An interacting theory for multiple partially massless spin-2 fields

Based on arXiv:1906.03868, with N. Boulanger, C. Deffayet and S. Garcia-Saenz

Lucas Traina

"Physics of the Universe, Fields and Gravitation" unit
University of Mons, Belgium

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Plan

What is a partially massless field?

Wigner's classification of the irreducible representations of the Poincaré group :

$\Longrightarrow$ Fields are either massless or massive.

Classification of the irreducible representations of $(A)dS$:

$\Longrightarrow$ Fields can also be partially massless (PM) [Deser, Nepomechie (1983)].

- They have a non-zero mass parameter

$$m^2 = \frac{2\Lambda}{(D-1)(D-2)}(s-t-1)(s+t)\,, \quad t = 0, \ldots, s-2\,.$$

- They possess a gauge symmetry

$$\delta\varphi_{\mu_1 \ldots \mu_s} = \bar{\nabla}_{(\mu_{t+1}} \ldots \bar{\nabla}_{\mu_s}\xi_{\mu_1 \ldots \mu_t)} + \ldots$$

Why studying PM spin-2 fields?

Simplest case of PM field :

$$m^2 = \frac{4\Lambda}{(D-1)(D-2)} \text{ and } \delta h_{\mu\nu} = \bar{\nabla}_\mu \bar{\nabla}_\nu \epsilon + \frac{2\Lambda}{(D-1)(D-2)}\, \bar{g}_{\mu\nu}\, \epsilon$$

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- Solve problems of massive gravity related to the scalar mode [van Dam, Veltman (1970), Zakharov (1970)], [Vainshtein (1972)]

  Stueckelberg decomposition :

$$h_{\mu\nu} \to h_{\mu\nu} + \frac{1}{m}\bar{\nabla}_\mu A_\nu + \frac{1}{m}\bar{\nabla}_\nu A_\mu + \frac{1}{m^2}\bar{\nabla}_\mu \bar{\nabla}_\nu \varphi$$

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Our results :

- Enhancement of the above no-go results.
- First interacting theory of PM fields consistent at the mathematical level.

Plan

Deformation procedure

Goal: Introduction of interactions preserving the number of gauge symmetries of the free theory.

Deformation procedure

- Perturbative deformation of the initial action S_0 and gauge transformation δ_0 :

$$
\begin{aligned}
S &= S_0 + \alpha S_1 + \alpha^2 S_2 + O(\alpha^3)\,, \\
\delta &= \delta_0 + \alpha \delta_1 + \alpha^2 \delta_2 + O(\alpha^3)\,.
\end{aligned}
$$

- Gauge invariance $\delta S = 0$ order by order in perturbation implies the conditions

$$
\begin{aligned}
0 &= \delta_0 S_0\,, \\
0 &= \delta_0 S_1 + \delta_1 S_0\,, \\
0 &= \delta_0 S_2 + \delta_1 S_1 + \delta_2 S_0\,, \\
&\vdots
\end{aligned}
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Deformation procedure in the BV formalism

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Interactions = Deformation of the BV functional of the initial theory
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- Information for free on all the gauge structure of the deformed theory.
- Possibility to have strong results without any restrictions on the number of derivatives and at higher-orders in the fields.

Plan

Starting point

Sum of N decoupled Fierz-Pauli actions around $(A)dS_D$ in the PM limit.
Invariant under the PM gauge symmetry

$$\delta_0 h^a_{\mu\nu} = \bar{\nabla}_\mu \bar{\nabla}_\nu \epsilon^a + \frac{2\Lambda}{(D-1)(D-2)} \bar{g}_{\mu\nu} \, \epsilon^a$$

- $a = 1, \ldots, N$ is a "color index".
- The tensors $F^a_{\mu\nu\rho} := \bar{\nabla}_\mu h^a_{\nu\rho} - \bar{\nabla}_\nu h^a_{\mu\rho}$ are abelian field strengths in the sense that they are invariant under the PM gauge symmetry.

