Gauge hierarchy problem and scalegenesis

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Alexander von Humboldt Stiftung / Foundation

Workshop on Connecting Insights in Fundamental Physics: Standard Model and Beyond in Corfu

Plan

- Revisit gauge hierarchy problem
- Classically scale invariance and scalegenesis
- Who makes the universe critical?

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- Revisit gauge hierarchy problem
- Classically scale invariance and scalegenesis
- Who makes the universe critical?

Gauge hierarchy problem

- Renormalized Higgs mass

$$\int^{\Lambda} d^4p \, \frac{1}{p^2} \sim \Lambda^2$$

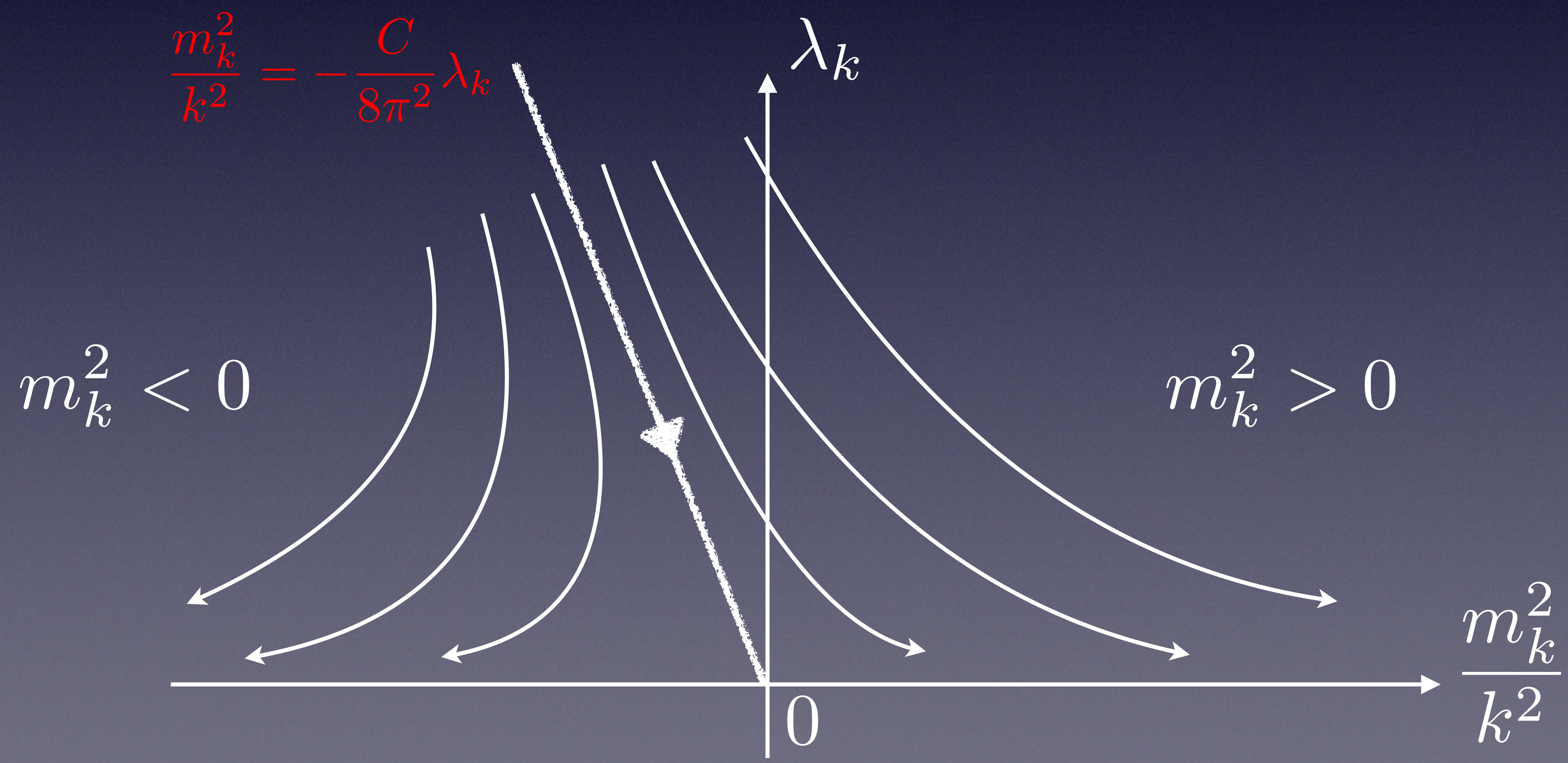
$$m_R^2 = m_\Lambda^2 + \frac{\Lambda^2}{16\pi^2}(\lambda + \cdots)$$

- $O(10^2\ \mathrm{GeV})^2 = -O(10^{19}\ \mathrm{GeV})^2 + O(10^{19}\ \mathrm{GeV})^2$

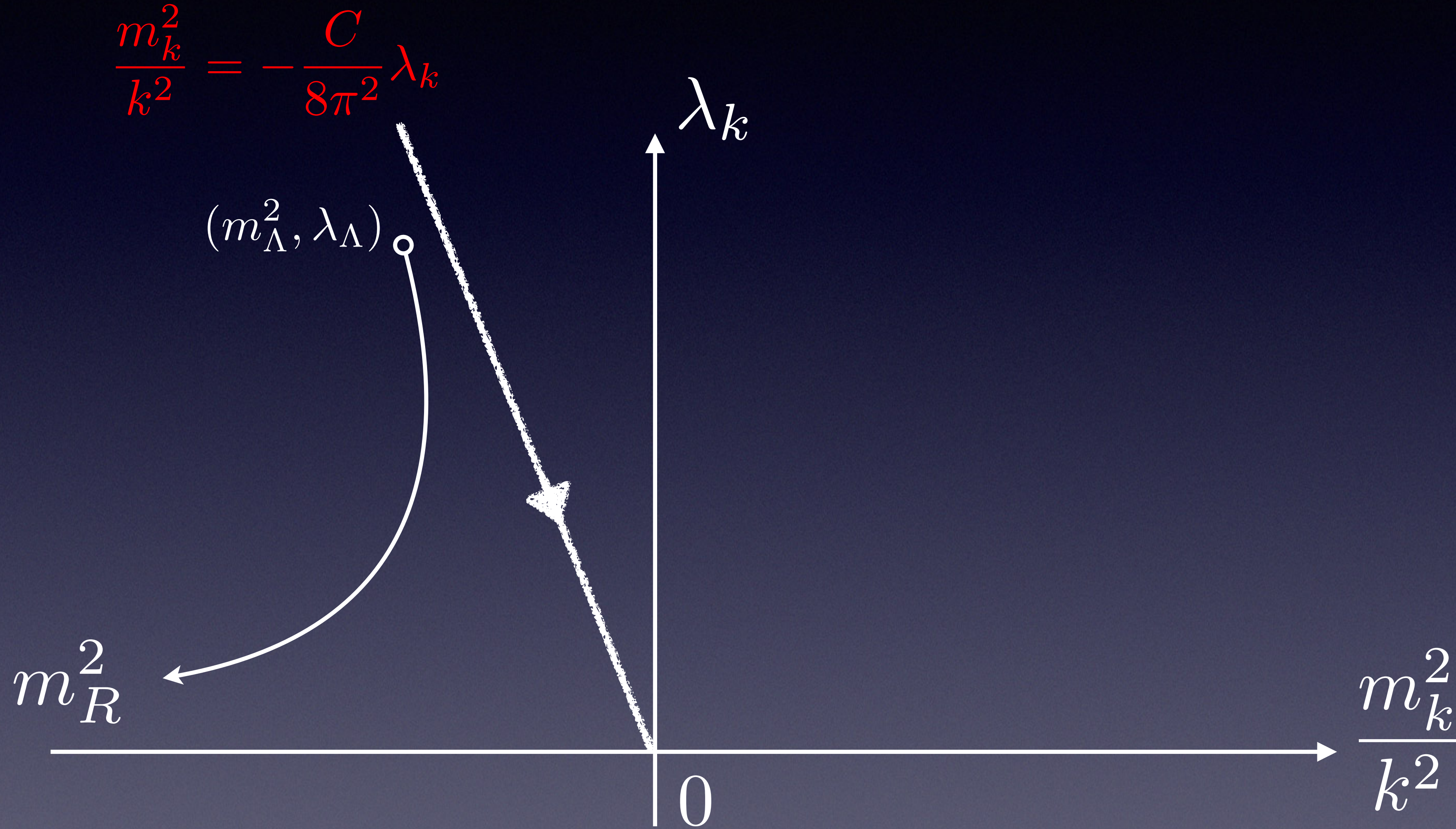
- $m_R^2 \ll m_\Lambda^2$

Gauge hierarchy problem from the viewpoint of the Wilson RG

$$\Gamma_k = \int d^4x \left[\frac{1}{2}(\partial_\mu \phi)^2 - \frac{m_k^2}{2}\phi^2 - \frac{\lambda_k}{4}\phi^4 \right]$$



