Polygonal bounces

and false vacuum decay

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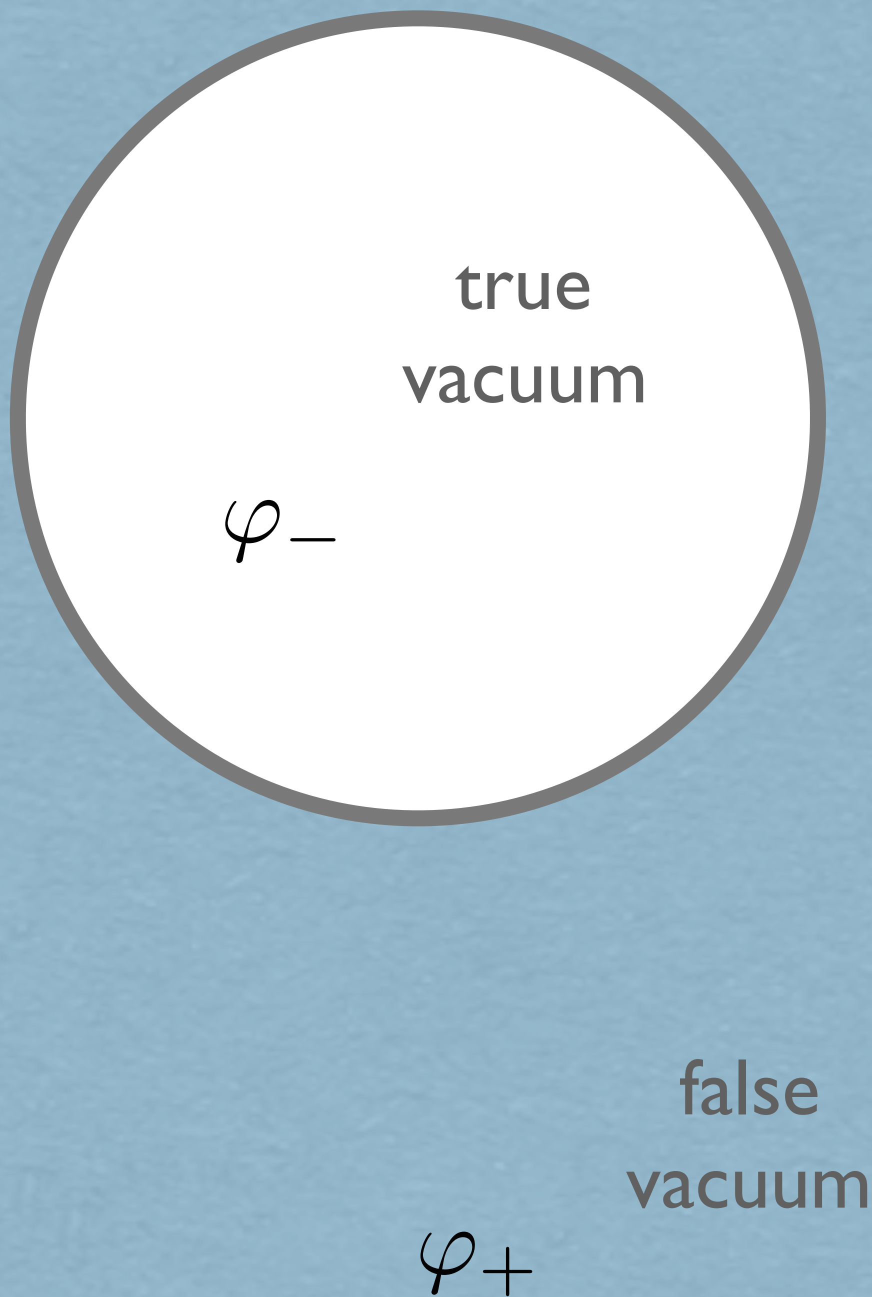
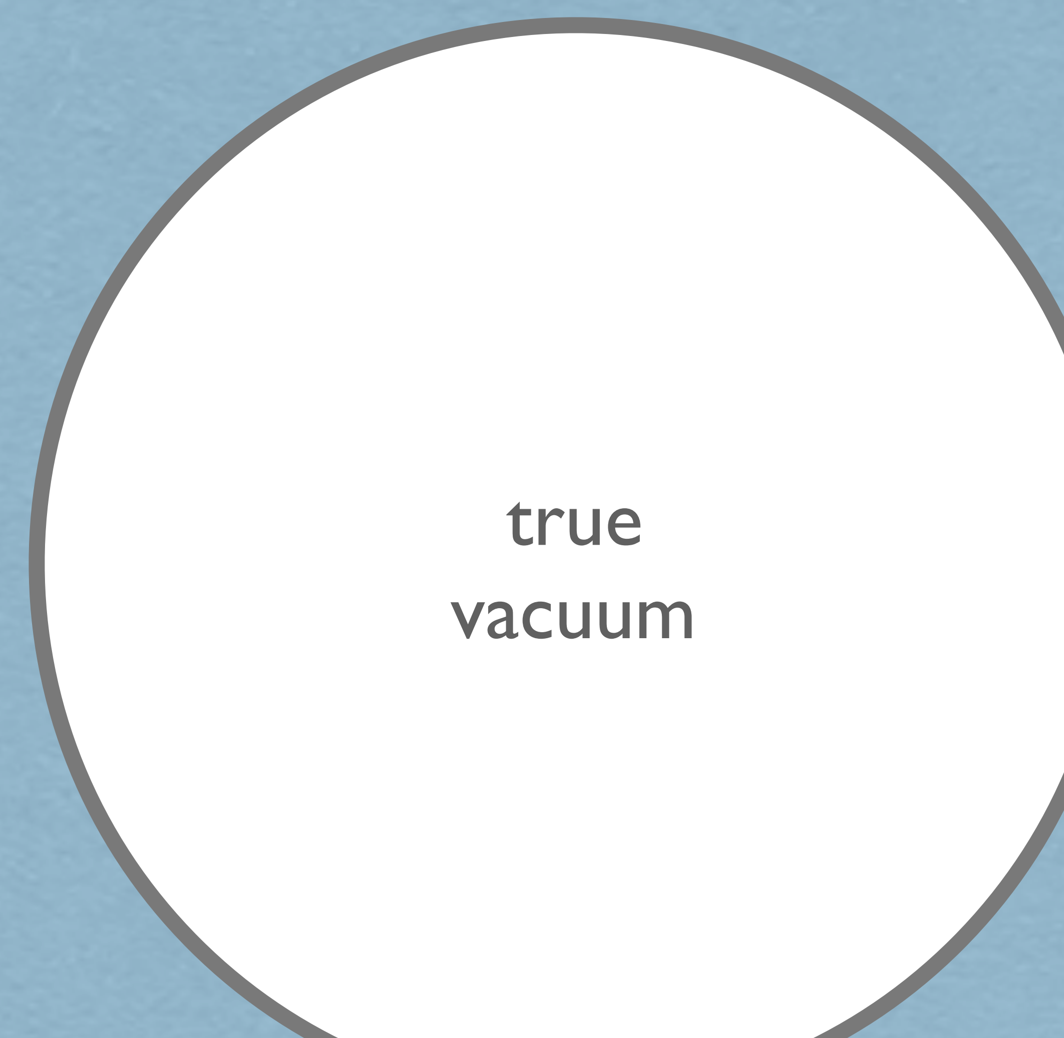
Workshop on Connecting Insights in Fundamental Physics: Standard Model and Beyond

Corfu, September 5th 2019

1st order phase transitions

- cosmology, EWPhTr, GWs,
- baryogenesis, B-fields,
- model parameter space,
- solid state, chemistry, ...

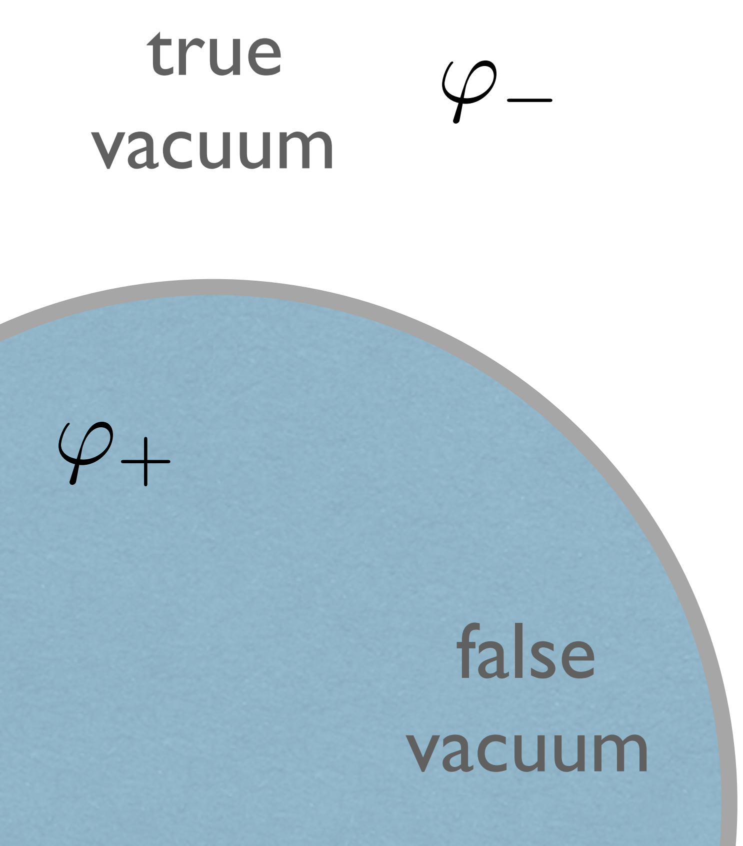


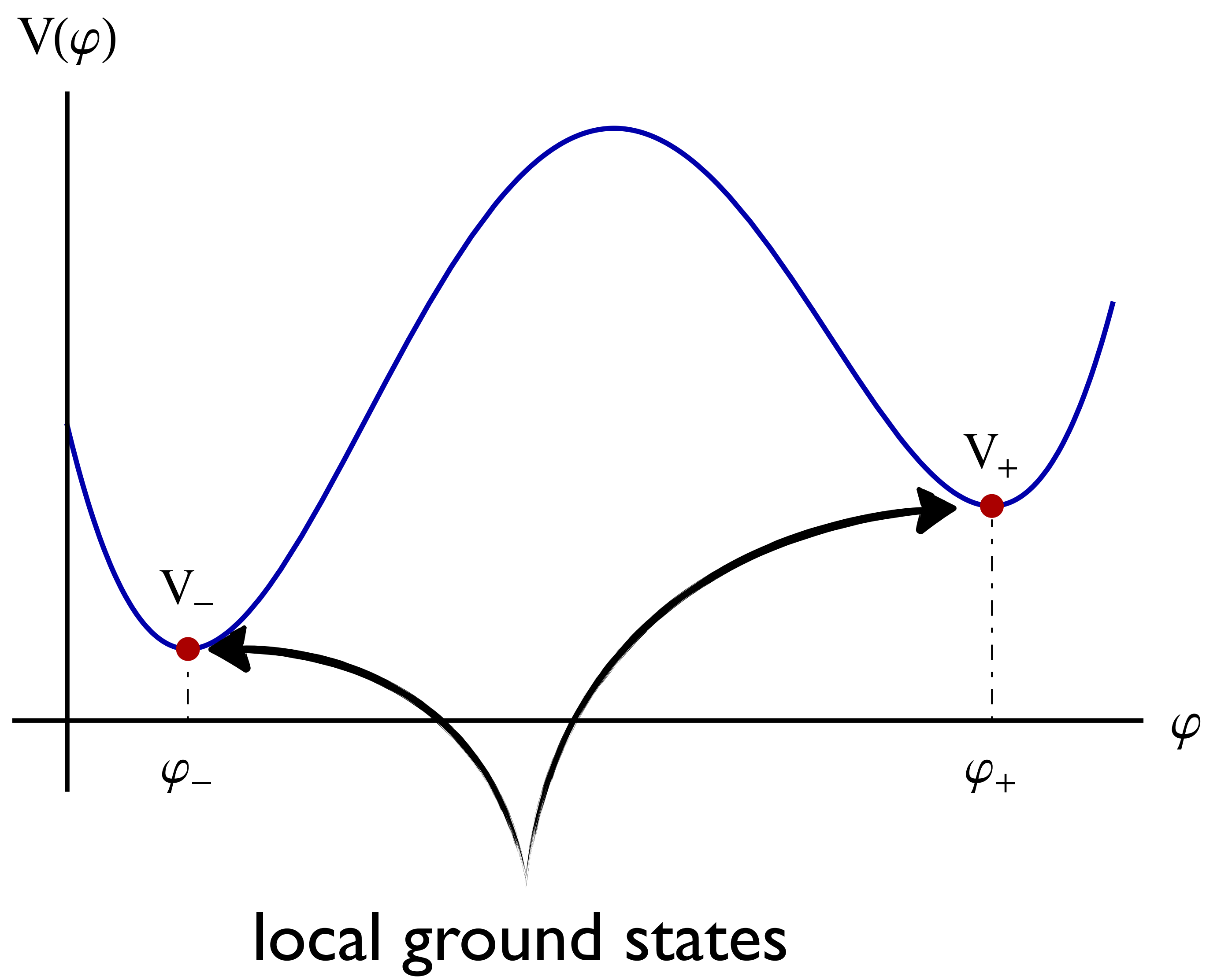


Phase Transitions

Local minima may be meta-stable and long lived — Kobzarev, Okun, Voloshin ’74

Theory of false vacuum decay — Coleman ’77





The bounce

Computing the transition rate

$$\Gamma/V = A\,e^{-B/\hbar} + \mathcal{O}(\hbar)$$

Theory of B

Coleman '77

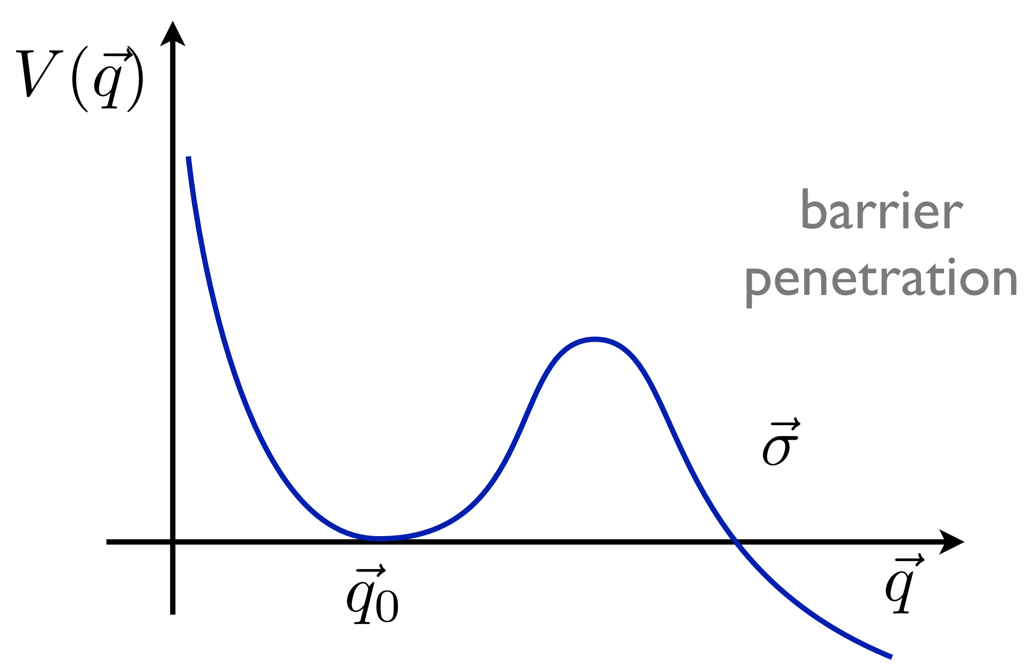
Theory of A

Callan, Coleman '77

Quantum tunneling, semi-classical approximation

1D QM $\quad L = \frac{1}{2}\dot{q}^2 - V(q) \qquad B = 2\int_{q_0}^{\sigma} \mathrm{d}q\sqrt{2V(q)}$ WKB '26

multi-D $\quad L = \frac{1}{2}\dot{\vec{q}}\cdot\dot{\vec{q}} - V(\vec{q}) \qquad B = 2\int_{\vec{q}_0}^{\vec{\sigma}} \mathrm{d}s\sqrt{2V(\vec{q})}$ Banks, Bender, Wu '73



Recast to variational principle

$$\delta\int_{\vec{q}_0}^{\vec{\sigma}} \mathrm{d}s\sqrt{2V} = 0$$

$$\delta\int_{\vec{q}_0}^{\vec{\sigma}} \mathrm{d}s\sqrt{2(E-V)} = 0$$

equivalent when

$E = 0$

$V \to -V$

The bounce

Generalize to single real scalar field theory

$$\mathcal{L} = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) \qquad B = S_E = \int d\tau d^3x \left(\frac{1}{2}\left(\frac{\partial\varphi}{\partial\tau}\right)^2 + \frac{1}{2}\left(\frac{\partial\varphi}{\partial_i x}\right)^2 \right) + V(\varphi)$$

Bounce solution Euclidean $O(4)$ symmetric

Coleman, Glaser, Martin '78

$$\rho^2 = t^2 + \sum x_i^2$$

Euclidean time =
radius of the bubble

The bounce

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Euclidean time = radius of the bubble

"...there always exists an *O(4)*-invariant bounce and it always has strictly lower action than any non-*O(4)* invariant bounce. The rigor of our proof is matched only by its tedium; I wouldn't lecture on it to my worst enemy."
Coleman, Erice lectures '77

Recently extended to multi-fields

Blum, Honda, Sato, Takimoto, Tobioka '16

The bounce

D dimensional $O(D)$ spherically symmetric Euclidean action

$$S_D = \frac{2\pi^{\frac{D}{2}}}{\Gamma\left(\frac{D}{2}\right)} \int_0^\infty \rho^{D-1}\, \mathrm{d}\rho \left(\frac{1}{2}\dot{\varphi}^2 + V(\varphi)\right)$$

