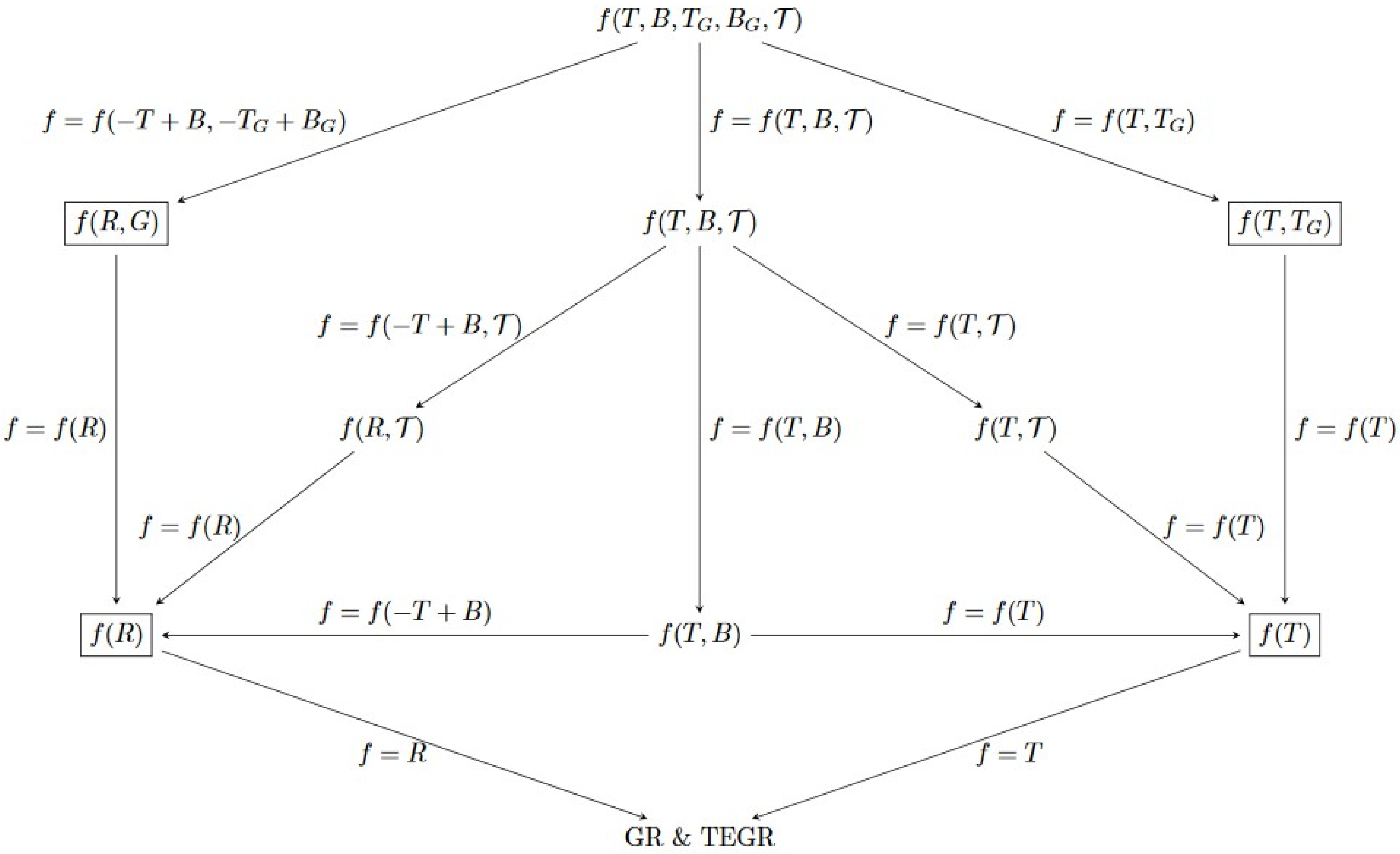
Torsional Gravity with higher derivatives

$$S = \frac{1}{2\kappa^2}\int d^4x\, e\, F\big(T,(\nabla T)^2,\Diamond T\big) + S_m(e^A_\mu,\Psi_m)$$



