# *Selected topics in QCD*

*Krzysztof Kutak*

*NCN*

# QCD review road map

*Monte Carlo*

*ITMD factorization*

*jet quenching*

*gluon saturation*

*hadron tomography*

*matrix elements*

*spin*

important but not addressed here

*fragmentation functions*

*transversal momentum dependent distributions*

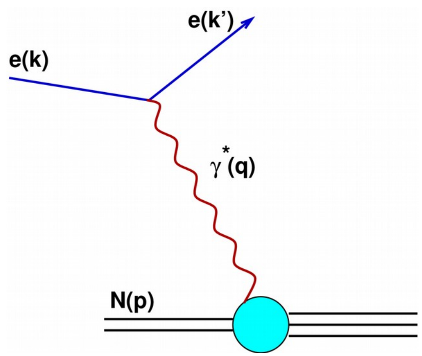
*lattice*

$$\frac{d\sigma}{dPS} \propto ISR \otimes \hat{\sigma}_{ab\to c_1\ldots c_n} \otimes FSR \otimes FF$$

## *Factorization for forward physics*

# *High energy limit and forward physics*

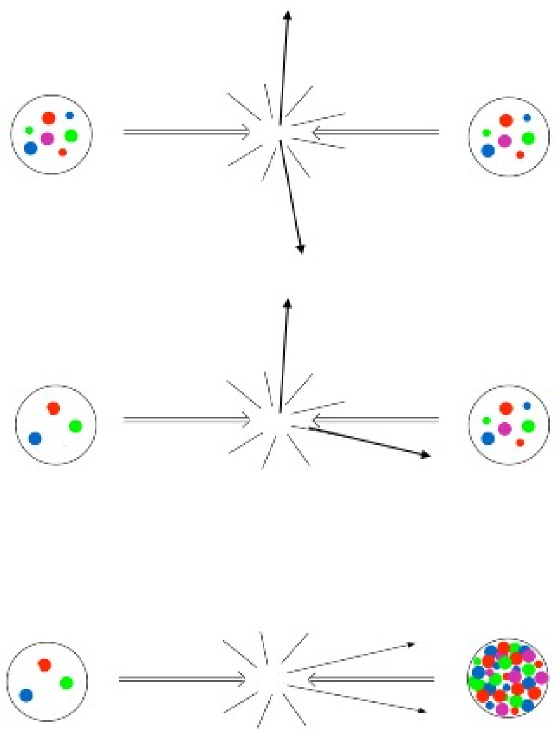
*In DIS*




$$x = \frac{Q^2}{2p\cdot q} = \frac{Q^2}{Q^2 + \hat{s}_{\gamma^* N}}$$

$$s_{(\gamma^* N)} \to \infty \qquad x = \frac{Q^2}{s_{(\gamma^* N)}} \to 0$$

*In p-p or p-A*



*central-central i.e. dilute dilute*

$x_1 \sim x_2$

*forward-central i.e.*
*dilute – moderately dense*

$x_1 > x_2$

*forward-forward i.e.*
*dilute -dense*

$x_1 >> x_2$

*from C. Marquet*

*low x*

*structure of hadrons at extreme conditions*

# High Energy Factorization

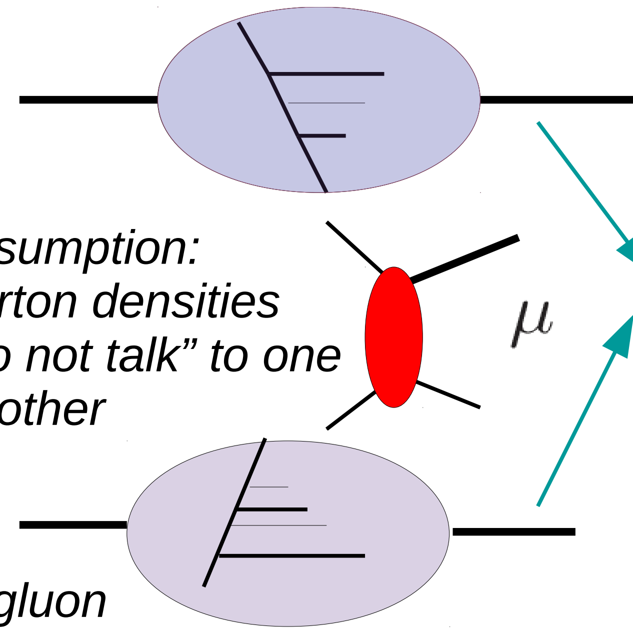
$$\frac{d\sigma}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \sum_{c,d}\int \frac{d^2k_{1t}}{\pi}\frac{d^2k_{2t}}{\pi}\frac{1}{16\pi^2(x_1x_2s)^2}\overline{|\mathcal{M}_{g^*g^*\to cd}|}^2\, \mathcal{F}_A(x_1,k_{1t}^2,\mu^2)\,\mathcal{F}_B(x_2,k_{2t}^2,\mu^2)\frac{1}{1+\delta_{cd}}$$

*originally derived for gg → QQ*

*developed for low x*

*rapidities and $p_t$ of produced jets*

*assumption: parton densities "do not talk" to one another*




*decreasing longitudinal momentum fractions of off-shell partons BFKL like*

*decreasing longitudinal momentum fractions of off-shell partons BFKL like*

*$\mu$   hard scale e.g. average of $p_t$ of jets*

*$k_{t1}$, $k_{t2}$   transversal momentum of incoming gluon*

*x   longitudinal momentum of incoming gluon In this framewoμrk of x of comparable values*

*L.V. Gribov, E.M. Levin, M.G. Ryskin Phys.Rept. 100 (1983) 1-150*

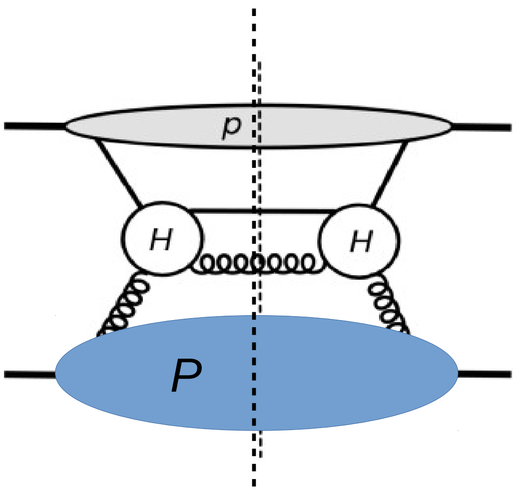
*S. Catani, M. Ciafaloni, F. Hautmann Nucl.Phys. B366 (1991) 135-188*

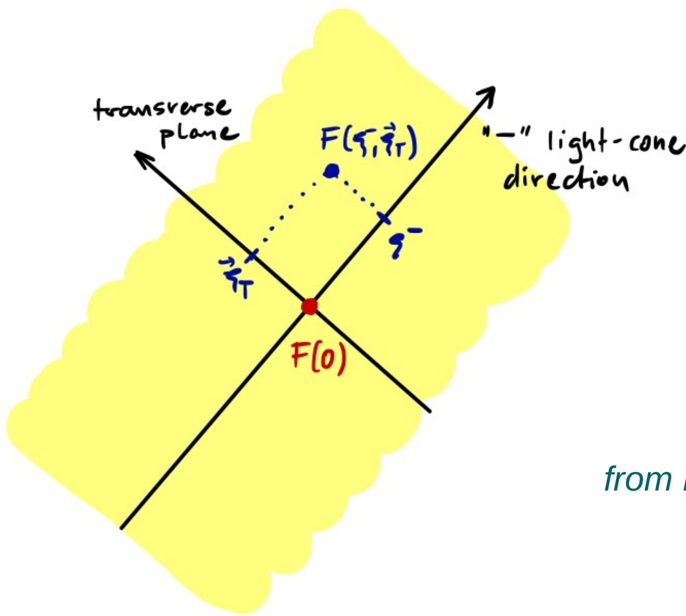
*Helicity based method for any process*

*A. van Hameren, P. Kotko, K. Kutak JHEP 1301 (2013) 078*

# Definition of TMD – gauge links

*The formula for HEF is strictly valid for large transversal  momentum and was obtained  in a specific gauge.  Ultimately we want to go beyond this.*




*from P. Kotko, Bialasówka 2019*

*Naive definition of gluon distribution*

$$\mathcal{F}(x,k_T) = 2\int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+}\, e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T}\ \langle P|\,\mathrm{Tr}\left\{\hat{F}^{i+}(0)\,\hat{F}^{i+}\left(\xi^+=0,\xi^-,\vec{\xi}_T\right)\right\}|P\rangle$$

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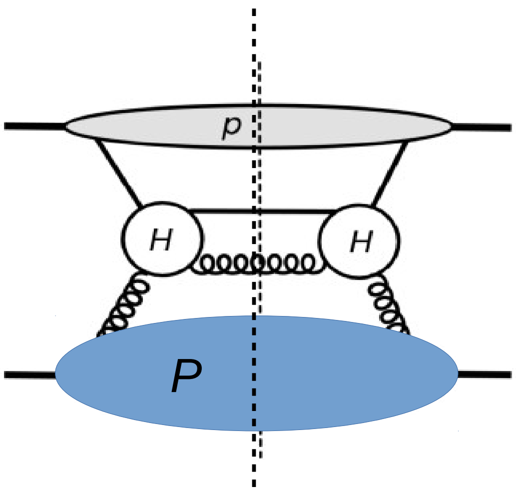
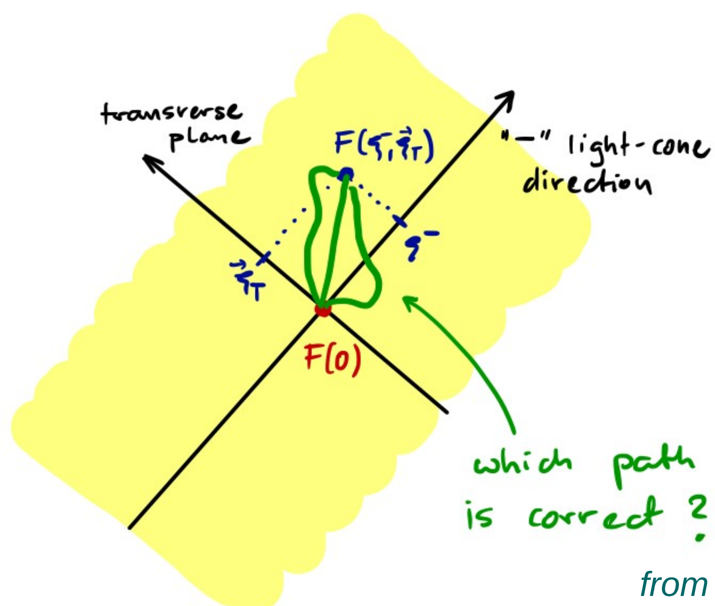


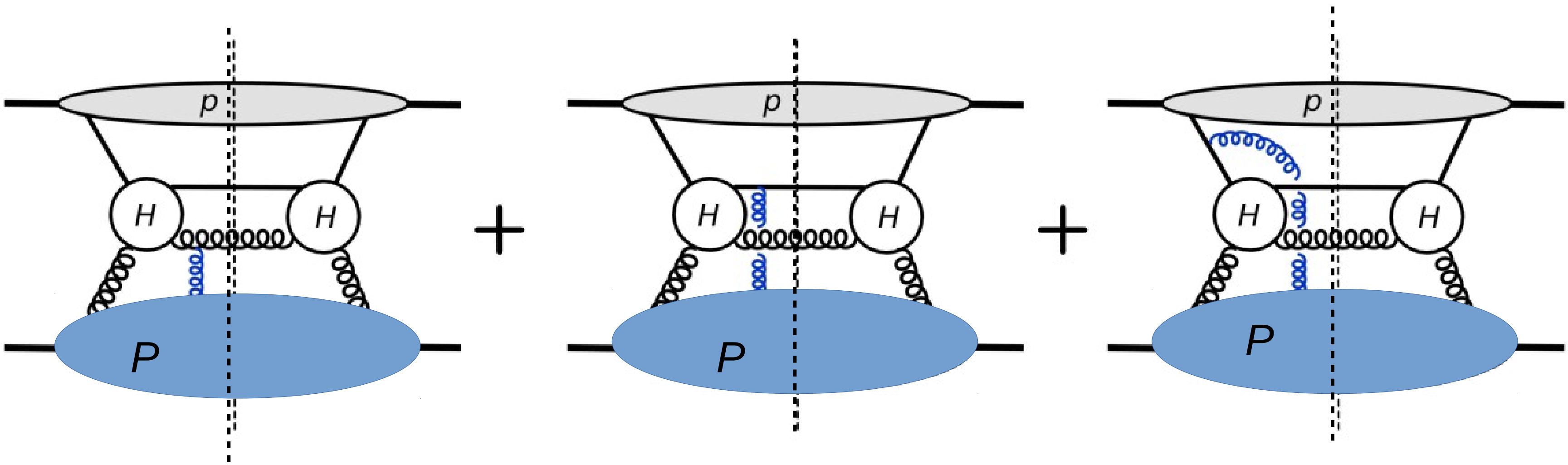




*from P. Kotko, Bialasówka 2019*

*Naive definition*

$$\mathcal{F}(x, k_T) = 2 \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T} \langle P| \operatorname{Tr}\left\{ \hat{F}^{i+}(0) \hat{F}^{i+}\left(\xi^+ = 0, \xi^-, \vec{\xi}_T\right)\right\} |P\rangle$$

# Definition of TMD – gauge links




*+ similar diagrams with 2,3,….gluon exchanges.*

*C.J. Bomhof, P.J. Mulders, F. Pijlman*
*Eur.Phys.J. C47 (2006) 147-162*

*All this need to be resummed*

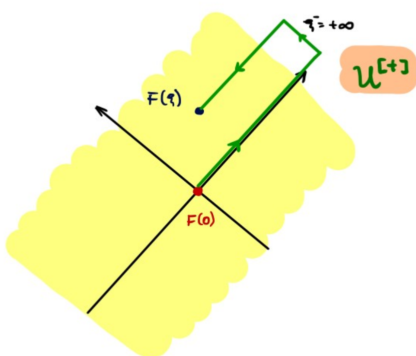
*This is achieved via gauge link which renders the gluon density gauge invariant*

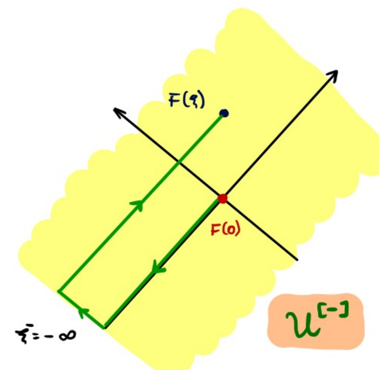
$$\mathcal{F}\left(x,k_T\right)=2\int\frac{d\xi^-d^2\xi_T}{(2\pi)^3\,P^+}\,e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T}\,\langle P|\,\mathrm{Tr}\left\{\hat{F}^{i+}\left(0\right)\mathcal{U}_{C_1}\hat{F}^{i+}\left(\xi\right)\mathcal{U}_{C_2}\right\}|P\rangle$$

