Wrapped Branes in Romans $F(4)$ Gauged Supergravity

Myungbo SHIM

Department of Physics
Kyung Hee University, Seoul, Korea

Humboldt Kolleg Frontiers in Physics, Corfu

mbshim1213@khu.ac.kr

September 17, 2019

Based on arXiv:1909.01534 [hep-th]

In collaboration with Nakwoo Kim

Overview

Wrapped Branes in Superstring/M-theory

A class of AdS solutions from wrapped branes

NS5, D5 branes wrapped on S^2 : Dual to 4d $\mathcal{N} = 1$ SYM as IR fixed point [Maldacena, Nunez 2000]

Criteria for permissible singularities in gravity sides [Maldacena, Nunez 2000, Gubser 2000]

M5 branes wrapped on supersymmetric cycles[Gauntlett, Kim, and Waldram 2000] and non-supersymmetric solutions[Gauntlett, Kim, Pakis, and Waldram 2002]

Review of wrapped branes in various supergravities[Naka 2002]

AdS from wrapped D3[Gauntlett, Mac Conamhna 2007]

D4-D8 wrapped on 4-cycles[Suh 2018]

Wrapped Branes and AdS/CFT

AdS_d solutions constructed by wrapped branes give us free energy on SCFT_{d-1}

Wrapped p-Branes with Different Topology

Consider the Branes wrapping on supersymmetric(calibrated) cycles

$$ds^2_{11/10} = ds^2_{AdS_{(p)+2-k}} + \underbrace{\# ds^2_{M_k} + \# ds^2_{X_{9/8-p}}}_{\text{Sasaki-Einstein Manifold}}$$

Calibration, Calibrated Cycles

Definition of Calibration

$\phi \in \Lambda^p$ satisfies $d\phi = 0, \int_{\xi_p} \phi \leq \int_{\xi_p} *1$

Definition of Calibrated Cycle

$Vol(M_p) = \int_{M_p} \phi \rightarrow Vol(\Sigma_p) \leq Vol(\Sigma'_p)$: Minimal cycles in manifolds

Manifolds of special holonomy have calibrated cycles

Construction by Killing spinor guarantees $d\phi = 0$

Calibrated Cycles in Manifolds of Special Holonomy

Calibrated Cycles in Manifolds of Special Holonomy

Calibration	*Manifold*	*Cycles*
Associative 3 form	G_2 *manifolds*	*Associative 3 cycle*
Co-associative 4 form	G_2 *manifolds*	*Co-associative 4 cycle*
Cayley 4 form	*Spin*(7) *manifolds*	*Cayley 4 cycle*
$\frac{1}{n!} J^n$	CY_N	*Kähler 2n cycle*
Holomorphic n form	CY_N	*SLAG n cylce*

Romans $F(4)$ Gauged Supergravity

Realization of $F(4)$ Superalgebra with 16 Supercharges [Romans 1985]

6d gauged supergravity with SU(2) gauge group. This theory is non-chiral, but there's various chiral versions of this theory.

Bosonic Action with Gravity, Gauge Fields, and Scalar

$$
\begin{aligned}
\mathcal{S}_{F(4)} = & \frac{1}{2\kappa_6^2} \int d^6x \sqrt{-g}\, [\frac{1}{4}R - \frac{1}{2}\partial_\mu \phi \partial^\mu \phi \\
& + \frac{1}{8}(g^2 e^{\sqrt{2}\phi} + 4gme^{-\sqrt{2}\phi} - m^2 e^{-3\sqrt{2}\phi}) \\
& - \frac{1}{4}e^{-\sqrt{2}\phi}(\mathcal{H}_{\mu\nu}\mathcal{H}^{\mu\nu} + F^I_{\mu\nu}F^{I\mu\nu}) - \frac{1}{12}e^{2\sqrt{2}\phi}G_{\mu\nu\rho}G^{\mu\nu\rho} \\
& - \frac{1}{8}\epsilon^{\mu\nu\rho\sigma\tau\kappa}B_{\mu\nu}(\mathcal{F}_{\rho\sigma}\mathcal{F}_{\tau\kappa} + mB_{\rho\sigma}\mathcal{F}_{\tau\kappa} + \frac{1}{3}m^2 B_{\rho\sigma}B_{\tau\kappa} + F^I_{\rho\sigma}F^I_{\tau\kappa})]
\end{aligned}
$$