[Otalora, Saridakis, PRD 94]

Torsional Modified Gravity



The Effective Field Theory (EFT) approach

- The EFT approach allows to ignore the details of the underlying theory and write an action for the perturbations around a time-dependent background solution.
- One can systematically analyze the perturbations separately from the background evolution. [Arkani-Hamed, Cheng JHEP0405 (2004)]

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$$S=\int d^4x\Big\{\sqrt{-g}\Big[\frac{M_P^2}{2}\Psi(t)R-\Lambda(t)-b(t)g^{00}$$ <- background

$$+M_2^4(\delta g^{00})^2-\bar{m}_1^3\delta g^{00}\delta K-\bar{M}_2^2\delta K^2-\bar{M}_3^2\delta K_\mu^\nu\delta K_\nu^\mu$$ <- linear evolution of perturbations

$$+m_2^2h^{\mu\nu}\partial_\mu g^{00}\partial_\nu g^{00}+\lambda_1\delta R^2+\lambda_2\delta R_{\mu\nu}\delta R^{\mu\nu}+\mu_1^2\delta g^{00}\delta R\Big]$$ <- linear evolution of perturbations

$$+\gamma_1 C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma}+\gamma_2\epsilon^{\mu\nu\rho\sigma}C_{\mu\nu}{}^{\kappa\lambda}C_{\rho\sigma\kappa\lambda}$$ <- linear evolution of perturbations

$$+\sqrt{-g}\Big[\frac{M_3^4}{3}(\delta g^{00})^3-\bar{m}_2^3(\delta g^{00})^2\delta K+\ldots\Big]\Big\},$$ <- 2^{nd}-order evolution of perturbations

The functions Ψ(t), Λ(t), b(t), are determined by the background solution

[Gubitosi, Piazza, Vernizzi, JCAP1302]

The (EFT) approach to torsional gravity

- Application of the EFT approach to torsional gravity leads to include terms:
- i) Invariant under 4D diffeomorphisms: e.g. R,T multiplied by functions of time.
- ii) Invariant under spatial diffeomorphisms: e.g. $g^{00},\ R^{00}$ and T^{0}
- ii) Invariant under spatial diffeomorphisms: e.g. ${}^{(3)}R_{\mu\nu\rho\sigma}$, ${}^{(3)}T^{\rho}{}_{\mu\nu}$, $K_{\mu\nu}$, $\hat{K}_{\mu\nu}$

  the extrinsic torsion is defined as

$$\hat{K}_{\mu\nu} \equiv h^{\sigma}_{\mu}\hat{\nabla}_{\sigma}n_{\nu} = K_{\mu\nu} - \mathcal{K}^{\lambda}{}_{\nu\mu}n_{\lambda} + n_{\mu}\frac{1}{g^{00}}T^{00}{}_{\nu}\ ;$$

with n_{μ} the orthogonal to t=cont. surfaces unitary vector $n_{\mu} = \frac{\delta^{0}_{\mu}}{\sqrt{-g^{00}}}$

[Cai, Li, Saridakis, Xue, PRD 97], [Li, Cai, Cai, Saridakis, JCAP1810]

[Yan, Zhang, Chen, Zhang, Cai, Saridakis, 1909.06388]

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Using the projection operator h^{μ}_{ν} we can express ${}^{(3)}R_{\mu\nu\rho\sigma} = h^{\alpha}_{\mu}h^{\beta}_{\nu}h^{\gamma}_{\rho}h^{\delta}_{\sigma}R_{\alpha\beta\gamma\delta} - K_{\mu\rho}K_{\nu\sigma} + K_{\nu\rho}K_{\mu\sigma}$

$$h^{d}_{a}h^{c}_{b}h^{f}_{e}T^{e}{}_{dc} = {}^{(3)}T^{f}{}_{ab}$$

[Cai, Li, Saridakis, Xue, PRD 97], [Li, Cai, Cai, Saridakis, JCAP1810]

[Yan, Zhang, Chen, Zhang, Cai, Saridakis, 1909.06388]