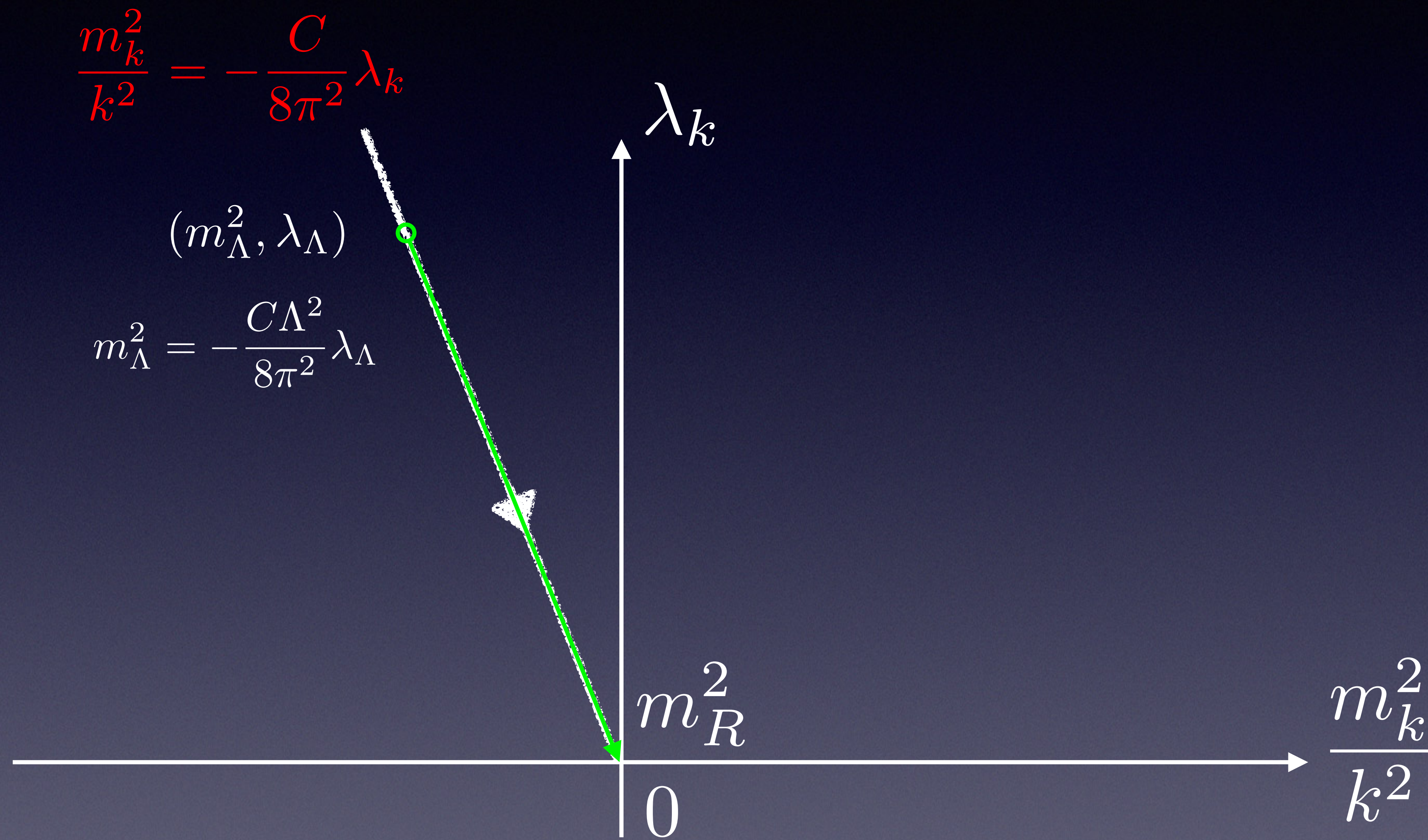
From the viewpoint of the Wilson RG



$$m_R^2 = m_\Lambda^2 + \frac{C\Lambda^2}{8\pi^2}\lambda_{k=0}$$

$$\lambda_\Lambda \simeq \lambda_{k=0}$$

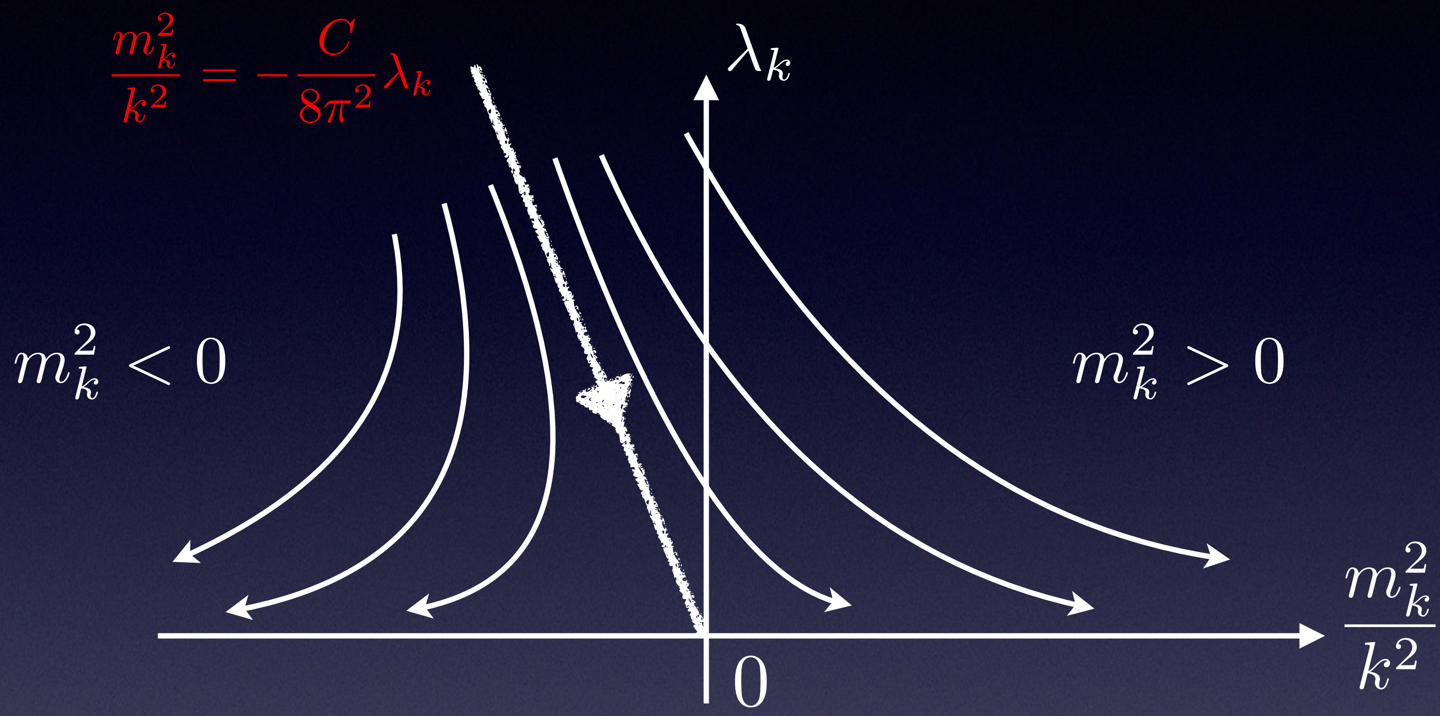
From the viewpoint of the Wilson RG



$$m_R^2 = m_\Lambda^2 + \frac{C\Lambda^2}{8\pi^2}\lambda_{k=0} = 0$$

$$\lambda_\Lambda \simeq \lambda_{k=0}$$

From the viewpoint of the Wilson RG



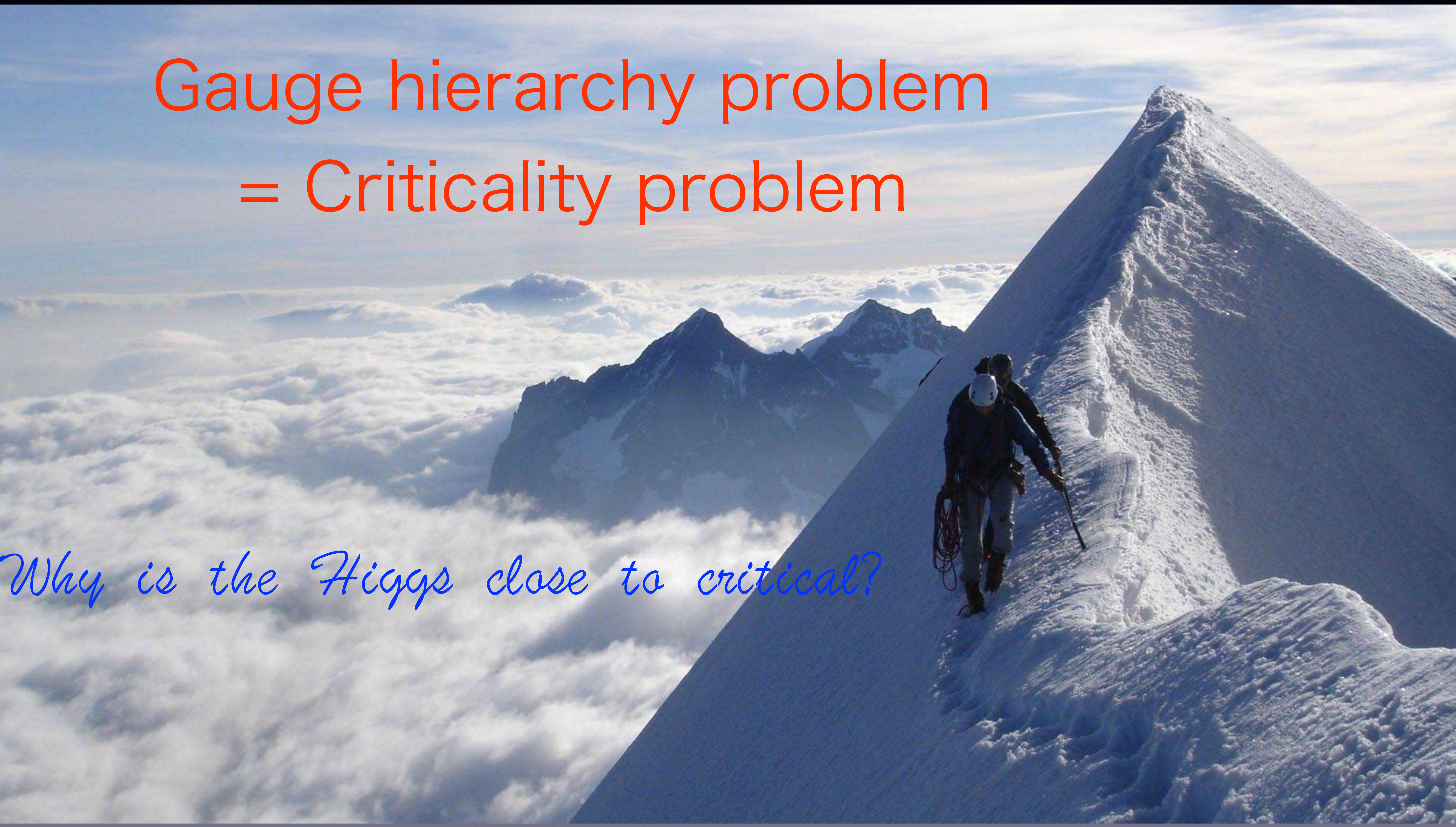
- Λ^2 determines the position of phase boundary (critical line).
- The phase boundary corresponds to the massless (critical) theory.
- To obtain small m_R, put the bare parameters close to the phase boundary.

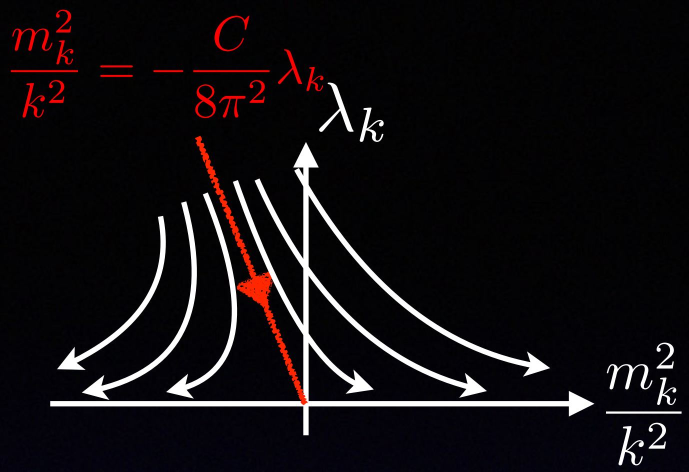
From the viewpoint of the Wilson RG

Gauge hierarchy problem

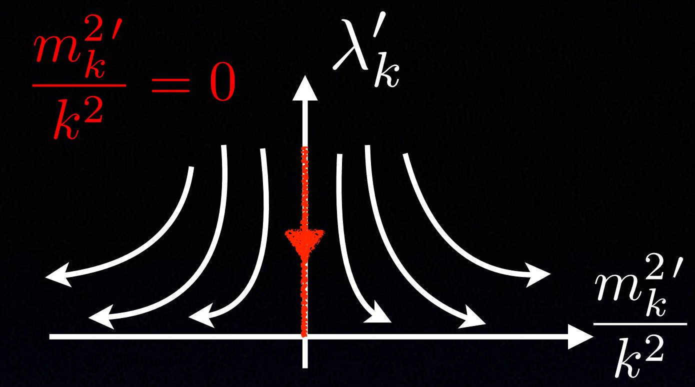
= Criticality problem

Why is the Higgs close to critical?





Discussion



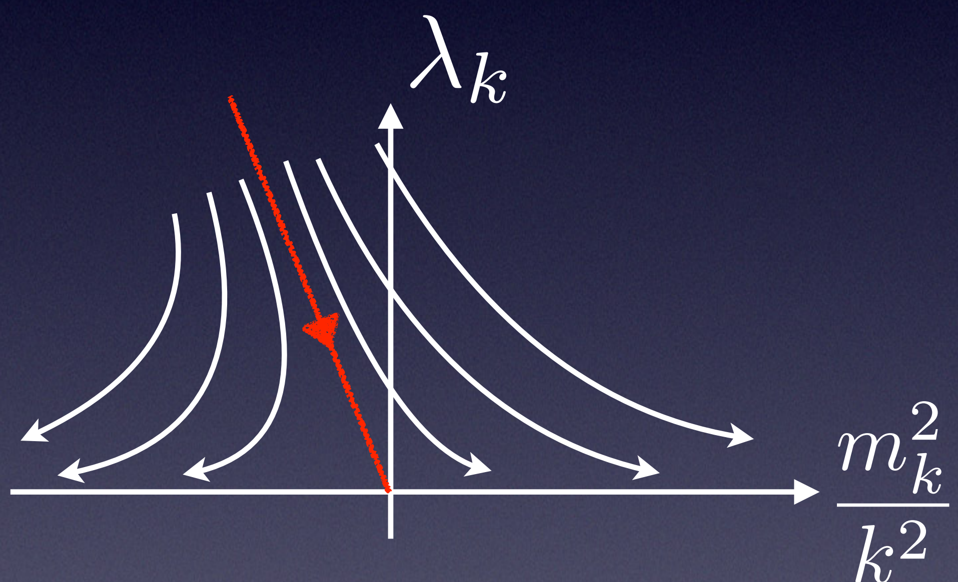
- Λ^2 is spurious?

C. Wetterich, 140B, 215
H. Aoki, S, Iso, Phys. Rev. D86, 013001

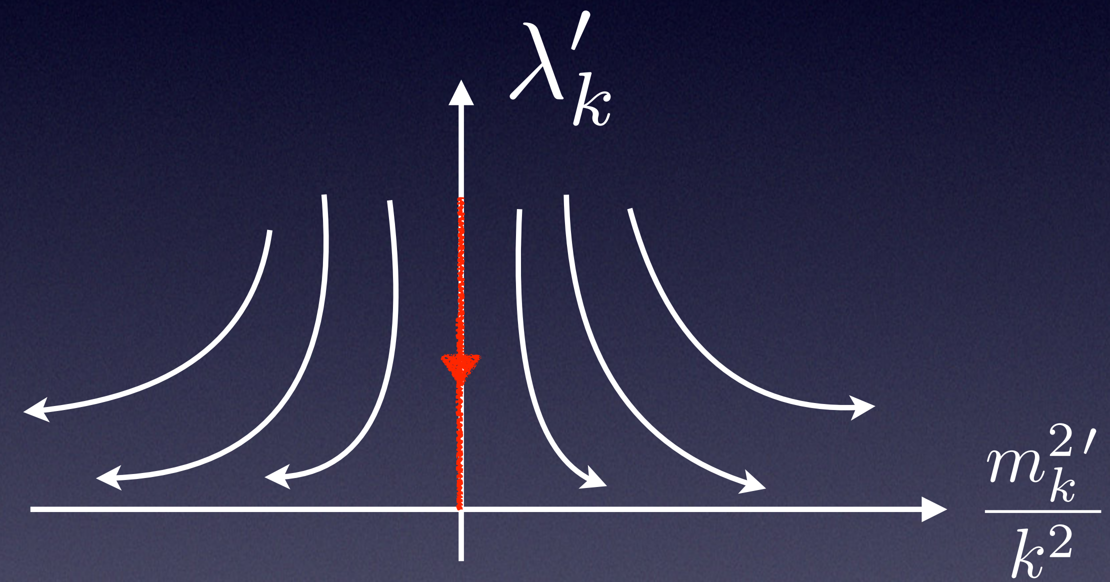
  - The **position** of phase boundary is physically meaningless.
- Choice of renormalization scheme ⇔ Choice of coordinate of theory space
  - Rotation of coordinate. ➡ $C = 0$ $\quad \frac{m_k^2}{k^2} = -\frac{C}{8\pi^2}\lambda_k \;\longrightarrow\; \frac{m_k^{2\prime}}{k^2} = 0$
- In perturbation theory, Λ^2 is always subtracted by the counter term or dimensional regularization.
- The distance between the flow and the boundary is physically meaningful.
- Gauge hierarchy problem ⇄ The bare theory of Higgs is almost scale (conformally) invariant.