Fermionic Degrees

Supersymmetry of the Theory

Fermions are a Symplectic-Majorana Spinor
Killing Spinor Equations: $\delta\psi = \delta\chi = 0$

Supersymmetric Transformations with the Mostly Positive Metric

$$
\begin{aligned}
\delta\psi_{\mu i} =& \partial_\mu \epsilon_i + \frac{1}{4}\omega_{\mu\nu\rho}\gamma^{\nu\rho}\epsilon_i + gA_\mu^{\hat{I}}(T^{\hat{I}})_i{}^j\epsilon_j + \frac{i}{8\sqrt{2}}(ge^{-\frac{\phi}{\sqrt{2}}} + me^{-3\frac{\phi}{\sqrt{2}}})\gamma_\mu\gamma_7\epsilon_i \\
&- \frac{i}{4\sqrt{2}}(\gamma_\mu{}^{\nu\rho} - 6\delta_\mu{}^\nu\gamma^\rho)e^{-\frac{\phi}{\sqrt{2}}}\gamma_7 F_{\nu\rho}^{\hat{I}} T_i^{\hat{I}\,j}\epsilon_j \\
&+ \frac{i}{8\sqrt{2}}e^{-\frac{\phi}{\sqrt{2}}}\mathcal{H}_{\nu\rho}(\gamma_\mu{}^{\nu\rho} - 6\delta_\mu{}^\nu\gamma^\rho)\epsilon_i - \frac{1}{24}e^{\sqrt{2}\phi}G_{\nu\rho\sigma}\gamma_7\gamma^{\nu\rho\sigma}\gamma_\mu\epsilon_i, \\
\delta\chi_i =& \frac{i}{\sqrt{2}}\gamma^\mu\partial_\mu\phi\epsilon_i - \frac{1}{4\sqrt{2}}(ge^{-\frac{\phi}{\sqrt{2}}} - 3me^{-3\frac{\phi}{\sqrt{2}}})\gamma_7\epsilon_i \\
&+ \frac{1}{2\sqrt{2}}e^{-\frac{\phi}{\sqrt{2}}}\gamma^{\nu\rho}\gamma_7 F_{\nu\rho}^{\hat{I}} T_i^{\hat{I}\,j}\epsilon_j - \frac{1}{4\sqrt{2}}e^{-\frac{\phi}{\sqrt{2}}}\mathcal{H}_{\nu\rho}\gamma^{\nu\rho}\epsilon_i + \frac{i}{12}G_{\mu\nu\rho}\gamma_7\gamma^{\mu\nu\rho}\epsilon_i.
\end{aligned}
$$

Embedding in Superstring/M-theory

Embedding in 10D

Massive Type IIA embedding [Cvetic, Lu, and Pope 1999]

Type IIB embedding [Jeong, Kelekci, and Colgain 2013, Hong, Liu, and Mayerson 2018, Apruzzi and Fazzi 2018]

Exceptional field theory formalism [Malek, Samtleben, and Camell 2018/19]

Parameters

$e^{-2\sqrt{2}\phi} = g/(3m)$ for supersymmetric vacua, $m_{10d} = \sqrt{2}m$ for 10d Romans mass.

Embedded in Massive IIA Superstring Theory

Wrapped D4-D8 Brane System

D4-D8 brane systems	$\leftrightarrow$	6D SUGRA
D4-D8 Worldvolume Theory[Brandhuber-Oz 1999]	$\leftrightarrow$	AdS_6
on SLAG 2 cycles[Naka 2002]	$\leftrightarrow$	$AdS_4 \times M_2$
on SLAG 3 cycles[Naka 2002]	$\leftrightarrow$	$AdS_3 \times M_3$
on Cayley and Kähler 4 cycles[Suh 2018]	$\leftrightarrow$	$AdS_2 \times M_4$
on two Riemann surfaces[Suh 2018]	$\leftrightarrow$	$AdS_2 \times \Sigma_{g_1} \times \Sigma_{g_2}$

Holographic RG Flow

IR AdS_2, AdS_3, AdS_4 fixed point solutions have RG flow connected to UV AdS_6 solution.