The (EFT) approach to torsional gravity

- We perturb the previous tensors, and we finally obtain:

$$
\begin{aligned}
R^{(0)}_{\mu\nu\rho\sigma} &= f_1(t)g_{\mu\rho}g_{\nu\sigma} + f_2(t)g_{\mu\rho}n_\nu n_\sigma + f_3(t)g_{\mu\sigma}g_{\nu\rho} \\
&\quad + f_4(t)g_{\mu\sigma}n_\nu n_\rho + f_5(t)g_{\nu\sigma}n_\mu n_\rho \\
&\quad + f_6(t)g_{\nu\rho}n_\mu n_\sigma, \\
T^{(0)}_{\rho\mu\nu} &= g_1(t)g_{\rho\nu}n_\mu + g_2(t)g_{\rho\mu}n_\nu, \\
K^{(0)}_{\mu\nu} &= f_7(t)g_{\mu\nu} + f_8(t)n_\mu n_\nu, \\
\hat{K}^{(0)}_{\mu\nu} &= 0 .
\end{aligned}
$$

where the time-dependent functions are determined by the background solution.

[Cai, Li, Saridakis, Xue, PRD 97], [Li, Cai, Cai, Saridakis, JCAP1810]
[Yan, Zhang, Chen, Zhang, Cai, Saridakis, 1909.06388]

The (EFT) approach to torsional gravity

- Finally, the EFT action of torsional gravity becomes:

$$S = \int d^4x\sqrt{-g}\Big[\frac{M_P^2}{2}\Psi(t)R - \Lambda(t) - b(t)g^{00} + \frac{M_P^2}{2}d(t)T^0\Big]$$
$$+S^{(2)} \ ,$$

- The perturbation part contains:

  i) Terms present in curvature EFT action

  ii) Pure torsion terms such as δT^2 , $\delta T^0\delta T^0$ and $\delta T^{\rho\mu\nu}\delta T_{\rho\mu\nu}$

  iii) Terms that mix curvature and torsion, such as $\delta T\delta R$, $\delta g^{00}\delta T$, $\delta g^{00}\delta T^0$ and $\delta K\delta T^0$

[Cai, Li, Saridakis, Xue, PRD 97], [Li, Cai, Cai, Saridakis, JCAP1810]

[Yan, Zhang, Chen, Zhang, Cai, Saridakis, 1909.06388]

The (EFT) approach to f(T) gravity: Background

- For the case of f(T) gravity, at the background level, we have:

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \big[- f_T(T^{(0)}) R + 2 \dot{f}_T(T^{(0)}) T^{(0)} - T^{(0)} f_T(T^{(0)}) + f(T^{(0)}) \big]$$

where by comparison:

$$\Psi(t) = -f_T(T^{(0)}) \ ,$$
$$\Lambda(t) = \frac{M_P^2}{2} \left[T^{(0)} f_T(T^{(0)}) - f(T^{(0)}) \right] \ ,$$
$$d(t) = -2 \dot{f}_T(T^{(0)}) \ ,$$
$$b(t) = 0 \ .$$

[Li, Cai, Cai, Saridakis, JCAP1810]

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$$d(t) = -2\dot{f}_T(T^{(0)}) \, ,$$
$$b(t) = 0 \, .$$

- Performing variation we obtain the background equations of motion (Friedmann Eqs):

$$b(t) = M_P^2 \Psi \left(-\dot{H} - \frac{\ddot{\Psi}}{2\Psi} + \frac{H\dot{\Psi}}{2\Psi} - \frac{\dot{d}}{4\Psi} + \frac{3Hd}{4\Psi} \right) - \frac{1}{2}(\rho_m + p_m),$$

$$\Lambda(t) = M_P^2 \Psi \left(3H^2 + \frac{5H\dot{\Psi}}{2\Psi} + \dot{H} + \frac{\ddot{\Psi}}{2\Psi} + \frac{\dot{d}}{4\Psi} + \frac{3Hd}{4\Psi} \right) - \frac{1}{2}(\rho_m - p_m),$$

[Li, Cai, Cai, Saridakis, JCAP1810]