D = 4 : FV decay at *T = 0* — Coleman '77

D = 3 : FV nucleation at finite *T* — Affleck '81, Linde '83

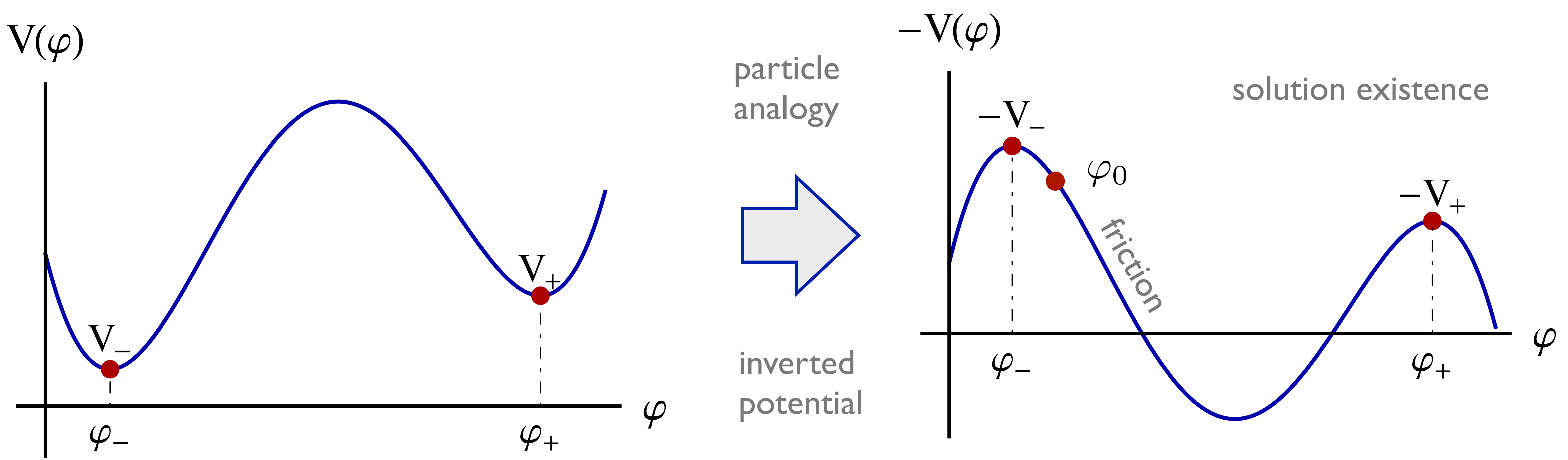
Bounce equation

$$\ddot{\varphi} + \frac{D-1}{\rho}\dot{\varphi} = dV$$

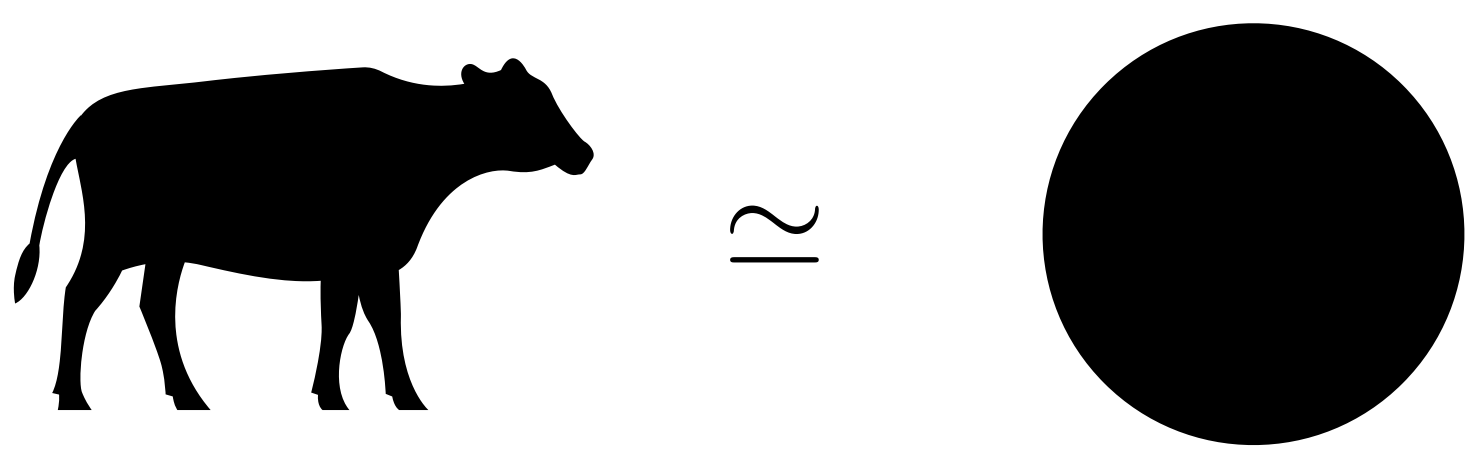
friction

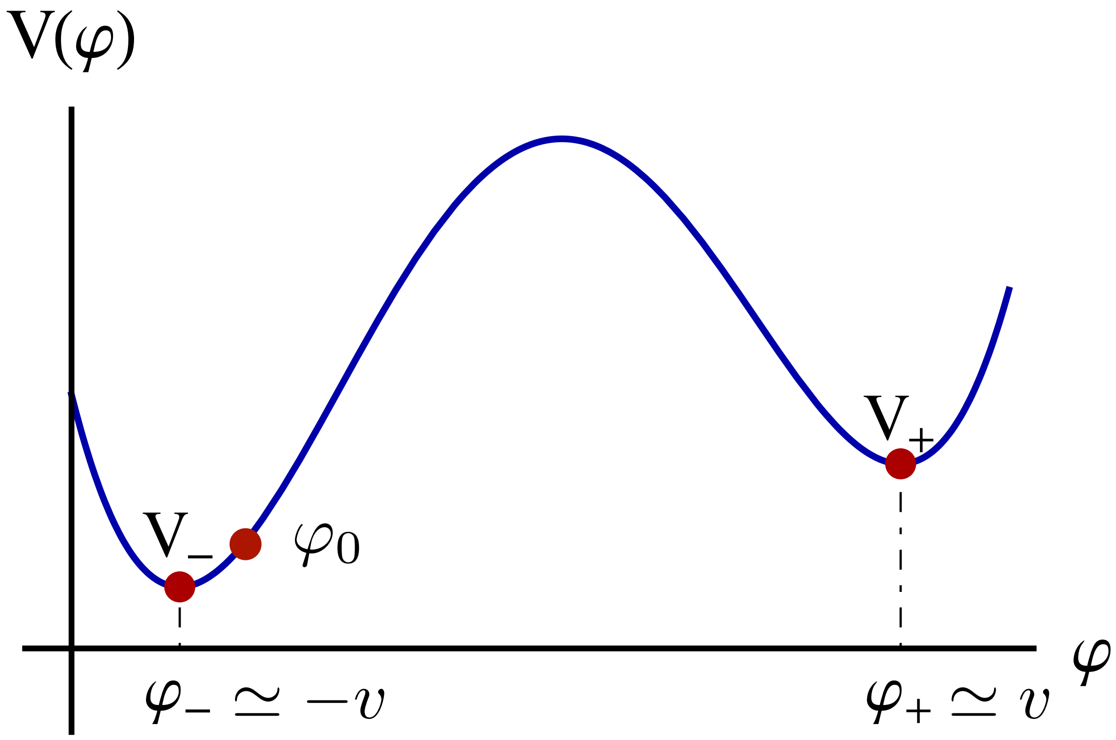
Boundary conditions

$$\varphi(0) = \varphi_0, \quad \varphi(\infty) = \varphi_+, \quad \dot{\varphi}(0,\infty) = 0$$



APPROXIMATING BOUNCES



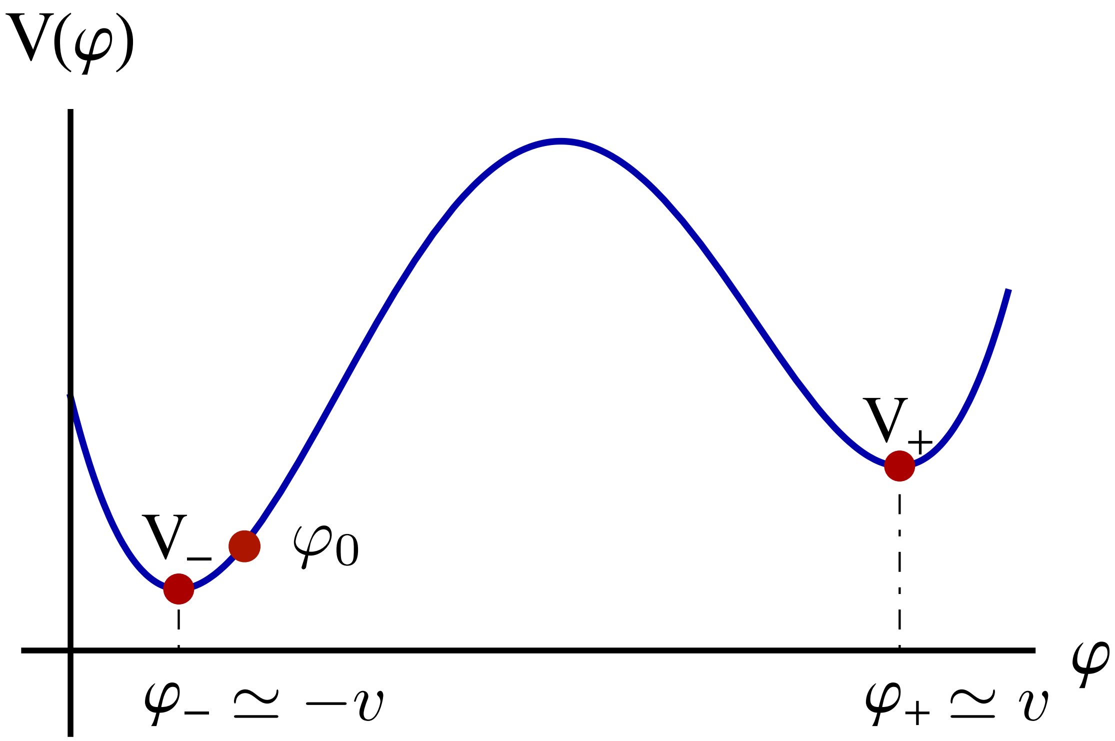


Thin wall approximation

Coleman '77

$$V(\varphi) = \frac{\lambda}{8}\left(\varphi^2 - v^2\right)^2 + \varepsilon\left(\frac{\varphi - v}{2v}\right), \qquad S_1 = \frac{v^3\sqrt{\lambda}}{3}$$

small ε limit $\varphi_0 \simeq \varphi_-$ until $\rho = R$



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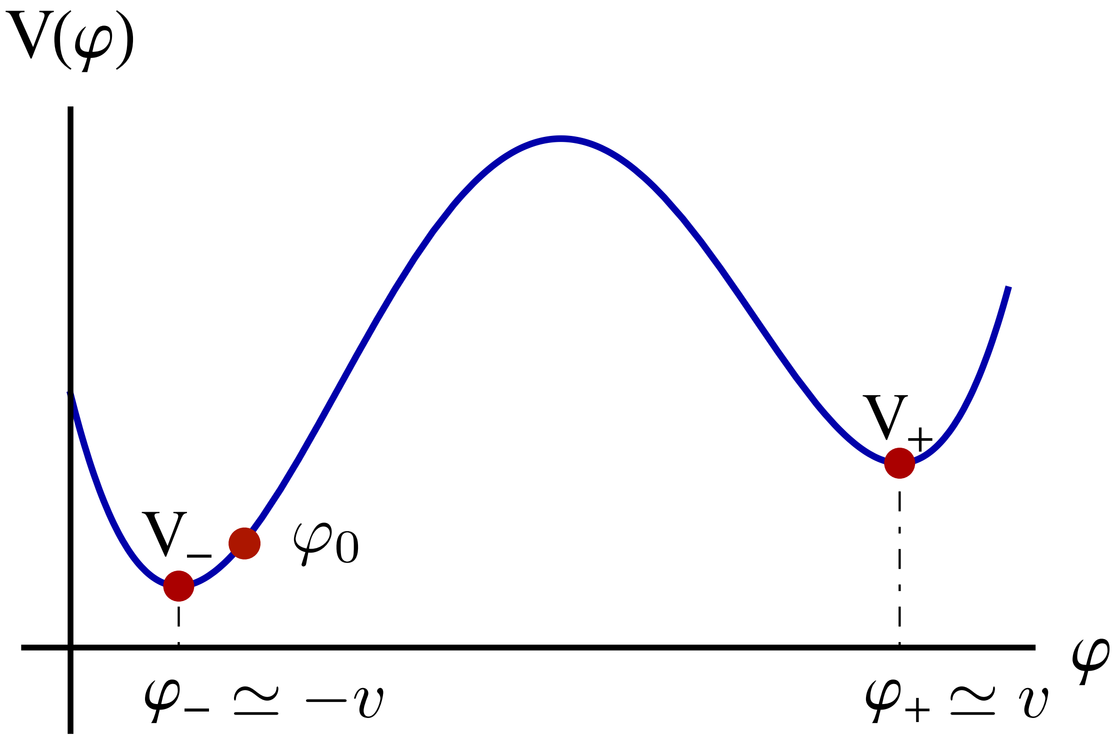
small ε limit $\varphi_0 \simeq \varphi_-$ **until** $\rho = R$

Field solution

$$\varphi(\rho) = \begin{cases} -v, & \rho \ll R \\ \varphi_1(\rho - R), & \rho \approx R \\ v, & \rho \gg R \end{cases}$$

$$\varphi_1(\rho) = v \tanh\left(\frac{\sqrt{\lambda}v}{2}\rho\right)$$

Extremize the action



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Bounce action

$$S_E = 2\pi^2 \int_0^\infty \rho^3 d\rho \left(\frac{1}{2}\dot{\varphi}^2 + V\right)$$
$$= -\frac{1}{2}\pi^2 R^4 \varepsilon + \pi^2 R^3 S_1$$

volume surface

Extremize the action

$$\frac{dS_E}{dR} = 0 \quad \Rightarrow \quad R = \frac{3S_1}{\varepsilon}$$

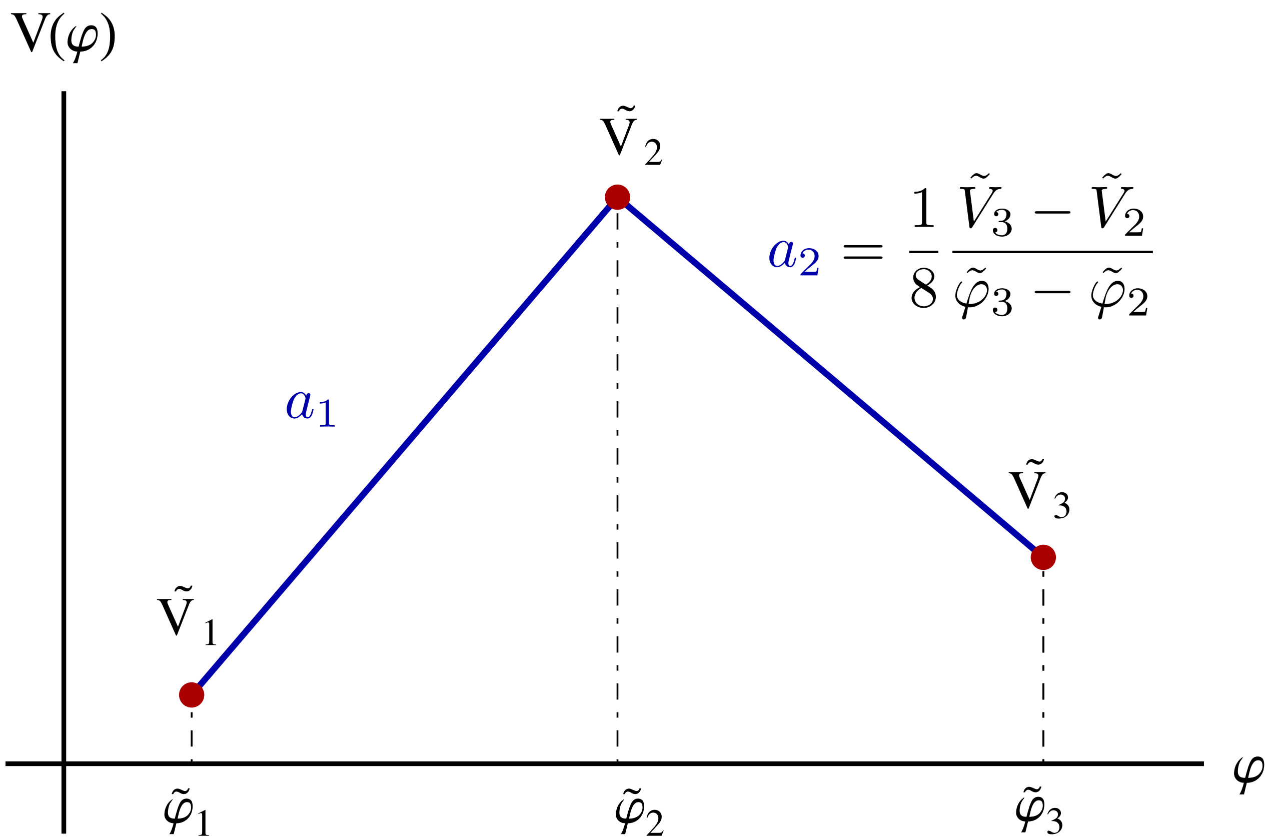
$$S_E = \frac{27\pi^2}{2}\frac{S_1^4}{\varepsilon^3}$$

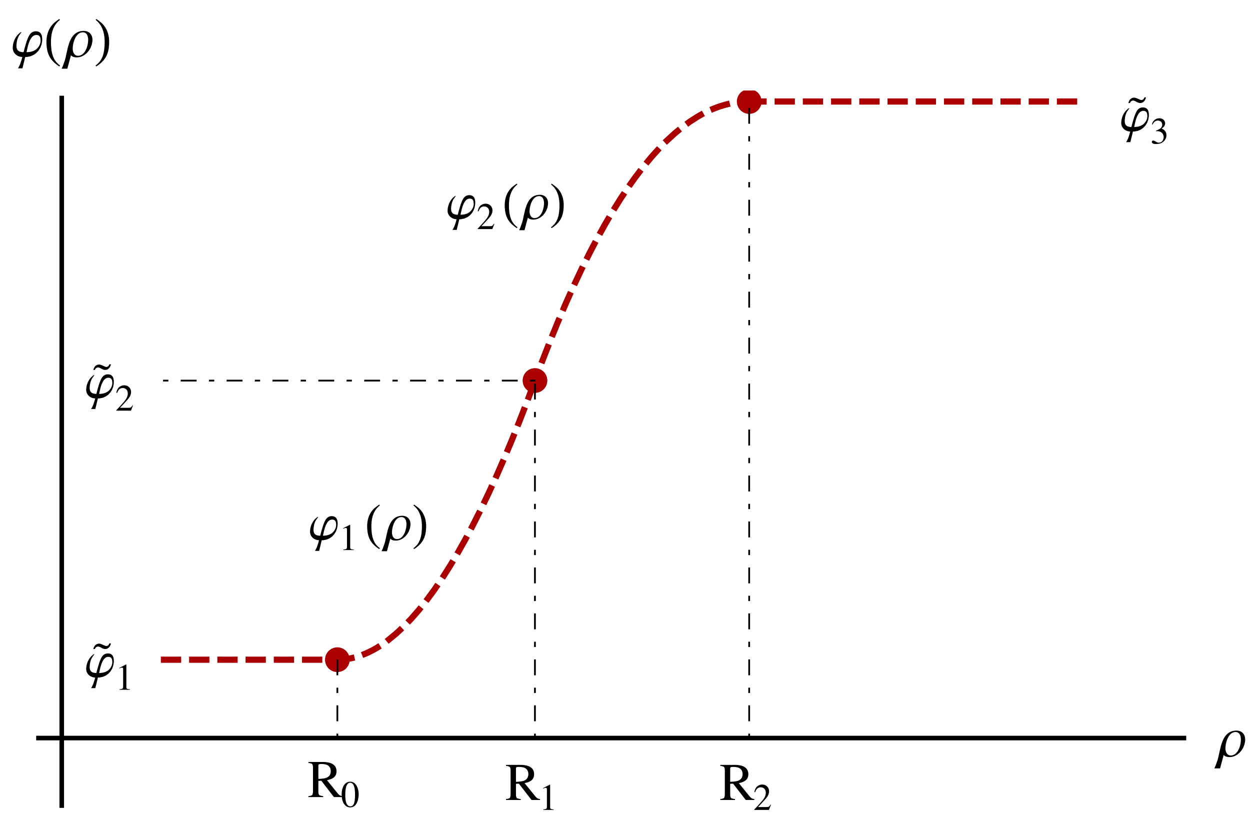
$$\frac{d^2 S_E}{dR^2} < 0$$

runaway

Coleman '77
Bödeker, Moore '09, '17

Triangle





Linear potentials

Duncan, Jensen '92

- triangle and box

Exact solution

$$\ddot\varphi + \frac{3}{\rho}\dot\varphi = dV = 8\,a$$

$$\varphi = v + a\,\rho^2 + \frac{b}{\rho^2}$$

Initial conditions @ R_0

shoot in φ_0 or R_0

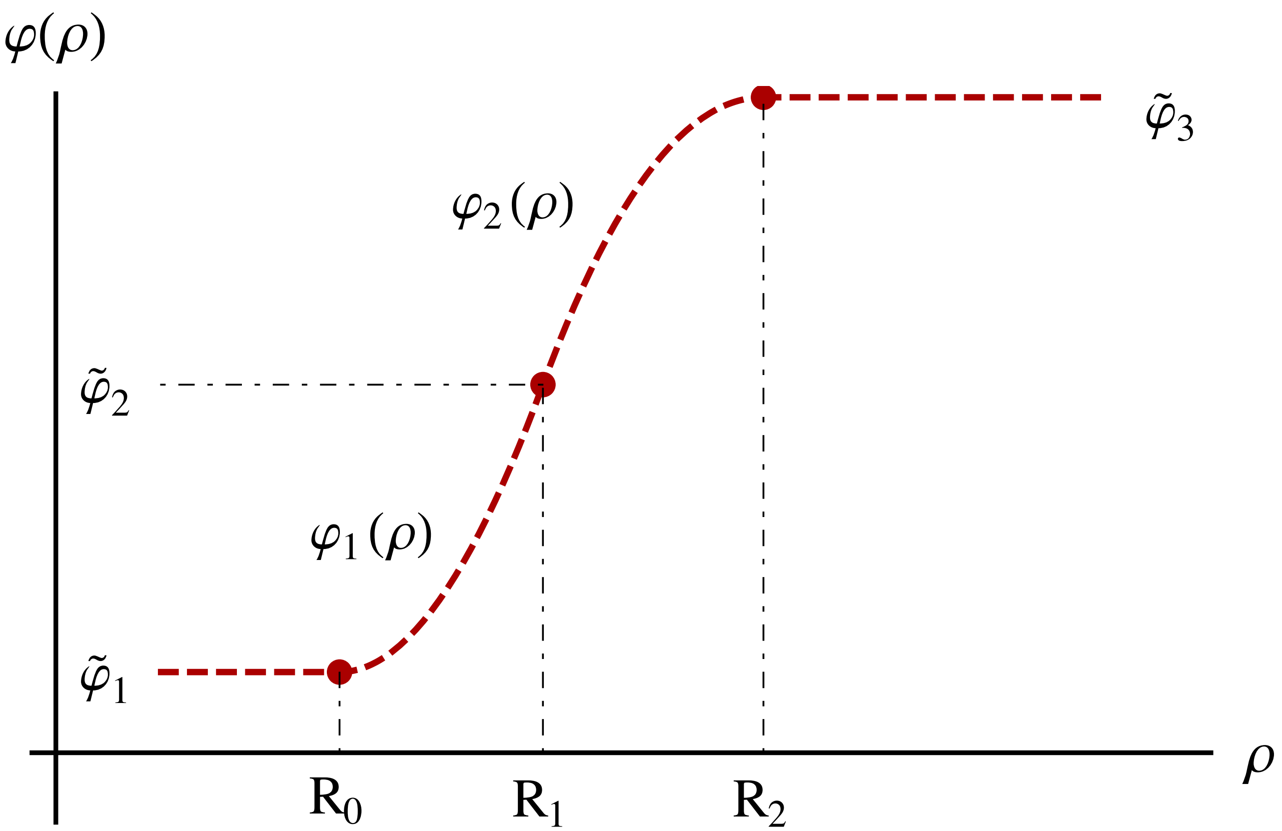
- a) $\varphi_1(0) = \varphi_0, \quad \dot\varphi_1(0) = 0$