Ansatz for BPS Solutions

Metric Ansatz

$$ds_6^2 = e^{2f}\left(-dt^2 + dr^2 + \sum_{\alpha=1}^{6-d-2} dx_\alpha^2\right) + \sum_i e^{2\lambda_i} ds^2_{M_{i,d}}$$

Gauge Field Ansatz

Cycles	$\mathcal{F}$	$F^{\hat{I}}_{\mu\nu}$	$B_{\mu\nu}$
2-Cycles	0	$F^{\hat{3}}_{45} = \frac{k\zeta}{g} e^{-2\lambda}$	0
3-Cycles	0	$F^{\hat{I}}_{\text{non-zero}} = \frac{k\zeta_I}{2g} e^{-2\lambda}$	0
Cayley 4-Cycles	0	$F^{\hat{I}}_{\text{non-zero}} = \frac{k\zeta_I}{3g} e^{-2\lambda}$	$B_{01} = -\frac{2}{3m^2g^2} e^{\sqrt{2}\phi-4\lambda}$
Kähler 4-Cycles	0	$F^{\hat{3}}_{23} = F^{\hat{3}}_{45} = \frac{k\zeta}{g} e^{-2\lambda}$	$B_{01} = -\frac{2}{m^2g^2} e^{\sqrt{2}\phi-4\lambda}$
Kähler $\Sigma_{\mathfrak{g}_1} \times \Sigma_{\mathfrak{g}_2}$	0	$F^{\hat{3}}_{23} = \frac{k_1\zeta}{g} e^{-2\lambda_1}$, $F^{\hat{3}}_{45} = \frac{k_2\zeta}{g} e^{-2\lambda_2}$	$B_{01} = -2\frac{k_1k_2}{m^2g^2} e^{\sqrt{2}\phi-2(\lambda_1+\lambda_2)}$

Topological Twist and Projection Condition

Projection Condition

$$\gamma_7 \epsilon_i = -i\gamma_r \epsilon_i$$

Topological Twist for 2, 3, 4 Cycles

2-cycles	$\omega_{45} = \zeta g A^{\hat{3}},$	$T^{\hat{3}}\epsilon = -\frac{1}{2}\zeta\gamma^{45}\epsilon$
3-cycles	$\omega_{34} = \zeta_1 g A^{\hat{1}},$	$T^{\hat{1}}\epsilon = -\frac{1}{2}\zeta_1\gamma^{34}\epsilon$
	$\omega_{53} = \zeta_2 g A^{\hat{2}},$	$T^{\hat{2}}\epsilon = -\frac{1}{2}\zeta_2\gamma^{53}\epsilon$
	$\omega_{45} = \zeta_3 g A^{\hat{3}},$	$T^{\hat{3}}\epsilon = -\frac{1}{2}\zeta_3\gamma^{45}\epsilon$

Kähler 4-cycle	$\omega_{23} \pm \omega_{45} = g\zeta A^{\hat{3}},$	$\frac{1}{2}\gamma_{23}\epsilon = \pm\frac{1}{2}\gamma_{45}\epsilon = -\zeta T^{\hat{3}}\epsilon$
Cayley cycle	$\omega_{23} \pm \omega_{45} = g\zeta_1 A^{\hat{1}},$	$\frac{1}{2}\gamma_{23}\epsilon = \pm\frac{1}{2}\gamma_{45}\epsilon = -\zeta_1 T^{\hat{1}}\epsilon$
$\gamma_{ij}^{\mp}\epsilon = 0,$	$\omega_{42} \pm \omega_{35} = g\zeta_2 A^{\hat{2}},$	$\frac{1}{2}\gamma_{42}\epsilon = \pm\frac{1}{2}\gamma_{35}\epsilon = -\zeta_2 T^{\hat{2}}\epsilon$
$(i, j = 2, \cdots, 5)$	$\omega_{34} \pm \omega_{52} = g\zeta_3 A^{\hat{3}},$	$\frac{1}{2}\gamma_{34}\epsilon = \pm\frac{1}{2}\gamma_{52}\epsilon = -\zeta_3 T^{\hat{3}}\epsilon$

For a Kähler 4-cycles of two Riemann surfaces, the sign is fixed.