SUSY cannot solve the criticality problem.

- Superpartners cancel the quartic divergence in arbitrary RG schemes.



$$\frac{m_k^2}{k^2} = -\frac{C}{8\pi^2}\lambda_k$$



$$\frac{m_k^{2\prime}}{k^2} = 0$$

- SUSY does not tell us why our universe is critical.

(μ problem)

Gauge hierarchy problem

- RG equation for dimensionless scalar mass

(distance between the phase boundary)

$$k\frac{d\bar{m}^2}{dk} = -\left(2-\gamma_m\right)\bar{m}^2 = -\theta_m \bar{m}^2$$

canonical dim.   anomalous dim.

$$\gamma_m = \frac{1}{16\pi^2}\left(2\lambda + 6y_t^2 - \frac{9}{2}g^2 - \frac{3}{2}g'^2\right)$$

$$\bar{m}^2 = \bar{m}_0^2\left(\frac{k}{\Lambda}\right)^{-\theta_m}$$

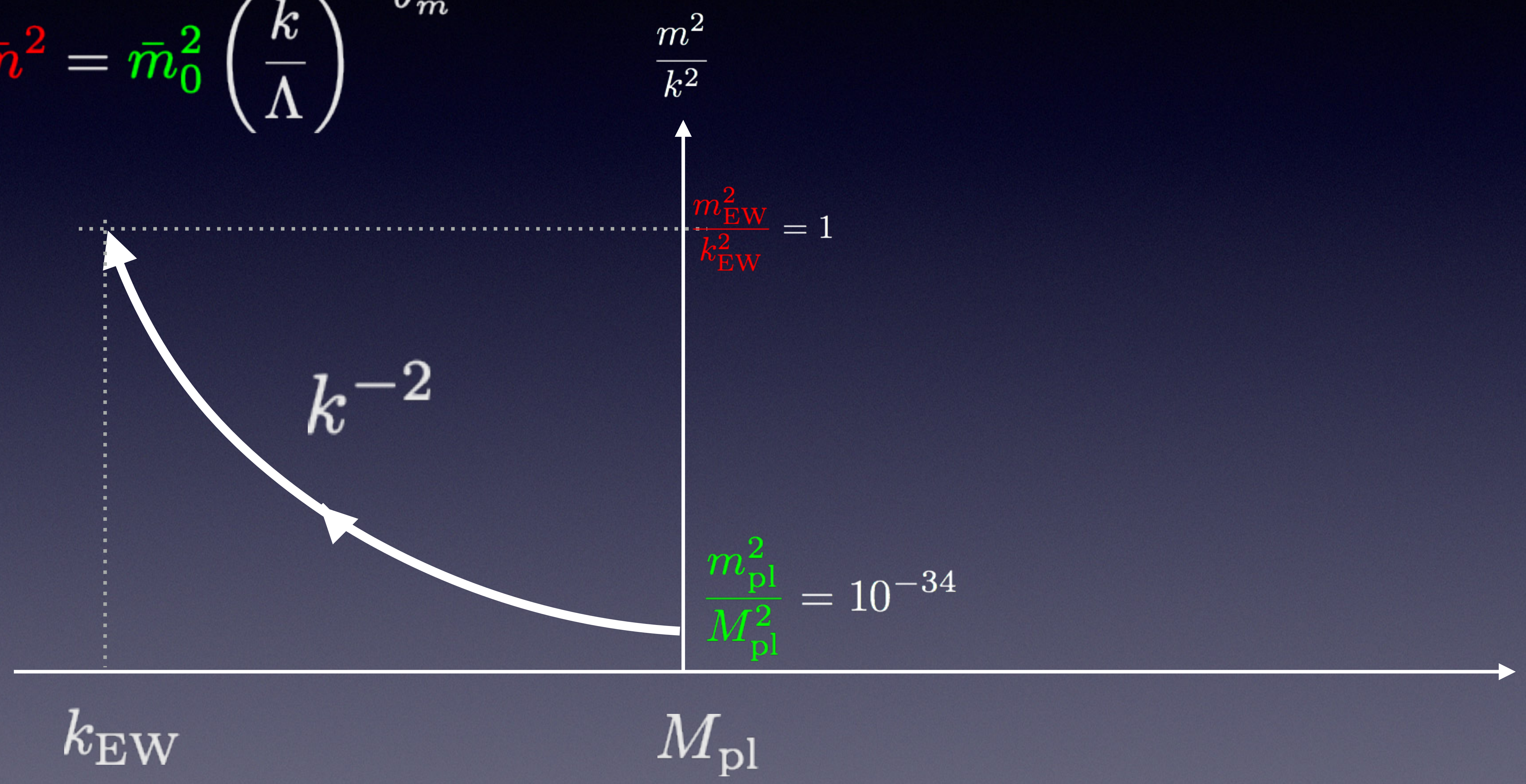
$$\bar{m}^2 = \frac{m^2}{k^2}$$

From observations

$$\theta_m \simeq 2 \qquad \bar{m}^2 = \frac{m_{\rm EW}^2}{k_{\rm EW}^2} \simeq 1 \qquad \bar{m}_0^2 = \frac{m_{\rm pl}^2}{M_{\rm pl}^2} \simeq 10^{-34} \qquad \Lambda = M_{\rm pl}$$

RG flow of scalar mass

$$\bar{m}^2 = \bar{m}_0^2 \left(\frac{k}{\Lambda}\right)^{-\theta_m}$$



Summary so far

- The quadratic divergence may be spurious.
- The gauge hierarchy problem is the criticality problem.

Why is the Higgs close to critical?

Plan

- Revisit gauge hierarchy problem
- Classical scale invariance and scalegenesis
- Who makes the universe critical?

Classical scale invariance

W. A. Bardeen, FERMILAB-CONF-95-391-T

- Only the Higgs mass is dimensionful in the standard model.
- The bare action of the SM might be scale invariant at the Planck scale.
- Classical scale invariance prohibits the bare Higgs mass.
- The universe is critical (theory on the critical line).

Classical scale invariance

- There is no scale corresponding to the electroweak scale.
- Scalegenesis: How to generate the scale?

Scalegenesis in the SM

- Coleman-Weinberg potential

$$V_{\text{eff}}(v_h) = \frac{\lambda_H}{4} v_h^4 + \sum_\alpha \frac{N_\alpha M_\alpha^4}{64\pi^2} \left(\log\left(\frac{M_\alpha^2}{\mu^2}\right) - C_\alpha \right)$$

- A scale μ is generated.
- $(\lambda_H$, y, g) →(μ, y, g): dimensional transmutation
- The Higgs potential becomes unstable for

$$m_H = 126\,\text{GeV},\ m_t = 173\,\text{GeV}$$

New D.o.F. effect

- Consider a new scalar field.

$$V = \frac{\lambda_H}{4} h^4 + \frac{\lambda_S}{4} S^4 + \frac{\lambda_{HS}}{4} S^2 h^2$$

- The Higgs obtains a radiative mass.

$$m_H^2 = M^2 \log \frac{M^2}{\mu^2} \qquad M^2 = \lambda_{HS} \langle S \rangle^2$$

- Spontaneous breaking of scale symmetry

Gauge hierarchy problem?

- When <S> is a GUT scale (~10^{16} GeV), λ_{HS} should be of order 10^{-28}.

$$M^2 = \lambda_{HS}\langle S\rangle^2$$

- Is this the gauge hierarchy problem or not?
  - Because λ_{HS} is log-running, its smallness is preserved under the RG running. (technically natural, so no problem!)

    Recall T. Kugo's talk
  - Why is λ_{HS} so much small? (Problem!?)

    The real hierarchy problem by G. Ross
    - <S> should be TeV order?

Summary so far

- The classical scale invariance makes the universe critical.
- Different notions for the gauge hierarchy problem.

Plan

- Revisit gauge hierarchy problem
- Classically scale invariance and scalegenesis
- Who makes the universe critical?

Our universe

Why not

Gravitational corrections to scalar mass

- RG equations

$$k\frac{d\bar{m}^2}{dk} = -(2-\gamma_m)\bar{m}^2$$

- Contributions from graviton fluctuation

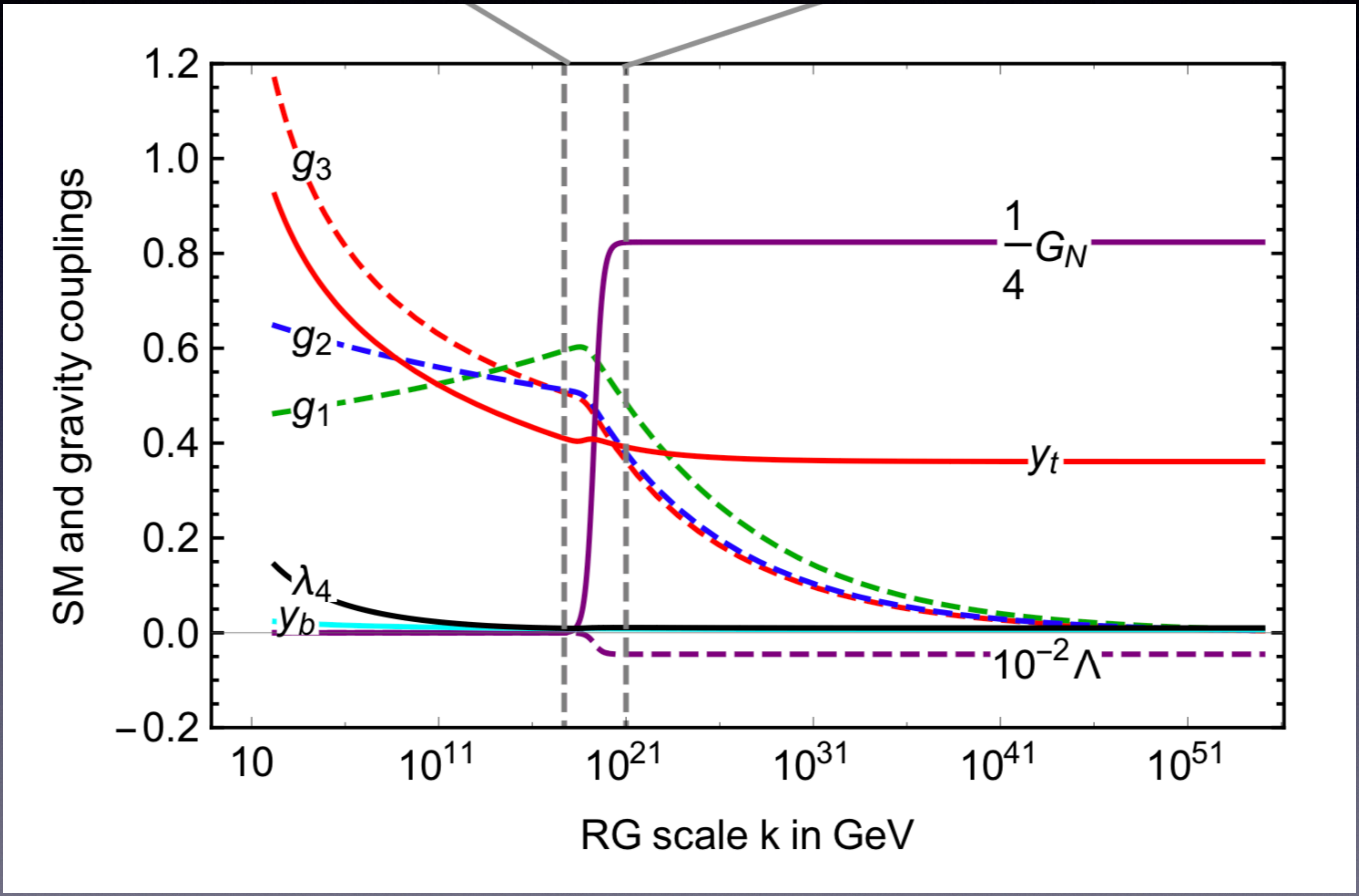
$$\gamma_m = \gamma_m|_{\mathrm{SM}} + \frac{3G_N}{\pi(1-2\Lambda)} \qquad \gamma_m|_{\mathrm{SM}} \simeq 0.027$$

- Asymptotically safe gravity could induce a large anomalous dimension.