  $v_1 = \varphi_0, \quad b_1 = 0$

- b) $\varphi_1(R_0) = \tilde\varphi_1, \quad \dot\varphi_1(R_0) = 0$

  $v_1 = \tilde\varphi_1 - 2\,a_1\,R_0^2, \quad b_1 = a_1\,R_0^4$

Triangle



Matching conditions @ R_1

$$\varphi_1(R_1) = \varphi_2(R_1) = \tilde\varphi_2, \quad \dot\varphi_1(R_1) = \dot\varphi_2(R_1)$$

Final conditions @ R_2

$$\varphi_2(R_2) = \tilde\varphi_3, \quad \dot\varphi_2(R_2) = 0$$

$$v_2 = \tilde\varphi_3 - 2\, a_2\, R_2^2, \quad b_2 = a_2\, R_2^4$$

Complete solution

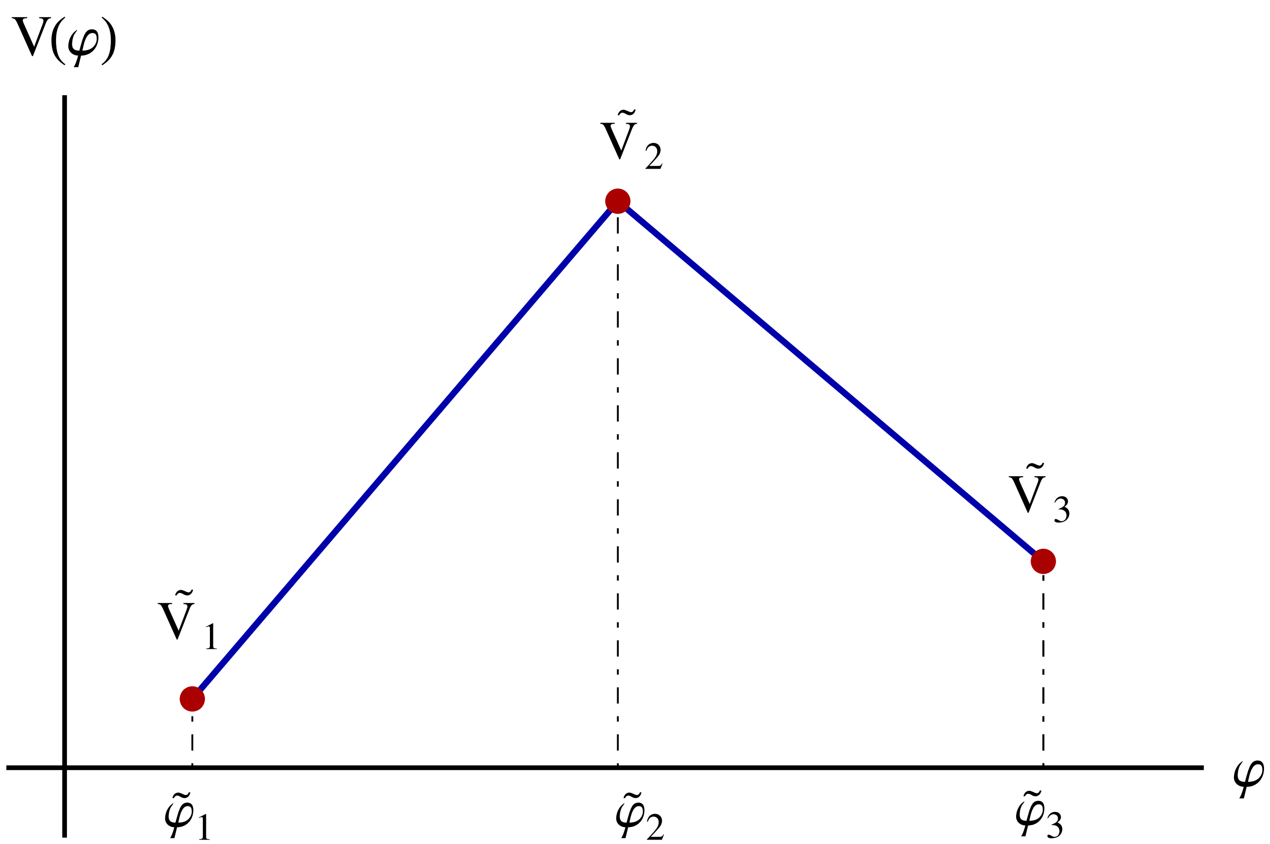
- a) $\varphi_0 = \dfrac{\tilde\varphi_3 + c\,\tilde\varphi_2}{1+c}, \qquad c = 2\dfrac{a_2 - a_1}{a_1}\left(1 - \sqrt{\dfrac{a_2}{a_2 - a_1}}\right) \qquad R_1 = \sqrt{\dfrac{D}{4}\left(\dfrac{\tilde\varphi_2 - \varphi_0}{a_1}\right)}$

- b) $R_1 = \dfrac{1}{2}\dfrac{\tilde\varphi_3 - \tilde\varphi_1}{\sqrt{a_1(\tilde\varphi_2 - \tilde\varphi_1)} - \sqrt{-a_2(\tilde\varphi_3 - \tilde\varphi_2)}}$

$$R_0^2 = R_1\left(R_1 - \sqrt{\frac{\tilde\varphi_2 - \tilde\varphi_1}{a_1}}\right)$$

$$R_2^2 = R_1\left(R_1 + \sqrt{\frac{\tilde\varphi_3 - \tilde\varphi_2}{-a_2}}\right)$$

Summary



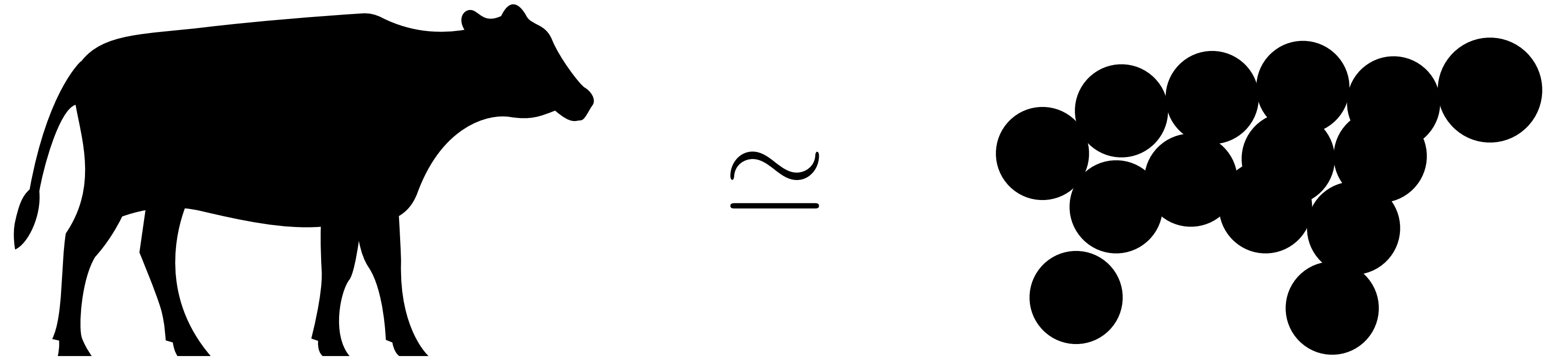
Complete exact analytic solution

Solved in terms of Euclidean radius

Stable in thin wall, goes over to TWA

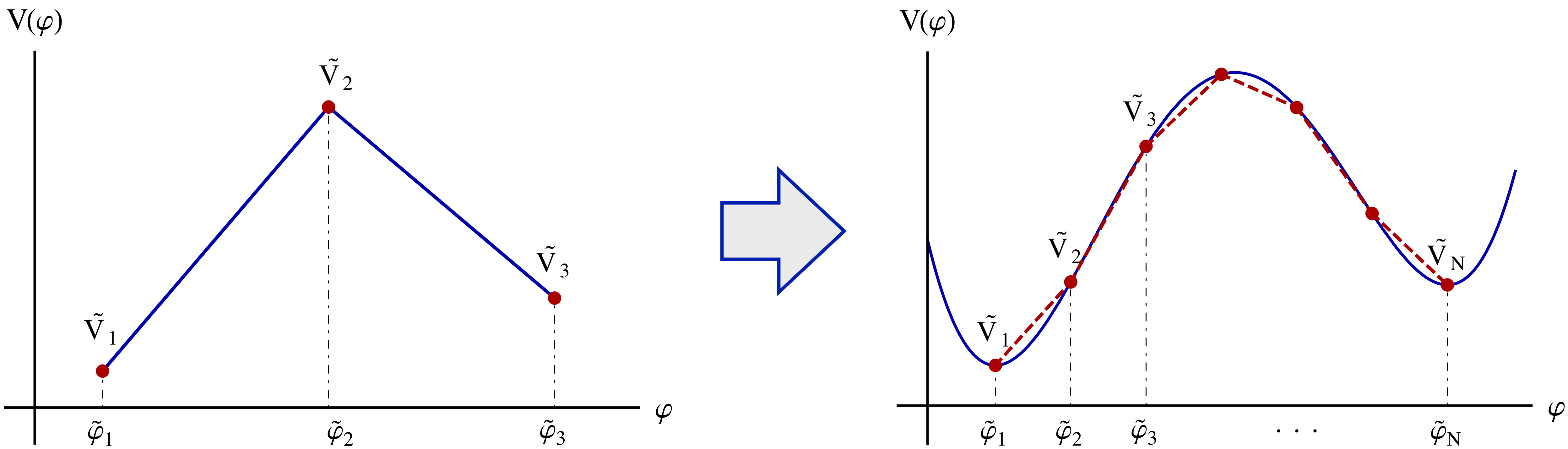
Limited validity outside the TW

APPROXIMATING BOUNCES



Polygonal bounces

Extend to more segments and *D* dimensions

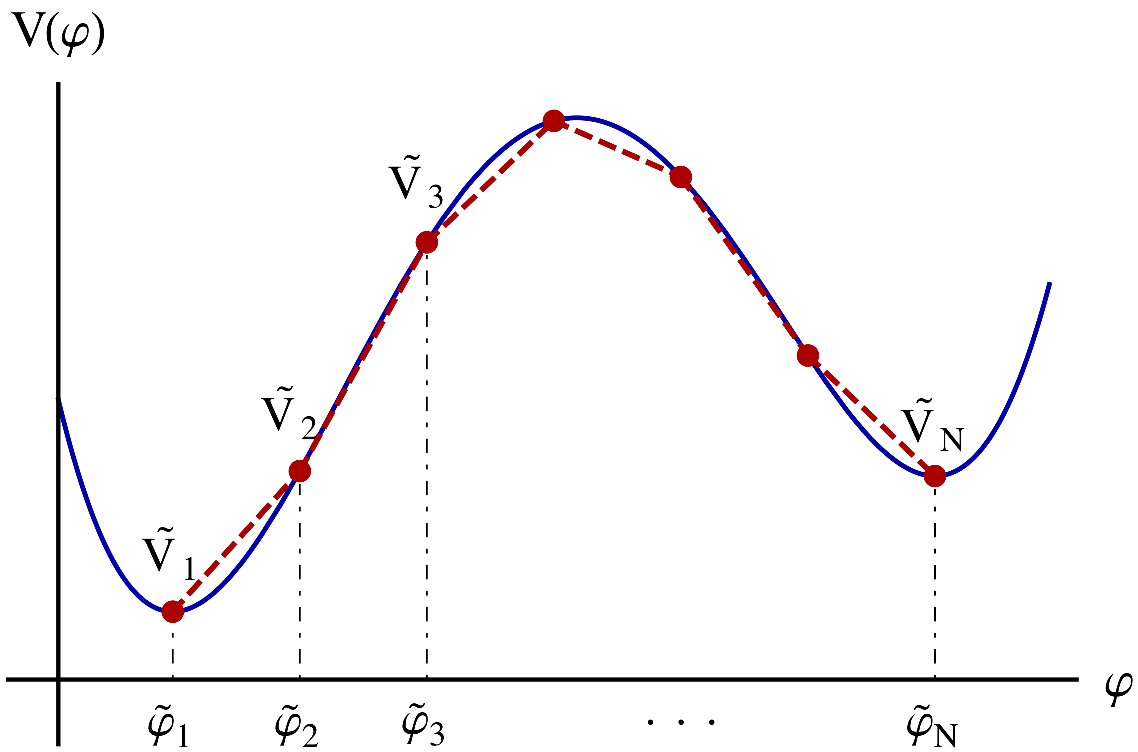


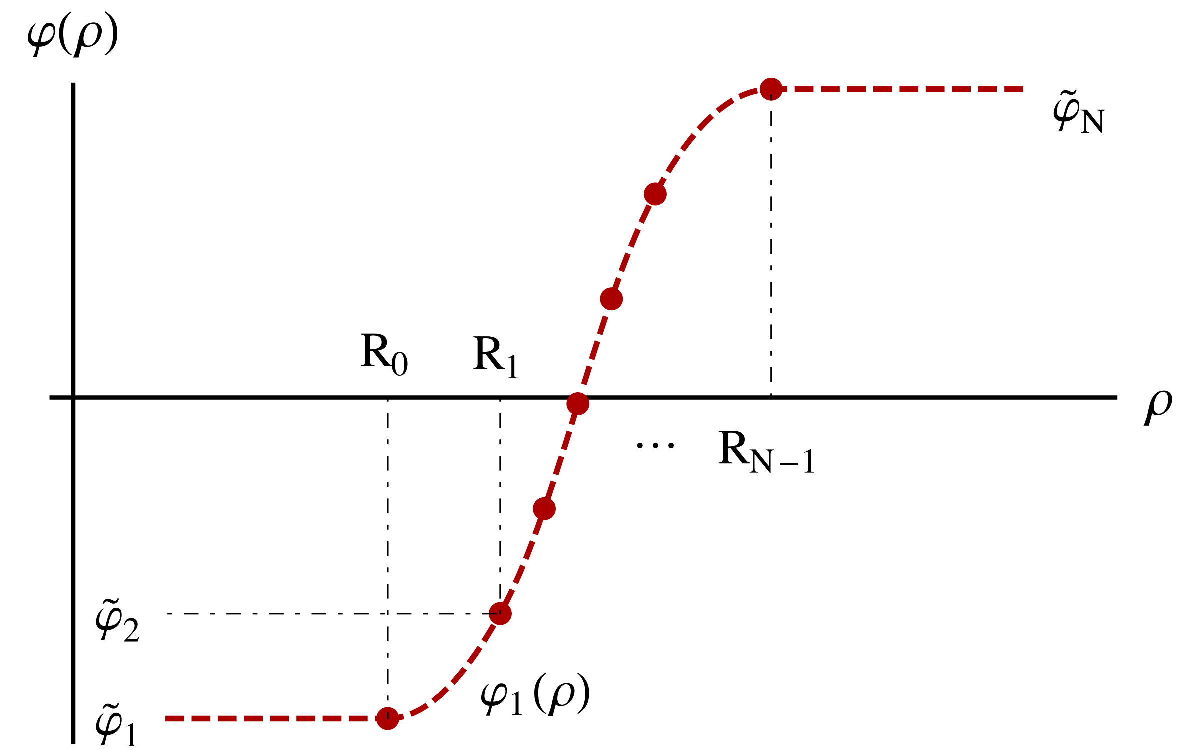
Approximates any V when $N \to \infty$, controlled precision

Geometric insight of segmentation, cover non-trivial features/unstable *V*s

Semi-analytic solution for algebraic manipulation/deformation

Polygonal bounces



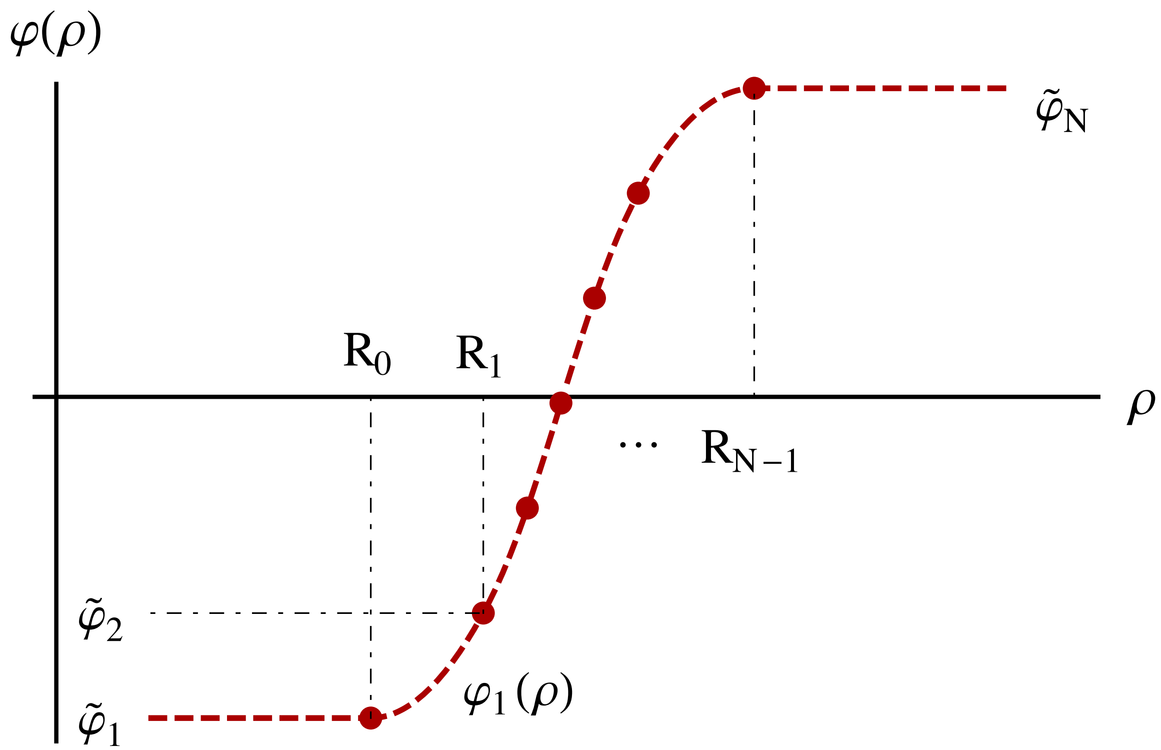


$$V_i(\varphi) = \underbrace{\left(\frac{\tilde{V}_{i+1} - \tilde{V}_i}{\tilde{\varphi}_{i+1} - \tilde{\varphi}_i}\right)}_{8\,a_i}(\varphi - \tilde{\varphi}_i) + \tilde{V}_i - \tilde{V}_N\,, \qquad dV_i = 8\,a_i.$$