*Hard part defines the path of the gauge link* $\mathcal{U}^{[C]}(\eta;\xi)=\mathcal{P}\exp\left[-ig\int_C \mathrm{d}z\cdot A(z)\right]$

# Definition of TMD – gauge links

*Two basic structures arise:*

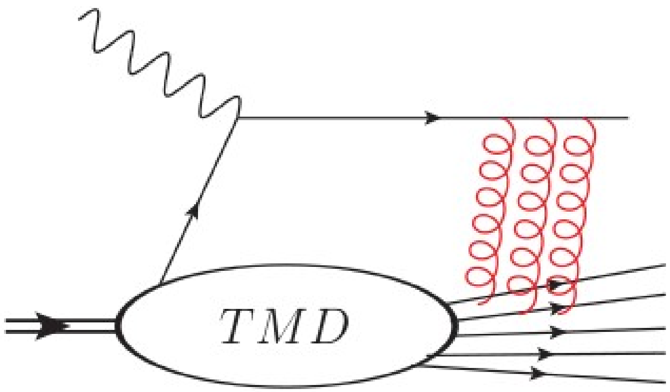


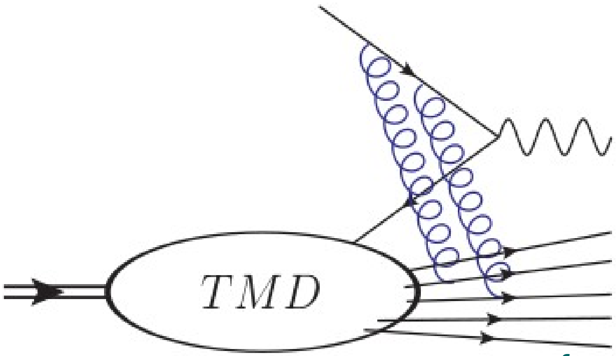

*from P. Kotko, Bialasówka 2019*

*Semi Inclusive DIS*

*Drell-Yan*




final state interactions




initial state interactions

*from R. Boussarie Initial Stages 2019*

$$\Phi_q^{[+]}(x, p_T) = \int \frac{\mathrm{d}(\xi \cdot P)\mathrm{d}^2\xi_T}{(2\pi)^3}\, e^{ip\cdot\xi}\, \langle H|\, \overline{\psi}(0)\, \mathcal{U}^{[+]}\, \psi(\xi)\, |H\rangle$$

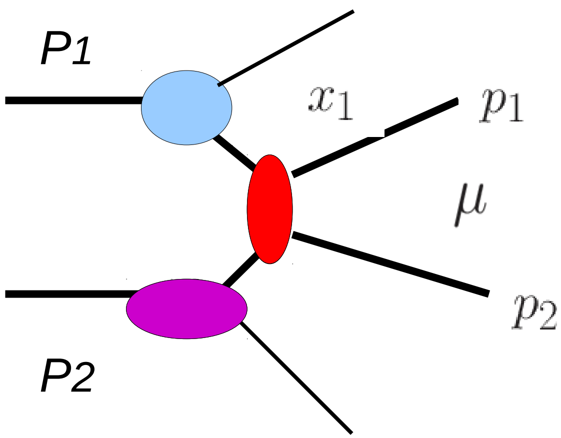
*C.J. Bomhof, P.J. Mulders, F. Pijlman Eur.Phys.J. C47 (2006) 147-162*

$$\mathcal{U}^{[\pm]} = U^n_{[(0^-,\mathbf{0}_T);(\pm\infty^-,\mathbf{0}_T)]} U^T_{[(\pm\infty^-,\mathbf{0}_T);(\pm\infty^-,\boldsymbol{\infty}_T)]} U^T_{[(\pm\infty^-,\boldsymbol{\infty}_T);(\pm\infty^-,\boldsymbol{\xi}_T)]} U^n_{[(\pm\infty^-,\boldsymbol{\xi}_T);(\xi^-,\boldsymbol{\xi}_T)]}$$

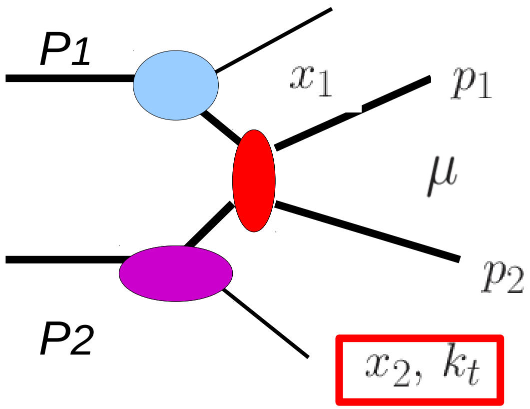
# Improved Transversal Momentum Dependent Factorization

$$\frac{d\sigma_{\rm SPS}^{P_1P_2\to{\rm dijets}+X}}{dy_1dy_2dp_{1t}dp_{2t}d\Delta\phi}=\frac{p_{1t}p_{2t}}{8\pi^2(x_1x_2s)^2}\sum_{a,c,d}x_1f_{a/P_1}(x_1,\mu^2)\,|\overline{\mathcal{M}_{ag^*\to cd}}|^2\quad\mathcal{F}_{g/P_2}(x_2,k_t^2)\frac{1}{1+\delta_{cd}}$$

M. Deak, F. Hautmann, H. Jung, K. Kutak JHEP 0909 (2009) 121




can be be used for estimates of saturation effects




Generalization but no possibility to calculate fully decorelations since no kt in ME

F. Dominguez, C. Marquet, Bo-Wen Xiao, F. Yuan Phys.Rev. D83 (2011) 105005

A method has been found to include kt in ME and express the factorization formula in terms of gauge invariant sub amplitudes

Conjecture P. Kotko K. Kutak , C. Marquet , E. Petreska , S. Sapeta, A. van Hameren, JHEP 1509 (2015) 106

Proof Tolga Altinoluk, Renaud Boussarie, Piotr Kotko JHEP 1905 (2019) 156

gauge invariant amplitudes and TMDs

$$\frac{d\sigma^{pA\to ggX}}{d^2P_td^2k_tdy_1dy_2}=\frac{\alpha_s^2}{(x_1x_2s)^2}\;x_1f_{g/p}(x_1,\mu^2)\sum_{i=1}^{6}\mathcal{F}_{gg}^{(i)}H_{gg\to gg}^{(i)}$$

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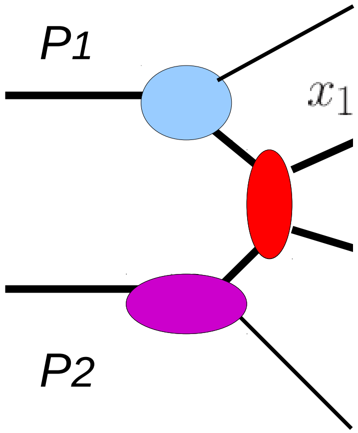


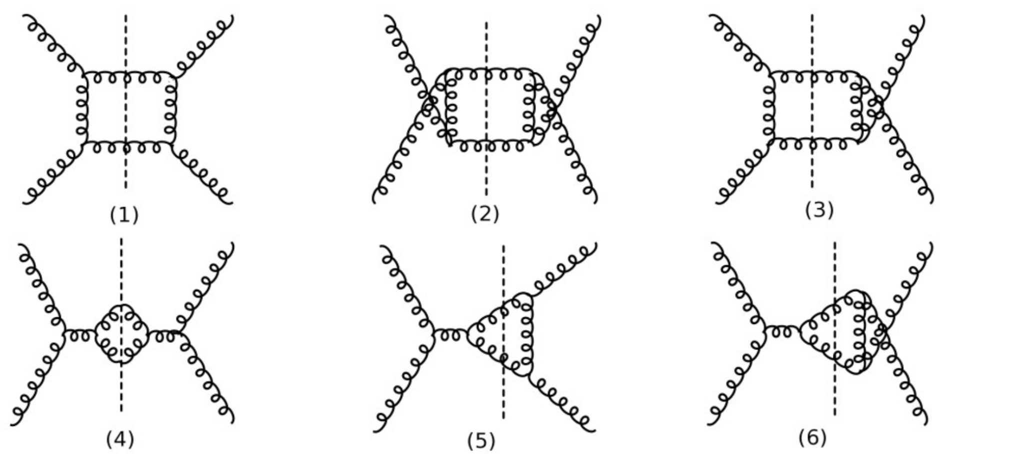


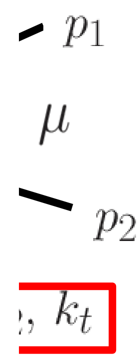


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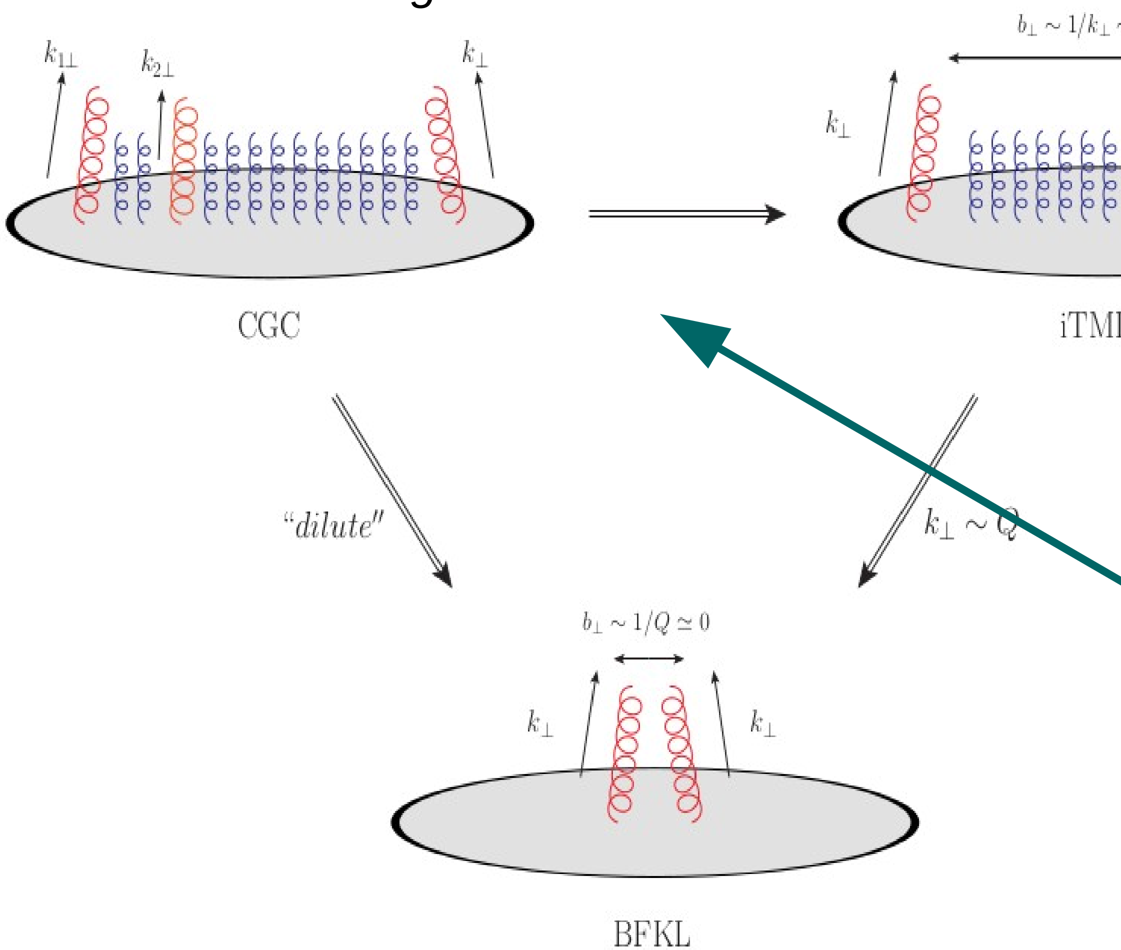
# *ITMD from CGC*

*T. Altinoluk, R. Boussarie, P. Kotko JHEP 1905 (2019) 156*

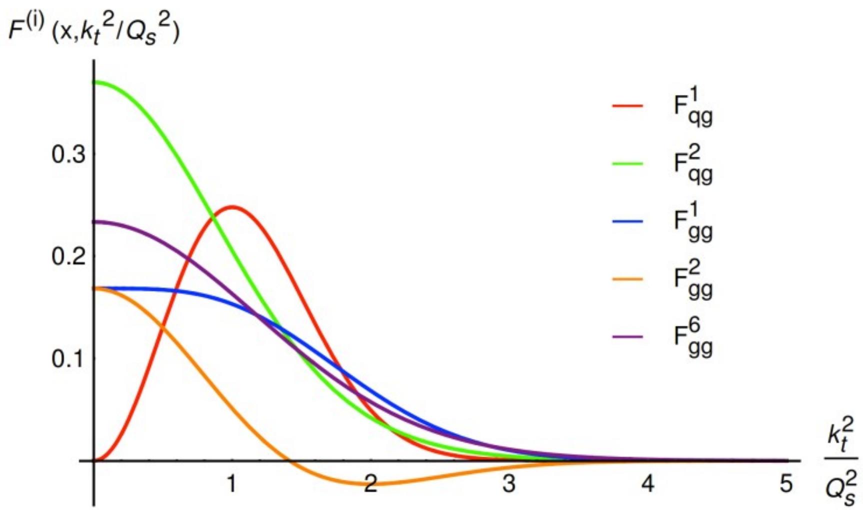
*Expansion in distance - parameter entering as argument  Wilson lines appearing in generic CGC amplitude i.e. amplitude for propagation in strong color field of target*

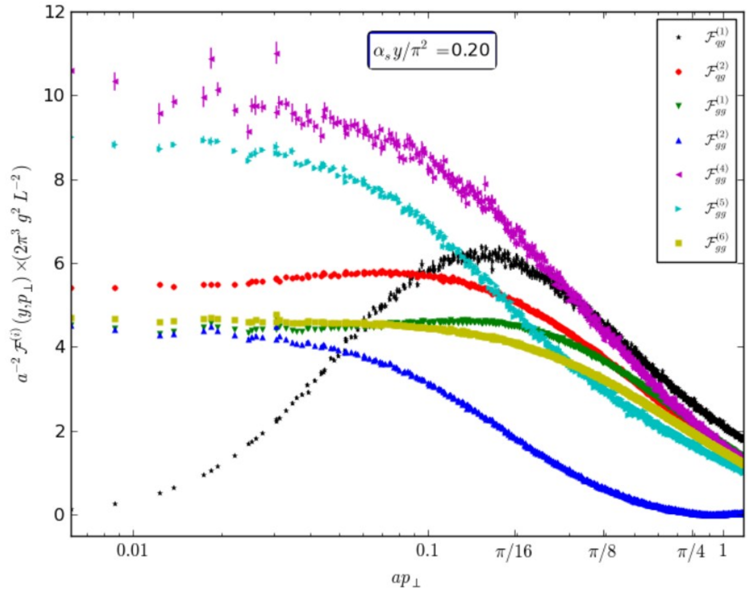



*from R. Boussarie Initial Stages 2019*

*Wandzura-Wilczek approximmation ITMD neglects higher genuine twist contributions i.e. hard gluon exchanges  between the target and the amplitude, while it resums all kinematic twist*