Recall A. Held and G. Ross talks.

Asymptotic safety scenario

A. Eichhorn and A. Held, PL777; PRL121



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@EW

- Asymptotically safe gravity could induce a large anomalous dimension.

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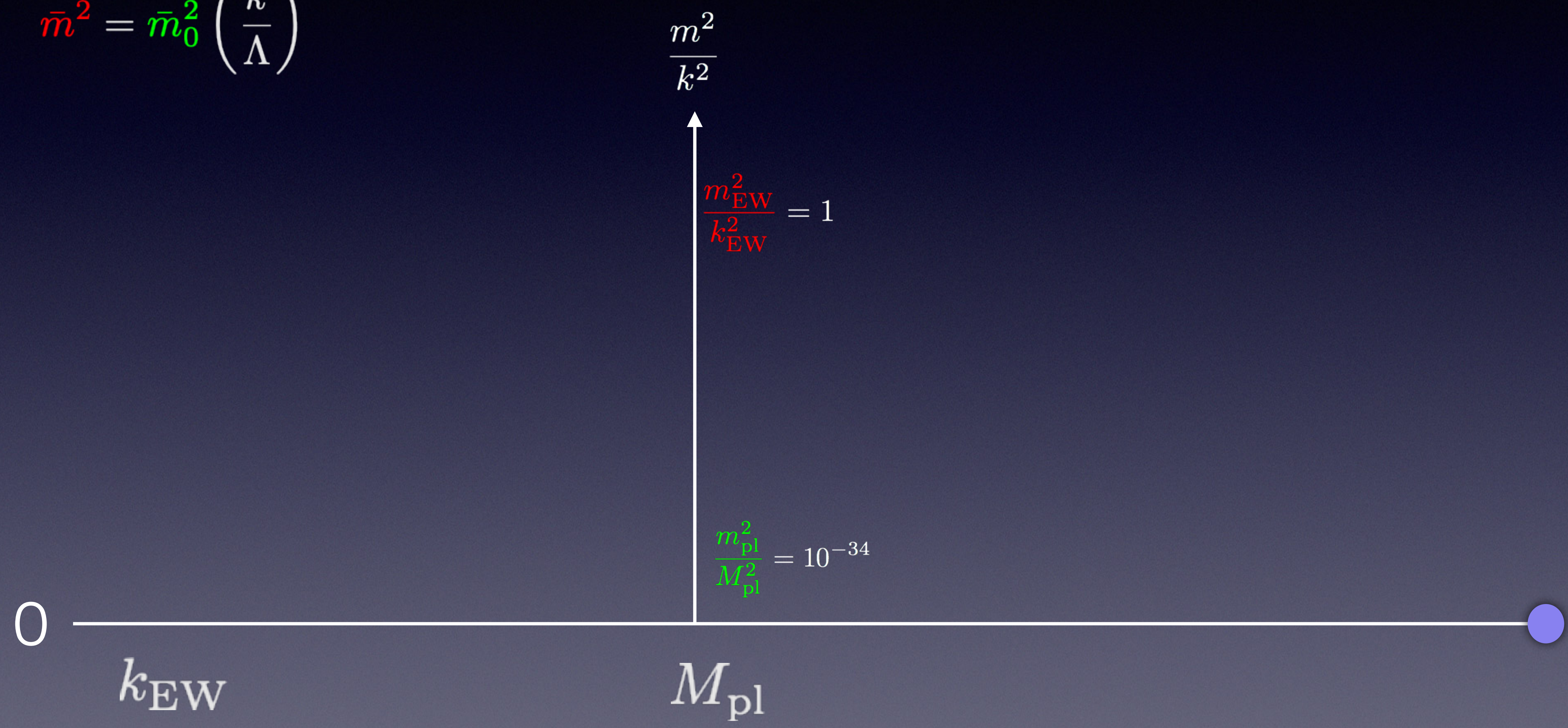
RG flow of scalar mass

$$\bar{m}^2 = \bar{m}_0^2 \left(\frac{k}{\Lambda}\right)^{-2+\gamma_m}$$



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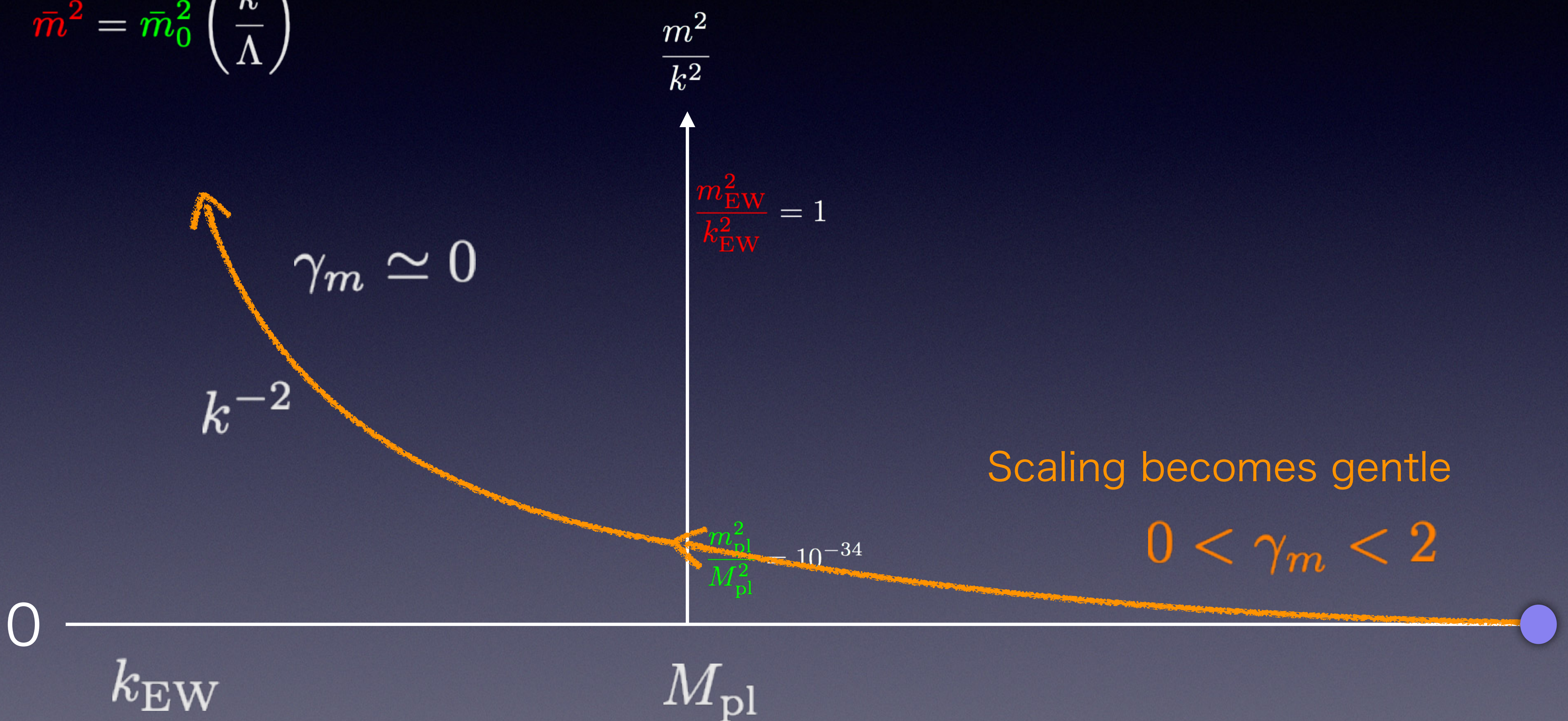

QG is decoupled

RG flow of scalar mass

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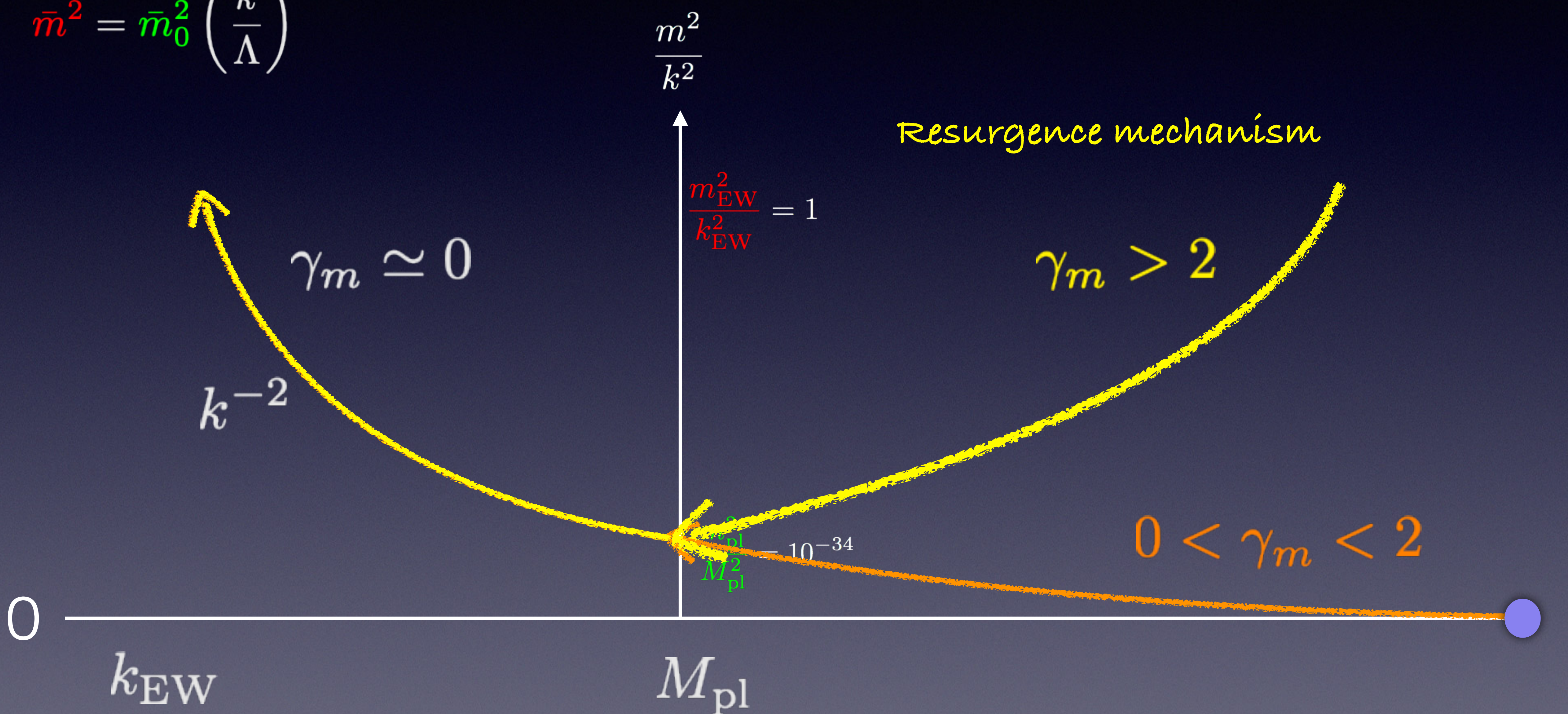