No free parameters, one segment three unknowns $v_i,\ b_i,\ R_i$

Generalize case b), solve R_0 or R_i a), retrieve φ_0

Constructing polygonal bounces



$$\ddot{\varphi}_i + \frac{D-1}{\rho}\dot{\varphi}_i = dV_i = 8\,a_i$$

$$\varphi_i = v_i + \frac{4}{D}a_i\,\rho^2 + \frac{2}{D-2}\frac{b_i}{\rho^{D-2}}$$

Initial/final conditions remain the same

Matching conditions @ R_i 3 parameters and 3 unknowns/segment

$$\varphi_i(R_1) = \varphi_{i+1}(R_i) = \tilde{\varphi}_{i+1}, \quad \dot{\varphi}_i(R_i) = \dot{\varphi}_{i+1}(R_i)$$

The bounce defined recursively

- a) $R_0 = 0$

$$v_n = \varphi_0 - \frac{4}{D-2}\left(a_1 R_0^2 + \sum_{i=1}^{n-1}(a_{i+1} - a_i)\,R_i^2\right)$$

- b) $\varphi_0 = \tilde{\varphi}_1$

$$b_n = \frac{4}{D}\left(a_1 R_0^D + \sum_{i=1}^{n-1}(a_{i+1} - a_i)\,R_i^D\right)$$

Constructing PB

Radii computed at each segment from matching the fields $\varphi_n(R_n) = \tilde{\varphi}_{n+1}$

fewnomial

$$R_n^D - \frac{D}{4}\frac{\delta_n}{a_n}R_n^{D-2} + \frac{D}{2(D-2)}\frac{b_n}{a_n} = 0$$

$\delta_n = \tilde{\varphi}_{n+1} - v_n$

require real positive roots

Constructing PB

Radii computed at each segment from matching the fields $\varphi_n(R_n) = \tilde{\varphi}_{n+1}$

fewnomial $$R_n^D - \frac{D}{4}\frac{\delta_n}{a_n}R_n^{D-2} + \frac{D}{2(D-2)}\frac{b_n}{a_n} = 0 \qquad \delta_n = \tilde{\varphi}_{n+1} - v_n$$

require real positive roots

radii solutions $$D = 3: \quad 2R_n = \frac{1}{\sqrt{a_n}}\left(\frac{\delta_n}{\xi} + \xi\right), \qquad \xi^3 = \sqrt{36a_n b_n^2 - \delta_n^3} - 6\sqrt{a_n}b_n,$$

simple cubic

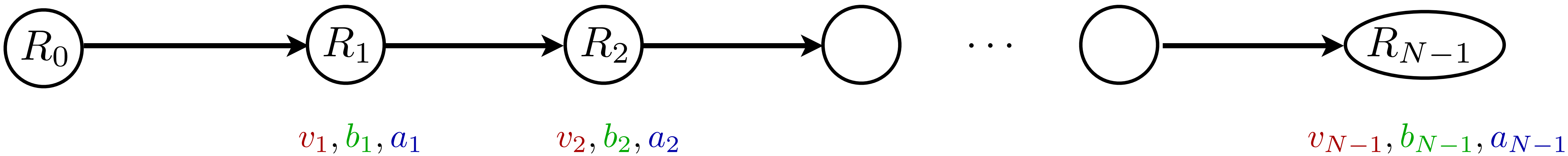
$$D = 4: \quad 2R_n^2 = \frac{1}{a_n}\left(\delta_n + \sqrt{\delta_n^2 - 4a_n b_n}\right)$$ quadratic

$D = 2, 6, 8$ in the paper, other *D*s possible numerically

Constructing PB

- b) $\varphi_0 = \tilde{\varphi}_1$

highly non-linear

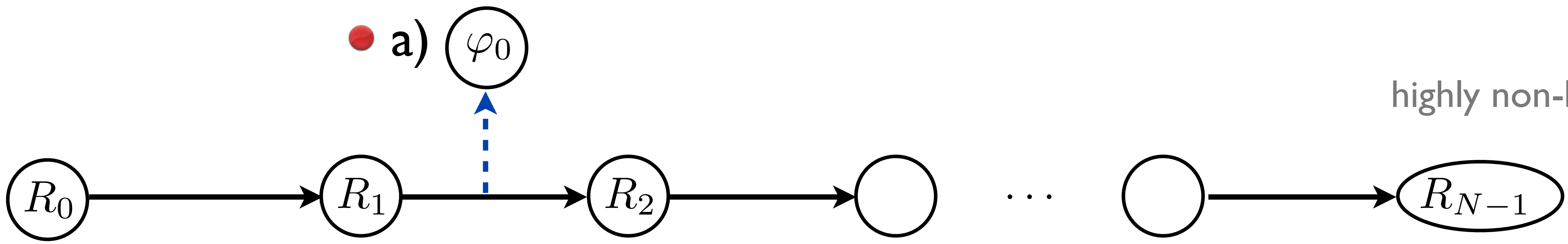


Matching

$$a_1 R_0^D + \sum_{i=1}^{N-2} \left(a_{i+1} - a_i\right) R_i^D - a_{N-1} R_{N-1}^D = 0$$

Constructing PB

highly non-linear

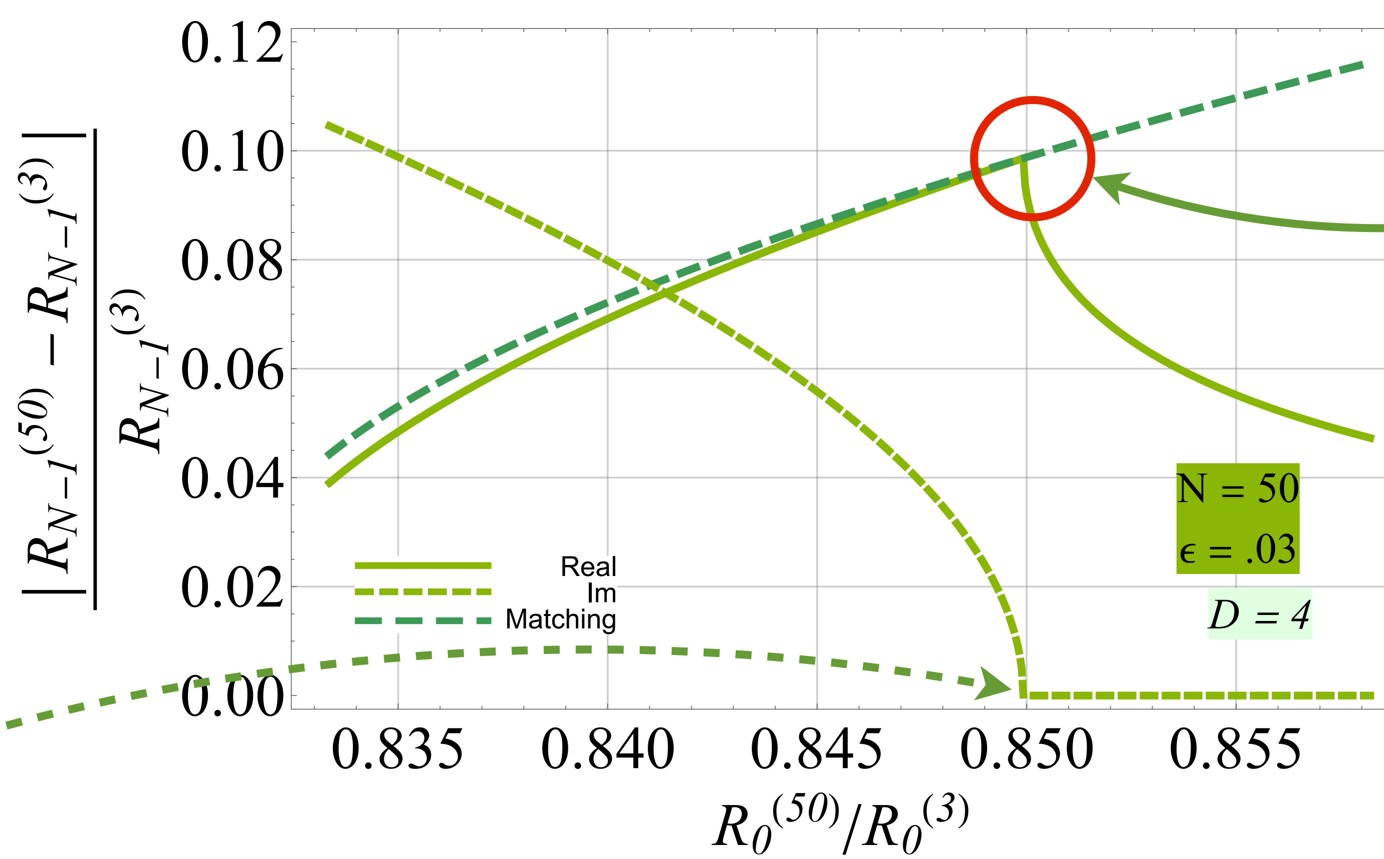


- b)

Matching

$$a_1 R_0^D + \sum_{i=1}^{N-2} (a_{i+1} - a_i)\, R_i^D - a_{N-1} R_{N-1}^D = 0$$

case b)



bounce solution

stiff behaviour

becomes imaginary, coincides with the solution

Bounce action

Euclidean action

$$S_D = \frac{2\pi^{\frac{D}{2}}}{\Gamma\left(\frac{D}{2}\right)} \int_0^\infty \rho^{D-1} \, \mathrm{d}\rho \left(\frac{1}{2}\dot{\varphi}^2 + V(\varphi)\right)$$

PB action

$$S_{>2} = \frac{2\pi^{\frac{D}{2}}}{\Gamma\left(\frac{D}{2}\right)} \left\{ \frac{R_0^D}{D}\left(\tilde{V}_1 - \tilde{V}_N\right) + \sum_{i=1}^{N-1} \left[\rho^2 \left(\frac{32 a_i^2 (D+1)\rho^D}{D^2(D+2)} + \frac{16 a_i b_i}{D(D-2)} - \frac{2 b_i^2}{\rho^D (D-2)} \right) + \frac{\rho^D}{D}\left(8 a_i \left(v_i - \tilde{\varphi}_i\right) + \tilde{V}_i - \tilde{V}_N\right) \right]_{R_{i-1}}^{R_i} \right\}$$

Total

$$S = \mathcal{T} + \mathcal{V}$$

$$\mathcal{T} \propto \int_0^\infty \rho^{D-1} \mathrm{d}\rho \, \dot{\varphi}^2, \qquad \mathcal{V} \propto \int_0^\infty \rho^{D-1} \mathrm{d}\rho \, \mathrm{V}(\varphi)$$

kinetic potential

Derrick's theorem

Non-existence of non-trivial static solutions of KG equation, no solitonic scalar 'particles'

Derrick '64

Unstable under re-scaling $\varphi(\rho) \to \varphi(\rho/\lambda)$

$$\lambda \times 0 = 0,$$
$$\lambda \times \infty = \infty$$

$$S_D^{(\lambda)} = \lambda^{D-2}\mathcal{T} + \lambda^D \mathcal{V}$$

change of variables...remain the same

action is extremized at non-scaled values for true solutions

$$\left.\frac{dS_D^{(\lambda)}}{d\lambda}\right|_{\lambda=1} = 0$$

$$(D-2)\mathcal{T} + D\mathcal{V} = 0$$

relation between kinetic and potential

$$\left.\frac{d^2 S_D^{(\lambda)}}{d\lambda^2}\right|_{\lambda=1} < 0$$

Caveat for PB $R \to \lambda R$

Works for $N \gg 1$

Benchmarks

the good, the bad and the ugly

Linearly off-set quartic potential

Coleman '77

$$V(\varphi) = \frac{\lambda}{8}\left(\varphi^2 - v^2\right)^2 + \varepsilon\left(\frac{\varphi - v}{2v}\right)$$

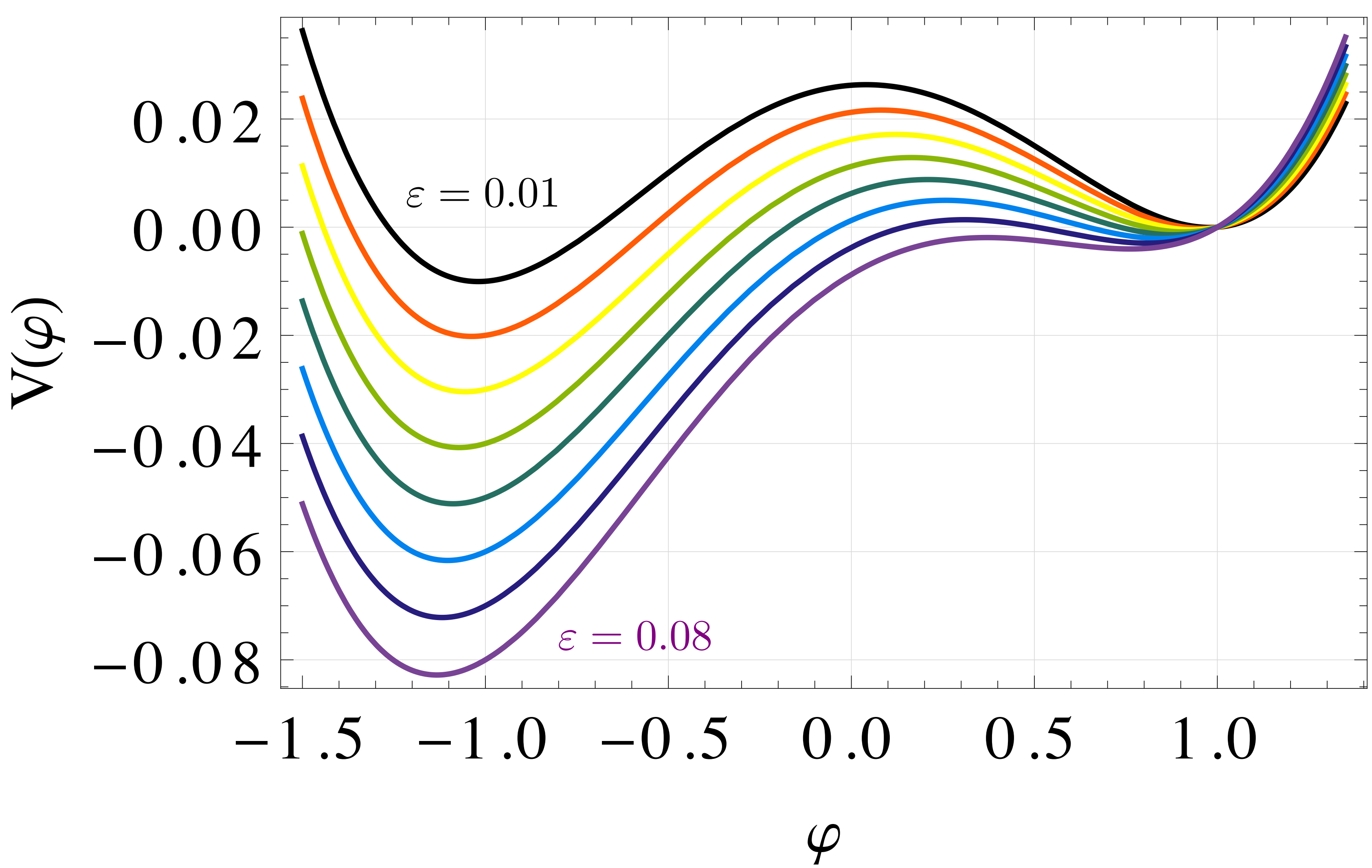
Benchmark for testing $\lambda = 0.25, \quad v = 1$

rescaling

Sarid '98

$\varepsilon \to 0$

thin wall



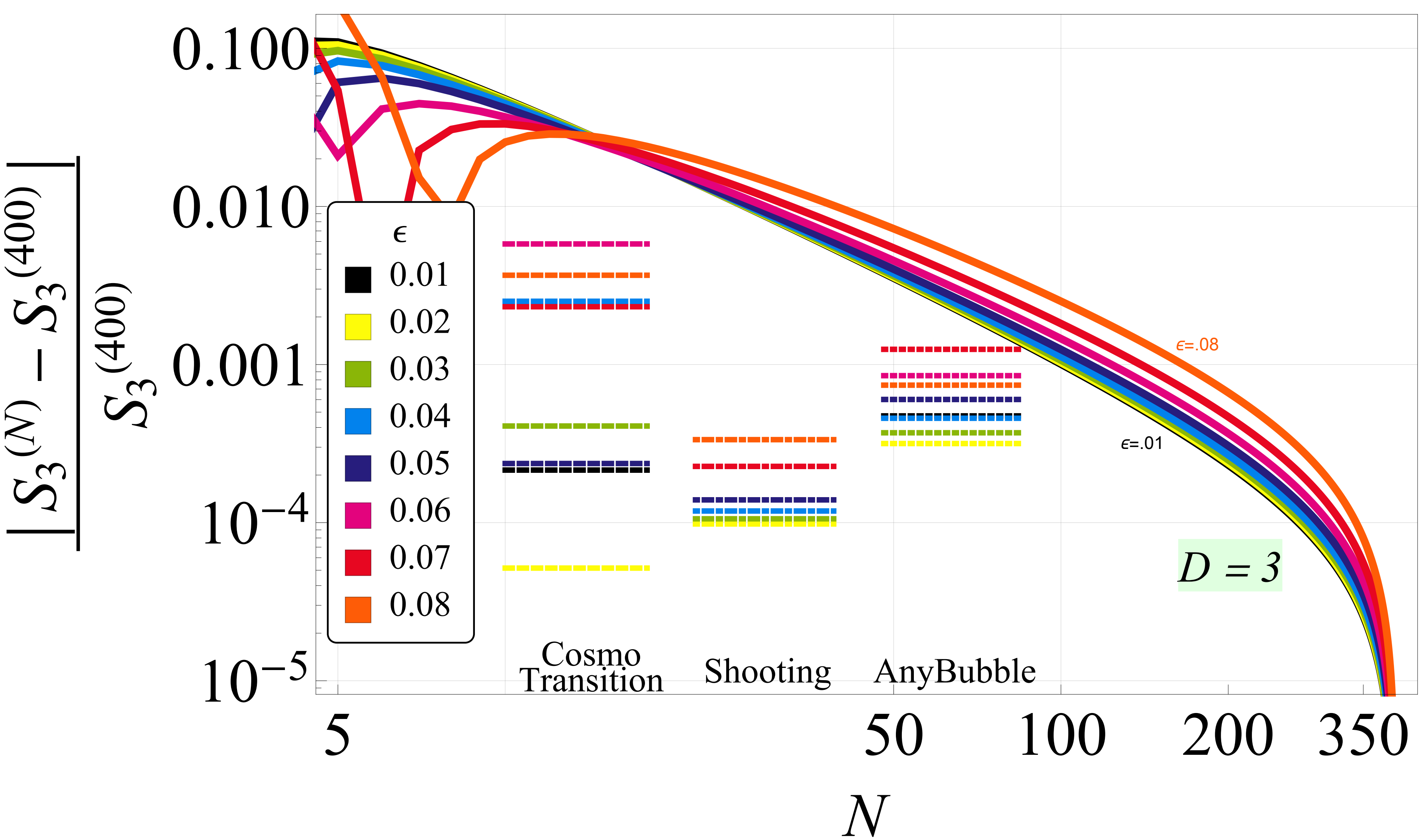
thick wall

$\varepsilon > 0.8$

2nd minimum disappears

Euclidean action, comparisons

- **CosmoTransitions** Runge-Kutta PDE solver, initial value approximations — Wainwright '11
  discontinued
- **AnyBubble** multiple shooting, damping approximations — Masoumi, Olum, Shlaer '16
- **Shooting** Mathematica, precise setting of initial values, issues with 0, infinity