The (EFT) approach to f(T) gravity: Background

- These can be written as:

$$H^2 = \frac{1}{3M_P^2}(\rho_m + \rho_{DE}^{\text{eff}}),$$

$$\dot{H} = -\frac{1}{2M_P^2}(\rho_m + \rho_{DE}^{\text{eff}} + p_m + p_{DE}^{\text{eff}})$$

with

$$\rho_{DE}^{\text{eff}} = b + \Lambda - 3M_P^2\left[H\dot{\Psi} + \frac{dH}{2} + H^2(\Psi - 1)\right]$$

$$p_{DE}^{\text{eff}} = b - \Lambda + M_P^2\Big[\ddot{\Psi} + 2H\dot{\Psi} + \frac{\dot{d}}{2} + (H^2 + 2\dot{H})(\Psi - 1)\Big].$$

and thus:

$$\rho_{DE}^{\text{eff}} = \frac{M_P^2}{2}\left[T^{(0)} - f(T^{(0)}) + 2T^{(0)} f_T(T^{(0)})\right]$$

$$p_{DE}^{\text{eff}} = -\frac{M_P^2}{2}\Big[4\dot{H}(1 + f_T(T^{(0)}) + 2T^{(0)} f_{TT}(T^{(0)})) - f(T^{(0)}) + T^{(0)} + 2T^{(0)} f_T(T^{(0)})\Big]$$

- The same equations with standard approach!

[Li, Cai, Cai, Saridakis, JCAP1810]

The (EFT) approach to f(T) gravity: Tensor Perturbations

- For tensor perturbations: $g_{00} = -1\ , \quad g_{0i} = 0\ ,$ $\qquad g_{ij} = a^2\big(\delta_{ij} + h_{ij} + \frac{1}{2} h_{ik} h_{kj}\big)$ i.e.

$$\bar{e}^0_\mu = \delta^0_\mu\ ,$$
$$\bar{e}^a_\mu = a\delta^a_\mu + \frac{a}{2}\delta^i_\mu \delta^{aj} h_{ij} + \frac{a}{8}\delta^i_\mu \delta^{ja} h_{ik} h_{kj}\ ,$$
$$\bar{e}^\mu_0 = \delta^\mu_0\ ,$$
$$\bar{e}^\mu_a = \frac{1}{a}\delta^\mu_a - \frac{1}{2a}\delta^{\mu i}\delta^j_a h_{ij} + \frac{1}{8a}\delta^{i\mu}\delta^j_a h_{ik} h_{kj}$$

- We obtain:

$$^{(3)}R \approx -\frac{1}{4} a^{-2} (\partial_i h_{kl} \partial_i h_{kl})\ ,$$
$$K^{ij} K_{ij} \approx 3H^2 + \frac{1}{4}\dot{h}_{ij}\dot{h}_{ij}\ ,$$
$$K \approx 3H\ ,$$
$$T = T^{(0)} + O(h^2) = 6H^2 + O(h^2)$$

- And finally:

$$S = \frac{M_P^2}{2}\int d^4x\sqrt{-g}\Big[\frac{f_T}{4}\big(a^{-2}\vec{\nabla} h_{ij}\cdot\vec{\nabla} h_{ij} - \dot{h}_{ij}\dot{h}_{ij}\big) + 6H^2 f_T - 12H\dot{f}_T - T^{(0)} f_T + f(T^{(0)})\Big]$$

[Cai, Li, Saridakis, Xue, PRD 97]

The (EFT) approach to f(T) gravity: Gravitational Waves

- Varying the action and going to Fourier space we get the equation for GWs:

$$\ddot{h}_{ij} + 3H(1-\beta_T)\dot{h}_{ij} + \frac{k^2}{a^2}h_{ij} = 0$$

with $\beta_T \equiv -\frac{\dot{f}_T}{3Hf_T}$

- An immediate result: The speed of GWs is equal to the speed of light!
- GW170817 constraints that

$$|c_g/c - 1| \leq 4.5 \times 10^{-16}$$

are trivially satisfied.

[Cai, Li, Saridakis, Xue, PRD 97]

The (EFT) approach to f(T) gravity: Gravitational Waves

- We can express: $\beta_T = \frac{d\ln f_T}{d\ln T}(1+w_{tot})$

- In GR and TEGR β_T is zero. Thus, if a non-zero β_T is measured in future observations, it could be the smoking gun of modified gravity.