# Plots of ITMD gluons




*Calculation – in large Nc approximation with analitic model for dipole gluon density – all gluons can be calculated from the dipole one*

*Kotko, K.K, Marquet, Petreska, Sapeta, van Hameren '16 JHEP 1612 (2016) 034*

*Standard HEF gluon density*

*Obtained from solutions of evolution equation which accounts for finite Nc. JIMWLK equation used to obtain Evolved gluon densities.*

*The JIMWLK equation is a renormalization group equation for the Wilson lines, obtained by integrating out the quantum fluctuations at smaller and smaller Bjorken-x.*

*C. Marquet, E. Petreska, C. Roiesnel JHEP 1610 (2016) 065*

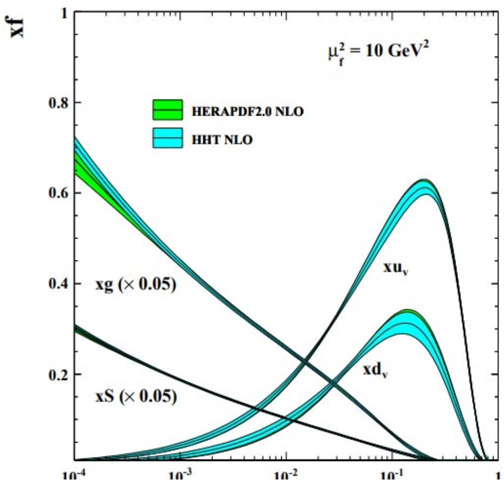
*The other densities are flat at low $k_t$ → less saturation*

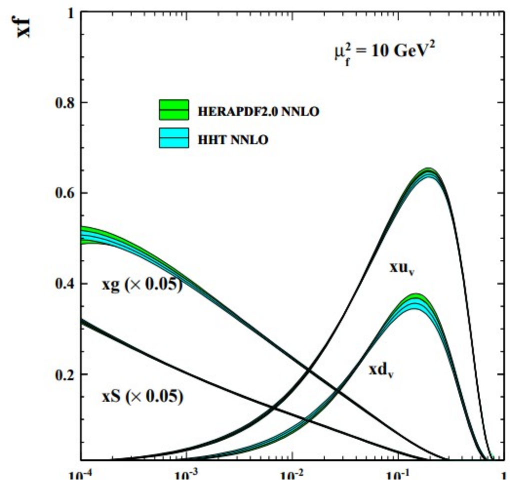
*Not negligible differences at large $k_t$ → differences at small angles*

## *Saturation*

# *Parton densities NLO, NNLO, nuclear*

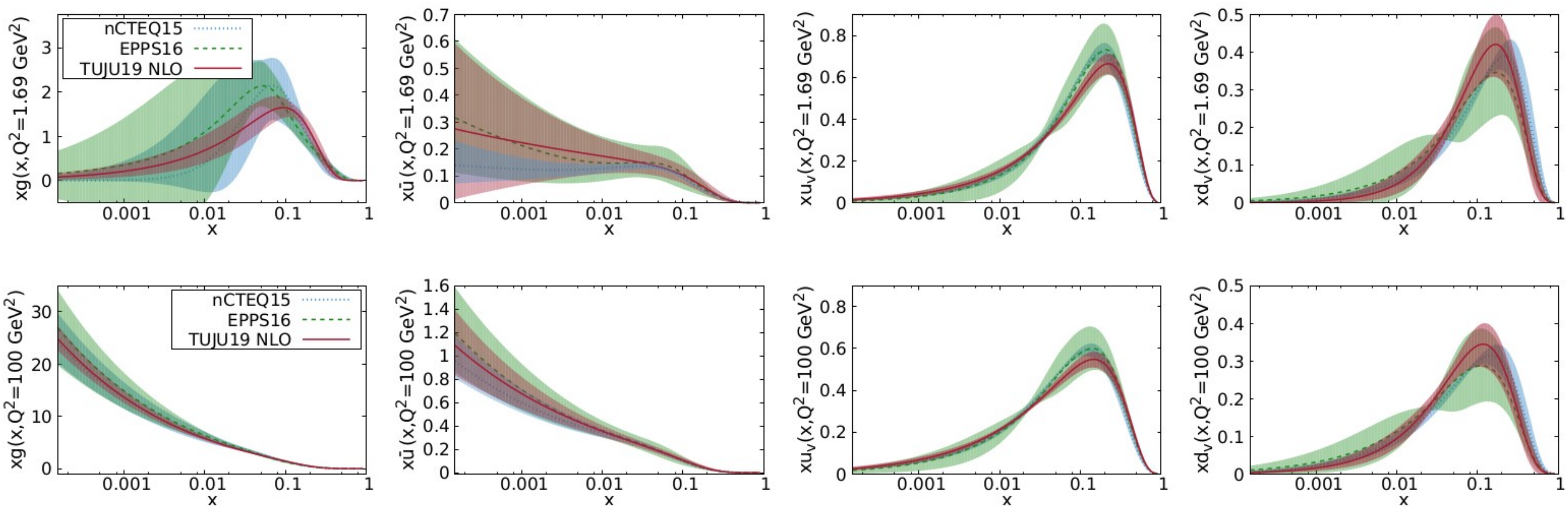
*proton pdfs*




*Fits end extraction of parton pdfs suggests that at low x structure of hadrons is still not fully understood. Large higher order corrections at low x and uncertainity asks for better understanding*

*lead pdfs –*
*large gluon uncertainity at low x*



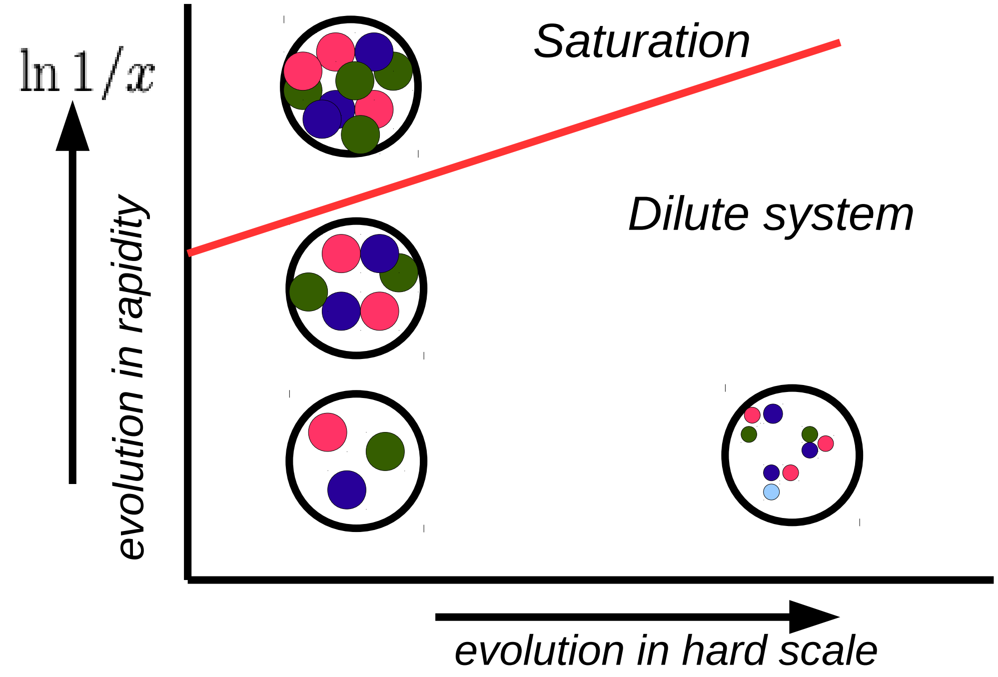

*Walt, Helenius, Vogelsang, 190804983*

# Saturation

Saturation – *state where number of gluons stops growing due to high occupation number. Way to fulfill unitarity requirements in high energy limit of QCD*.

*L.V. Gribov, E.M. Levin, M.G. Ryskin*
*Phys.Rept. 100 (1983) 1-150*

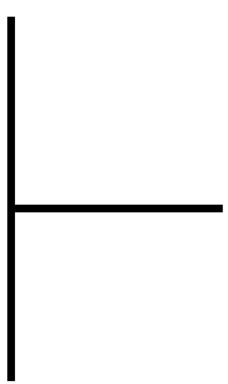
*Larry D. McLerran, Raju Venugopalan*
*Phys.Rev. D49 (1994) 3352-3355*



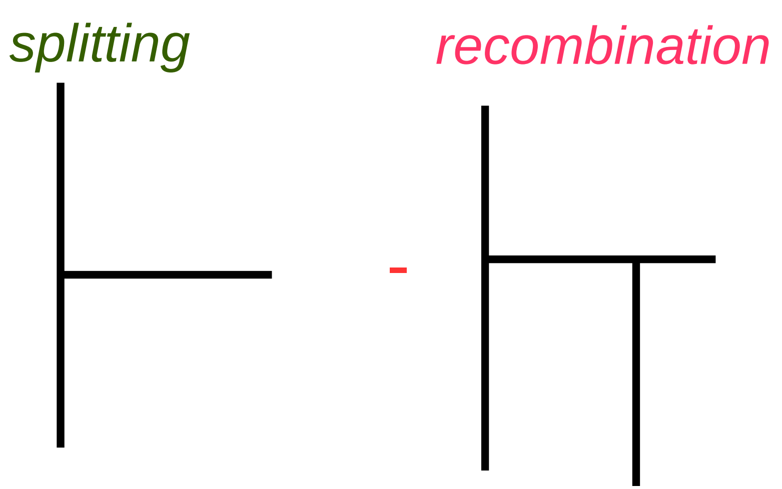

*On microscopic level it means that gluon apart splitting recombine*

*splitting*

*Linear evolution*
*Equation*
*BFKL*



*Nonlinear evolution equations*



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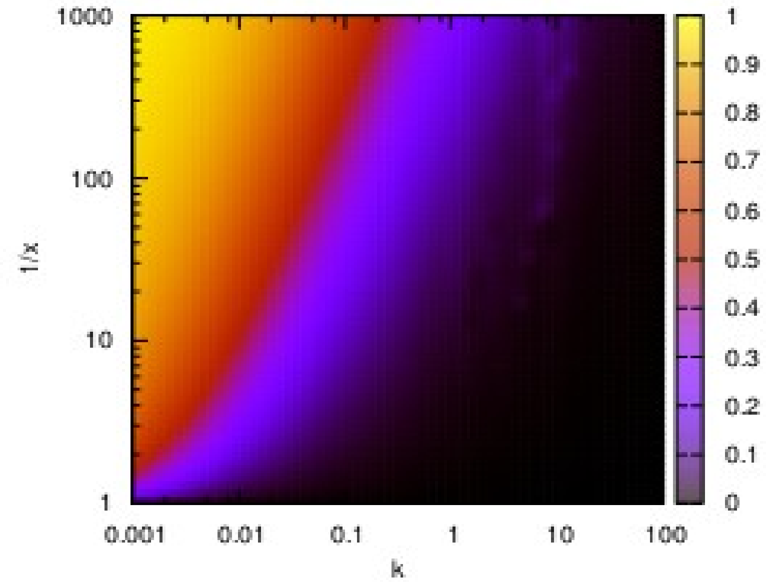


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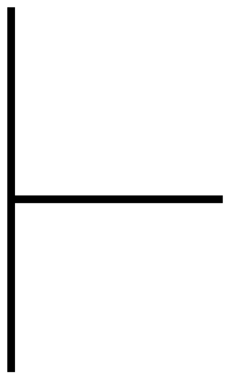
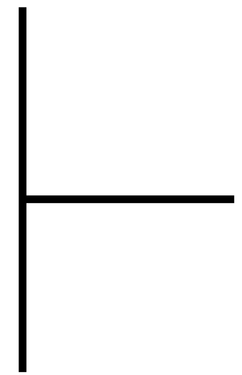


*Linear evolution equation BFKL*

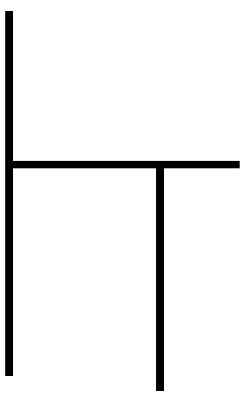
*Nonlinear evolution equations*

*splitting*



-

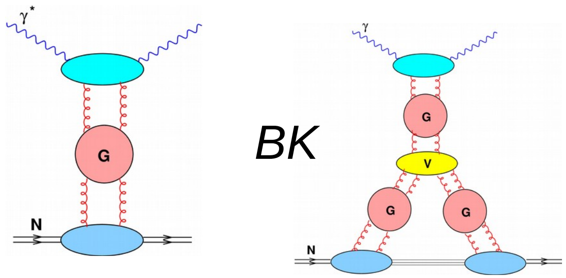
*recombination*



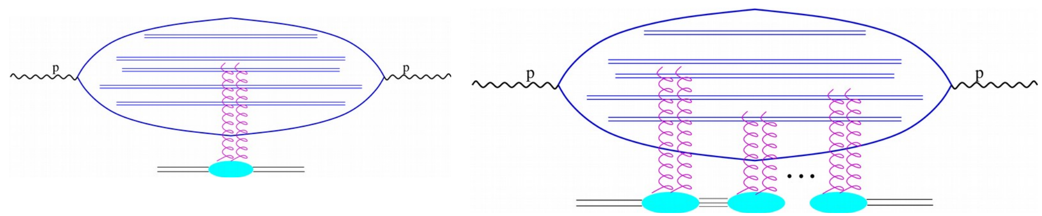
# Balitky Kovchegov equation in m- and p-space

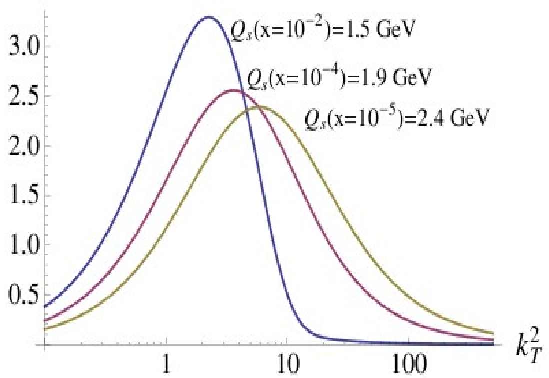
*momentum space - Bjorken frame*

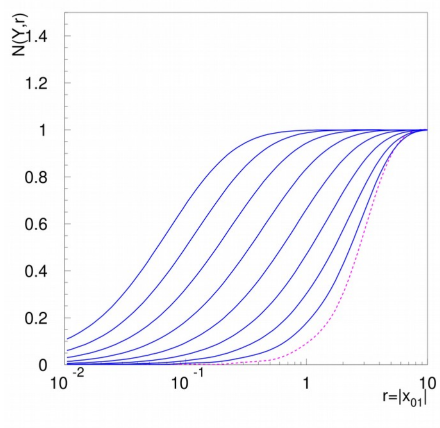
*BFKL* *BK*




*position space - Mueller frame*




from *A. Stasto Acta Phys.Polon. B35 (2004) 3069-3102*

$$\mathcal{F}(x,k) = \mathcal{F} + K_{ms} \otimes \mathcal{F}(x,k) - \frac{1}{R^2} TPV \otimes \mathcal{F}(x,k) \quad N(x,r,b) = N_0 + K_{ps} \otimes (N(x,r,b) - N(x,r,b)^2)$$