BPS Equations for Wrapped Branes

BPS Equations

$$f'e^{-f} = -\frac{1}{4\sqrt{2}}\left[ge^{\frac{1}{\sqrt{2}}\phi} + me^{-\frac{3}{\sqrt{2}}\phi} - \sum_i \frac{\Delta_i k_i}{g} e^{-\frac{1}{\sqrt{2}}\phi - 2\lambda_i(r)}\right] + 3\Upsilon e^{\frac{1}{\sqrt{2}}\phi - \sum_i \Delta_i \lambda_i(r)}$$

$$\lambda_i' e^{-f} = -\frac{1}{4\sqrt{2}}\left[ge^{\frac{1}{\sqrt{2}}\phi} + me^{-\frac{3}{\sqrt{2}}\phi} + \sum_i \frac{\Delta_i k_i}{g} e^{-\frac{1}{\sqrt{2}}\phi - 2\lambda_i(r)}\right] - \Upsilon e^{\frac{1}{\sqrt{2}}\phi - \sum_i \Delta_i \lambda_i(r)}$$

$$\frac{\phi'}{\sqrt{2}} e^{-f} = -\frac{1}{4\sqrt{2}}\left[-ge^{\frac{1}{\sqrt{2}}\phi} + 3me^{-\frac{3}{\sqrt{2}}\phi} + \sum_i \frac{\Delta_i k_i}{g} e^{-\frac{1}{\sqrt{2}}\phi - 2\lambda_i(r)}\right] + \Upsilon e^{\frac{1}{\sqrt{2}}\phi - \sum_i \Delta_i \lambda_i(r)}$$

$\sum_i \Delta_i = d$ and Υ is zero for $d = 2, 3$, while non-zero for $d = 4$. Fixed points, *i.e.* lower-dimensional AdS spaces, arise when the radii of cycles λ_i and the scalar ϕ are constants.

Summary of All Fixed Point Solutions

Table of All Possible Fixed Point Solutions

Cycles	k	BPS solution	Non-BPS solution	Does non-BPS solution violate the BF Bound?
2-Cycles	1	X	X	-
	-1	O	O	Yes
3-Cycles	1	X	X	-
	-1	O	O	Yes
$\mathbb{H}_2 \times \mathbb{H}_2$	$(-1,-1)$	O	X	-
$S^2 \times S^2$	$(1,1)$	X	O	No
$S^2 \times \mathbb{H}_2$	$(1,-1)$	X	X	-
Kähler 4-Cycles	1	X	O	No
	-1	O	X	-
Cayley 4-Cycles	1	X	X	-
	-1	O	X	-

Solutions with 2 and 3 Cycles

Parametrization of Solutions

$$e^f = \frac{\alpha}{g}\frac{1}{r}e^{-\frac{1}{\sqrt{2}}\phi}, \qquad e^{\lambda_i} = \frac{\beta_i}{g}e^{-\frac{1}{\sqrt{2}}\phi}, \qquad \Lambda(r) = e^{-\sqrt{2}\phi}, \qquad \gamma = \Lambda^2$$

2-Cycles with $k = -1$

$$\beta^2_{BPS} = 4, \qquad \alpha^2_{BPS} = 8, \qquad \gamma_{BPS} = \frac{g}{2m}$$

$$\beta^2_{non-BPS} \approx 3.47593, \qquad \alpha^2_{non-BPS} \approx 6.61921, \qquad \gamma_{non-BPS} \approx 0.694146\frac{g}{m}$$

3-Cycles with $k = -1$

$$\beta^2_{BPS} = 3, \qquad \alpha^2_{BPS} = \frac{9}{2}, \qquad \gamma_{BPS} = \frac{2g}{3m}$$

$$\beta^2_{non-BPS} \approx 3.41324, \qquad \alpha^2_{non-BPS} \approx 5.27966, \qquad \gamma_{non-BPS} \approx 0.507683g/m$$

Solutions with 4 Cycles : Cayley and Kähler

Cayley 4-Cycles with $k = -1$

$$\beta^2_{BPS} = 8/3, \qquad \alpha^2_{BPS} = 2, \qquad \gamma_{BPS} = \frac{3}{4}\frac{g}{m}$$

Kähler 4-Cycles with $k = 1$

$$\beta^2_{non-BPS} = \frac{4}{5}\left(4 \pm \sqrt{6}\right), \quad \alpha^2_{non-BPS} = \frac{1}{5}\left(4 \pm \sqrt{6}\right), \quad \gamma_{non-BPS} = \frac{1}{4}\left(2 \mp \sqrt{6}\right)\frac{g}{m}$$

Kähler 4-Cycles with $k = -1$

$$\beta^2_{BPS} = 4, \qquad \alpha^2_{BPS} = 2, \qquad \gamma_{BPS} = \frac{g}{2m}$$

Solutions with 4 Cycles : Two Riemann Surfaces

Near Horizon of AdS_6 Black Holes

These are applicable to calculate entropy of supersymmetric black holes with $\mathbb{H}_2 \times \mathbb{H}_2$ horizon or non-supersymmetric black holes with $S_2 \times S_2$ horizon.