PB within permille after 100 iterations

stable for thin/thick

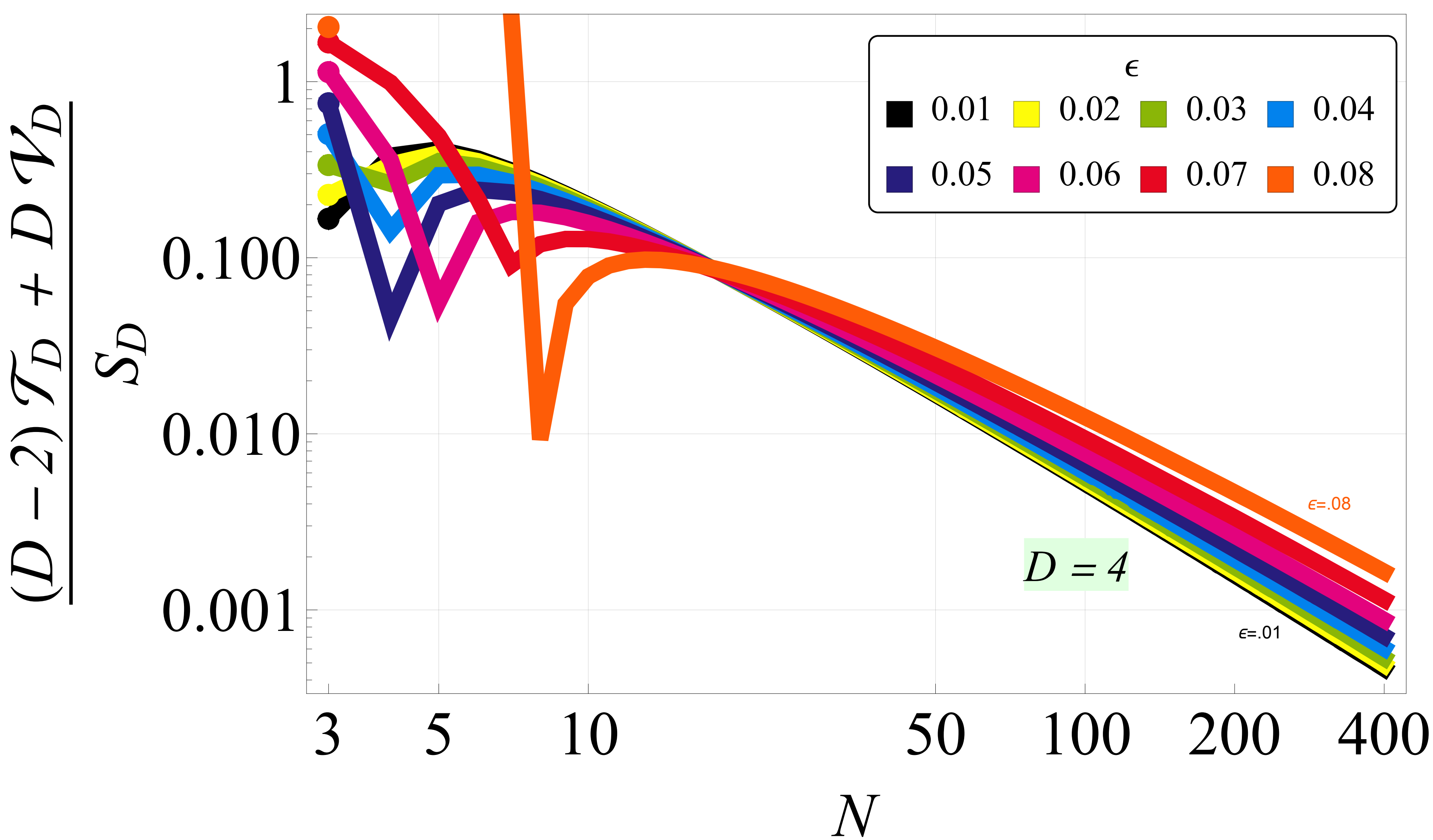
faster convergence for thin wall

Derrick's theorem $(D-2)\mathcal{T} + D\mathcal{V} \to 0$

finite part corrections up to $N \simeq 10$

independent measure of goodness of approximation

above relation 'exact' for the PB potential



Rescaling

$R_0 = 0$ $\quad$ $R_1 = 0$

Use Derrick's theorem to find the solution

estimate the initial value of R_i

compute the kinetic $\mathcal{T}(R_i)$ and potential pieces $\mathcal{V}(R_i)$

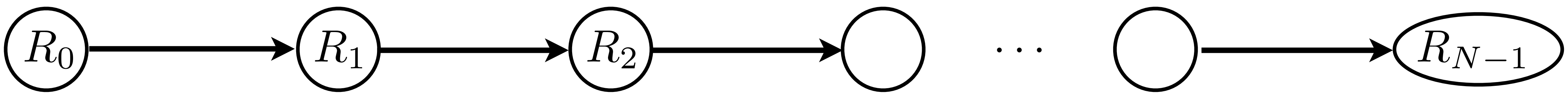
rescale the radius R_i by $\lambda = \sqrt{\dfrac{(2-D)\mathcal{V}(R_i)}{D\mathcal{V}(R_i)}}$

iterate until $|\lambda - 1| \lesssim \dfrac{1}{N}$

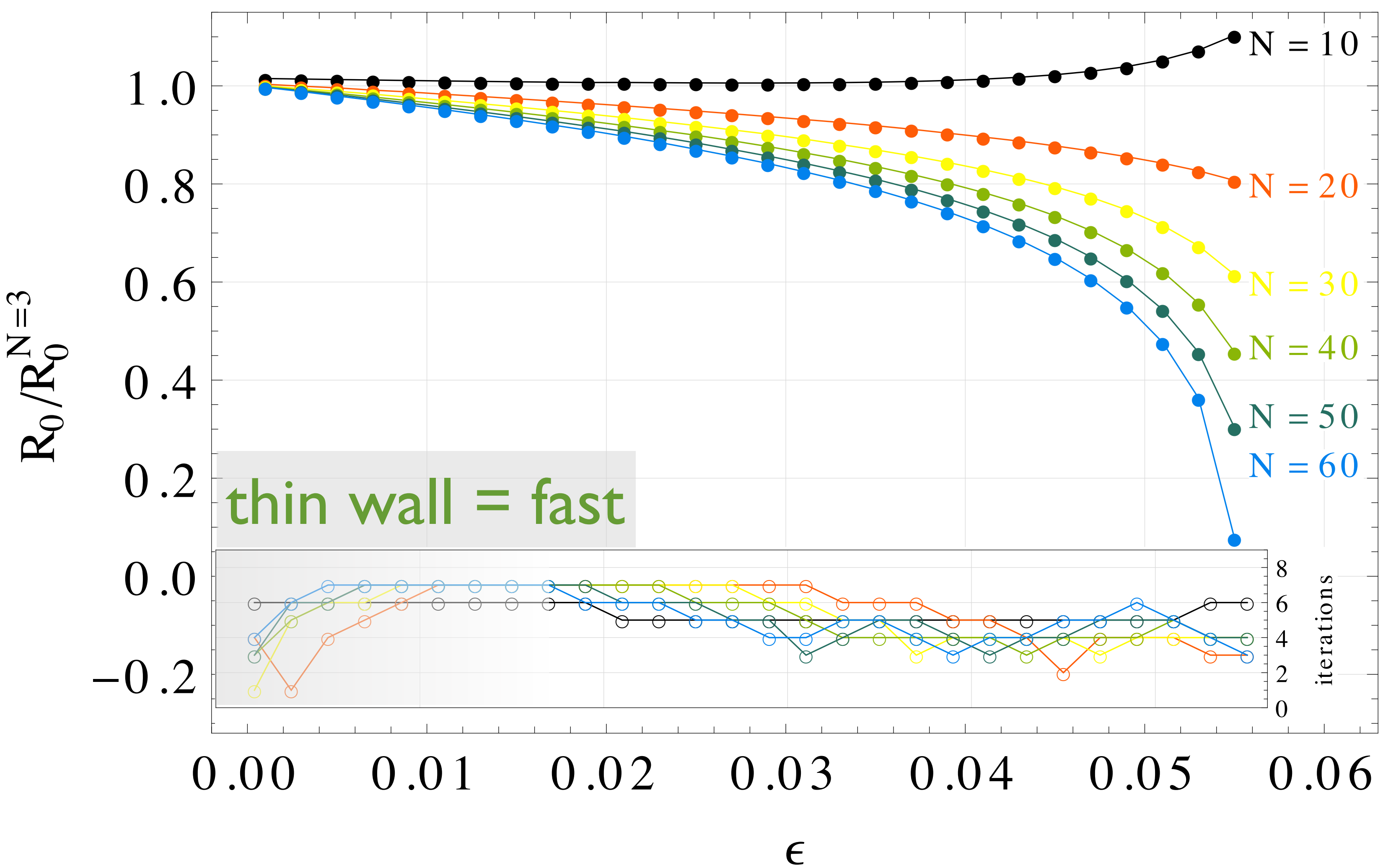
retrieve φ_0 from $R_i(\varphi_0) = \sqrt{\dfrac{\tilde{\varphi}_{i+1} - \varphi_0}{a_i}}$

Rescaling

- b)



Rescaling converged to permille level



~200 bounces in 2 secs

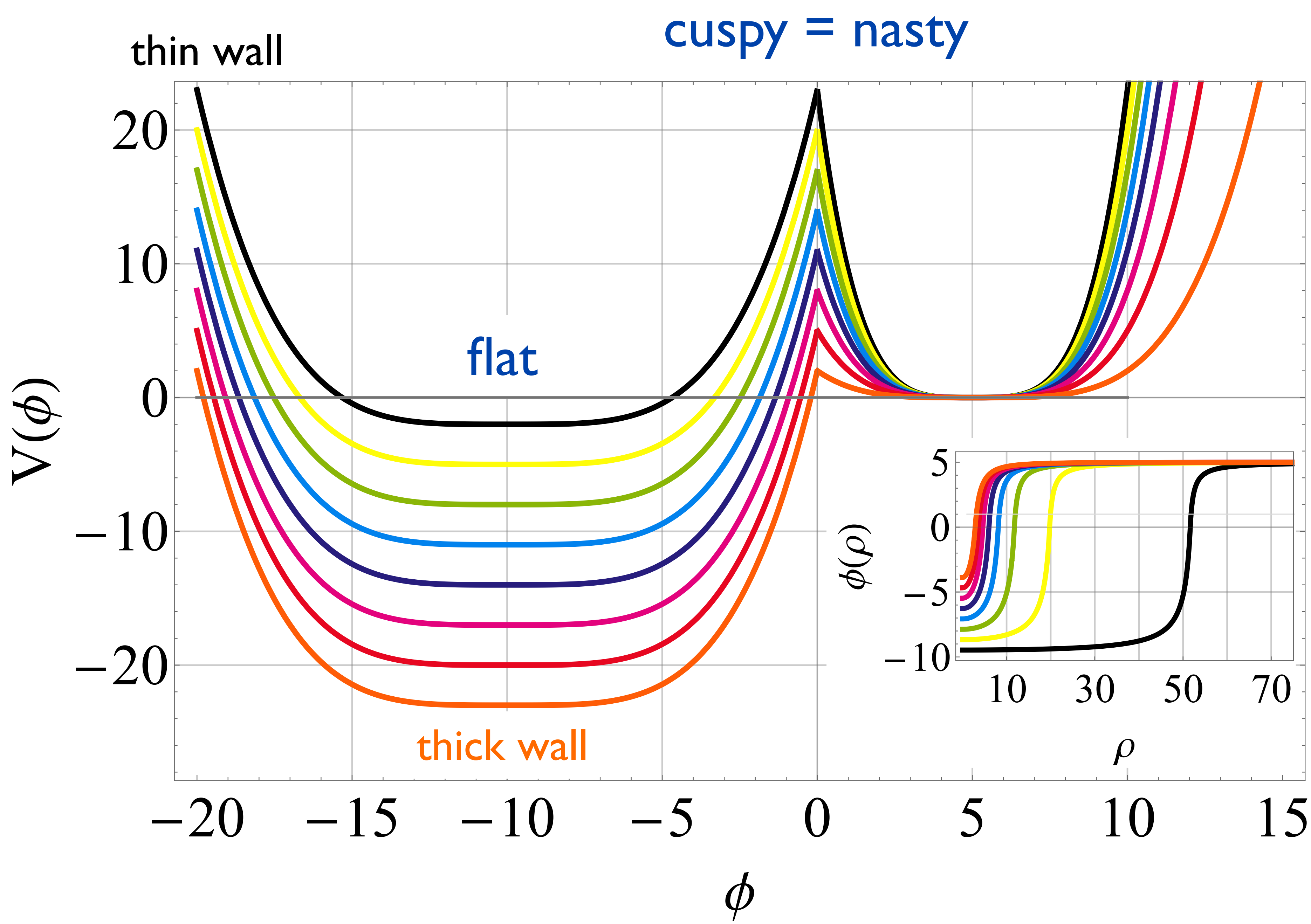
failing for small radii, expansion

Bi-quartic

Other exact *N=3* potentials, quartic-linear, quartic-quartic

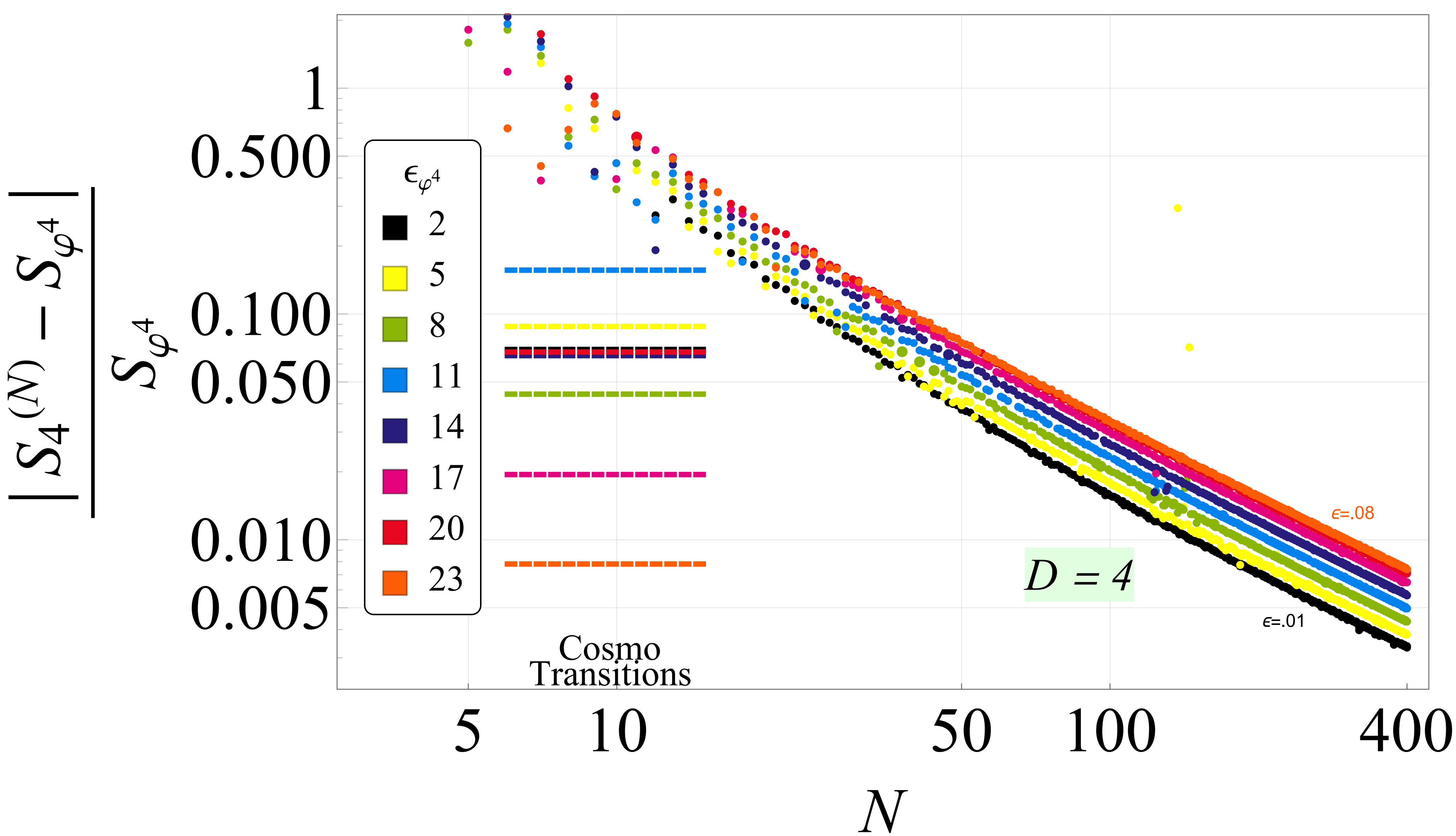
Dutta, Hector, Vaudrevange, Westphal '11

known exact solution, 'fair' comparison and test for the PB method



Bi-quartic

- **CosmoTransitions** fails with the action, possible to repair by hand, precise from 20% to 0.5%
- **AnyBubble** fails to compute
- **Polygonal bounce** works smoothly with a bi-homogeneous segmentation