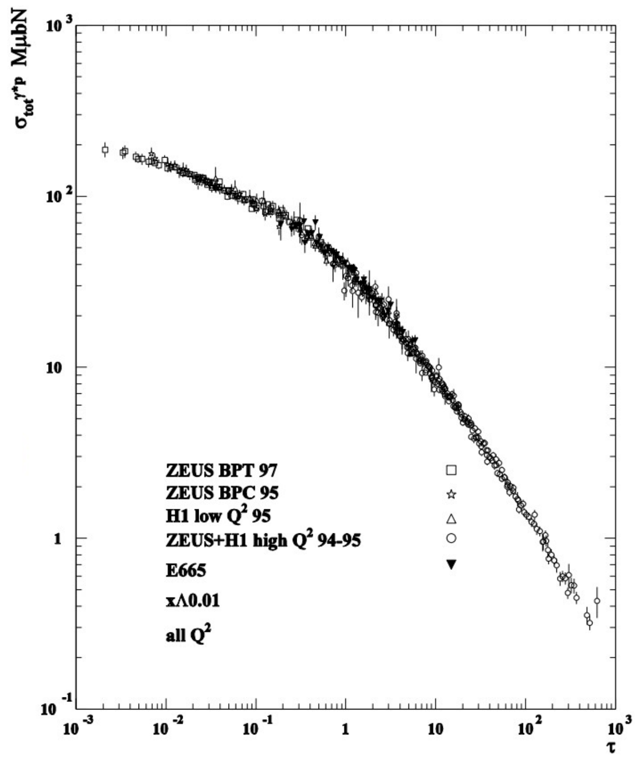
*Dipole unintegrated gluon density*

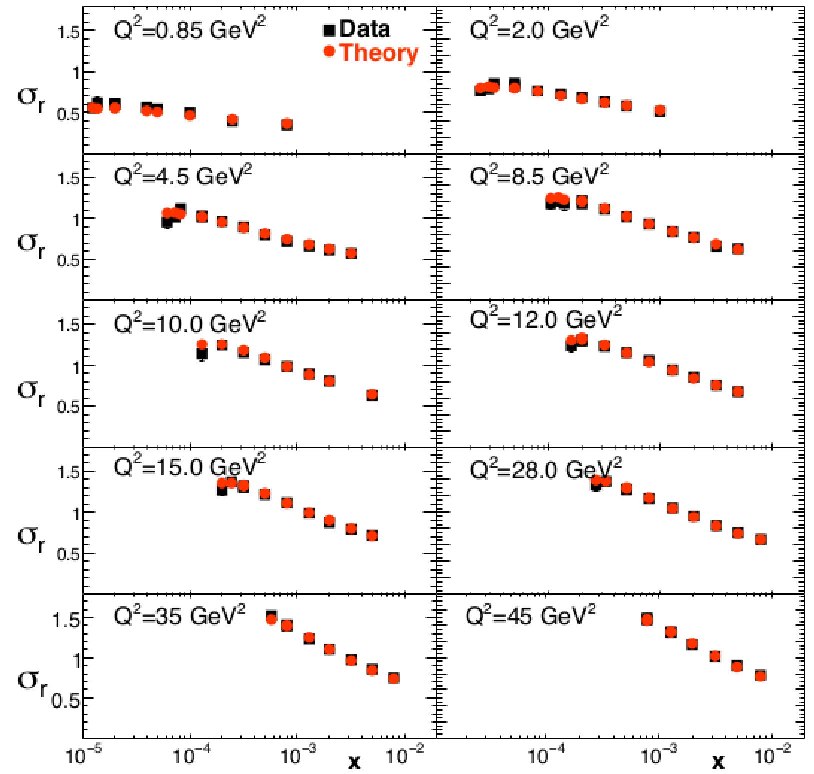
*Evolved with BK dipole amplitude – expectation value of product of Wilson lines in fundamental representation*

*Related by Fourier transform*

# *Observables which hint for saturation*

*J. L. Albacete, at al Eur.Phys.J. C71 (2011) 1705*




*K.Golec-Biernat, J. Kwieciński, A. Staśto Phys.Rev.Lett.86:596-599,2001*

$$N(x,r) = \; g\,(r\,Q_s(x))$$

$$\sigma_r = F_2 - \frac{y^2}{1+(1-y)^2}\,F_L$$

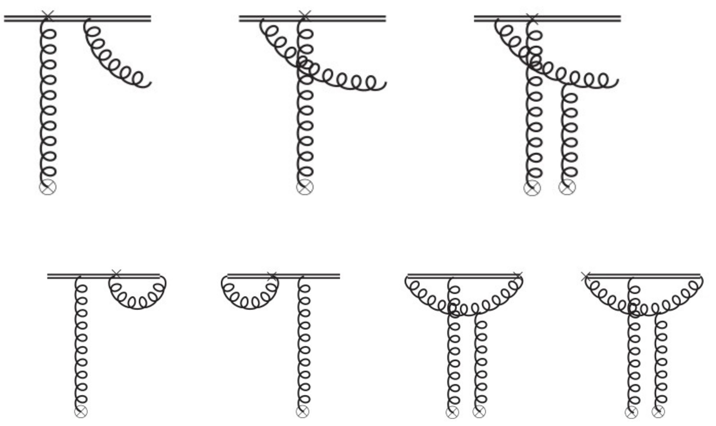
*Saturation introduce relation between longitudinal momentum and transversal momentum cross-section is a function of one parameter*

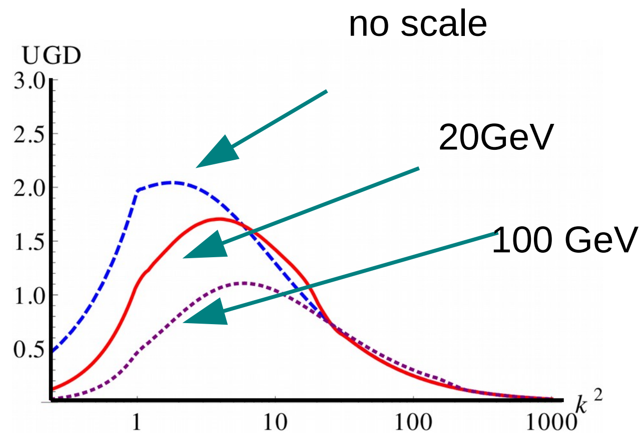
# More exclusive observables -di-jets and Sudakov form factor

*Low kt gluons are suppressed. The conservation of probability leads to change of shape of gluon density which depends on the hard scale*

*Sudakov - no emision probability. Resumes unresloved real and virtual emissions. Standard thing in Monte Carlo.*

*Example of diagrams contributing to Sudakov*






*Nucl.Phys. B921 (2017) 104-126*
*B. Xiao, F. Yuan, J. Zhou.*

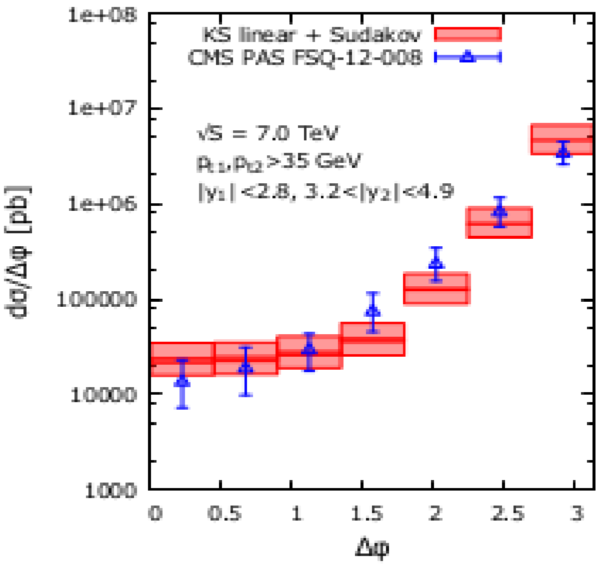
*Phys. Rev. D 88, 114010 (2013)*
*A. H. Mueller, Bo-Wen Xiao, Feng Yuan*

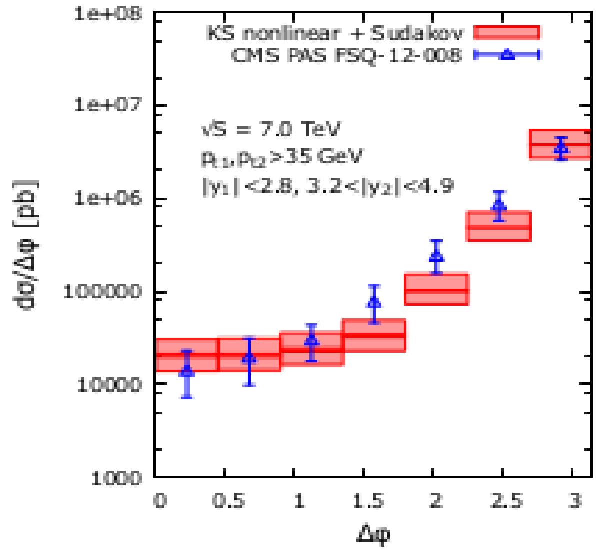
*Phys.Lett. B737 (2014) 335-340*
*A. van Hameren, P. Kotko, K. Kutak, S. Sapeta*

*K. Kutak*
*Phys.Rev. D91 (2015) no.3, 034021*

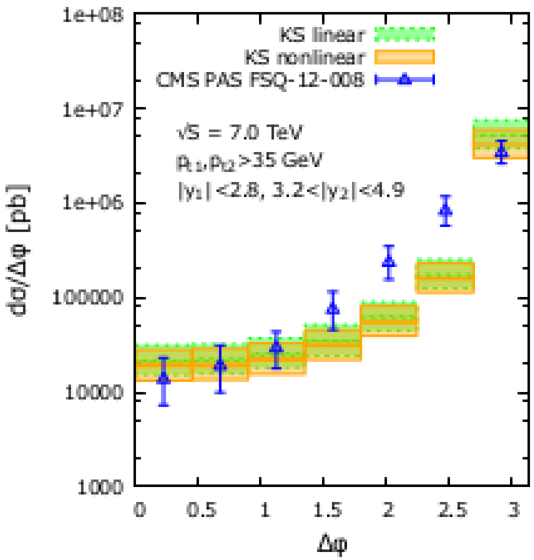
$$\mathcal{F}(x,k^2,\mu^2) = \frac{N_c}{2\pi^2\alpha_s} S_\perp k^2 \int d^2r \left[1 - N(x,r^2)\right] e^{-ik\cdot r} e^{-S_{sud}(x,r,\mu^2)}$$

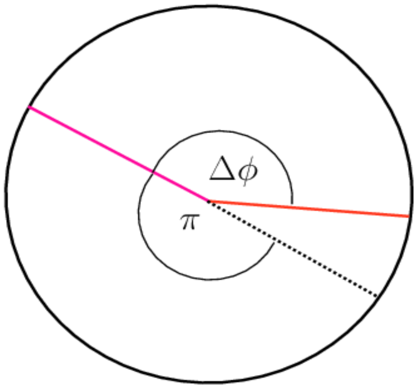
# *Decorelations and forward-central dijets*



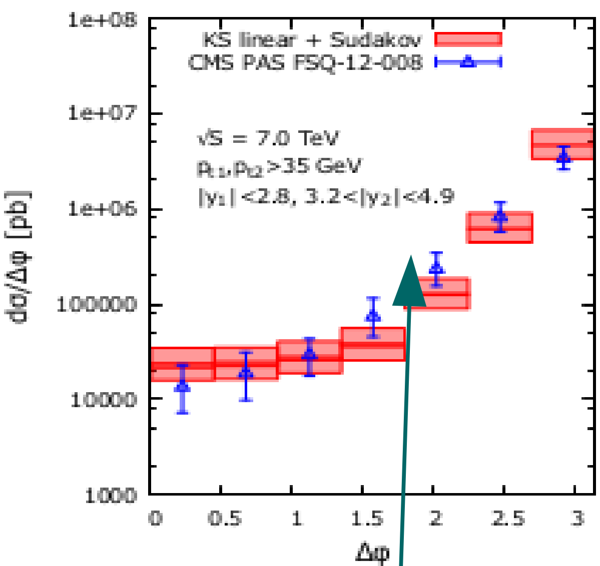

*No saturation but visible Sudakov effects*

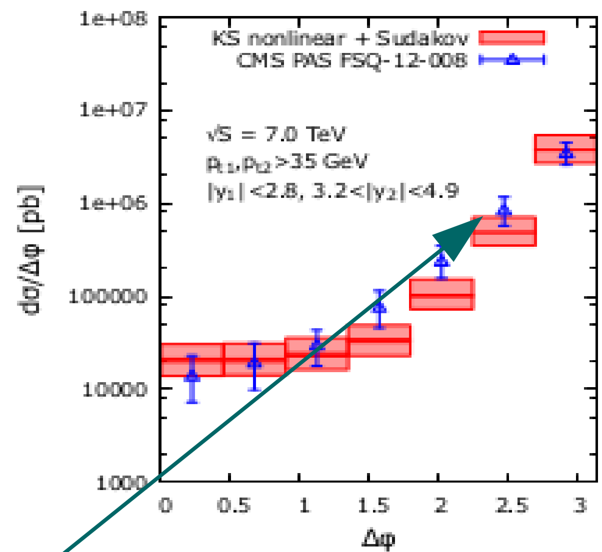



*Phys.Lett. B737 (2014) 335-340*
*A. van Hameren, P. Kotko, K. Kutak, S. Sapeta*

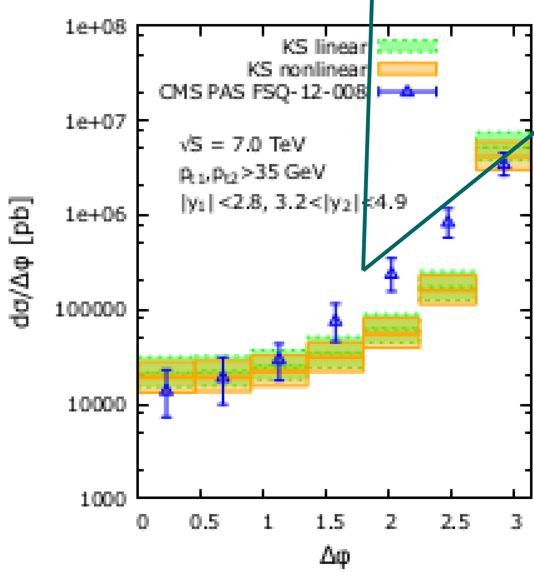
# Decorelations and forward-central dijets