Two Riemann Surfaces with $k_1 = -1,\ k_2 = -1 \Leftrightarrow \mathrm{AdS}_2 \times \mathbb{H}_2 \times \mathbb{H}_2$

$$\alpha_{BPS} = \sqrt{2}, \qquad \beta_1^{BPS} = \beta_2^{BPS} = 2, \qquad \gamma_{BPS} = \frac{g}{2m},$$

Non-BPS Two Riemann Surfaces with $k_1 = 1,\ k_2 = 1 \Leftrightarrow \mathrm{AdS}_2 \times S_2 \times S_2$

$$\alpha^2 = \frac{1}{5}\left(4 \pm \sqrt{6}\right), \qquad \beta_1^2 = \beta_2^2 = \frac{4}{5}\left(4 \pm \sqrt{6}\right), \qquad \gamma = \frac{g}{4m}(2 \mp \sqrt{6})$$

RG Analysis for Wrapped Branes

Flow Equation with New Variables

$x = e^{2\lambda - 2\phi/\sqrt{2}}$ and $F = xe^{\frac{4}{\sqrt{2}}\phi}$

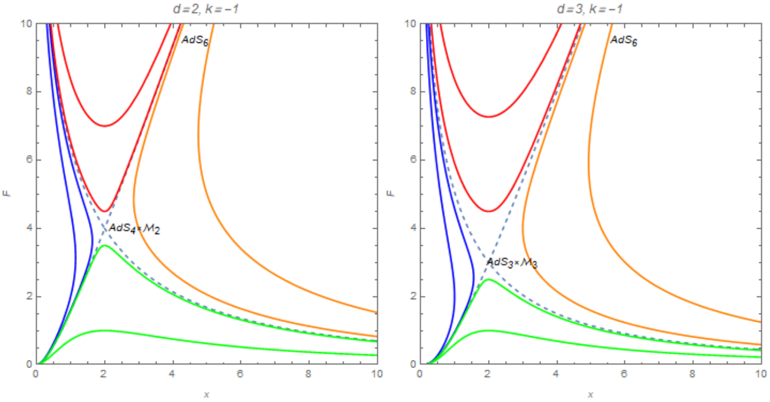
Flow Equations and UV Series

$$\frac{dF}{dx} = \frac{F(4kx + 2mgx^2)}{x[x(g^2F - mgx + (4-d)k) + 4\sqrt{2}g\Upsilon]}, \quad F = 3\frac{m}{g}x + \frac{3dk}{g^2} + \sum_{n=1}^{\infty} \mathcal{C}_n^{(F)} x^{-\frac{n}{2}}$$

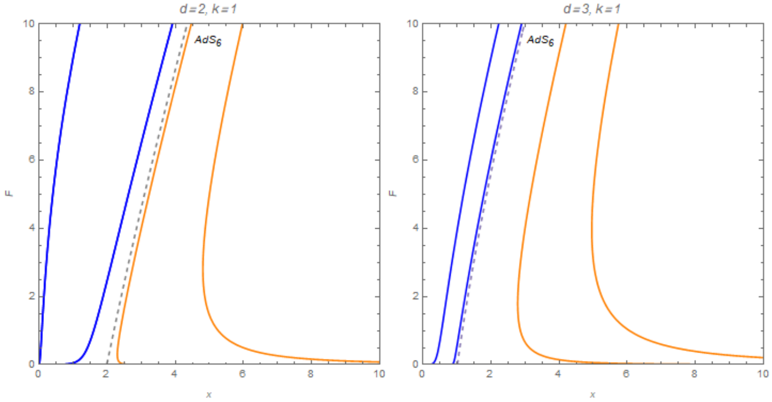
Instanton Densities for Each Cases

$$\Upsilon_{d=2,3} = 0, \qquad \Upsilon_{\mathrm{Cayley}} = -\frac{1}{3\sqrt{2}g^2m}, \qquad \Upsilon_{\mathrm{Kähler}} = -\frac{1}{\sqrt{2}g^2m}$$