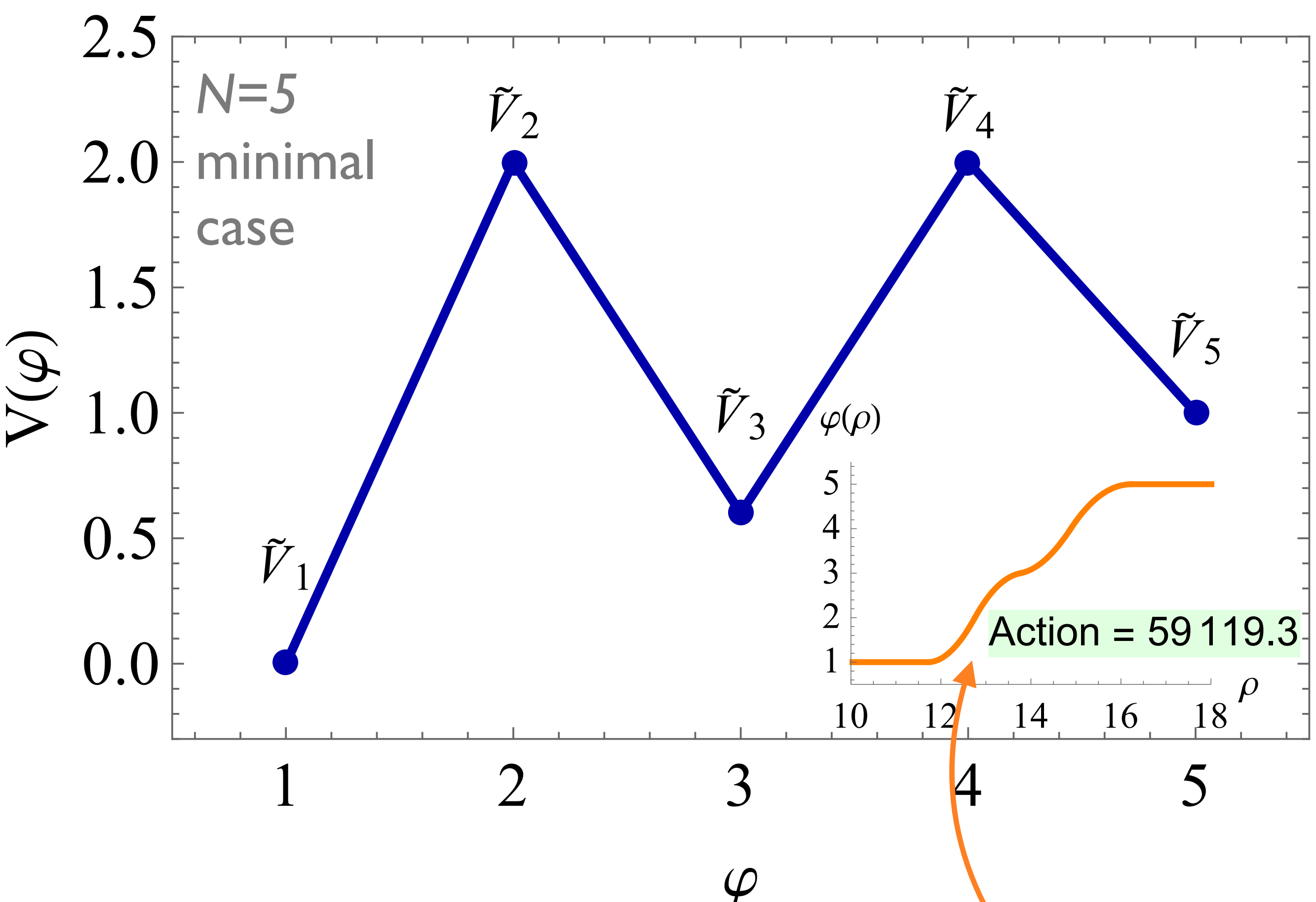
Disappearing instanton

Intermediate minima, multi-step transitions

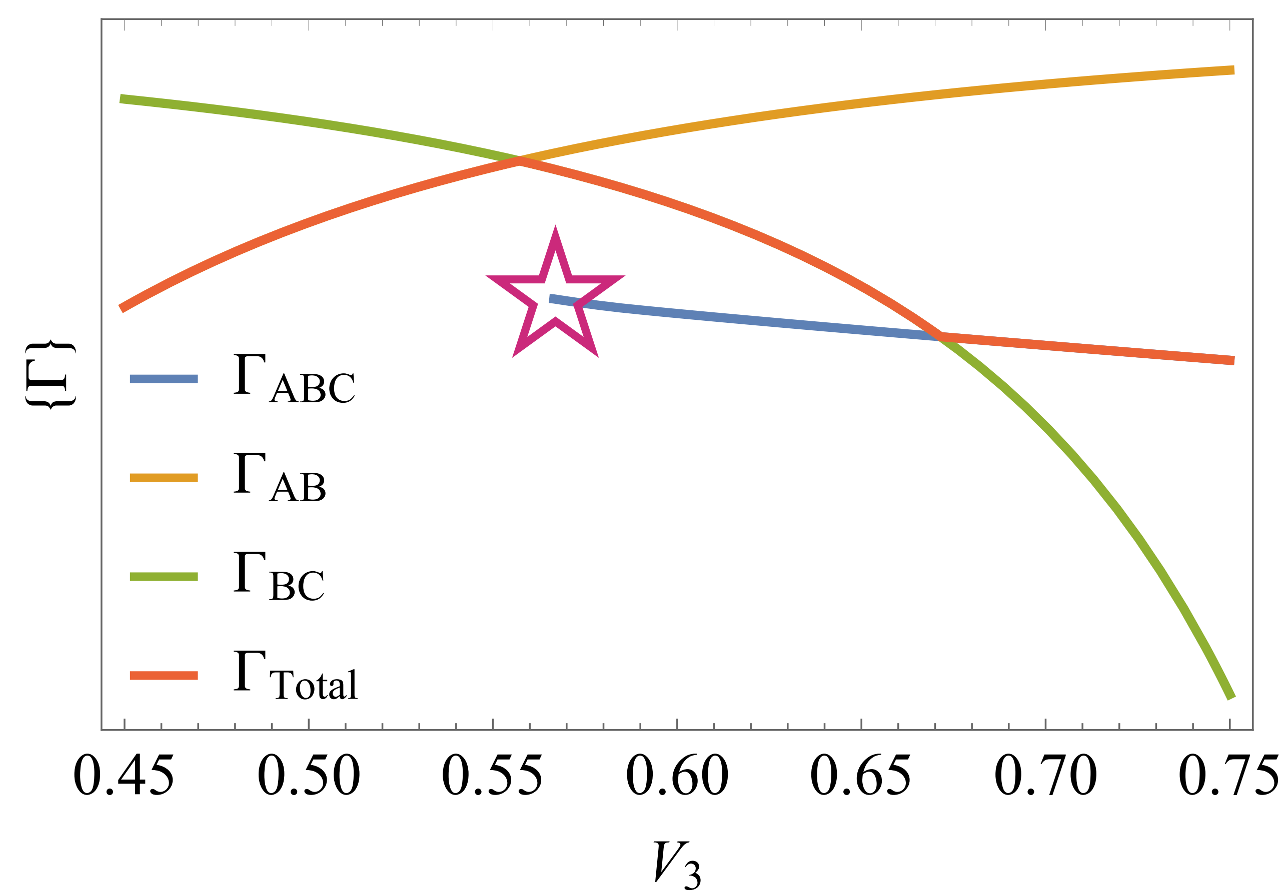
Dahlen, Brown '11

appear in theories with many fields, relaxion-type potentials

Q: which transition wins, direct tunneling or two subsequent transitions?

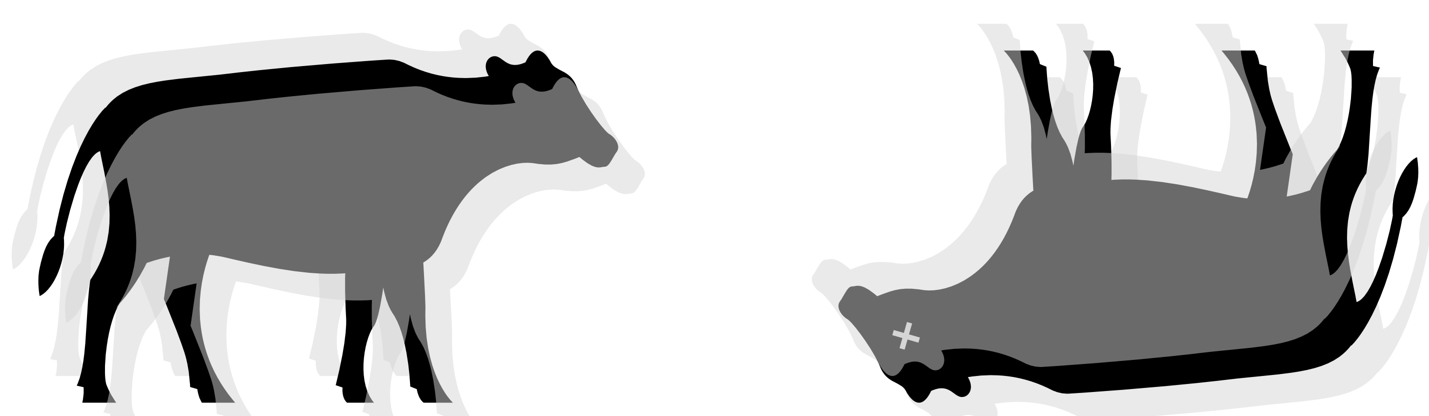


bubble with two walls



Direct tunneling impossible when intermediate minimum too low

QUANTUM FLUCTUATIONS



Quantum fluctuations

- Total decay rate in *D=4* Callan, Coleman '77

$$\Gamma = \left(\frac{S_4}{2\pi}\right)^2 \left|\frac{\det'(-\partial^2 + V''(\varphi(\rho)))}{\det(-\partial^2 + V''(\varphi_-))}\right|^{-1/2} e^{-S_4-\delta_4}$$

- So far we focused on S_4 - the semiclassical bounce

Quantum fluctuations

- Total decay rate in *D=4* — Callan, Coleman '77

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large l infinites cancel

- So far we focused on S_4 - the semiclassical bounce
- Prefactor typically quite involved, few analytics (TW) — Konoplich '86
- Precise numerical calculation — Dunne, Min '05

spherical symmetry
= angular separation

$$\mathcal{O}_l = -\frac{d^2}{d\rho^2} - \frac{3}{\rho}\frac{d}{d\rho} + \frac{l(l+1)}{\rho^2} + V''(\rho) + 1$$

Quantum fluctuations

$$\frac{\det \mathcal{O}_l}{\det \mathcal{O}_l^{\text{free}}} = \mathcal{R}_l(\rho = \infty)^{(l+1)^2}$$

Gel'fand, Yaglom '59

$$\mathcal{R}_l(\rho) = \frac{\psi_l(\rho)}{\psi_l^{\text{free}}(\rho)}$$

ratio of $\mathcal{O}_l$ eigenfunctions

- angular momentum threshold

$l < L$

$$-\ln \Gamma_{\text{lo}} = \frac{1}{2} \sum_{l=0}^{L} (l+1)^2 \ln |\mathcal{R}_l(\infty)|$$

direct calculation, straightforward integration, possibly analytical with PB? (open issue)

$l > L$

$$-\ln \Gamma_{\text{hi}} = -\frac{(L+1)(L+2)}{8} \mathcal{I}_1 + \frac{\ln L}{16} \mathcal{I}_2 - \frac{\mathcal{I}_2 + \mathcal{I}_3}{16}$$

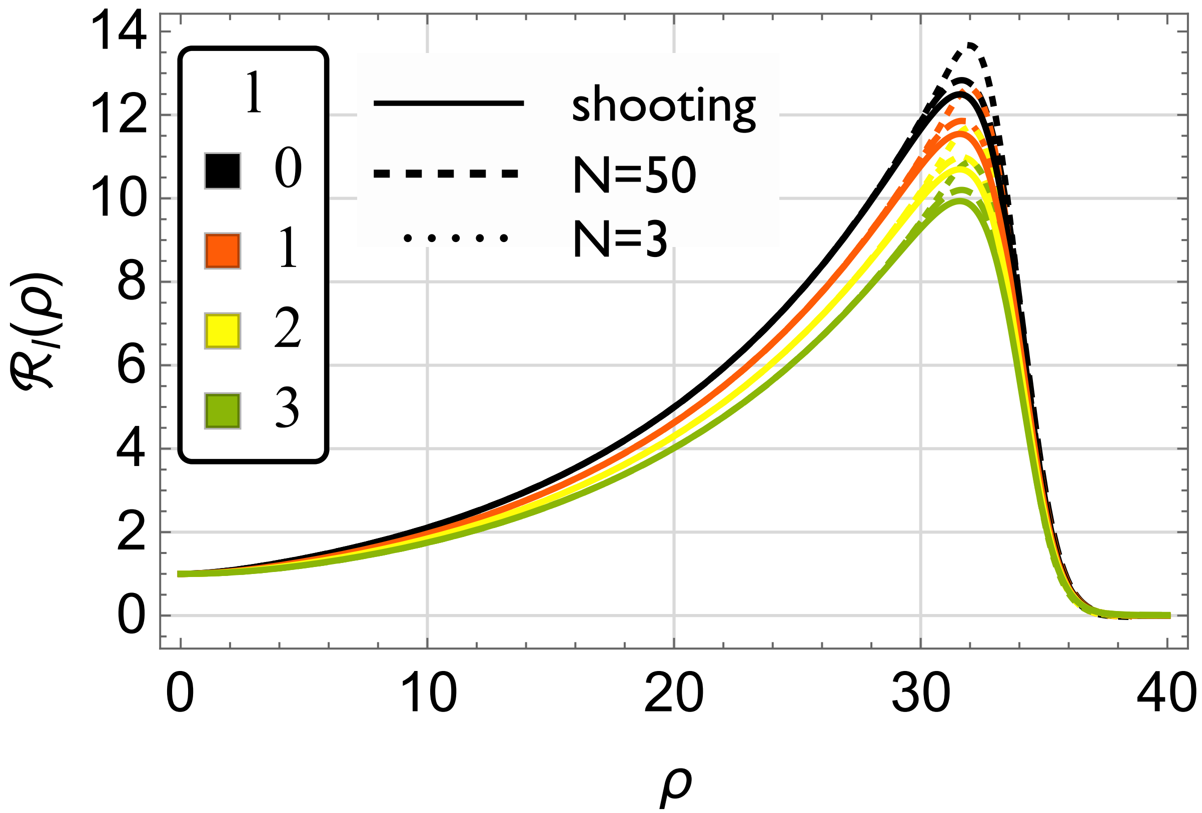
WKB approximation

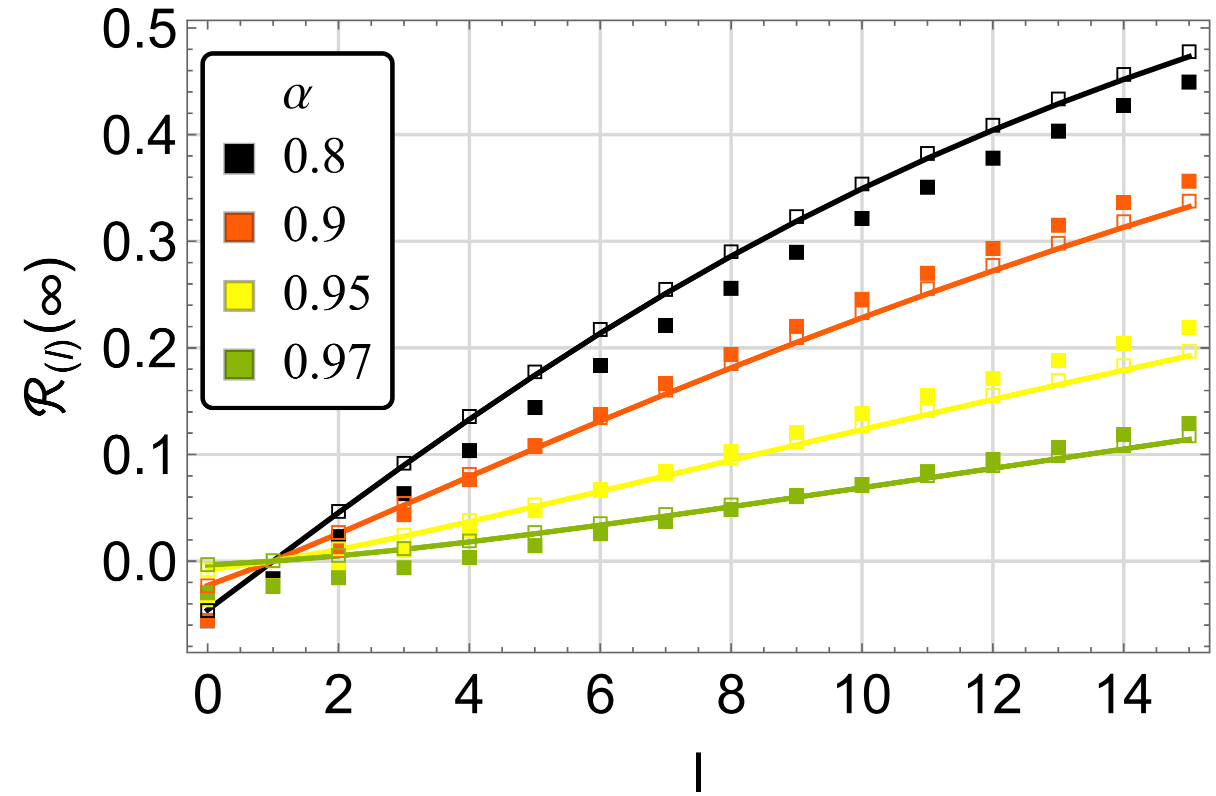
cancels the δ_4 counterterm

computed analytically with the PB approach

Quantum fluctuations

$$V''(\rho) = -3\varphi(\rho) + \frac{3\alpha}{2}\varphi^2(\rho)$$





$\alpha \to 1$ approaches thin wall, PB works well

several negative eigenvalues - imprecise bounce

nevertheless, a fairly good approximation

α	shooting	$N=3$	$N=10$	$N=50$	$N=100$
0.8	0.36	0.30	0.31	0.31	0.30
0.9	0.30	0.24	0.27	0.27	0.28
0.95	0.24	0.20	0.22	0.23	0.23
0.97	0.22	0.18	0.20	0.21	0.21

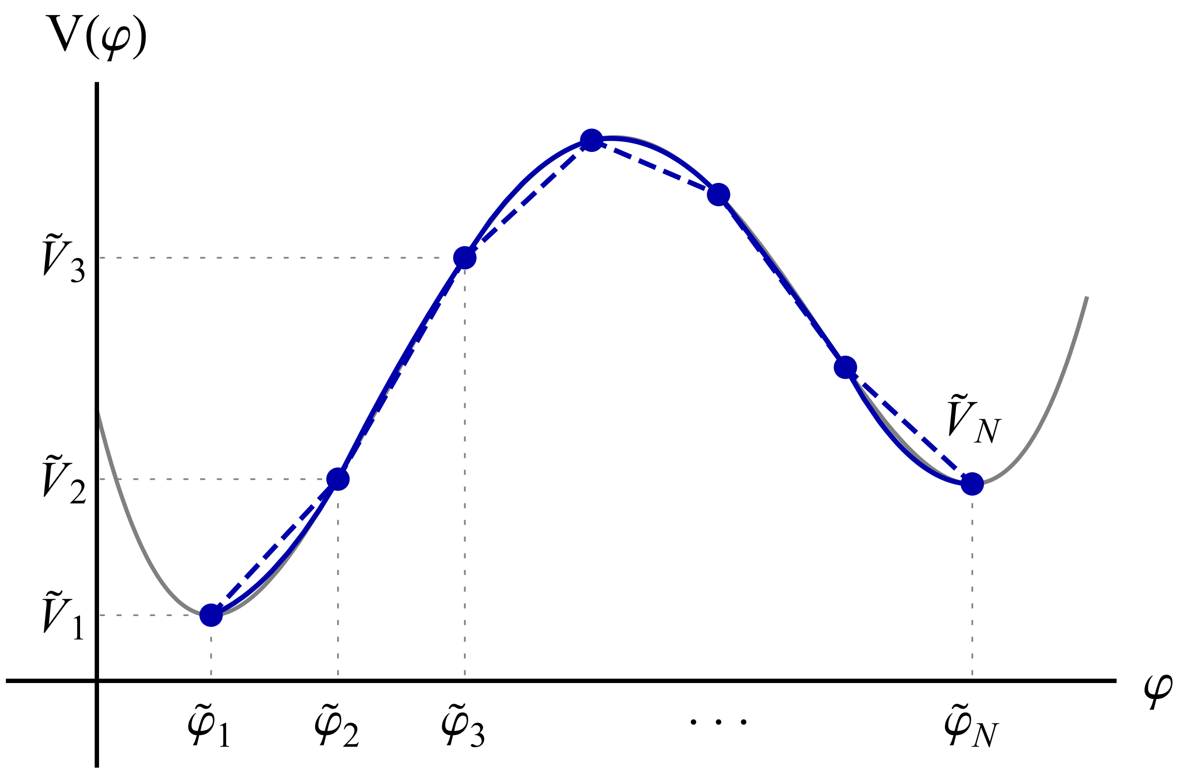
thin wall: $\frac{9}{32}\left(1 - \frac{2\pi}{9\sqrt{3}}\right) \sim 0.17$ Konoplich '86