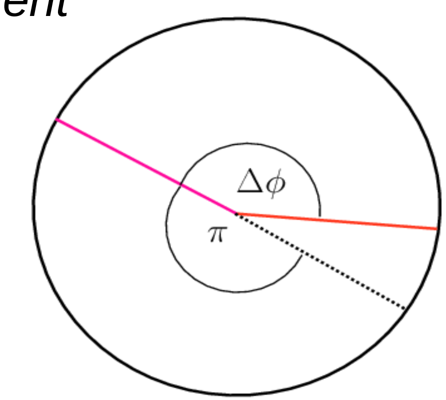



*No saturation but visible Sudakov effects*

*enhancement*

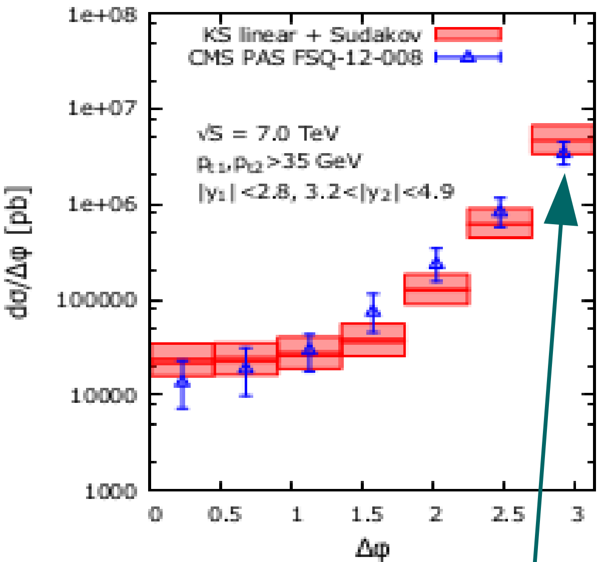


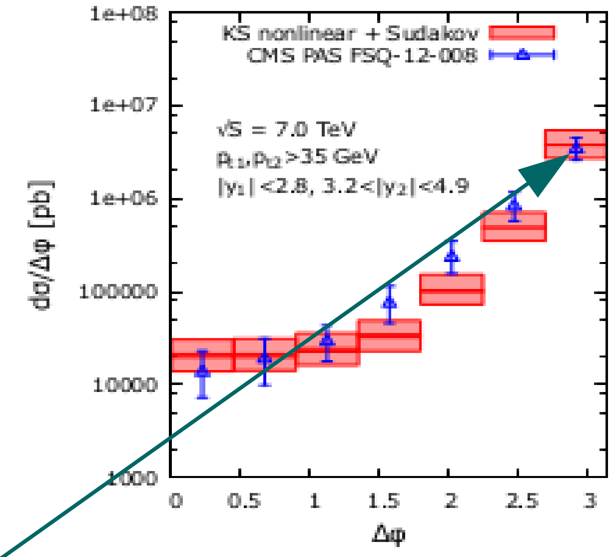





*Phys.Lett. B737 (2014) 335-340*
*A. van Hameren, P. Kotko, K. Kutak, S. Sapeta*

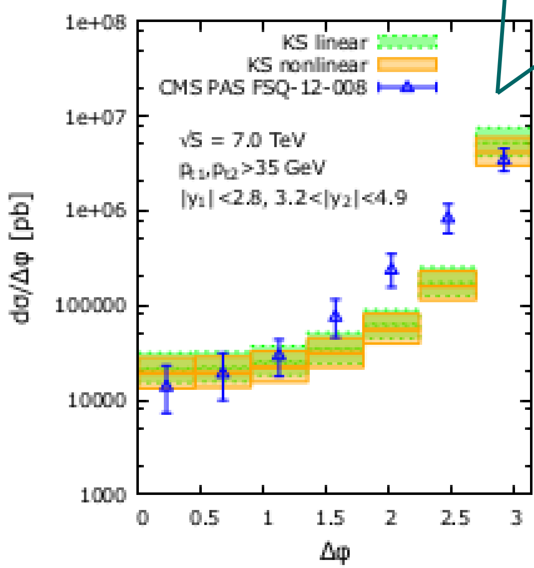
# Decorelations and forward-central dijets

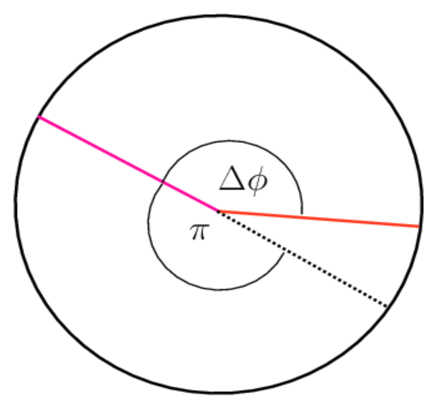



*No saturation but visible Sudakov effects*

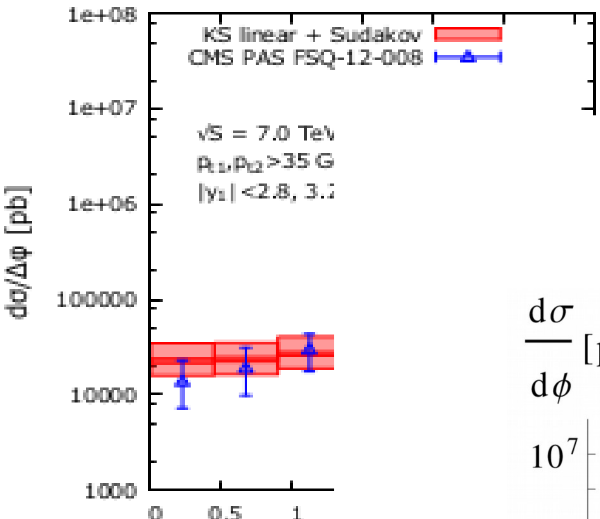
*suppression*

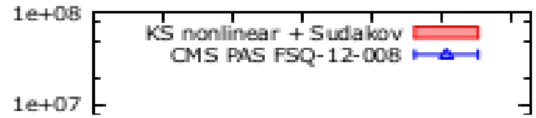



*Phys.Lett. B737 (2014) 335-340*
*A. van Hameren, P. Kotko, K. Kutak, S. Sapeta*

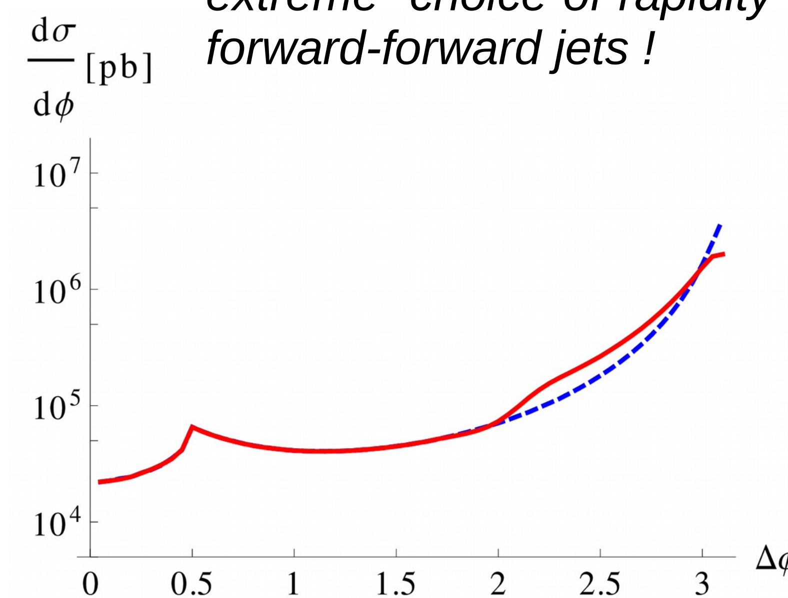
# *Decorelations and forward-central dijets*



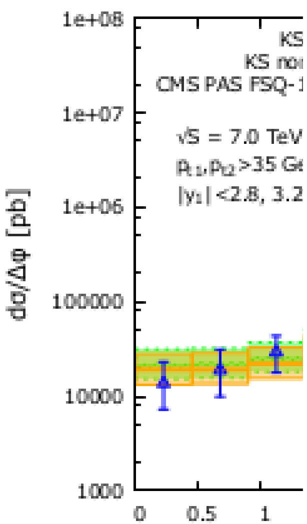




*aturation but visible*
*Sudakov effects*

*This is we can we expect  with more extreme  choice of rapidity cuts. Go for forward-forward jets !*



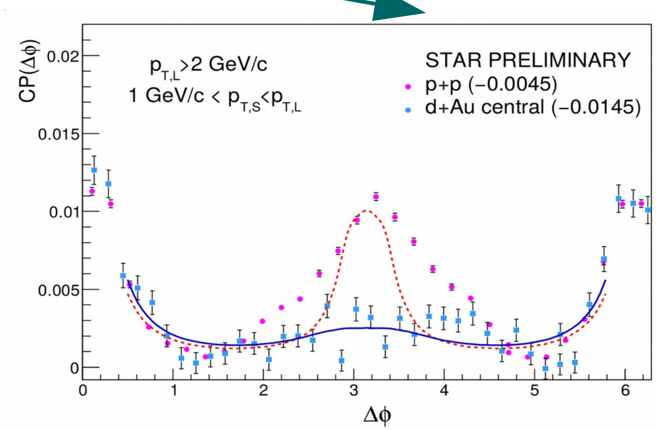

*K. Kutak*
*Phys.Rev. D91 (2015) no.3, 034021*



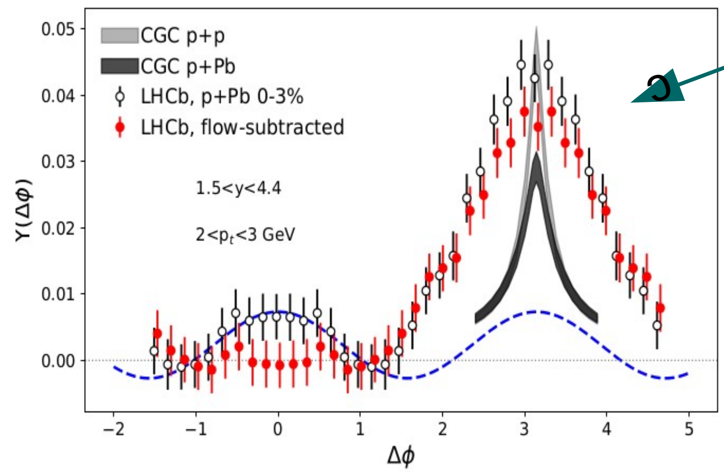

*5-340*
*K. Kutak, S. Sapeta*

# *Di-hadrons production – hints of saturation*

*HEF and BK for gluon*




*J. Albacete, C, Marquet*
*Phys.Rev.Lett. 105 (2010) 162301*

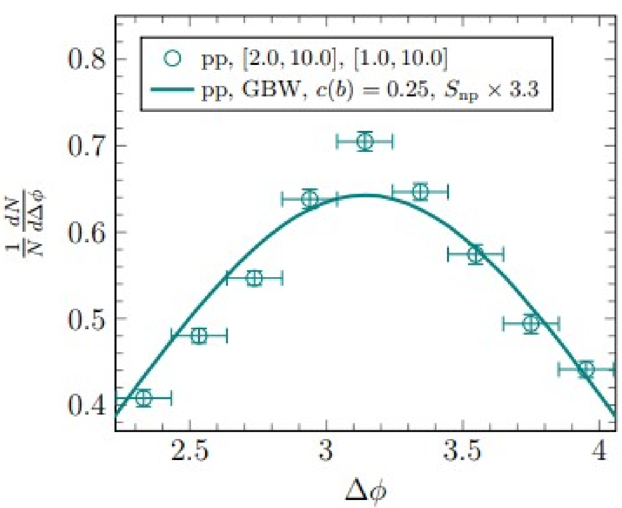


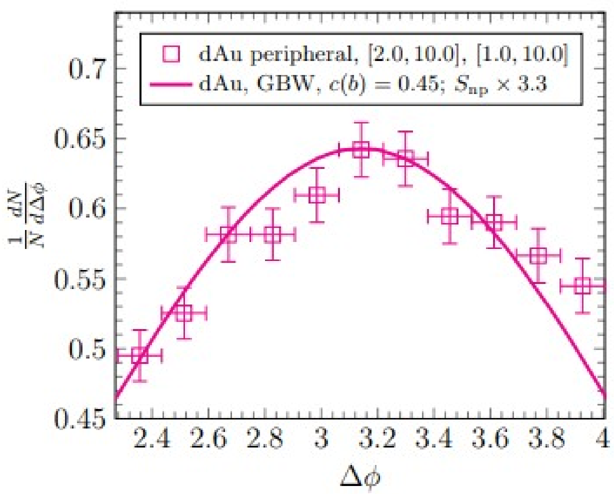

*ITMD no Sudakov*

*Expectation: Sudakov will broaden the distribution*

*G. Giacalone, C. Marquet, M. Matas*
*Phys.Rev. D99 (2019) no.1, 014002*

1.1<$y_1$<1.9    2.5<$y_2$<4



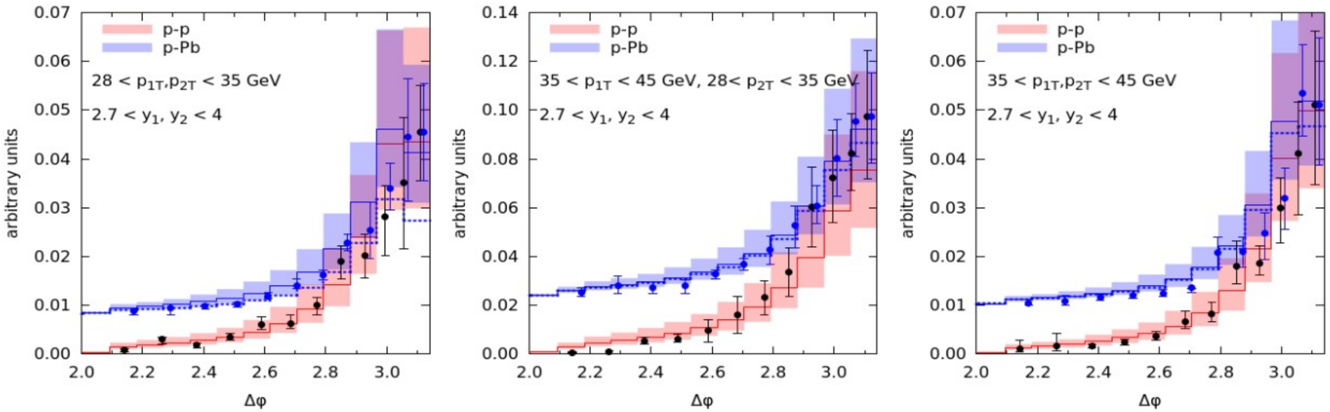

*Correlation limit of CGC + Sudakov*

*A.Stasto, S. Wei, B. Xiao, F. Yuan*
*Phys.Lett. B784 (2018) 301-306*

RHIC data

# Signature of saturation in forward-forward dijets

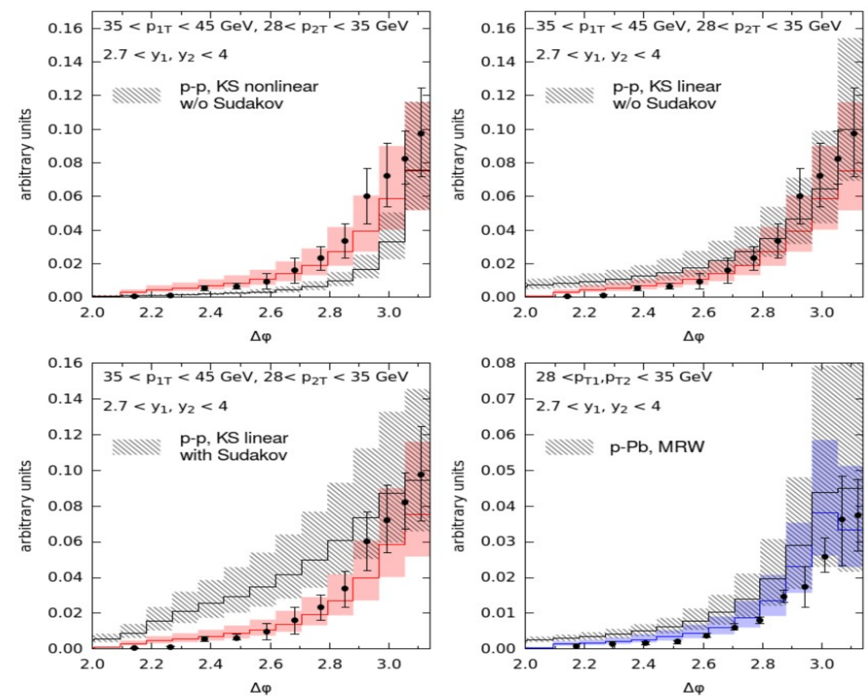
*A. Hameren, P. Kotko, K. Kutak, S. Sapeta*
*Phys.Lett. B795 (2019) 511-515*