Diagrams for $d = 2, 3$ and $k = -1$



Diagrams for $d = 2, 3$ and $k = 1$



Singularity Structure of the Flows in $d = 2, 3$

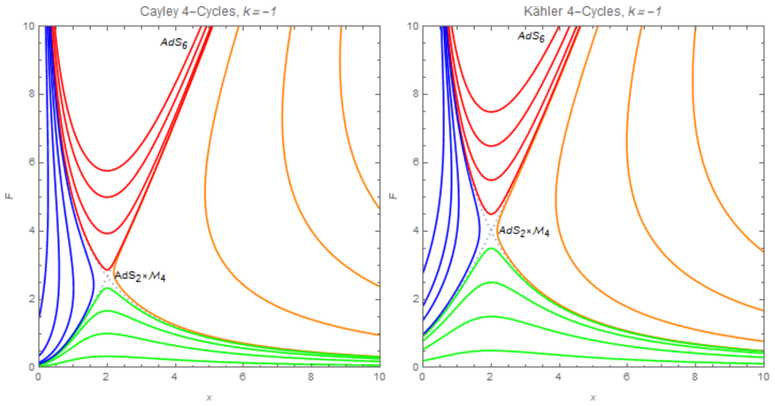
The Flows between AdS_6 and AdS_{6-d}

$c_1 = 9.1296$ for $d = 2$ and $c_1 = 13.951$ for $d = 3$

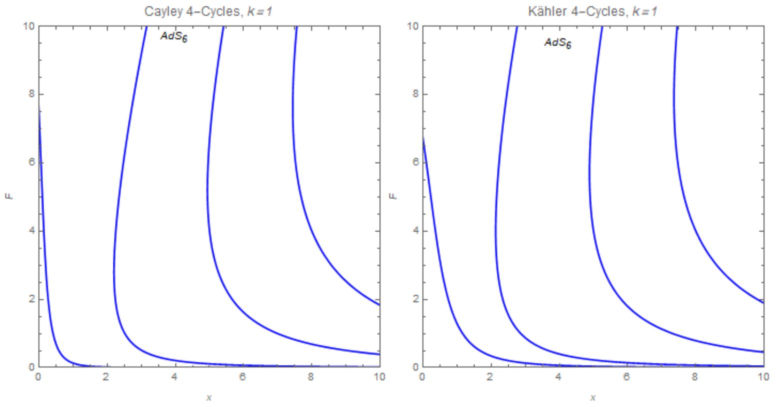
IR Singularities

k	x	F	e^{2f}	$\lvert g_{tt}^{10d} \rvert$	$V(\phi)$	Type
± 1	∞	0	0	0	∞(bad)	-
± 1	0	0	∞	∞	∞(bad)	Bad
-1	0	∞	0	0	$-\infty$	Good

Diagrams for $d = 4$ and $k = -1$



Diagrams for $d = 4$ and $k = 1$



Singularity Structure of the Flows in $d = 4$

Behavior of Solutions with 4-cycles

BPS Solutions with Cayley and Kähler 4-cycles behave in the same way. Just a fixed point slightly differs.

The Flows between AdS_6 and AdS_2

$c_1 = 23.538$ for Kähler and $c_1 = 19.7959$ for Cayley

IR Singularities

k	x	F	e^{2f}	g_{tt}^{10d}	$V(\phi)$	Type
± 1	∞	0	0	0	∞(Bad)	-
± 1	0	0	∞	∞	∞(Bad)	Bad
± 1	0	Finite	∞	∞	∞(Bad)	Bad
-1	0	∞	0	0	∞(Bad)	-

RG Analysis for Two Riemann Surfaces

New Variables

$x_1 := e^{2\lambda_1 - \sqrt{2}\phi}$, $x_2 := e^{2\lambda_2 - \sqrt{2}\phi}$, $u := e^{2\sqrt{2}\phi} x_1 x_2 = e^{2\lambda_1 + 2\lambda_2}$

Flow Equations for Two Riemann Surfaces

$$\frac{dx_1}{du} = \frac{x_1}{u}\left[\frac{g^3 mu - g^2 m^2 x_1 x_2 + 2gm(k_1 x_2 - k_2 x_1) - 4}{g^3 mu + g^2 m^2 x_1 x_2 + 2gm(k_1 x_2 + k_2 x_1) - 4}\right]$$

$$\frac{dx_2}{du} = \frac{x_2}{u}\left[\frac{g^3 mu - g^2 m^2 x_1 x_2 - 2gm(k_1 x_2 - k_2 x_1) - 4}{g^3 mu + g^2 m^2 x_1 x_2 + 2gm(k_1 x_2 + k_2 x_1) - 4}\right]$$

Relation with Kähler 4-Cycles

With $k_1 = k_2$ and $x_1 = x_2$, the equations are reduced to the case of Kähler 4-cycles.