BACK

TO

semi CLASSICS

Higher orders



Expand to higher orders

- improves convergence
- important @ extrema

$$V_i \simeq \tilde{V}_i - \tilde{V}_N + \partial\tilde{V}_i\left(\varphi_i - \tilde{\varphi}_i\right)$$

$$+\frac{\partial^2\tilde{V}_i}{2}\left(\varphi_i - \tilde{\varphi}_i\right)^2$$

Perturbative expansion $\varphi = \varphi_{PB} + \xi$

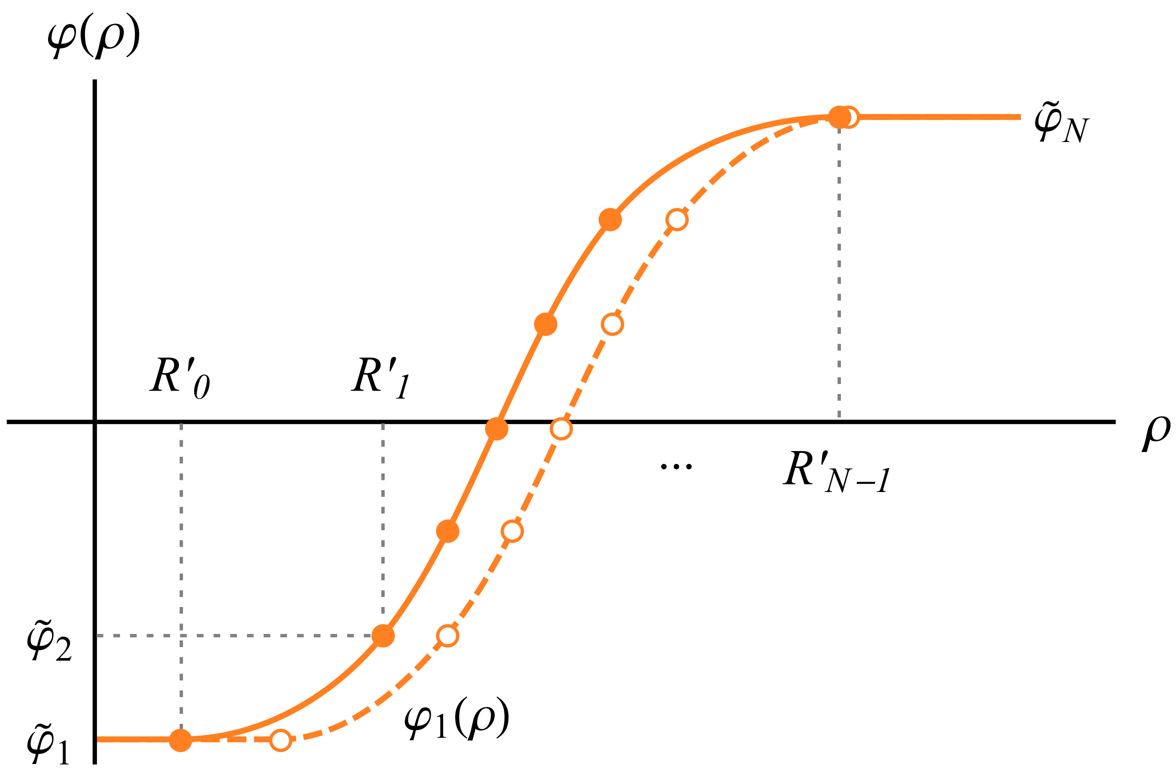
$$\ddot{\varphi} + \frac{D-1}{\rho}\dot{\varphi} = 8\left(a + \alpha\right) + \delta dV\left(\varphi_{PB}(\rho)\right)$$

$$\ddot{\xi} + \frac{D-1}{\rho}\dot{\xi} = 8\alpha + \delta dV(\rho)$$

$$\xi = \nu + \frac{2}{D-2}\frac{\beta}{\rho^{D-2}} + \frac{4}{D}\alpha\rho^2 + \mathcal{I}(\rho)$$

$$\mathcal{I}_s^{D=4} = \partial^2\tilde{V}_s\left(\frac{v_s - \tilde{\varphi}_s}{8}\rho^2 + \frac{b_s}{2}\ln\rho + \frac{a_s}{24}\rho^4\right)$$

Higher orders



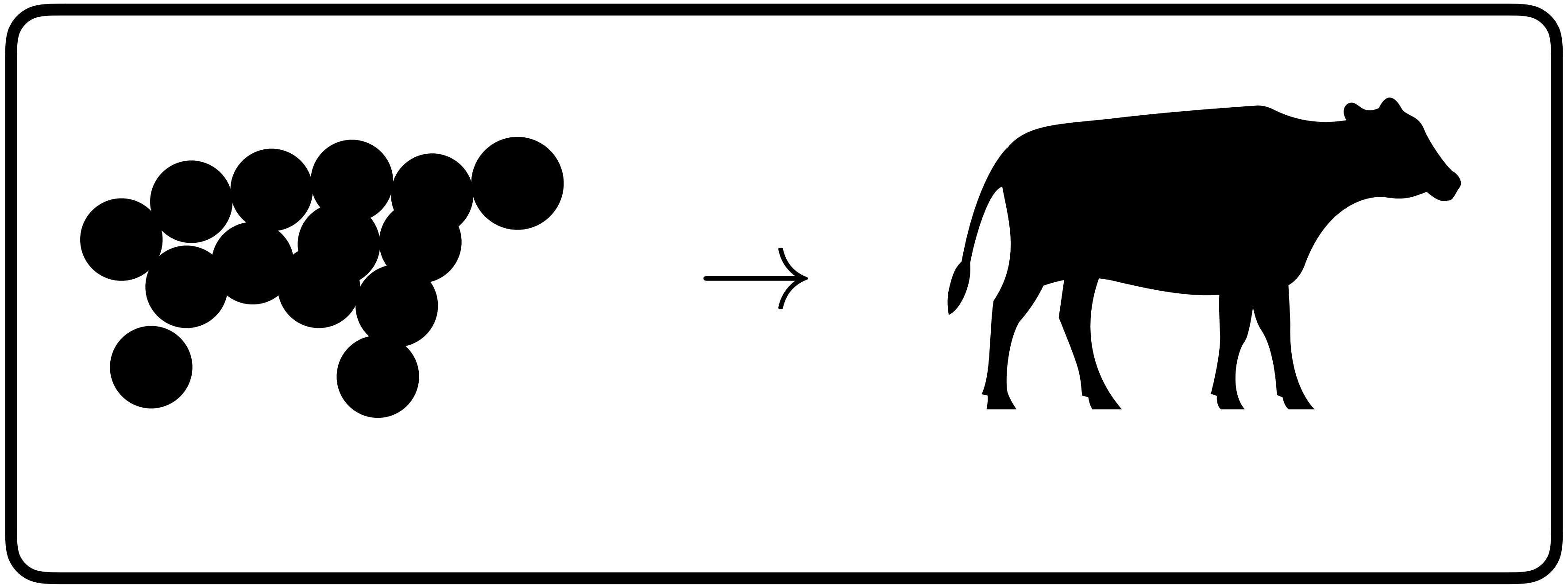
Match at perturbed radii

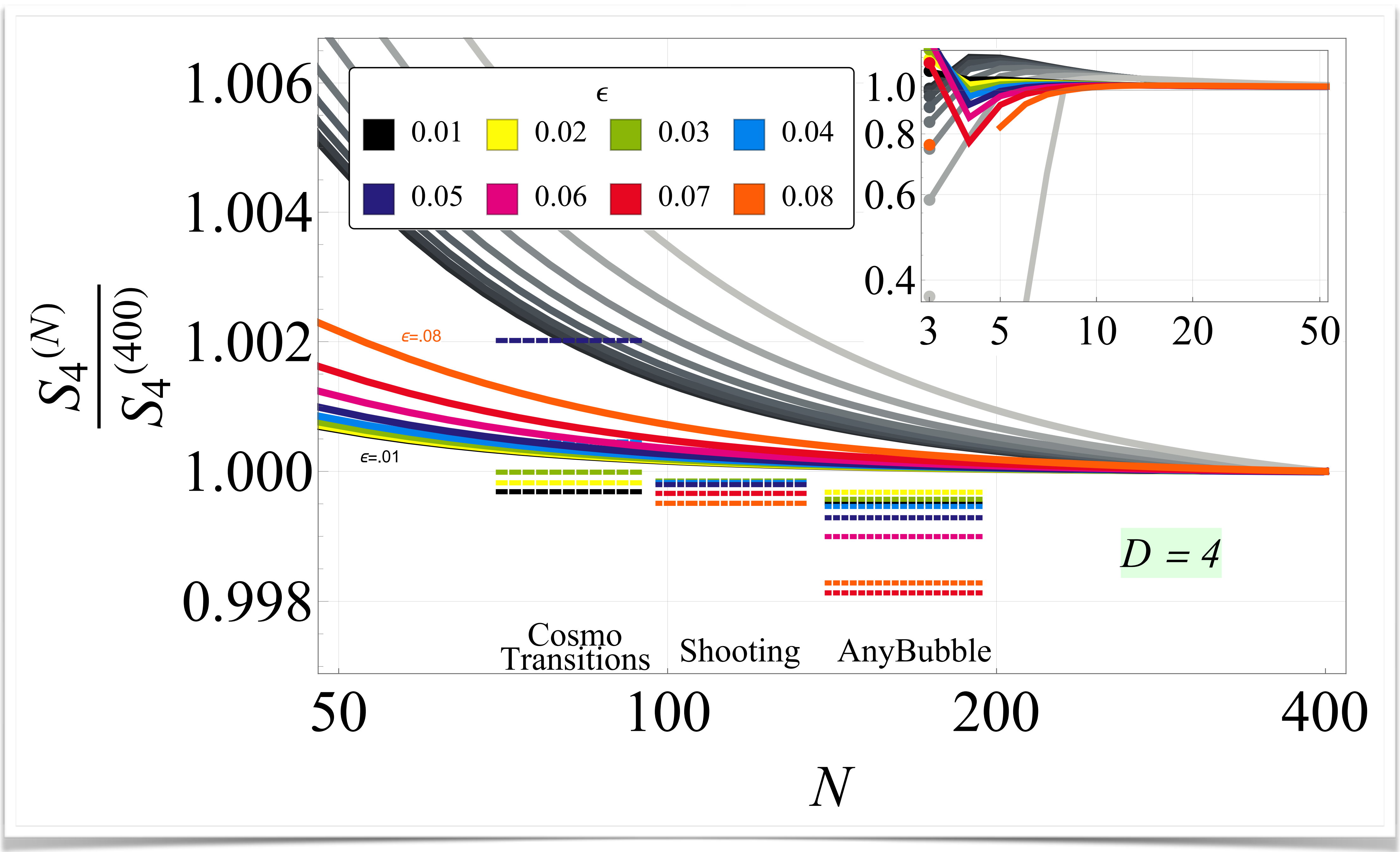
$$R_s \to R_s\,(1 + r_s)\,, \qquad r_s \ll 1$$

Rederive the matching conditions

$$r_s = \frac{\beta_s + \frac{D-2}{2}\left(\nu_s + \mathcal{I}_s + \frac{4}{D}\alpha_s R_s^2\right) R_s^{D-2}}{(D-2)\left(b_s - \frac{4}{D} a_s R_s^D\right)}$$

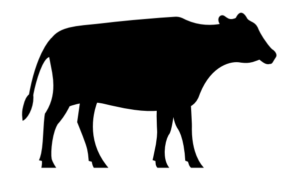
A single linear equation = very fast

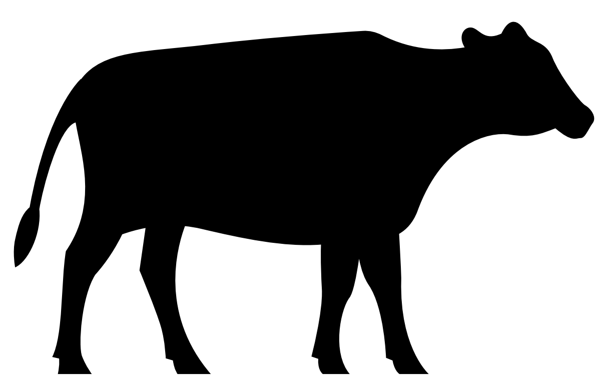


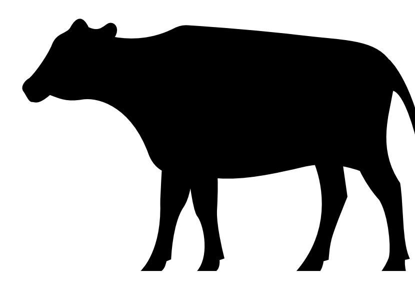


ϵ
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
ϵ=.08
ϵ=.01
1.006
1.004
1.002
1.000
0.998
$\frac{S_4(N)}{S_4(400)}$
Cosmo Transitions
Shooting
AnyBubble
D = 4
50
100
200
400
N
1.0
0.8
0.6
0.4
3
5
10
20
50

MULTI-FIELD BOUNCES





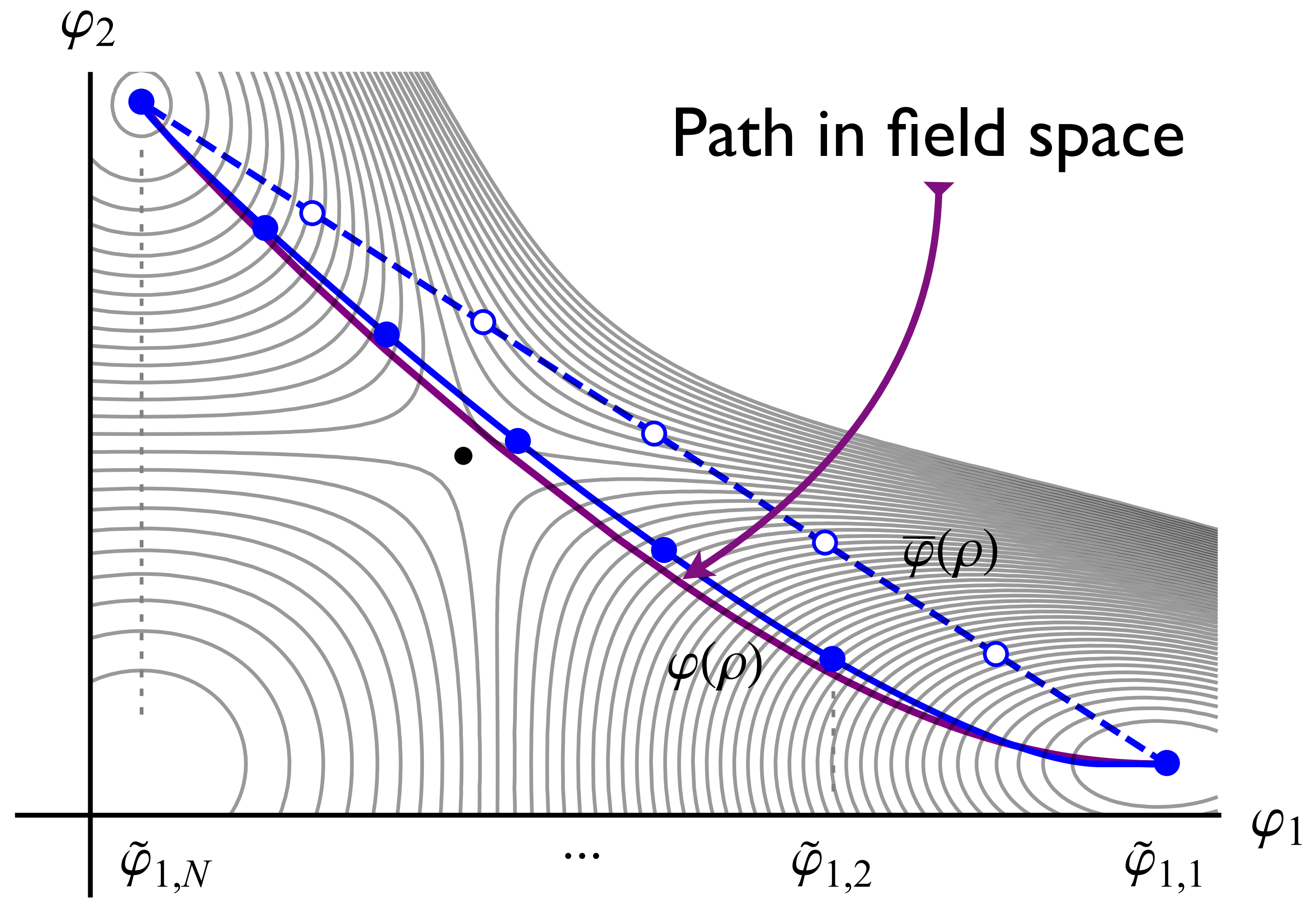


Multi-fields

$$\ddot{\varphi}_i + \frac{D-1}{\rho}\dot{\varphi}_i = \frac{dV}{d\varphi_i} \qquad \varphi_i(0) = \varphi_{0i}$$

Shooting method impractical

- highly non-linear
- multi-dimensional field space



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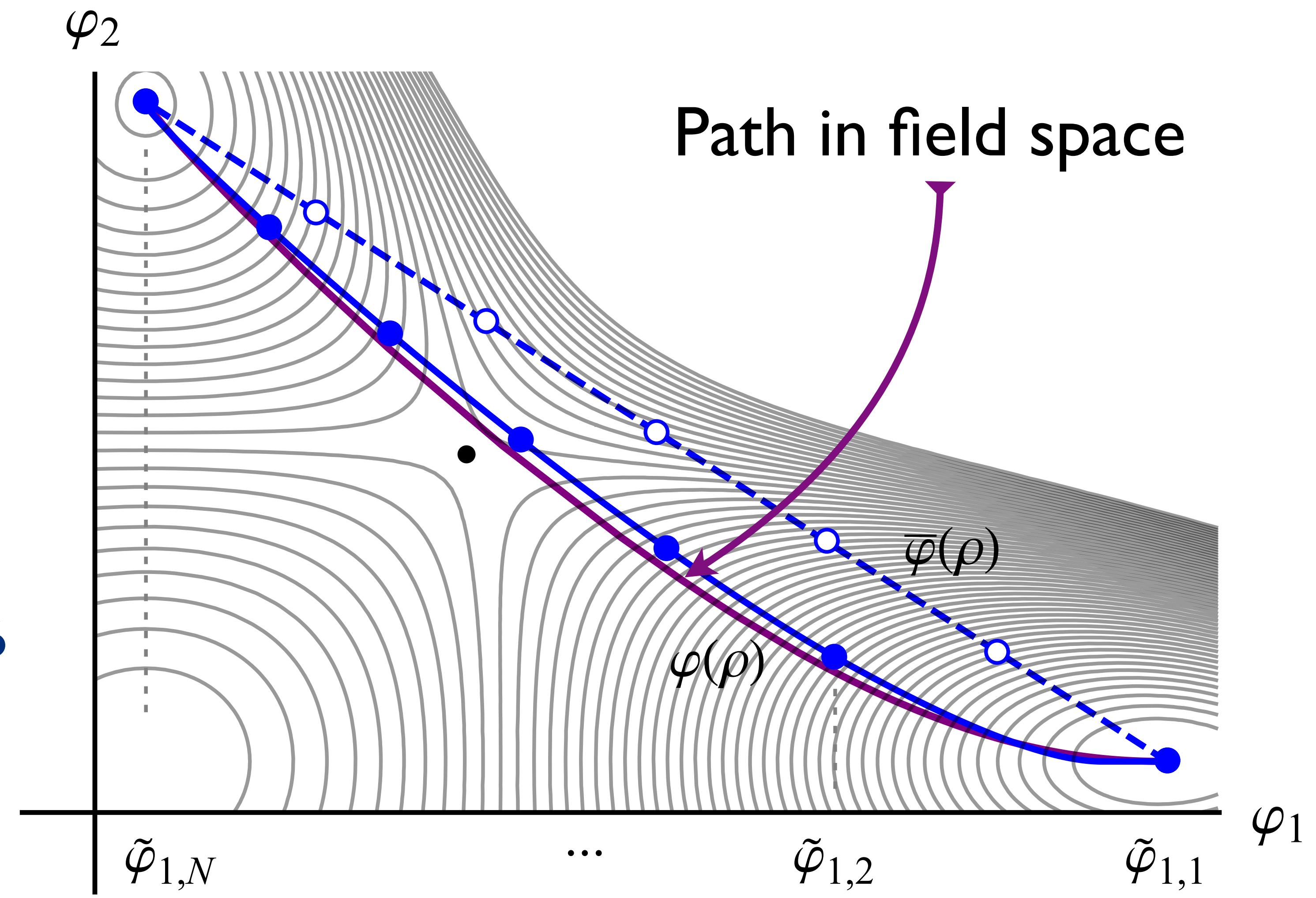