*broadening = ITMD+Sudakov*

*Data: number of dijets normalized to number of single inclusive jets. We can not calculate that. We can compare shapes.Procedure:   fit normalization to p-p data. Use that both for p-p and p-Pb. Shift p-Pb data*

*The procedure allows for visualization of broadening*

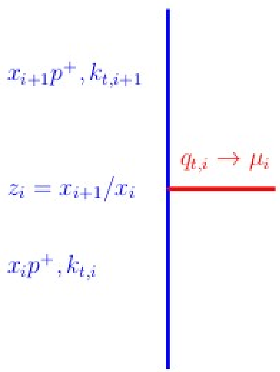


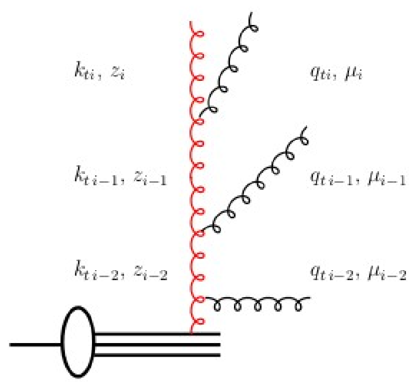
*Monte Carlo*

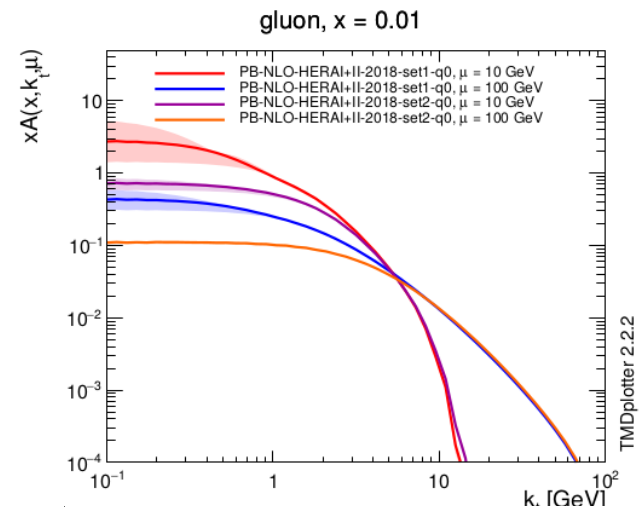
# Monte Carlo – parton branching method

*The idea: construct such parton shower that gives also  TMD dependent paton density.*
*On integrated level the pdf obeys DGLAP equation.*

*A. Bermudez Martinez, P. Connor, H. Jung, A. Lelek, R. Žlebčík, F. Hautmann, V. Radescu  Phys.Rev. D99 (2019) no.7, 074008*

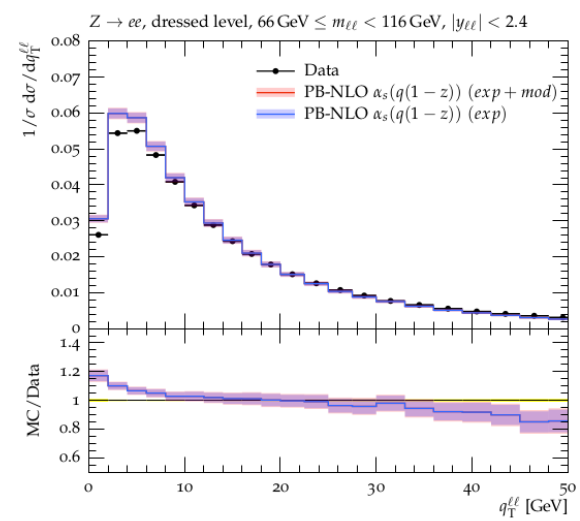



$$\mathcal{A}_a(x,\mathbf{k},\mu^2) = \Delta_a(\mu^2)\,\mathcal{A}_a(x,\mathbf{k},\mu_0^2) + \sum_b \int \frac{d^2\mathbf{q}'}{\pi\mathbf{q}'^2}\,\frac{\Delta_a(\mu^2)}{\Delta_a(\mathbf{q}'^2)}\,\Theta(\mu^2-\mathbf{q}'^2)\,\Theta(\mathbf{q}'^2-\mu_0^2)$$

$$\times \int_x^{z_M} \frac{dz}{z}\,P_{ab}^{(R)}(\alpha_s,z)\,\mathcal{A}_b\left(\frac{x}{z},\mathbf{k}+(1-z)\mathbf{q}',\mathbf{q}'^2\right)$$




*The method provides first consistent and complete set of TMD's  which are applicable in Monte Carlo simulations and can be generalized to account for small x effects at least in linear regime. Applies in the regime above saturation scale.*

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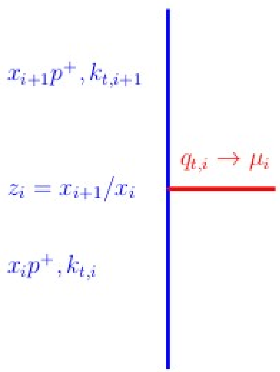
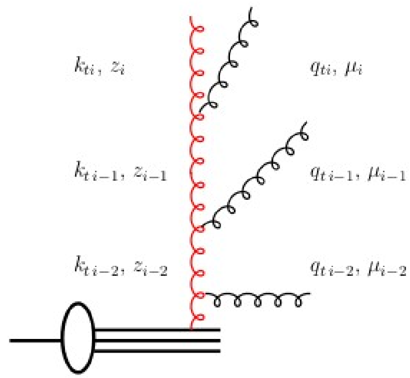
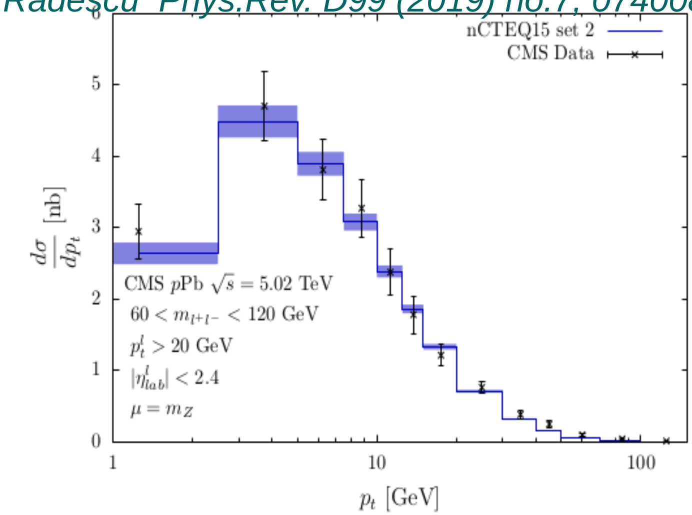
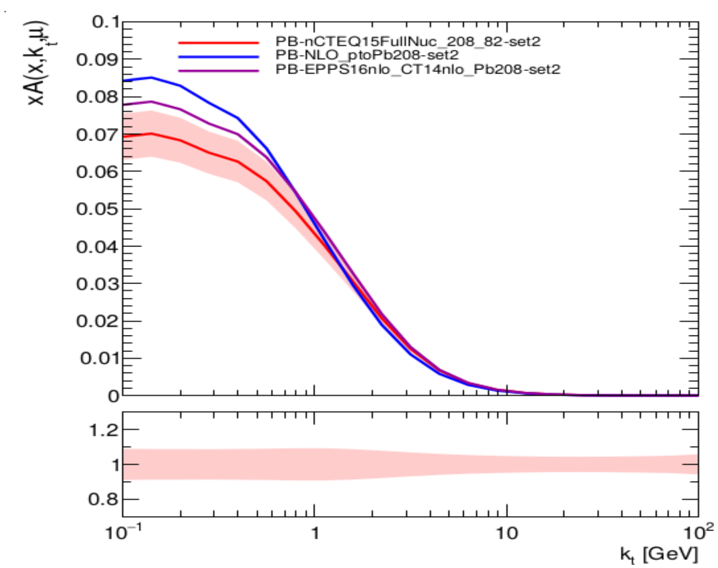



$$\mathcal{A}_a(x,\mathbf{k},\mu^2) = \Delta_a(\mu^2)\,\mathcal{A}_a(x,\mathbf{k},\mu_0^2) + \sum_b \int \frac{d^2\mathbf{q}'}{\pi \mathbf{q}'^2}\,\frac{\Delta_a(\mu^2)}{\Delta_a(\mathbf{q}'^2)}\,\Theta(\mu^2-\mathbf{q}'^2)\,\Theta(\mathbf{q}'^2-\mu_0^2)$$

$$\times \int_x^{z_M} \frac{dz}{z}\, P_{ab}^{(R)}(\alpha_s, z)\,\mathcal{A}_b\left(\frac{x}{z}, \mathbf{k}+(1-z)\mathbf{q}', \mathbf{q}'^2\right)$$

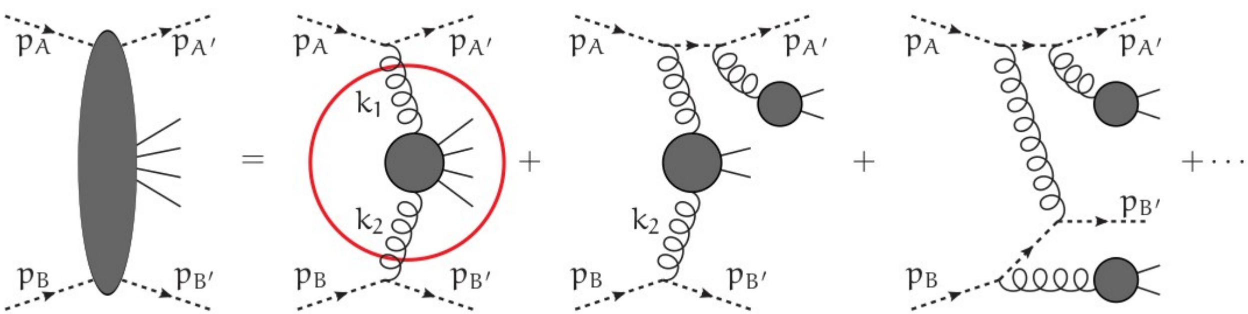



*The method provides first consistent and complete set of TMD's which are applicable in Monte Carlo simulations and can be generalized to account for small x effects at least in linear regime. Applies in the regime above saturation scale.*

# TMD matrix elements and Monte Carlo

*KaTie is a Monte Carlo program for tree-level calculations of any process within the Standard Model any initial-state partons on-shell or off-shell employs numerical Dyson-Schwinger recursion to calculate helicity amplitudes*

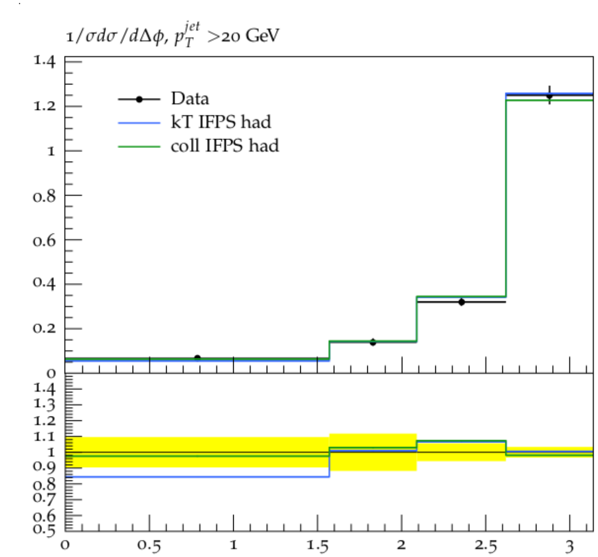
A. van Hameren ,Comput.Phys.Commun. 224 (2018) 371-380

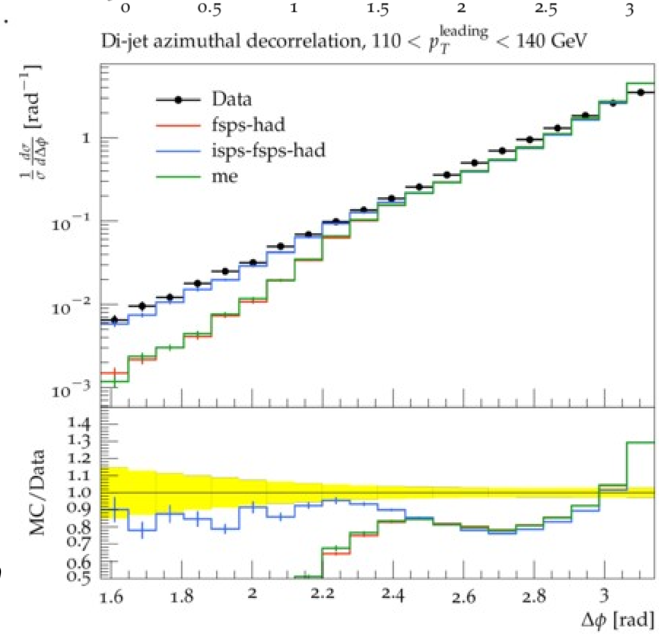



*There has been a proof of the concept analysis of possible calculation of any NLO process In HEF. The difficult part is caused is the linear denominator*

$$\int d^{4-2\varepsilon}\ell \, \frac{\mathcal{N}(\ell)}{p\cdot(\ell+K_0)\,(\ell+K_1)^2\,(\ell+K_3)^2\,(\ell+K_4)^2} = ?$$