Some Difficulties and Flow from UV

Difficulties for Analysis

Due to the parametrization, there's a line of singularities from the denominator.

What can we do?

At least, we can analyze the flow connected from AdS_6 UV by UV expansion.

$$x_1 = \sqrt{g/(3m)}\sqrt{u} - \frac{2k_1}{gm} + \sum_{n=1}^{\infty} \mathcal{C}_n^{(1)} u^{-n/4}$$

$$x_2 = \sqrt{g/(3m)}\sqrt{u} - \frac{2k_2}{gm} + \sum_{n=1}^{\infty} \mathcal{C}_n^{(2)} u^{-n/4}$$

$\mathcal{C}_1^{(1)} = \mathcal{C}_1^{(2)} = \mathcal{C}_1$ is an integration constant

Flows from AdS_6

$k_1 = k_2$: The Same as Single Cycles

One can immediately notice $x_1 = x_2$ when $k_1 = k_2$. They are the same as single Kähler 4-cycles.

$k_1 \neq k_2$: One Parameter Family of Solutions

There's one parameter, the integration constant $\mathcal{C}_1$, family of solutions, but there's no supersymmetric fixed point.

UV and IR Singularities

UV Behavior

UV Behavior of the BPS solutions with two Riemann surfaces are the same as before

Types of IR Singularities

x_1	x_2	F	e^{2f}	g_{tt}^{10d}	$V(\phi)$	Type
∞	0	0	0	0	∞(Bad)	-
0	∞	0	0	0	∞(Bad)	-
∞	∞	0	0	0	∞(Bad)	-
0	0	0	∞	∞	∞(Bad)	Bad

Entropy of Black Objects in 6 Dimensions

Non-Supersymmetric AdS Black Hole Entropy

$$S_{BH}^{non-BPS} = \frac{2(3\sqrt{6}-2)}{25g^3 m G_N^{(6)}} \times \begin{cases} 16\pi^2 & S^2 \times S^2 \text{ horizon} \\ 18\pi^2 & \mathbb{CP}^2 \text{ horizon} \end{cases}$$

Black $1, 2$-brane Entropy Density

$$s^6_{\mathrm{BS,BPS}} = \frac{2\pi R A_H^5}{\kappa_6^2} = \frac{6\sqrt{6}\pi}{g^3 m \kappa_6^2}\mathrm{Vol}(M_3),$$

$$s^6_{\mathrm{BB,BPS}} = \frac{2\pi R^2 A_H^4}{\kappa_6^2} = \frac{32\pi}{g^3 m \kappa_6^2}\mathrm{Vol}(M_2)$$

Holographic Relations with 5d Seiberg Theory

Entropy Density and Partition Functions[Bobev, Crichigno 2015]

$$s^{6}_{\mathrm{BS,BPS}} = \frac{3\sqrt{6}}{4g^3 m G_N^6}\mathrm{Vol}(M_3) \Rightarrow \frac{\sqrt{6}}{144 G_N^6}\mathrm{Vol}(M_3) = \frac{1}{6}c_{2d}$$

$$s^{6}_{\mathrm{BB,BPS}} = \frac{4}{g^3 m G_N^6}\mathrm{Vol}(M_2) \Rightarrow \frac{4\pi(\mathfrak{g}-1)}{27 G_N^6} = -\frac{4(\mathfrak{g}-1)}{9\pi}\mathcal{F}_{S^5} = \frac{\mathcal{F}_{S^3\times\Sigma_{\mathfrak{g}}}}{2\pi}$$

Field Theory Relations[Hosseini, Yaakov, and Zaffaroni 2018, Crichigno, Jain, and Willett 2018]

$$c_{2d} = -\frac{\sqrt{6}\mathrm{Vol}(\Sigma_3)}{8\pi^2}\mathcal{F}_{S^5}$$

$$\mathcal{F}_{S^3\times\Sigma_{\mathfrak{g}}} = -\frac{8(\mathfrak{g}-1)}{9}\mathcal{F}_{S^5} = \mathcal{F}_{S^3}$$