- **CosmoTransitions** Wainwright '11

  bounce and path deformation separate, oscillations, Runge-Kutta PDE solver

- **AnyBubble** Masoumi, Olum, Shlaer '16

  multiple shooting, damping approximations

- **Other recent approaches**

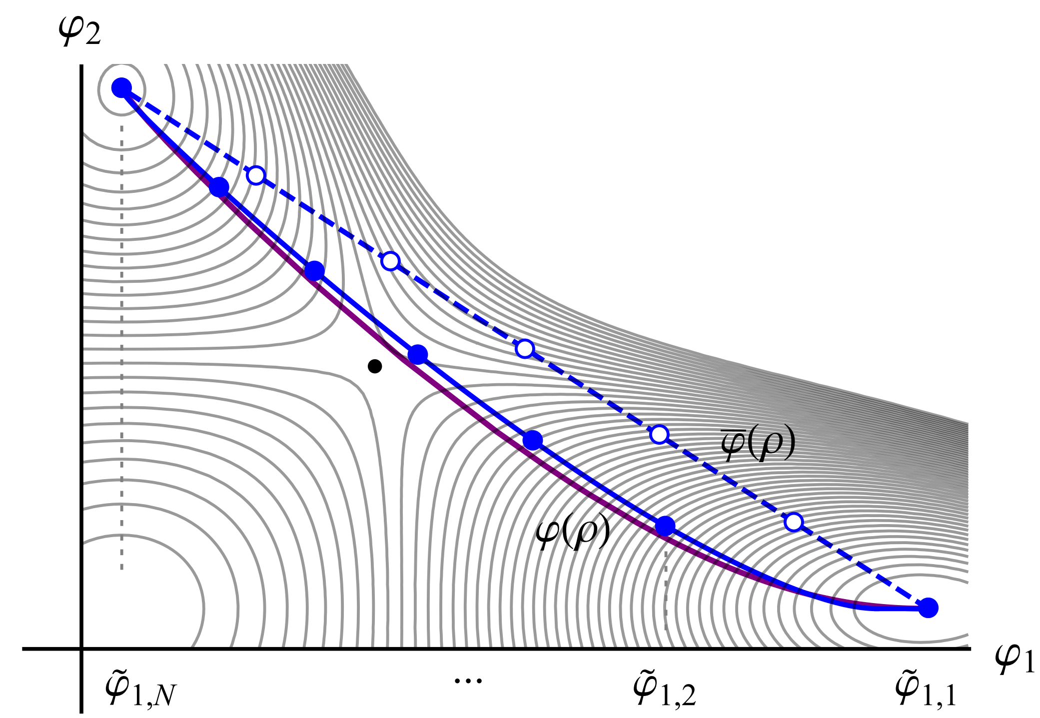
  tunneling potential — Espinosa, Konstandin '18

  machine learning — Piscopo, Spannowsky, Waite '19

Multi-fields

Polygonal approach with many fields

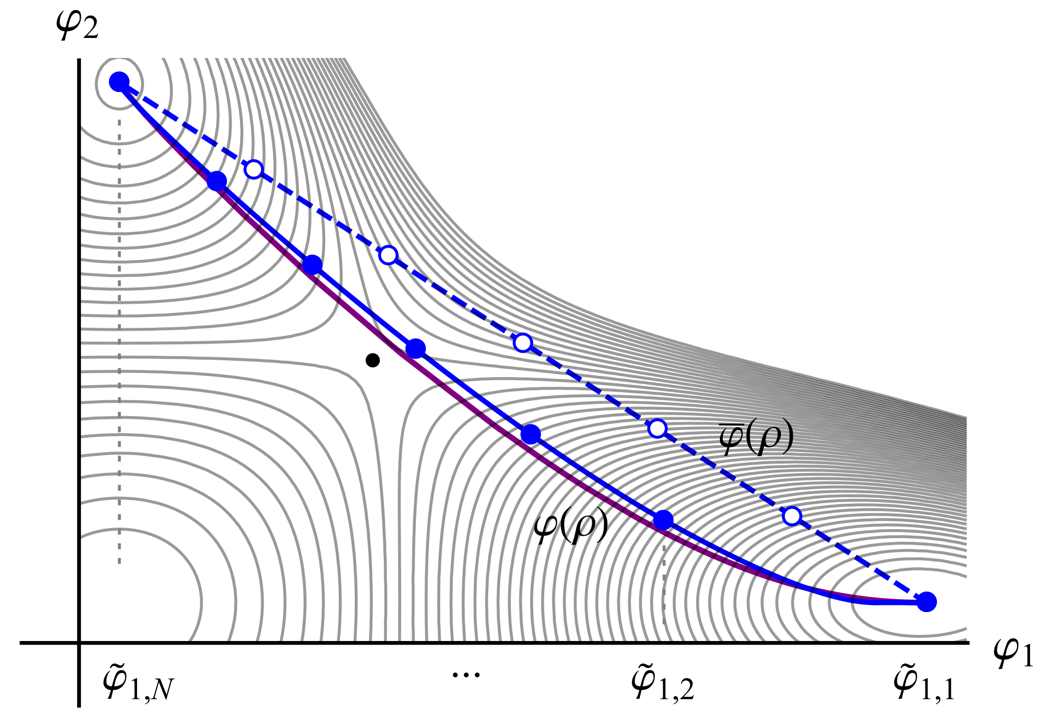
- Initial ansatz straight line, via saddle, custom segmentation $\tilde{\varphi}_{is}$
- Initial solution longitudinal single field PB $\bar{\varphi}_{is}(\rho)$



Multi-fields

Polygonal approach with many fields

- Initial ansatz — straight line, via saddle, custom segmentation — $\tilde{\varphi}_{is}$
- Initial solution — longitudinal single field PB — $\bar{\varphi}_{is}(\rho)$



Crucial idea #1

- perturbation up to linear term in V, keeps the PB

$$\underbrace{\ddot{\bar{\varphi}}_{is} + \frac{D-1}{\rho}\dot{\bar{\varphi}}_{is}}_{8\bar{a}_{is}} + \underbrace{\ddot{\zeta}_{is} + \frac{D-1}{\rho}\dot{\zeta}_{is}}_{8a_{is}} = \frac{dV}{d\varphi_i}(\bar{\varphi}+\zeta)$$

$$\zeta_{is} = v_{is} + \frac{2}{D-2}\frac{b_{is}}{\rho^{D-2}} + \frac{4}{D}a_{is}\rho^2$$

Multi-fields

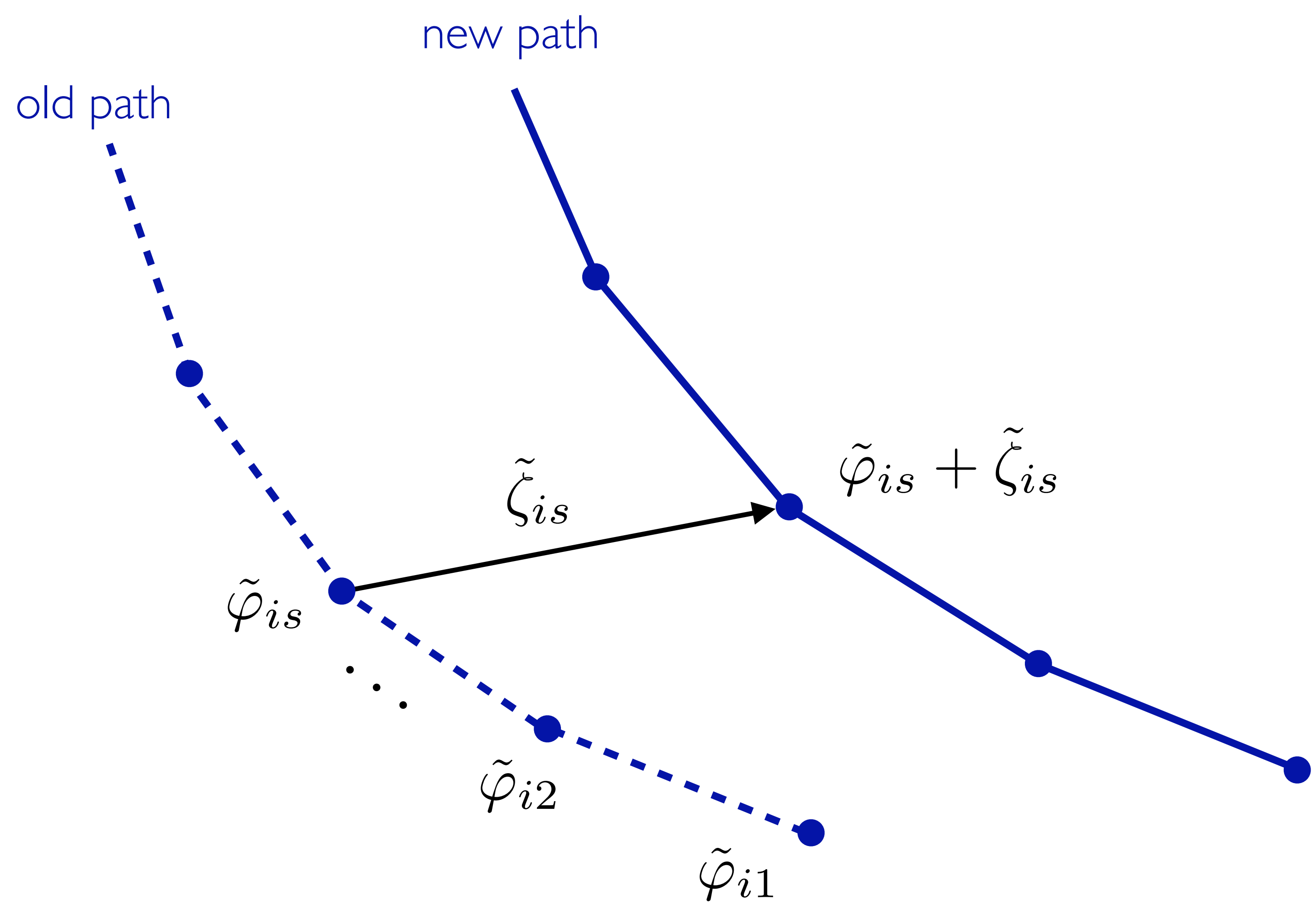
$$\underbrace{\ddot{\bar{\varphi}}_{is} + \frac{D-1}{\rho}\dot{\bar{\varphi}}_{is}}_{8\bar{a}_{is}} + \underbrace{\ddot{\zeta}_{is} + \frac{D-1}{\rho}\dot{\zeta}_{is}}_{8a_{is}} = \frac{dV}{d\varphi_i}\left(\bar{\varphi} + \zeta\right)$$

$$8a_{is} \simeq \frac{dV}{d\varphi_i}\left(\tilde{\varphi}_{is} + \tilde{\zeta}_{is}\right) - 8\bar{a}_{is}$$

Crucial idea #2

$$\frac{dV}{d\varphi_i} \simeq \frac{1}{2}\left(d_i\tilde{V}_s + d_i\tilde{V}_{s+1} + d^2_{ij}\tilde{V}_s\tilde{\zeta}_{js} + d^2_{ij}\tilde{V}_{s+1}\tilde{\zeta}_{js+1}\right)$$

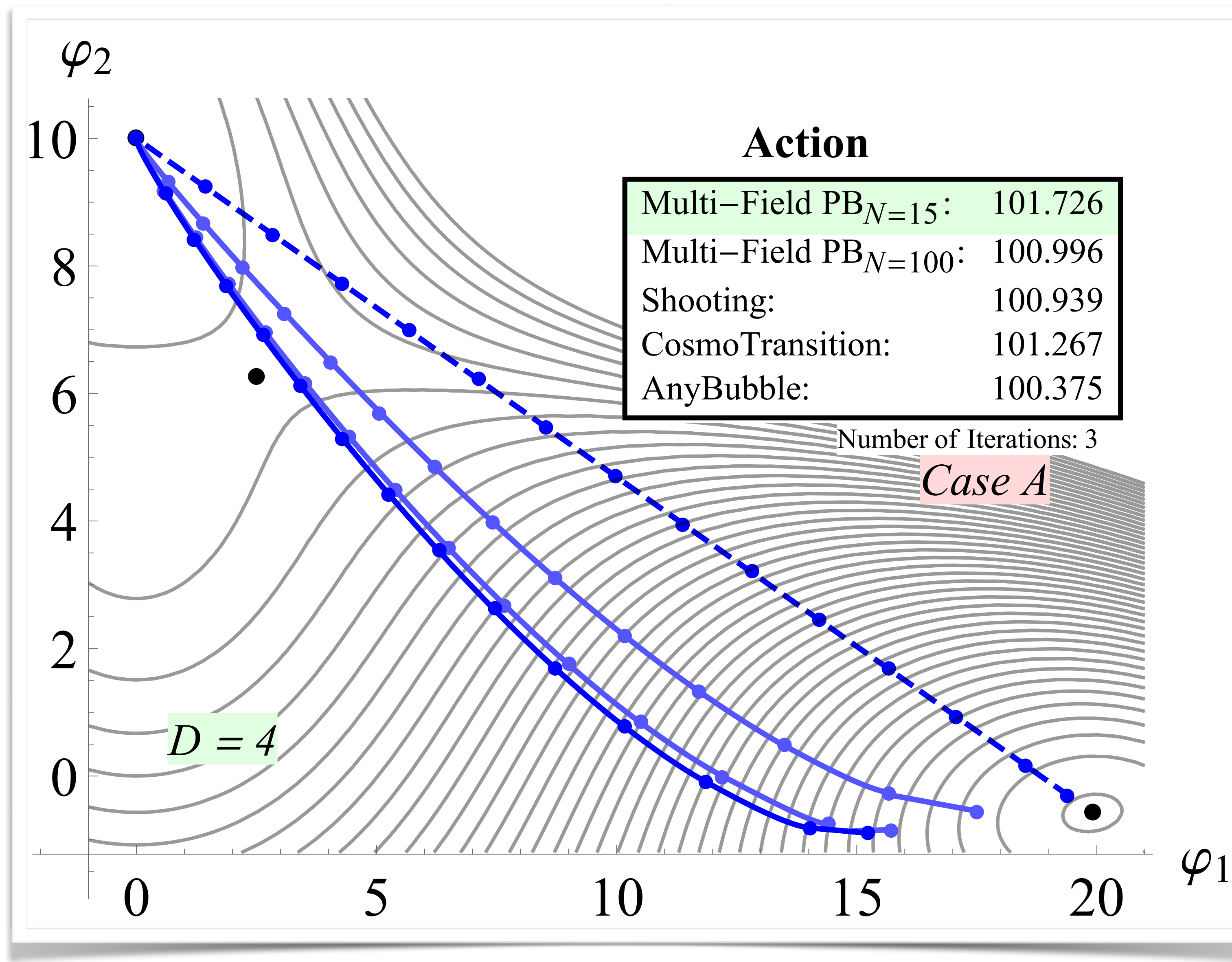
- simultaneous solution for the bounce and path deformation
- linear system for r_{i0} (as in the single field expansion) and $\tilde{\zeta}_{is}$
- iterate until $\tilde{\zeta}_{is} < \varepsilon_{\Delta\varphi}$



Multi-fields

$$V(\varphi_i) = \sum_{i=1}^{2} \left(-\mu_i^2 \varphi_i^2 + \lambda_i^2 \varphi_i^4\right) + \lambda_{12} \varphi_1^2 \varphi_2^2 + \tilde{\mu}^3 \varphi_2$$

- no oscillations
- converges in a few iterations
- works for thin wall
- works for *D=3* and *4*
- tested for up to 20 fields



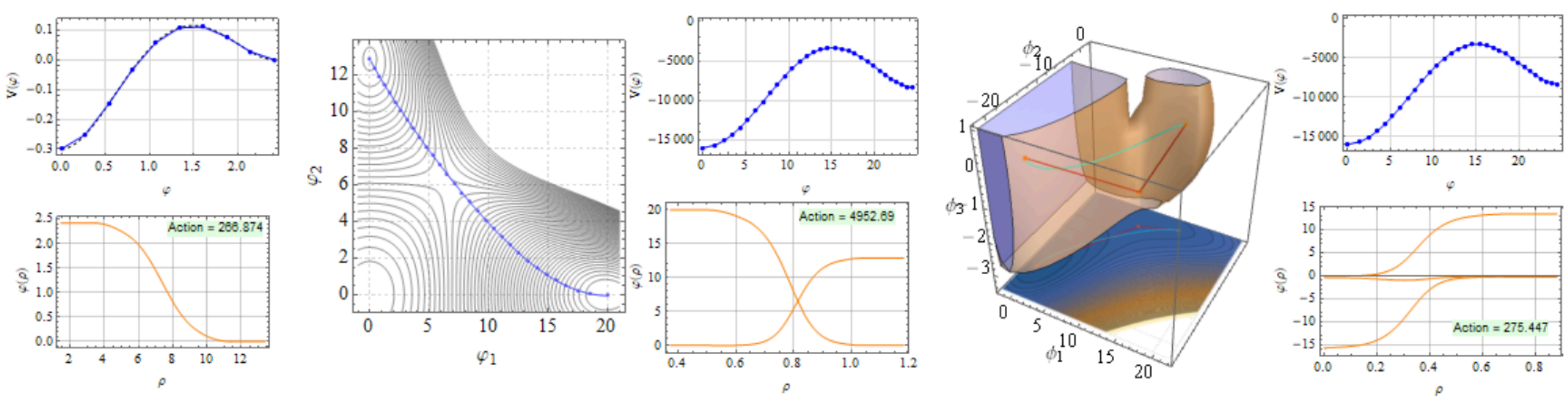
FindBounce

~ Mathematica package for fast multifield bounce evaluation ~

README.md

FindBounce

FindBounce is a Mathematica package that computes the bounce configuration needed to compute the false vacuum decay rate with multiple scalar fields.
The physics background is described in the paper by Guada, Maiezza and Nemevšek (2019).



Installation

To use the *FindBounce* package you need Mathematica version 10 or later. The package is released in the `.paclet` file format that contains the code, documentation and other necessary resources. Download the latest `.paclet` file from the repository "releases" page to your computer and install it by evaluating the following command in the Mathematica:

```
(* This built-in package is usually loaded automatically at kernel startup. *)
Needs["PacletManager`"]

(* Path to .paclet file downloaded from repository "releases" page. *)
PacletInstall["full/path/to/FindBounce-X.Y.Z.paclet"]
```

Usage

After installing the paclet, load it in the Mathematica session with `Needs`. To access the documentation, open the notebook interface help viewer and search for "FindBounce".

```
Needs["FindBounce`"]
```

Coming soon...

To begin, let us define a single field potential, find its extrema and plot it.

```
potential[x_] := 0.5 x^2 - 0.5 x^3 + 0.1 x^4;

extrema = Block[{x}, x /. NSolve[D[potential[x], x] == 0, x]]
(* {0., 0.867218, 2.88278} *)
```

```
bf = FindBounce[potential[x], {x}, { extrema[[1]], extrema[[3]] }]
(* BounceFunction[...]*)

bf["Action"]
(* 1515.5 *)

bf["Dimension"]
(* 4 *)

bf["Properties"]
(* {"Action", "Coefficients", "Dimension", "Domain", "InitialSegment",  "Path", "Potential", "Radii",...} *)
```

The field configuration can also be easily plotted.

```
BouncePlot[bf]
```

Closing remarks

Conclusions

Polygonal bounces give a quick, reliable and robust result for FV decay

Algebraical manipulation of Euclidean radius, analytical control

Solved by different methods, easy to deform & extend

Useful for multiple scalar fields

Outlook

Package to appear, applications to thermal field theory and gravitational waves

Additional insight into quantum corrections (theory of A), gravity?

Thank you