*Triangles, boxes,…. have been studied and regularization method has been proposed in*
1710.07609 van Hameren. *Threat the above integral as limit of known results and choose a special parametrization of momenta in the process*

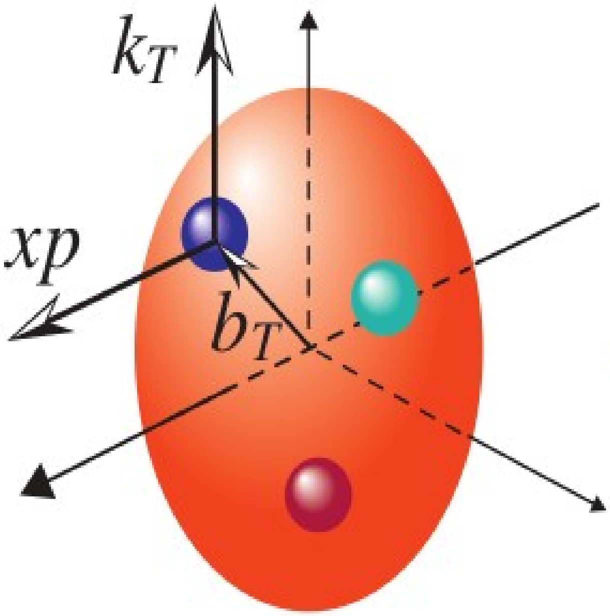


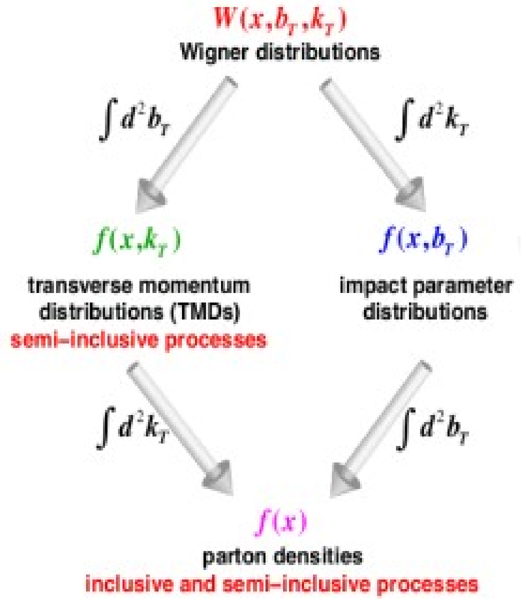



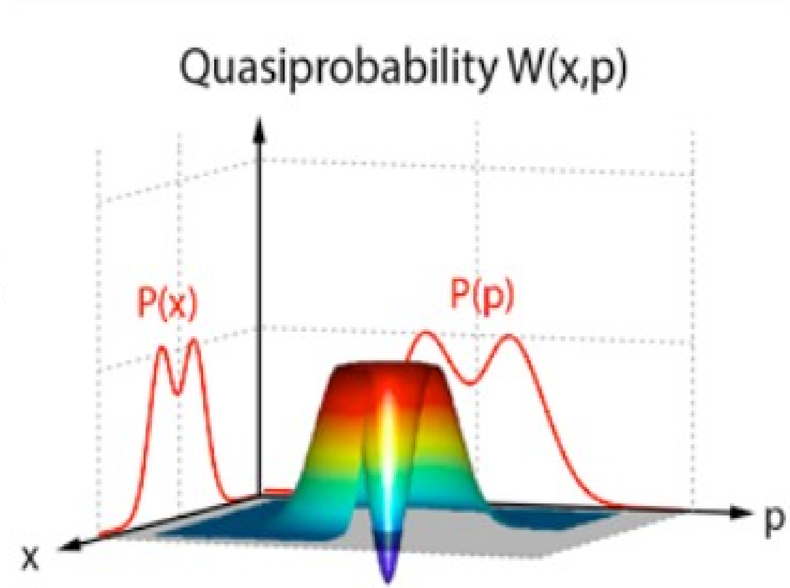

*NLO for HEF also addresse in*
*M. A. Nefedov Nucl.Phys. B946 (2019) 114715*

## *Hadron tomography*

# 5D – tomography of hadrons Wigner distribution



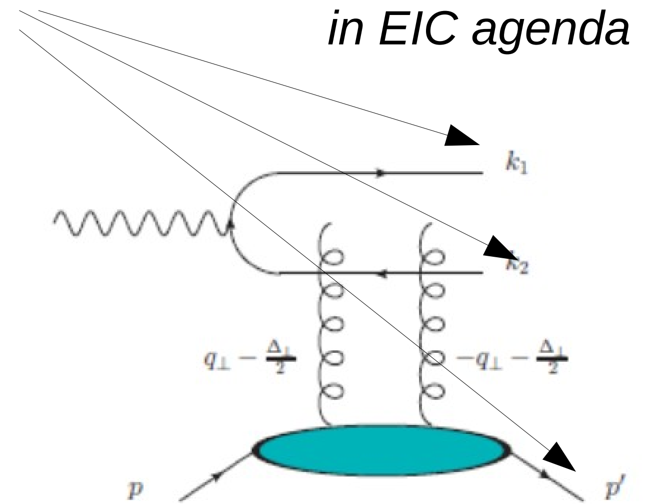

- *Wigner distributions  encode all here quantum information of how partons are distributed inside proton [Ji, 03; Belitsky, Ji, Yuan, 03]*
- *In condensed matter Wigner distributions of photons can be measured.*
- *Can we measure the gluon Wigner at small-x?*

*From B.Xiao. Initial Stages 2019*

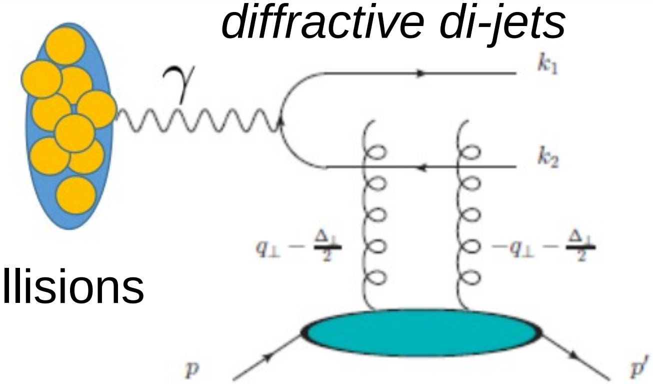
# *5D – tomography-Wigner distribution*

*measure*

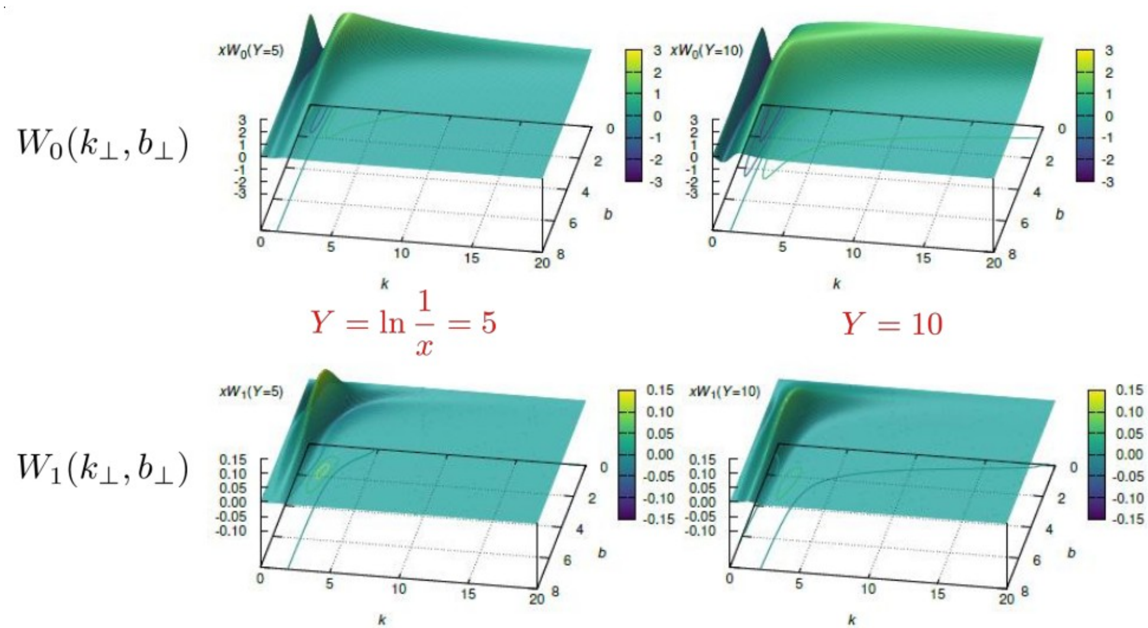
*in EIC agenda*




*diffractive di-jets*




UPC collisions




*From solution of BK equation in small x limit*

$$xW(x, \boldsymbol{k}, \boldsymbol{b}) = -\frac{2N_c}{\alpha_S} \int \frac{d^2\boldsymbol{r}}{(2\pi)^2} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \left(\frac{1}{4}\nabla_{\boldsymbol{b}}^2 + \boldsymbol{k}^2\right) T_Y(\boldsymbol{r}, \boldsymbol{b})$$

*Ji, Yuan, Zhao,Hatta, Nakagawa, Yuan, Zhao, 16, Bhatttacharya, Metz,Zhou, 17,*

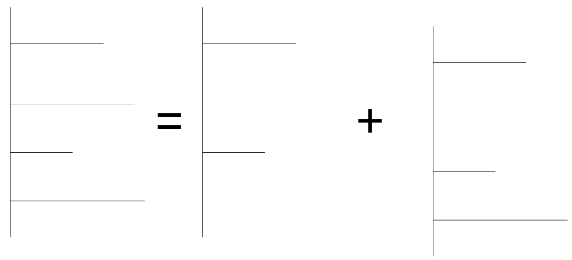
*Hagiwara, Hatta, Pasechnik, Tasevsky, Teryaev, 17*

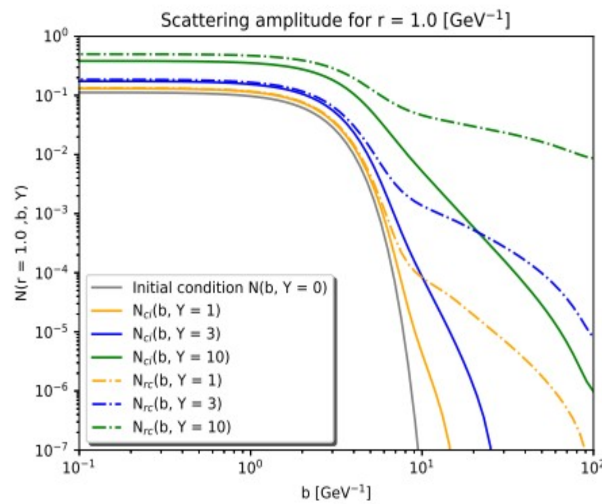
*Mueller, and Schenke, 19*

*Gauge links play a role since one can define different Wigner distributions*

# *b dependence from collinearly improved BK*

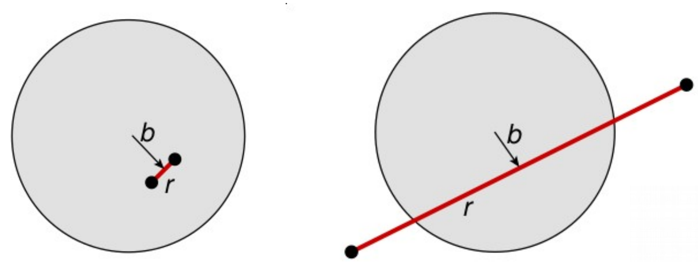
*Include a kinematical constraint in the BK kernel to account for the finite energy corrections. Suppression of anticollinear configuration.*



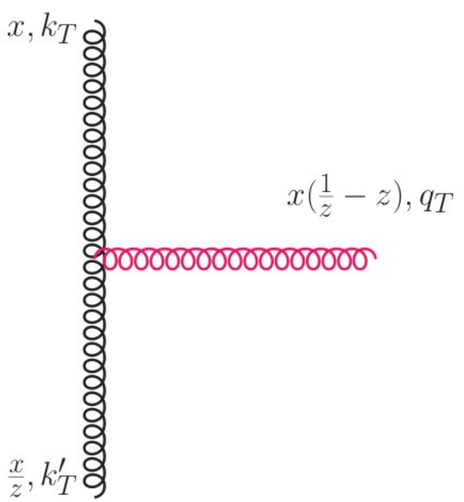




*Long standing problem diffusion into large dipole sizes*

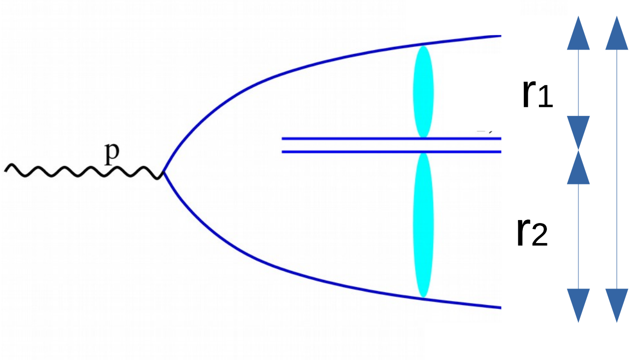
J. Cepila, J.G. Contreras, M. Matas
Phys.Rev. D99 (2019) no.5, 051502




*In the momentum space*




In the coordinate space



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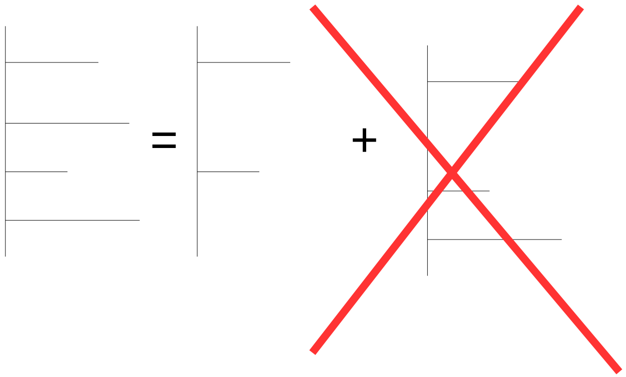