- Very important since f(T) gravity has the same polarization modes with GR.
- The effect of f(T) gravity on GWs comes through its effect on the background solutions itself, since at linear perturbation order f(T) gravity is effectively TEGR.

[Cai, Li, Saridakis, Xue, PRD 97]

The (EFT) approach to f(T) gravity: Scalar Perturbations

- For scalar perturbations:

$$g_{00} = -1 - 2\phi\ ,$$
$$g_{0i} = 0\ ,$$
$$g_{ij} = a^2[(1-2\psi)\delta_{ij} + \partial_i\partial_j F]$$

i.e

$$e^0_\mu = \delta^0_\mu + \delta^0_\mu\phi + a\delta^i_\mu\partial_i\chi\ ,$$
$$e^a_\mu = a\delta^i_\mu\delta^a_i + \delta^0_\mu\delta^a_i\partial^i\mathcal{E} + a\delta^i_\mu\delta^a_j\left[\epsilon_{ijk}\partial_k\sigma - \psi\delta_{ij} + \frac{1}{2}\partial_i\partial_j F\right]$$

- So $$T^0 = g^{0\mu}T^\nu{}_{\mu\nu} = -3H + 6H\phi + 3\dot\psi - 6H\phi^2 - 6\dot\psi\phi + \frac{1}{a}\partial_i\partial_i\chi - \frac{1}{2a}\partial_i\phi\partial_i\chi - \frac{3}{2a}\phi\partial_i\partial_i\chi - \frac{1}{2a}\partial_i\psi\partial_i\chi + \frac{1}{2a}\psi\partial_i\partial_i\chi$$

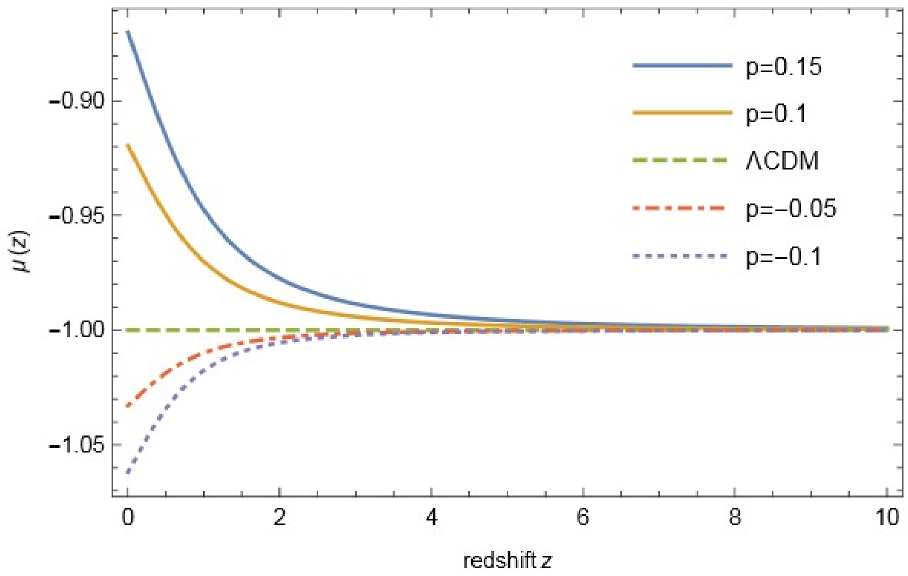
- Thus:

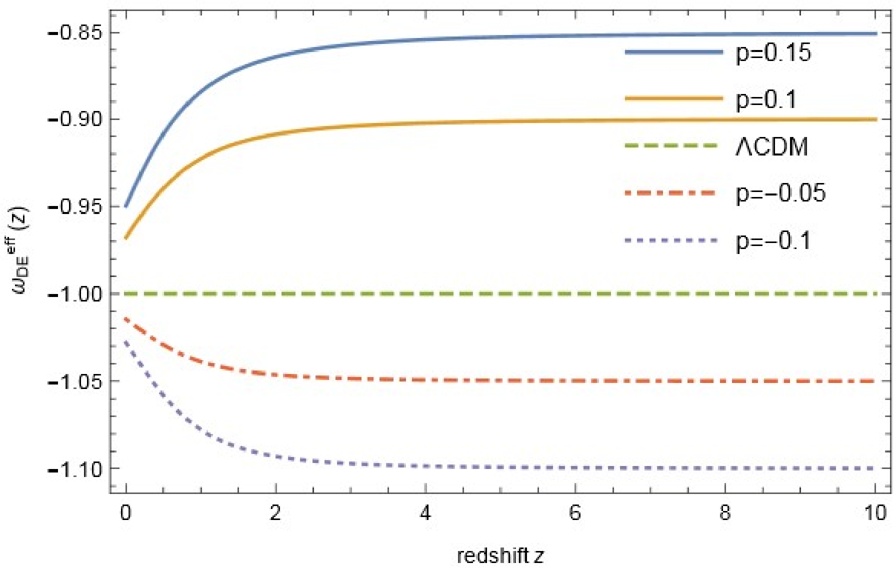
$$S = \int d^4x\Big[\frac{M_P^2}{2}\left(-2af_T\partial_i\psi\partial_i\psi + 4af_T\partial_i\phi\partial_i\psi + 4a^2\dot f_T\partial_i\psi\partial_i\chi + 4\dot f_T a^2 H\partial_i\pi\partial_i\chi\right) + a^3M^2\pi^2 - a^3\phi\delta\rho_m\Big]$$

[Li, Cai, Cai, Saridakis, JCAP1810]

The (EFT) approach to f(T) gravity: Tensor Perturbations

- Finally: $\mu(z) = \dfrac{2M_P^2 k^2 \phi (1+z)^2}{\delta \rho_m}$ with $\mu \equiv \dfrac{1}{f_T}$





$$f(T) = -T + \alpha T^p$$

$$\alpha = (6H_0^2)^{1-p} \frac{1-\Omega_{m0}}{2p-1}$$

[Li, Cai, Cai, Saridakis, JCAP1810]

Conclusions

- i) Many cosmological and theoretical arguments favor modified gravity.
- ii) Can we modify gravity based in its torsion formulation?
- iii) Simplest choice: f(T) gravity, i.e extension of TEGR
- iv) f(T) cosmology: Interesting phenomenology. Signatures in growth structure.
- v) Non-minimal coupled scalar-torsion theory: Quintessence, phantom or crossing behavior.
- vi) EFT approach allows for a systematic study of perturbations
- vii) Observational signatures in the dispersion relation of GWs
- viii) No further polarization modes.

Outlook

- Many subjects are open. Amongst them:
- i) Examine higher-order perturbations to look for further polarizations.
- ii) Extend the analysis to other torsional modified gravity.
- iii) Try to break the various degeneracies and find a signature of this particular class of modified gravity
- vi) Convince people to work on the subject!

THANK YOU!

Covariant formulation of f(T) gravity

- In standard f(T) gravity spin connection is set to zero.
- However vierbein transformations must be accompanied by connection ones:

$$e'^{A}_{\mu} = \Lambda^{A}_{B} e^{B}_{\mu}$$

$$\omega'^{A}_{B\mu} = \Lambda^{A}_{C} \omega^{C}_{D\mu} \Lambda^{D}_{B} + \Lambda^{A}_{C} \partial_{\mu} \Lambda^{C}_{B}$$

[Krssak, Pereira EPJC 75]

- Example: FRW geometry

$e^{A}_{\mu} = diag\,(1, a, a, a)$ or $e^{A}_{\mu} = diag\,(1, a, ra, ra\sin\theta)$

$\omega^{A}_{B\mu} = 0$ $\qquad$ $\omega^{1}_{2\theta} = -1,\ \omega^{1}_{3\phi} = -\sin\theta,\ \omega^{2}_{3\phi} = -\cos\theta$

- On the other hand, if one assumes/imposes $\omega'^{A}_{B\mu} = 0$ then only "peculiar" forms of vierbeins will be allowed.

- $\Longrightarrow$ Lorentz invariance has been restored in f(T) gravity

[Krssak, Saridakis CQG 33]