Starting point

Sum or difference of N decoupled Fierz-Pauli actions around $(A)dS_D$ in the PM limit :

$$S_0 = -\frac{1}{4}\int d^D x\sqrt{-\bar{g}}\, k_{ab}\left[F^{a\mu\nu\rho}F^b_{\mu\nu\rho} - 2F^{a\mu}F^b_\mu\right]$$

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- $F^a_\mu := \bar{g}^{\nu\rho}F^a_{\mu\nu\rho}$.
- k_{ab} is an internal metric that may be chosen to be diagonal, with entries $+1$ and -1.

Deformation of the gauge algebra

Consistency implies that the deformed gauge transformation must form an algebra

$$\left[\delta^{(\epsilon_1)}\,,\,\delta^{(\epsilon_2)}\right] h^a_{\mu\nu} = \delta^{(\chi)} h^a_{\mu\nu} + \text{trivial}\,.$$

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Unique candidate at first order :

$$\left[\delta^{(\epsilon_1)}\,,\,\delta^{(\epsilon_2)}\right] h^a_{\mu\nu} = \delta^{(\chi)}_0 h^a_{\mu\nu} + O(\alpha^2)$$
$$\chi = \alpha\left({m^a}_{bc}\,\epsilon^b_1\epsilon^c_2 + {n^a}_{bc}\,\bar{\nabla}^\mu\epsilon^b_1\bar{\nabla}_\mu\epsilon^c_2\right) + O(\alpha^2)\,,$$

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Result valid to all orders : abelian algebra

$${m^a}_{bc} = 0\,, \qquad {n^a}_{bc} = 0\,.$$

Extends the result of [Garcia-Saenz, Hinterbichler, Joyce, Mitsou, Rosen (2016)] without any assumption on the number of derivatives and at higher-orders in the fields.

First-order deformations

Result consistent with the gauge invariance at first order :

$$\delta_1 h^a_{\mu\nu} = \alpha\, f^a{}_{b,c} F^b_{\rho(\mu\nu)} \bar{\nabla}^\rho \epsilon^c\,, \quad D = 4\,,$$

$$\text{with } f_{ab,c} := k_{ad}\, f^d{}_{b,c} = f_{(ab),c}\,.$$

This leads to the cubic action $S_1 = \int d^4x \sqrt{-\bar{g}}\; h^a_{\mu\nu} J^{\mu\nu}_a$, where

$$J^{\mu\nu}_a := f_{bc,a} \left[F^{b\mu}{}_{\rho\sigma} F^{c\nu\rho\sigma} - F^{b\mu} F^{c\nu} - F^{b\rho(\mu\nu)} F^c_\rho - \tfrac{1}{4}\, \bar{g}^{\mu\nu} F^{b\rho\sigma\lambda} F^c_{\rho\sigma\lambda} + \tfrac{1}{2}\, \bar{g}^{\mu\nu} F^{b\rho} F^c_\rho \right]$$

Consistency condition implies on the current that it must be conserved in the sense

$$\bar{\nabla}_\mu \bar{\nabla}_\nu J^{\mu\nu}_a + \tfrac{2\Lambda}{(D-1)(D-2)}\, \bar{g}_{\mu\nu} J^{\mu\nu}_a \approx 0\,.$$

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- Extension of the cubic vertex of a single PM spin-2 field to several ones.
- Other possibilities were a priori possible for several fields.
- $f_{ab,c} = f_{(ab),c}$ not trivial $\longrightarrow$ # deformation parameters $= \frac{N^2(N+1)}{2}$.

Second-order deformation

Consistency of the deformed gauge symmetry :
Only possible solution is to impose the quadratic constraints

$$f_{ae,b}\, f^{e}{}_{c,d} := k^{ef}\ f_{ea,b}\, f_{fc,d} = 0\,,$$

which imply

$$\delta_2 = 0\,.$$

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Consistency at all orders :
The theory that stops at the cubic level $S = S_0 + S_1$ is fully consistent under the gauge symmetry $\delta = \delta_0 + \delta_1$.

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Conclusions and perspectives

- Enhancement of the known no-go results to multiple PM spin-2 fields and with less assumptions.
- First interacting theory for PM fields mathematically consistent.
- Physical problem because of the relative signs between kinetic terms in the action.
- Related to conformal multi-gravity theories.
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Thank you for your attention!