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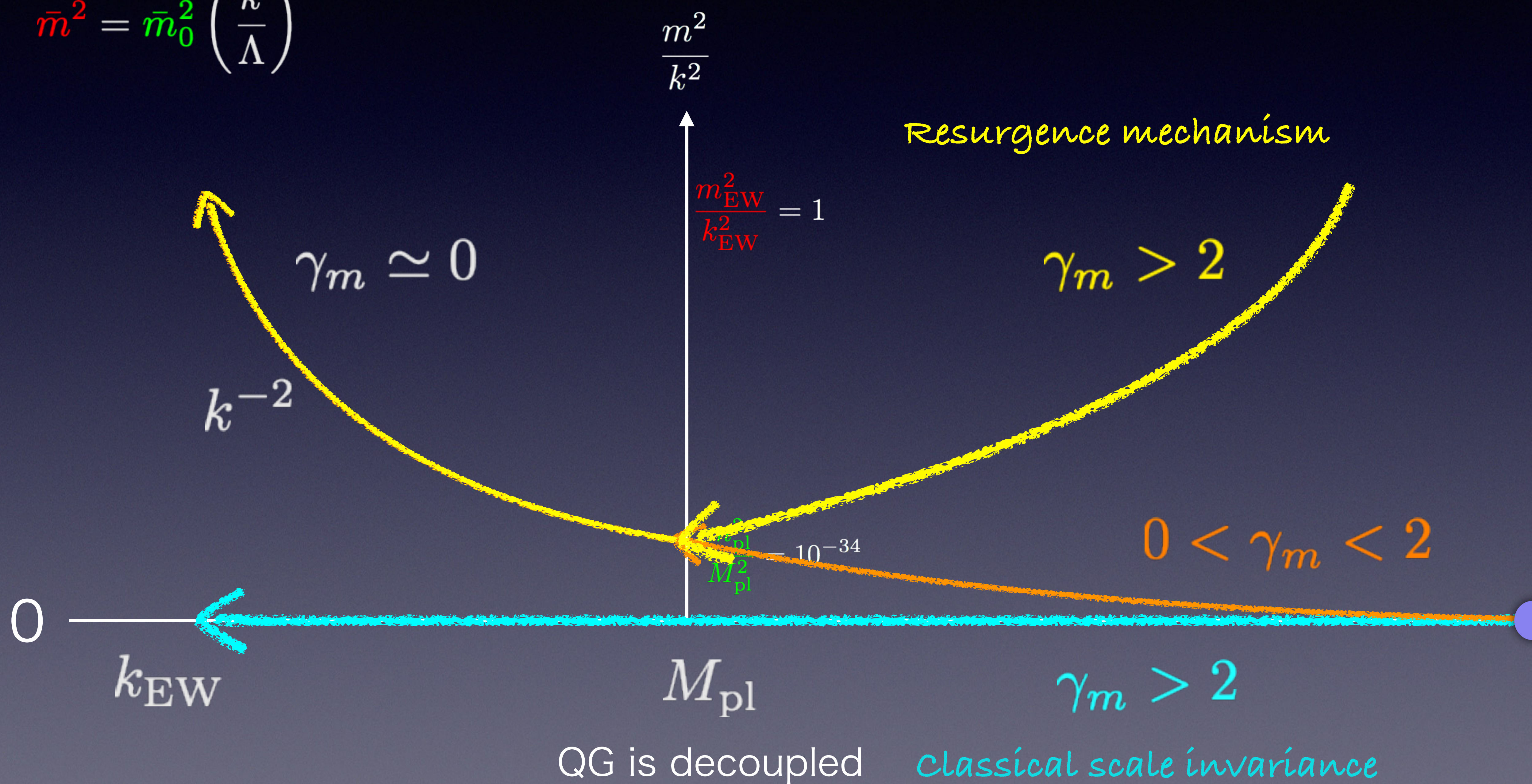


Above the Planck scale

Below the Planck scale

Summary

- Gauge hierarchy problem is criticality problem.
- Different definitions of gauge hierarchy problem.
- Scalegenesis:
  - Quantum dynamics generates a scale.
- Asymptotically safe gravity could make the universe critical.

Appendix

Effective potential

- Introduce the mean-field and integrate out S_i:

$$V_{\text{eff}}(\bar{S}, f, h) = M^2(\bar{S}_i^\dagger \bar{S}_i) + \frac{\lambda_H}{4} h^4 - N_f(N_f\lambda_S + \lambda'_S) f^2 + \frac{N_c N_f}{32\pi^2} M^4 \ln \frac{M^2}{\Lambda_H^2}$$

- "Scalegenesis"

$$M^2 = 2(N_f\lambda_S + \lambda'_S) f - \frac{\lambda_{HS}}{2} h^2$$

$$f = \langle S_i^\dagger S_i \rangle$$

Tr ln S_i

- Physical values

$$\langle h \rangle = \frac{N_f \lambda_{HS}}{G} \Lambda_H^2 \exp\left(\frac{32\pi^2 \lambda_H}{N_c G} - \frac{1}{2}\right)$$

=246 GeV

$$m_h^2 = \frac{\langle h \rangle^2}{2} \left(\frac{16\lambda_H^2 (N_f \lambda_S + \lambda'_S)}{G} + \frac{N_c N_f \lambda_{HS}^2}{8\pi^2}\right)$$

=126 GeV

$$\langle S^\dagger S \rangle = \langle f \rangle = \frac{2\lambda_H}{G} \Lambda_H^2 \exp\left(\frac{32\pi^2 \lambda_H}{N_c G} - \frac{1}{2}\right)$$

$$M_0^2 = \frac{G}{2\lambda_H} \langle S^\dagger S \rangle$$

$$G = 4N_f \lambda_H \lambda_S - N_f \lambda_{HS}^2 + 4\lambda_H \lambda'_S$$

Effective model

J. Kubo and **MY,** Phys.Rev**. D93,** no.7, 075016

- Scale invariant effective Lagrangian

$$\mathcal{L}_{\rm eff} = ([\partial^\mu S_i]^\dagger \partial_\mu S_i) - \lambda_S (S_i^\dagger S_i)(S_j^\dagger S_j) - \lambda'_S (S_i^\dagger S_j)(S_j^\dagger S_i) + \lambda_{HS}(S_i^\dagger S_i) H^\dagger H - \lambda_H (H^\dagger H)^2$$

$\Big\downarrow$ mean-field approx.

$$\langle S_i^\dagger S_j \rangle = f_0 \delta_{ij} + Z_\sigma \sigma \delta_{ij} + Z_\phi t^a_{ji} \phi^a$$

$$\langle H \rangle = (v_h + h)/\sqrt{2}$$

$$\mathcal{L}'_{\rm MFA} = ([\partial^\mu S_i]^\dagger \partial_\mu S_i) - M_0^2 (S_i^\dagger S_i) + N_f (N_f \lambda_S + \lambda'_S) Z_\sigma \sigma^2 + \frac{\lambda'_S}{2} Z_\phi \phi^a \phi^a - 2(N_f \lambda_S + \lambda'_S) Z_\sigma^{1/2} \sigma (S_i^\dagger S_i) - 2\lambda'_S Z_\phi^{1/2} (S_i^\dagger t^a_{ij} \phi^a S_j) + \frac{\lambda_{HS}}{2} (S_i^\dagger S_i)(2v_h + h) h - \frac{\lambda_H}{4} h^2 (6v_h^2 + 4 v_h h + h^2)$$

$$M_0^2 = 2(N_f \lambda_S + \lambda'_S) f_0 - \frac{\lambda_{HS}}{2} v_h^2$$

Minimum of potential

- Effective potential

$$V_{\rm eff}(\bar{S},f,h) = M^2(\bar{S}_i^\dagger \bar{S}_i) + \frac{\lambda_H}{4}h^4 - N_f(N_f\lambda_S + \lambda_S')f^2 + \frac{N_cN_f}{32\pi^2}M^4 \ln\frac{M^2}{\Lambda_H^2}$$

$$0 = \frac{\partial}{\partial \bar{S}_i^a}V_{\rm MFA} = \frac{\partial}{\partial f}V_{\rm MFA} = \frac{\partial}{\partial H_l}V_{\rm MFA}\ (l=1,2).$$

$$\langle S^a\rangle\langle M^2\rangle = 0$$

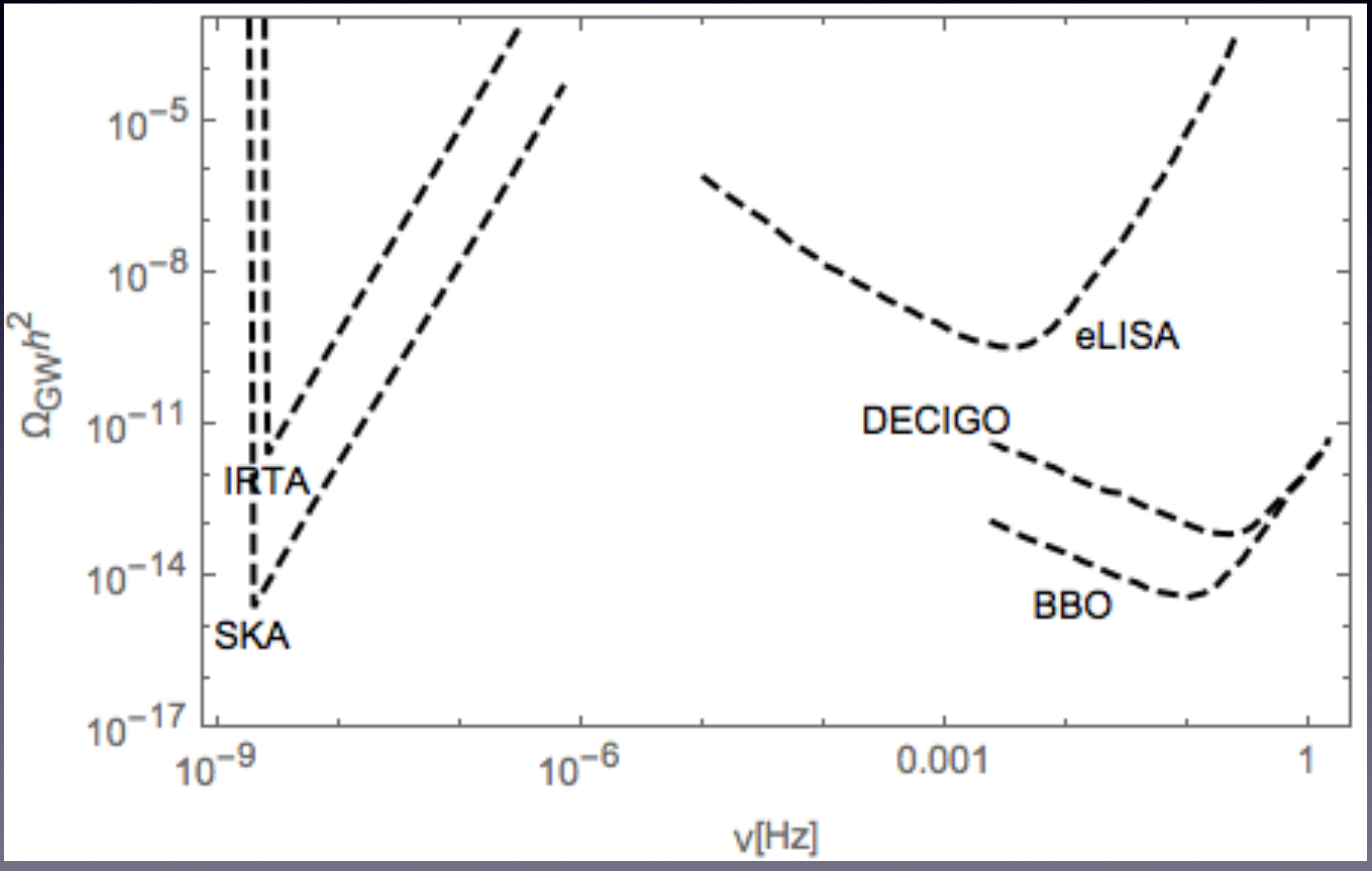
$$\text{(i)}\langle S^a\rangle \neq 0\ \langle M^2\rangle = 0 \qquad \langle V_{\rm eff}\rangle = 0$$

$$\text{(ii)}\langle S^a\rangle = 0\ \langle M^2\rangle = 0 \qquad \langle V_{\rm eff}\rangle = 0$$

$$\text{(iii)}\langle S^a\rangle = 0\ \langle M^2\rangle \neq 0$$

$$\langle V_{\rm MFA}\rangle = -\frac{N_cN_f}{64\pi^2}\Lambda_H^4 \exp\left(\frac{64\pi^2\lambda_H}{N_cG} - 1\right) < 0.$$

Future experiments



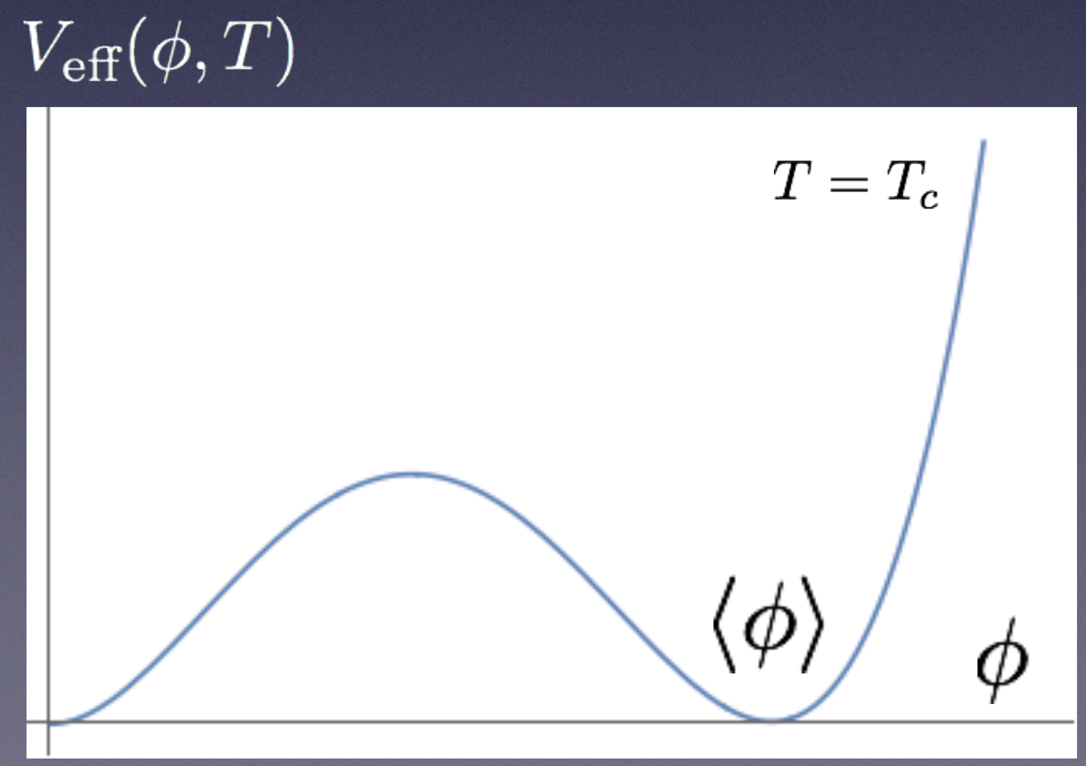
$V_{\rm eff}$ at finite T

J. Kubo and **MY,** PTEP **2015**, no.9, 093B01

- Using the Matsubara formalism, the effective potential is obtained. $V_{\rm eff}(h, f, T)$

- Note: ``Strong” 1st order means

$$\frac{\langle \phi \rangle}{T_c} \gtrsim 1$$



Two ways of scalegenesis

- **Perturbative**
- **Non-perturbative**

- Dark matter candidate (WIMP)
- Dark phase transitions
- Gravitational waves
- Neutrino mass
- Baryogengesis
- etc.