Lower Dimensional Theories

3, 4 Dimensional Theories

$$\mathcal{S}^{Ein}_{6-d} = \frac{\mathrm{Vol}(\mathcal{M}_d)}{2\kappa_6^2} \int d^{6-d}x \sqrt{-g_{6-d}} \Big[\frac{1}{4} R - \frac{d}{(4-d)} \partial_\mu \lambda \partial^\mu \lambda - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{kd}{4} e^{-\frac{8\lambda}{4-d}} + \frac{1}{8} e^{-\frac{2d\lambda}{4-d}} \big(g^2 e^{\sqrt{2}\phi} + 4gme^{-\sqrt{2}\phi} - m^2 e^{-3\sqrt{2}\phi} \big) - \frac{\tau_{\mathcal{M}_d}}{4g^2} e^{-\frac{2(8-d)\lambda}{4-d}} e^{-\sqrt{2}\phi} \Big]$$

Parameters and Metric

$$\tau_{\mathcal{M}_{d=2}} = 2, \qquad \tau_{\mathcal{M}_{d=3}} = 3/2, \qquad ds_6^2 = e^{-\frac{2d}{4-d}\lambda} ds^2_{6-d} + e^{2\lambda} ds^2_{\mathcal{M}_d}$$

2 Dimensional Theories

Bosonic Action

$$\begin{aligned}\mathcal{S}_2 = &\frac{\mathrm{Vol}(\mathcal{M}_4)}{2\kappa_6^2}\int d^2x\sqrt{-g_2}e^{2\lambda_1+2\lambda_2}\left[\frac{1}{4}R_2+\frac{1}{2}(e^{-2\lambda_1}k_1+e^{-2\lambda_2}k_2)\right.\\ &+\frac{1}{2}g^{\mu\nu}\partial_\mu\lambda_1\partial_\nu\lambda_1+\frac{1}{2}g^{\mu\nu}\partial_\mu\lambda_2\partial_\nu\lambda_2+2g^{\mu\nu}\partial_\mu\lambda_1\partial_\nu\lambda_2-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi\\ &+\frac{1}{8}(g^2e^{\sqrt{2}\phi}+4gme^{-\sqrt{2}\phi}-m^2e^{-3\sqrt{2}\phi})\\ &\left.-\frac{\tau_{\mathcal{M}_4}}{8g^2}e^{-\sqrt{2}\phi}(e^{-4\lambda_1}+e^{-4\lambda_2})-\frac{\tau^2_{\mathcal{M}_4}}{2m^2g^4}e^{\sqrt{2}\phi-4\lambda_1-4\lambda_2}\right]\end{aligned}$$

Parameters

$\tau_{\mathcal{M}_{\text{Cayley}}}=2/3$, $\tau_{\mathcal{M}_{\text{Kähler}}}=2$, and $\tau_{\Sigma_1\times\Sigma_2}=2$. Note that for Cayley and Kähler 4-cycles as *e.g.* $\mathbb{CP}^2$ we need to set $\lambda_1=\lambda_2$ and $k_1=k_2$.

Summary

(Non-)Supersymmetric Fixed Points

- We analyzed all the possible fixed point solutions with wrapped branes ansatz.
- In contrast to M5 branes(7d), IR singularities need further investigations in Euclidean signature.
- There are some difficulties for full analysis of the flows for two Riemann surfaces
- Numerical solutions for two Riemann surfaces have complicated singular structures in $x_i - u$ diagrams

Consistent Truncation

- These are not a bosonic part of the certain lower dimensional supergravity.

Leftovers

Euclidean Analysis

- Maldacena-Nuñez and Gubser criteria are a kind of short cut
- Existence of the regular solutions corresponding to each singularities determines whether the singularities are good or bad.

View from Dynamical Systems

- The flow equations describe a dynamical system
- Catastrophe theories are used to analyze RG flows to find and classify conformal fixed points[Vafa, Warner 1989]

Lower Dimensional Supergravity

- Supersymmetric completion of consistent truncation is not done yet.

감사합니다

$\epsilon\upsilon\chi\alpha\rho\iota\sigma\tau\acute{\omega}$

Thank you