How to formulate?

$\boxed{\textbf{NJL model}}$	$\boxed{\textbf{Our approach}}$

- Global symmetries

Anomalous

$$\boxed{\mathrm{SU}(N_f)_V \times \mathrm{SU}(N_f)_A \times \mathrm{U}(1)_V \times \cancel{\mathrm{U}(1)_A}}$$

Anomalous

$$\boxed{\mathrm{U}(N_f) \times \cancel{\text{Scale symmetry}}}$$

- Mean fields and excitations

$$\boxed{\Phi_{ij} := \bar{\psi}_i(1 - i\gamma^5)\psi_j \propto \delta_{ij}\sigma + t^a_{ji}\pi^a}$$

$$\boxed{S^\dagger_i S_j \propto \delta_{ij}\sigma + t^a_{ji}\phi^a}$$

- Effective Lagrangian

$$\boxed{\mathcal{L}_{\text{eff}} = \bar{\psi} i \partial\!\!\!/ \psi + 2G \,\mathrm{Tr}\, \Phi^\dagger \Phi + G_D(\det \Phi + \text{h.c.})}$$

describes **dynamical** **<u>SU(N$_f$)$_A$</u> breaking.**

$\boxed{\text{Next page}}$

describes

dynamical <u>scale symmetry</u> breaking.

Energy scale

Planck scale

Scale invariant SM & hidden sector

$\mathcal{L}_{\mathrm{SM}}|_{m_H \to 0} + \mathcal{L}_{\mathrm{H}}$

Described by the effective model

$\mathcal{L}_{\mathrm{SM}}|_{m_H \to 0} + \mathcal{L}_{\mathrm{eff}}$

TeV

Λ_{H}

Dynamical scale symmetry breaking $\langle S^\dagger S \rangle \neq 0$

Electroweak symmetry breaking $\langle h \rangle \neq 0$

Two ways of scalegenesis

- **Perturbative**
  - Radiative corrections
  - Hard breaking effects (scale anomaly)
  - Renormalization point μ
  - Coleman-Weinberg '73; Hempfling '96; Meissner, Nicolai; Chang, Ng, Wu; Foot, Kobakhideze, Volkas; '07; Iso, Okada, Orikasa '09; Endo, Sumino '15; etc.
- **Non-perturbative**
  - Strong dynamics. e.g. QCD
  - Dynamical symmetry breaking
  - $\Lambda_{\rm QCD}$
  - Nambu—Jona-Lasinio '61; Weinberg; Susskind '76, '79;Hur, Ko '11; Kubo, Lim, Lindner '14; Kubo, **M.Y** '15; Haba, Ishida, Kitazawa, Yamaguchi '17; Haba, T, Yamada '17

Our model

J. Kubo and **MY,** Phys.Rev**. D93,** no.7, 075016

- SU(Nc)×flavor sym.×scale inv. scalar gauge theory

$$\mathcal{L}_{\mathrm{H}} = -\frac{1}{2}\,\mathrm{tr}F^2 + ([D^\mu S_i]^\dagger D_\mu S_i) - \hat{\lambda}_S (S_i^\dagger S_i)(S_j^\dagger S_j)$$
$$- \hat{\lambda}'_S (S_i^\dagger S_j)(S_j^\dagger S_i) + \hat{\lambda}_{HS}(S_i^\dagger S_i) H^\dagger H - \lambda_H (H^\dagger H)^2$$

$$\mathcal{L}_{\mathrm{SM}}\big|_{m\to 0} \quad (H^\dagger H)(S_i^\dagger S_i) \quad \mathcal{L}_{\mathrm{H}}$$

- Due to the strong dynamics, $\langle S_i^\dagger S_i \rangle \neq 0$

Dynamical scale symmetry breaking

How to calculate?

- Strong dynamics is highly complicated.
- Effective model approaches have succeeded.
- Formulate the effective model for our model by following the idea of Nambu-Jona-Lasinio (NJL) model in QCD!

Effective model

J. Kubo and **MY,** Phys.Rev**. D93,** no.7, 075016

- Assume that

anomalous hard breaking << dynamical breaking

- Scale invariant effective Lagrangian

$$\mathcal{L}_{\rm eff} = ([\partial^\mu S_i]^\dagger \partial_\mu S_i) - \lambda_S (S_i^\dagger S_i)(S_j^\dagger S_j) - \lambda'_S (S_i^\dagger S_j)(S_j^\dagger S_i) + \lambda_{HS}(S_i^\dagger S_i) H^\dagger H - \lambda_H (H^\dagger H)^2$$

What we want to see is that

the scaleless theory dynamically generates a scale.

Effective potential

- Introduce the mean-field f and integrate out S_i:

$$V_{\rm eff}(\bar{S}, f, h) = M^2(\bar{S}_i^\dagger \bar{S}_i) + \frac{\lambda_H}{4}h^4 - N_f(N_f\lambda_S + \lambda'_S)f^2 + \frac{N_c N_f}{32\pi^2}M^4 \ln\frac{M^2}{\Lambda_H^2}$$

Tr ln S_i

- "Scalegenesis"

- Physical values

$$\langle h \rangle = \frac{N_f \lambda_{HS}}{G}\Lambda_H^2 \exp\left(\frac{32\pi^2\lambda_H}{N_c G} - \frac{1}{2}\right)$$

=246 GeV

$$m_h^2 = \frac{\langle h \rangle^2}{2}\left(\frac{16\lambda_H^2(N_f\lambda_S + \lambda'_S)}{G} + \frac{N_c N_f \lambda_{HS}^2}{8\pi^2}\right)$$

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$$\langle S^\dagger S \rangle = \langle f \rangle = \frac{2\lambda_H}{G}\Lambda_H^2 \exp\left(\frac{32\pi^2\lambda_H}{N_c G} - \frac{1}{2}\right)$$

$$M_0^2 = \frac{G}{2\lambda_H}\langle S^\dagger S \rangle$$

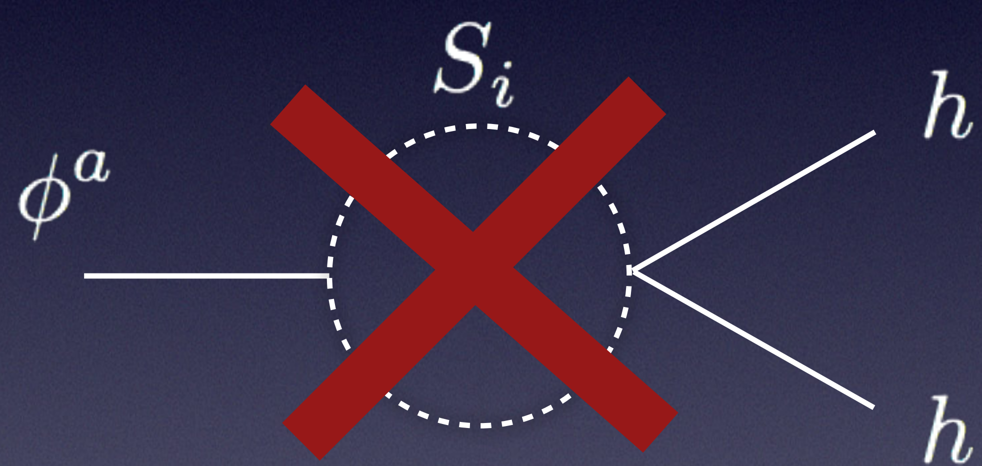
$$G = 4N_f\lambda_H\lambda_S - N_f\lambda_{HS}^2 + 4\lambda_H\lambda'_S$$

Dark matter candidate

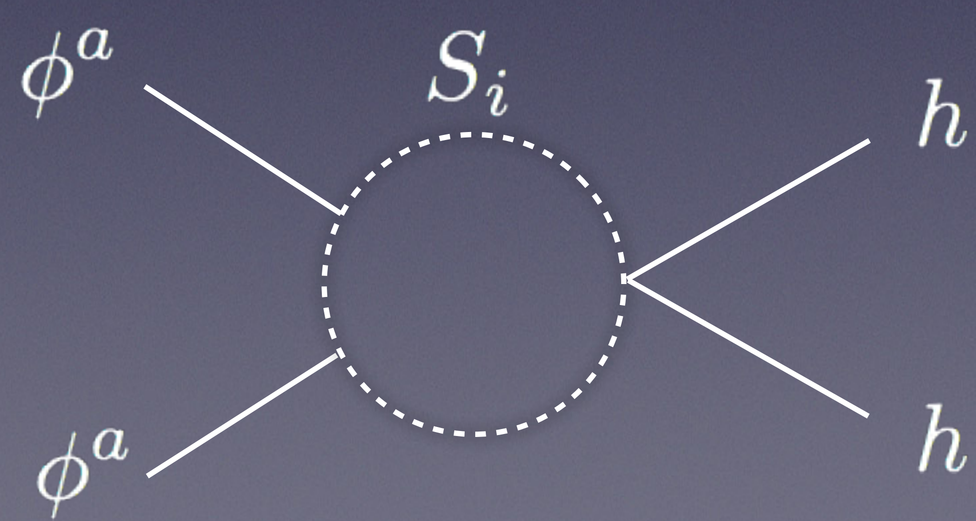
$$\boxed{S_i^\dagger S_j \propto \delta_{ij}\sigma + t^a_{ji}\phi^a}$$

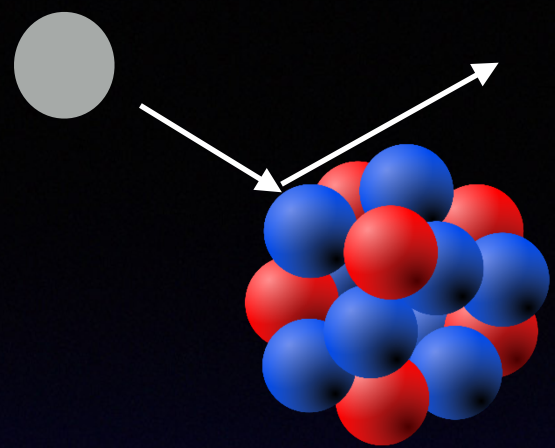
- If flavour symmetry is unbroken, ϕ^a is stable.

Decay



Annihilation



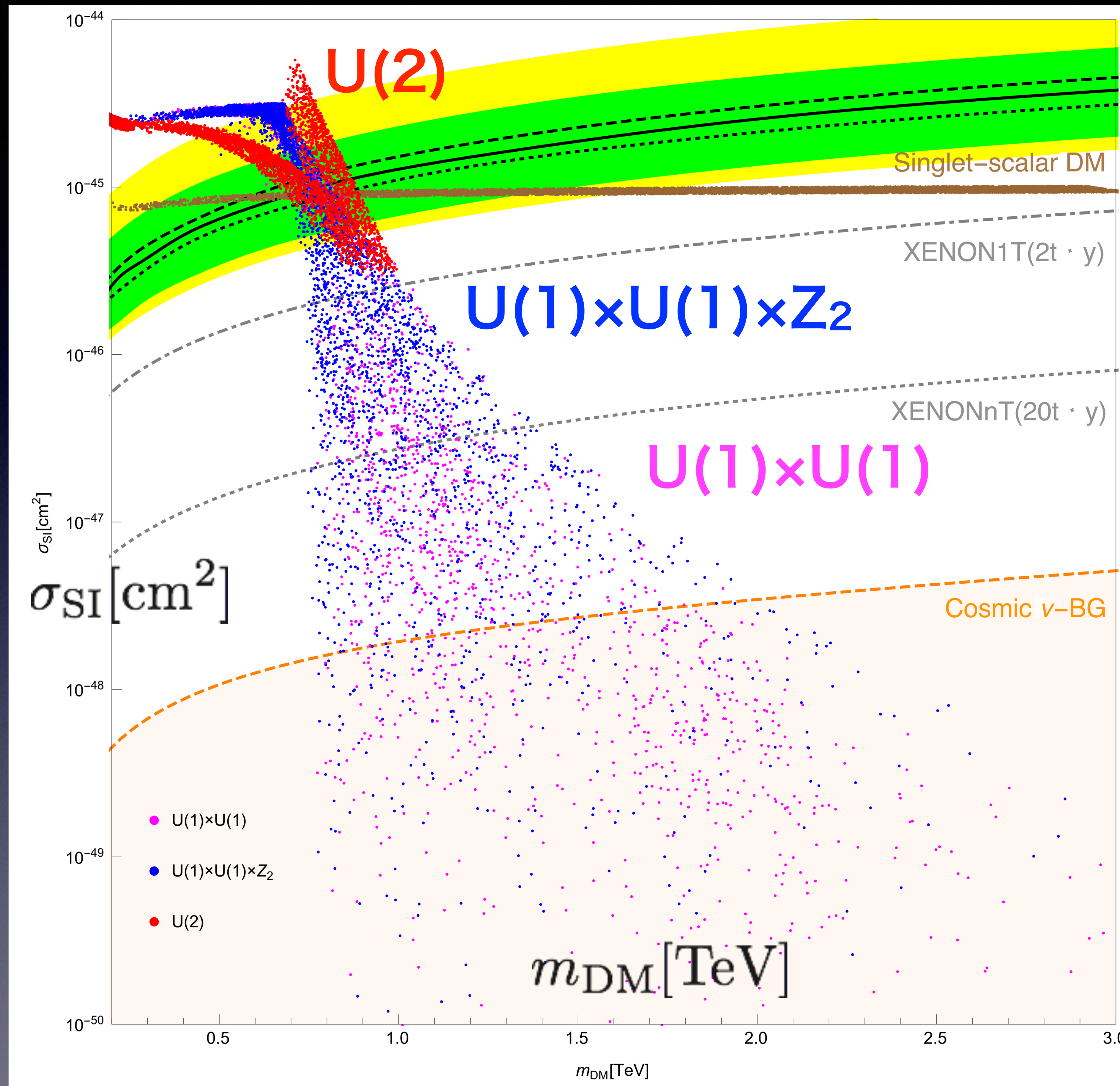


$$N_c = 6,\ \ N_f = 2$$

$$m_H = 126\ \mathrm{GeV}$$

$$v_h = 246\ \mathrm{GeV}$$

$$\Omega_{\mathrm{DM}} h^2 = 0.12$$



Phys.Rev. D93 no.7, 075016;
arXiv:1712.06324

Electroweak and scale phase transitions

- Two order parameters

EW PT	Scale PT
$\langle h \rangle$	$\langle S^\dagger S \rangle$

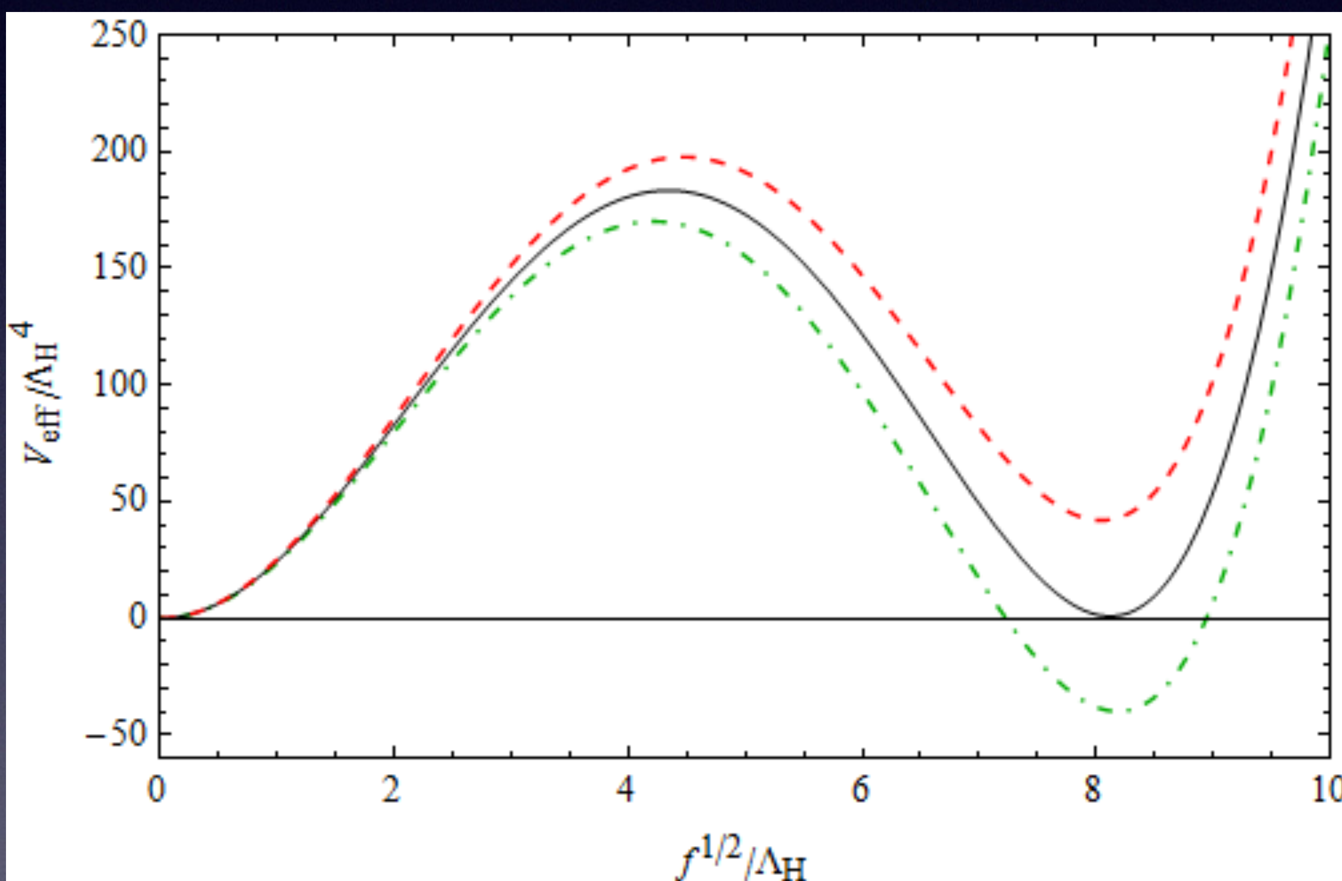
Thermal effect

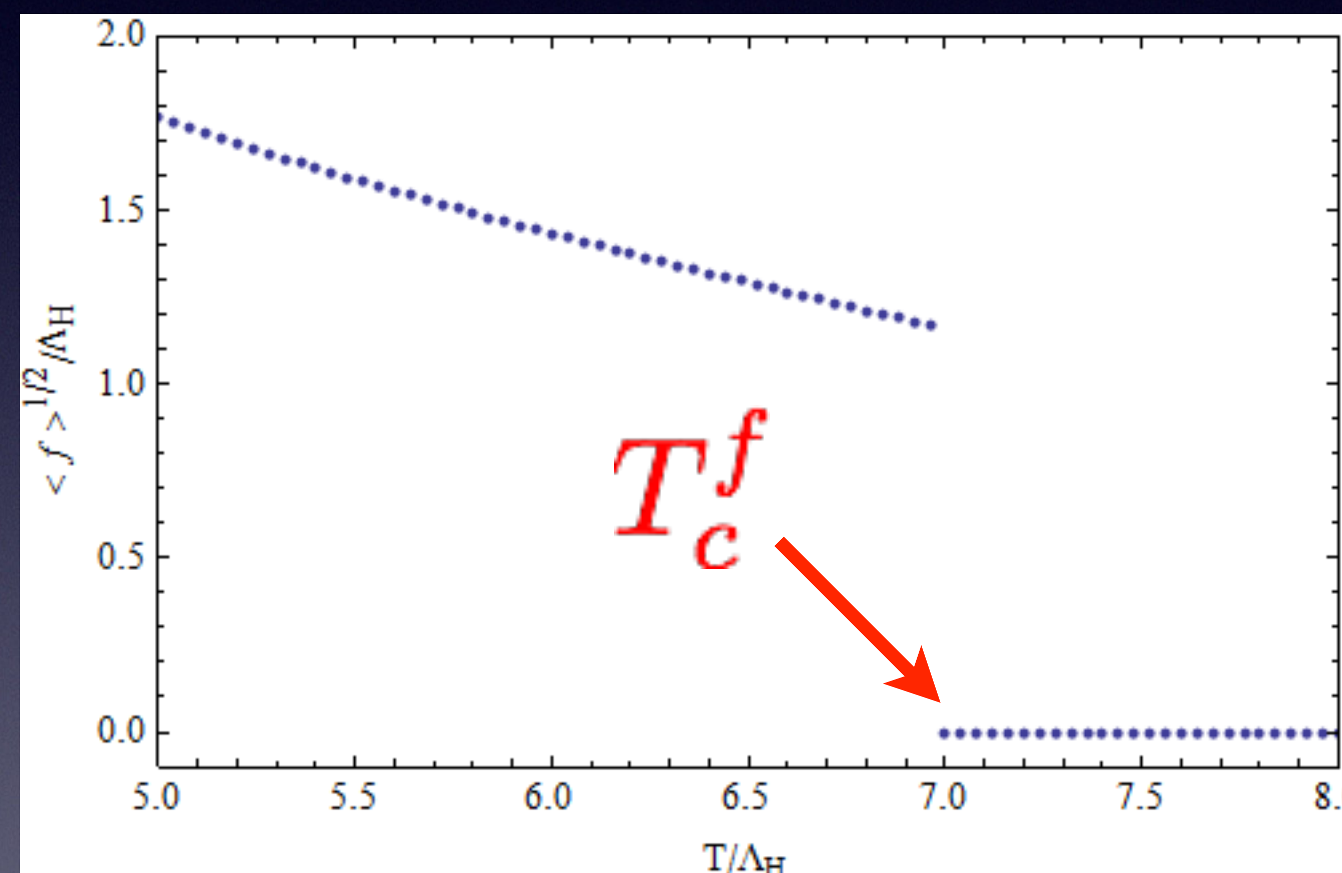
$$\langle h \rangle|_{T=T_c^h} = 0 \qquad \langle S^\dagger S \rangle|_{T=T_c^f} = 0$$

Scale PT could be 1st order

$V_{\rm eff}/\Lambda_{\rm H}^4$

$\langle S^\dagger S\rangle^{1/2}/T$



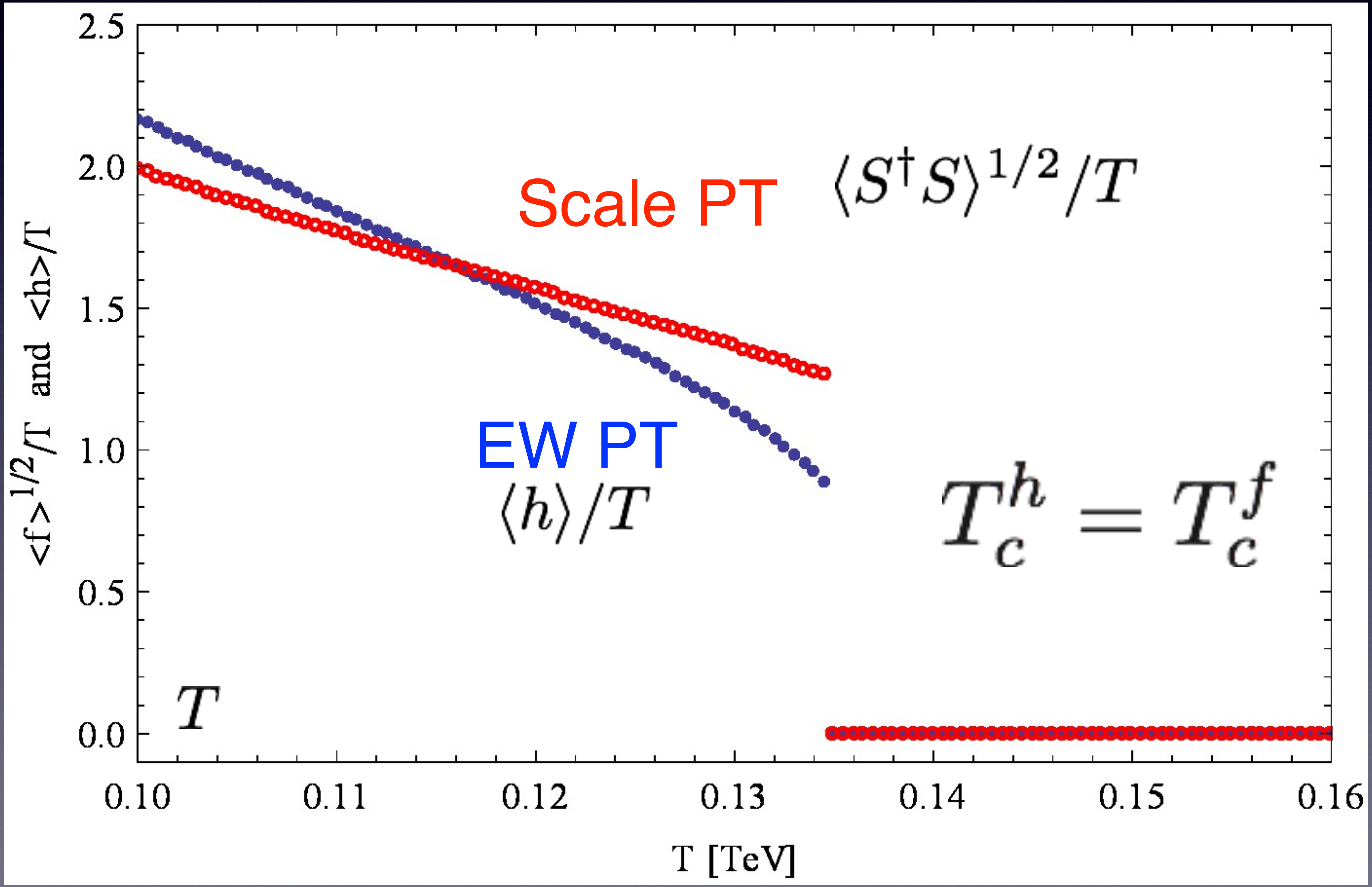


$\langle S^\dagger S\rangle^{1/2}/\Lambda_{\rm H}$

$T/\Lambda_{\rm H}$

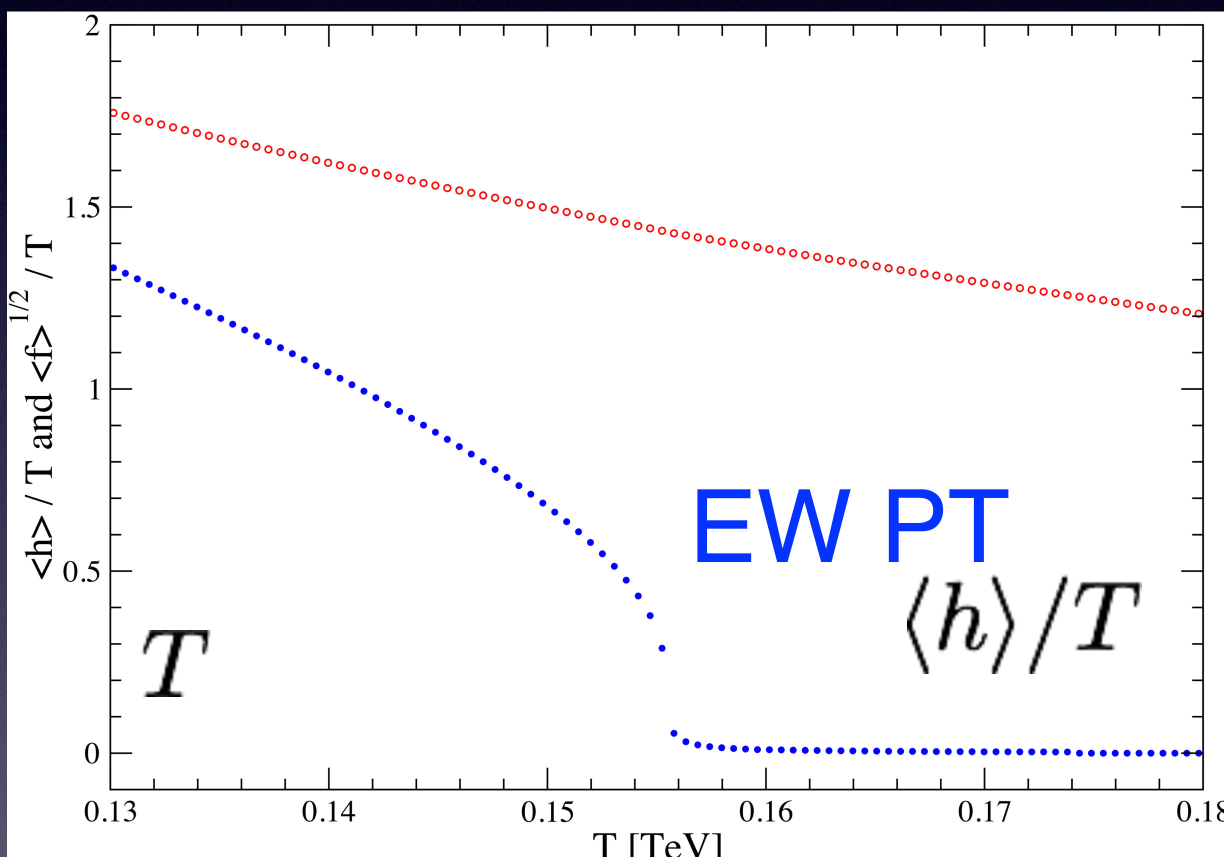
Electroweak and scale phase transitions

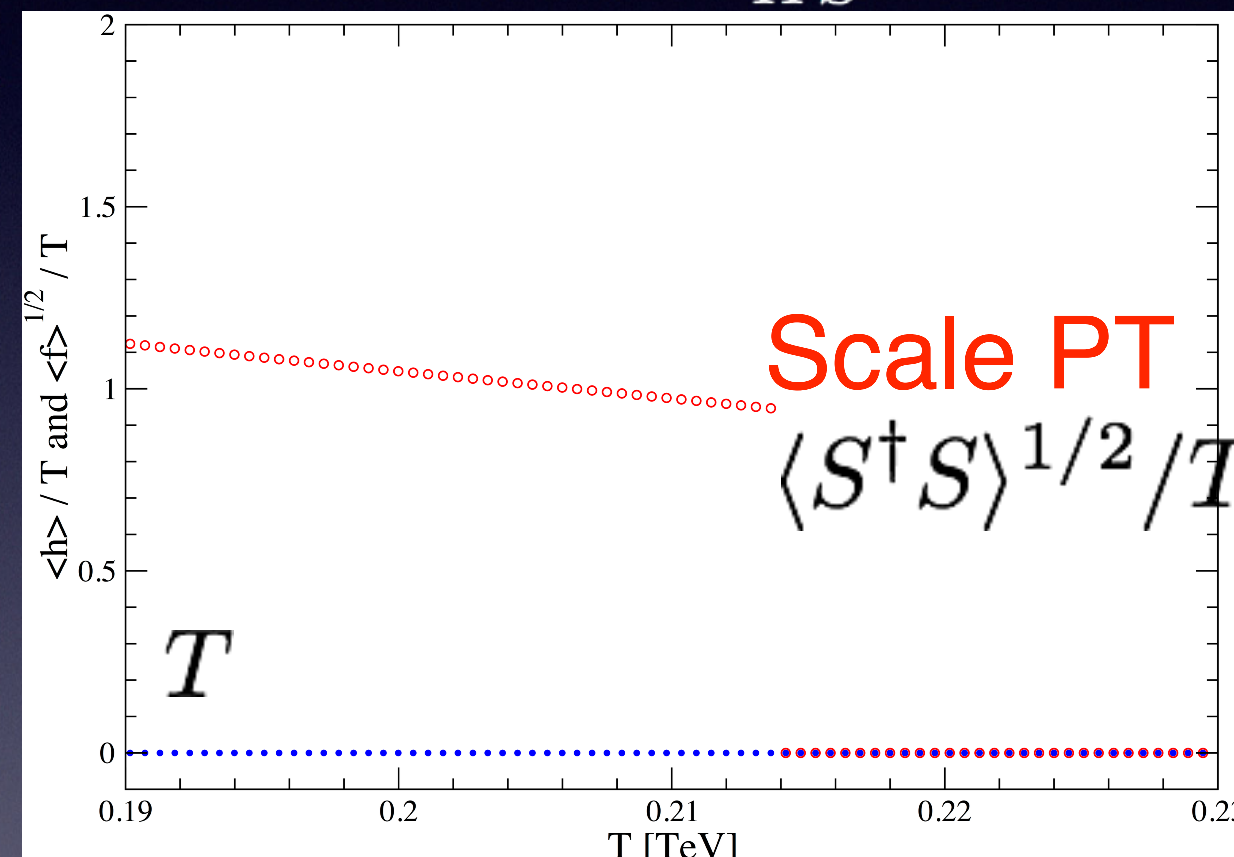
$\lambda_{HS} = 0.296$



Electroweak and scale phase transitions

$$T_c^h < T_c^f \qquad \lambda_{HS} = 0.086$$



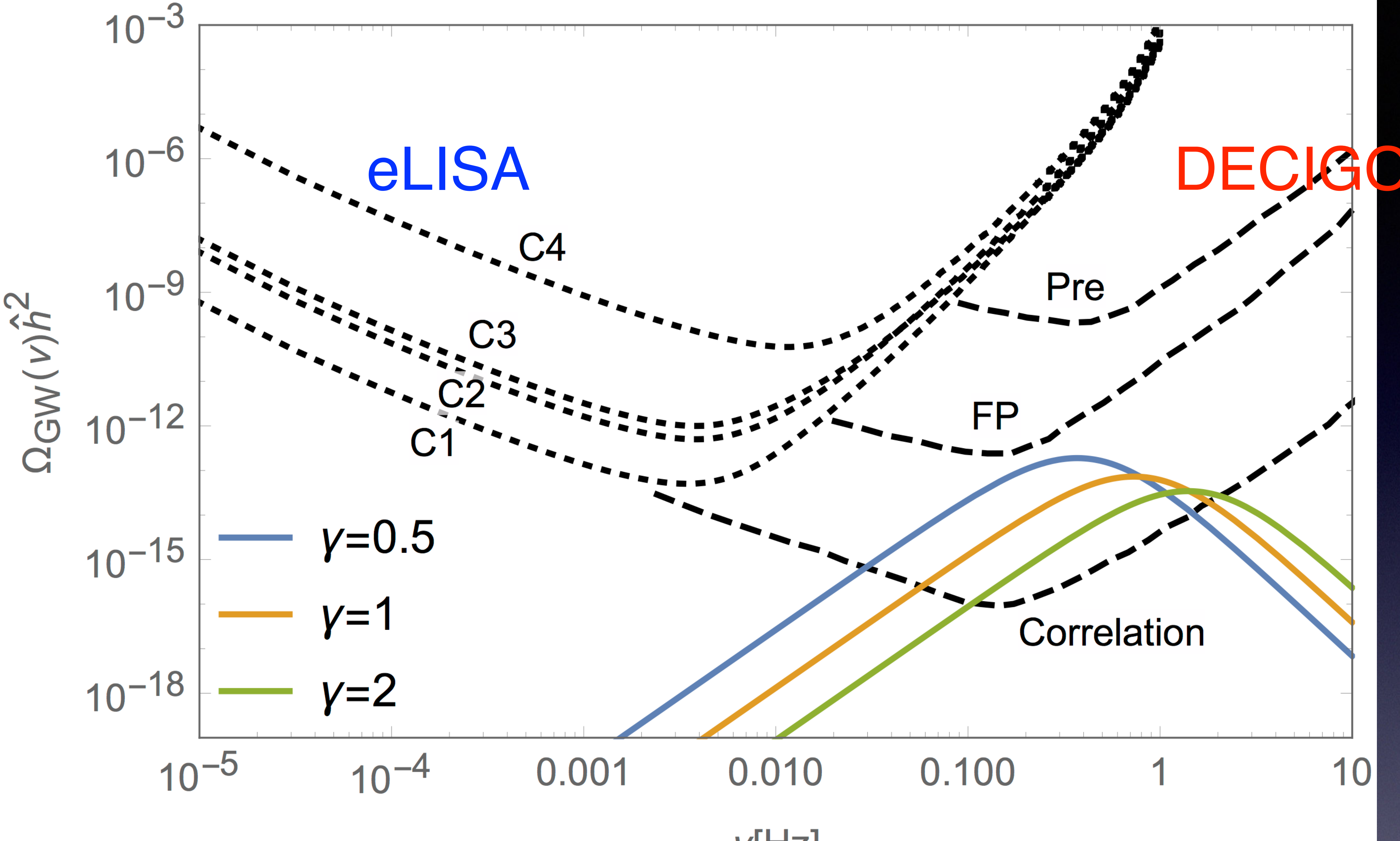


The EW PT depends on λ_{HS}.

$$\mathcal{L}_{\mathrm{SM}}\big|_{m\to 0} \qquad (H^\dagger H)(S_i^\dagger S_i) \qquad \mathcal{L}_{\mathrm{H}}$$

Gravitational waves from 1st order scale phase transition

- 1st order phase transition
  - latent heat $\Delta Q = T\Delta S$: Energy source of GWs
  - Gravitational waves could be produced.
- In the SM, the EW PT is weak 1st order.



J. Kubo and **MY,** JCAP 1612, no.12, 001

γ	T_t [TeV]	$S_3(T_t)/T_t$	α	$\tilde{\beta}$	$\tilde{\Omega}_{\rm sw}\hat{h}^2$	$\tilde{\nu}_{\rm sw}$ [Hz]
0.5	0.300	149	0.070	3.7×10^3	1.9×10^{-13}	0.37
1.0	0.311	145	0.062	7.0×10^3	7.4×10^{-14}	0.73
2.0	0.316	146	0.059	13×10^3	3.4×10^{-14}	1.4