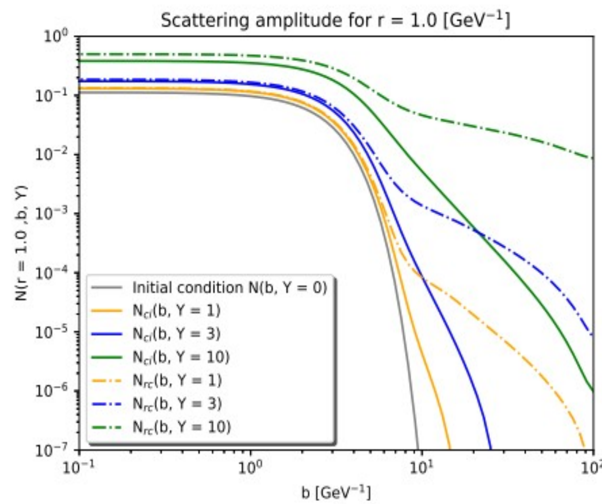


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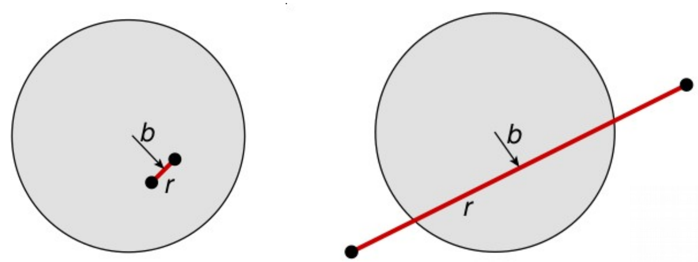
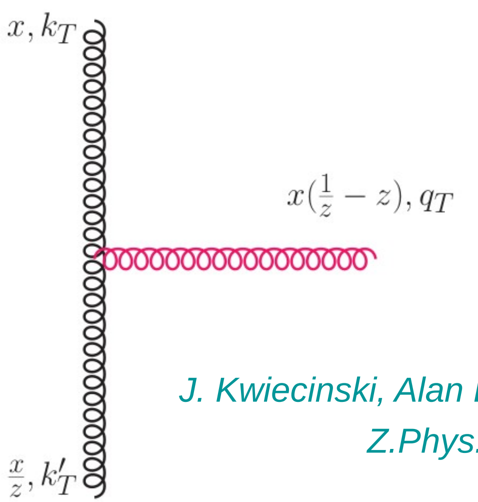
J. Cepila, J.G. Contreras, M. Matas Phys.Rev. D99 (2019) no.5, 051502

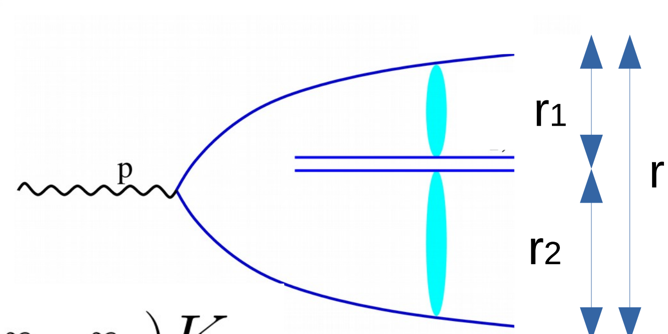



*In the momentum space*

In the coordinate space




$$q_T^2 < \frac{1-z}{z} k_T^2$$




J. Kwiecinski, Alan D. Martin, P.J. Sutton Z.Phys. C71 (1996) 585-594

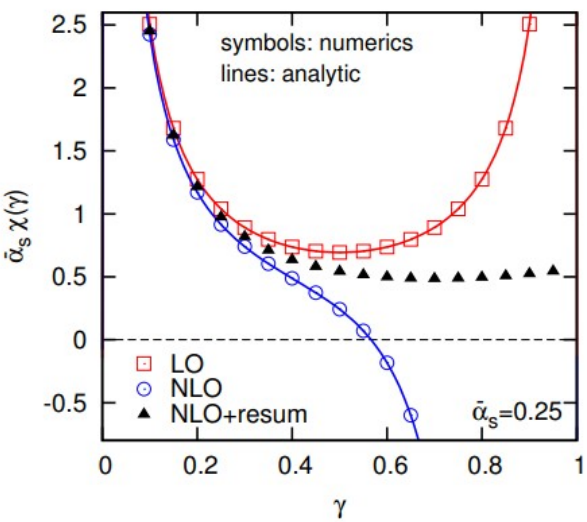
$$K_{p\,s} \longrightarrow f(r, r_1, r_2) K_{p\,s}$$

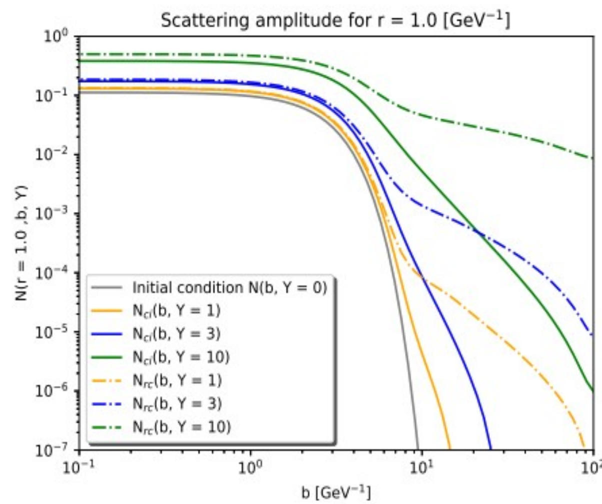
E. Iancu, J. Madrigal, A. Mueller G. Soyez, D. Triantafyllopoulos Phys. Lett. B744 (2015) 293-302

*position space* $\longrightarrow$

# Collinear improved BK

*Include a kinematical constraint in the BK kernel to account for the finite energy corrections. Suppression of anticollinear configuration.*

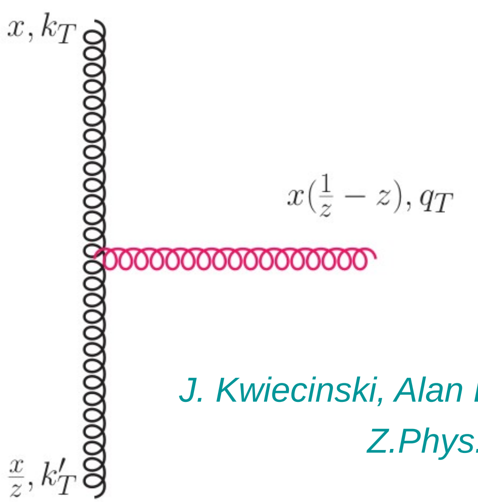



*Long standing problem diffusion into large dipole sizes*

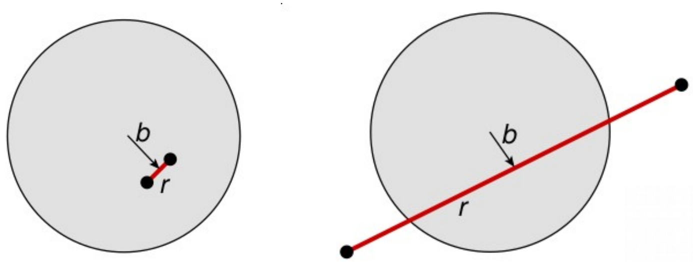
*J. Cepila, J.G. Contreras, M. Matas*
*Phys.Rev. D99 (2019) no.5, 051502*

*In the momentum space*

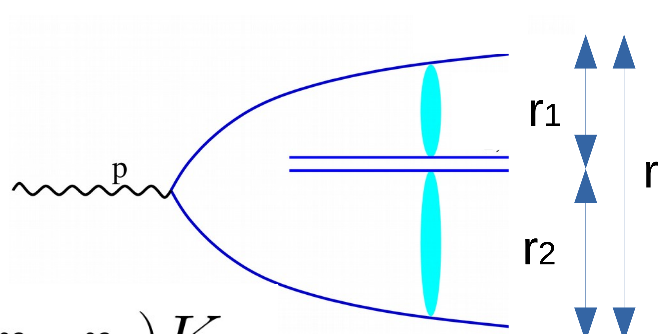



$$q_T^2 < \frac{1-z}{z} k_T^2$$

*J. Kwiecinski, Alan D. Martin, P.J. Sutton*
*Z.Phys. C71 (1996) 585-594*




In the coordinate space

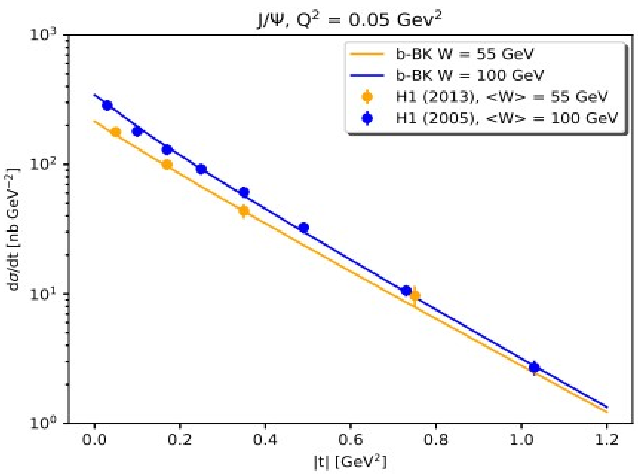


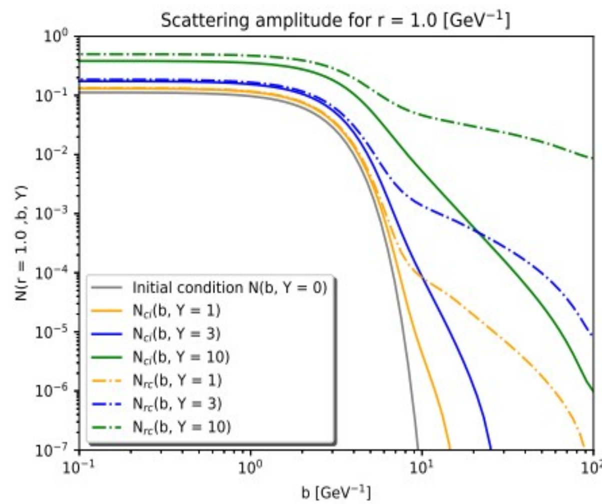

$$K_{p\,s} \longrightarrow f(r, r_1, r_2) K_{p\,s}$$

*E. Iancu, J. Madrigal, A. Mueller G. Soyez, D. Triantafyllopoulos*
*Phys. Lett. B744 (2015) 293-302*

# Collinear improved BK

*Include a kinematical constraint in the BK kernel to account for the finite energy corrections. Suppression of anticollinear configuration.*

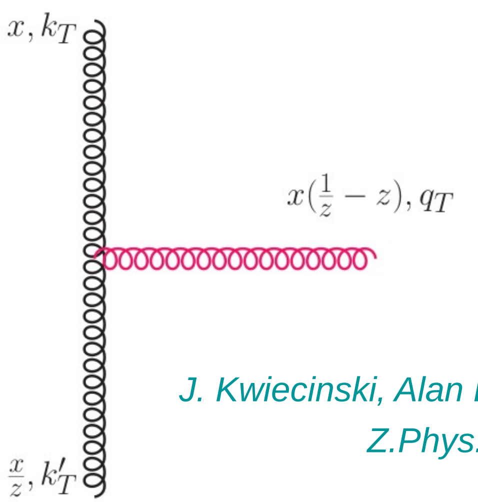


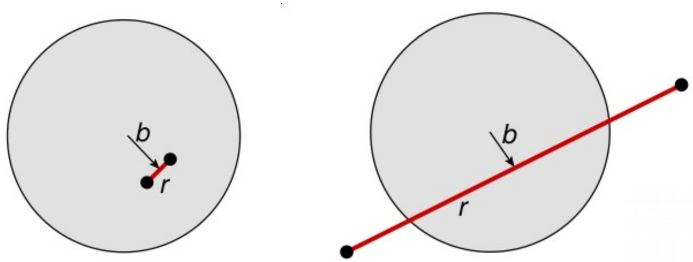




*Long standing problem diffusion into large dipole sizes*

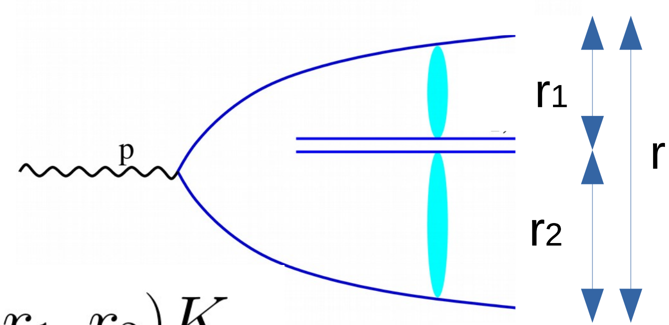
*J. Cepila, J.G. Contreras, M. Matas*
*Phys.Rev. D99 (2019) no.5, 051502*

*In the momentum space*




In the coordinate space




$$q_T^2 < \frac{1-z}{z} k_T^2$$

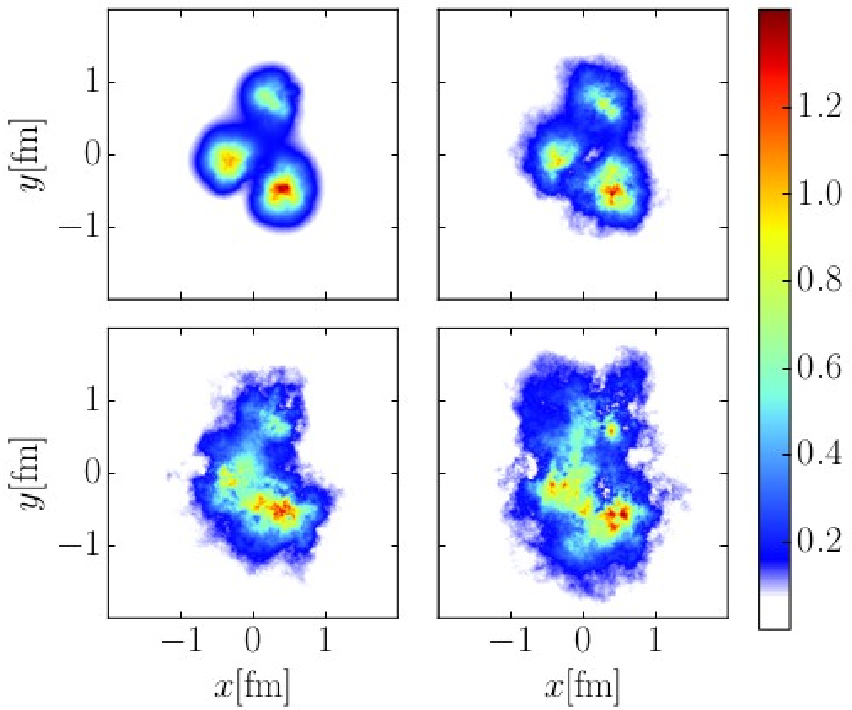
$$K_{p\,s} \longrightarrow f(r, r_1, r_2) K_{p\,s}$$

*J. Kwiecinski, Alan D. Martin, P.J. Sutton*
*Z.Phys. C71 (1996) 585-594*

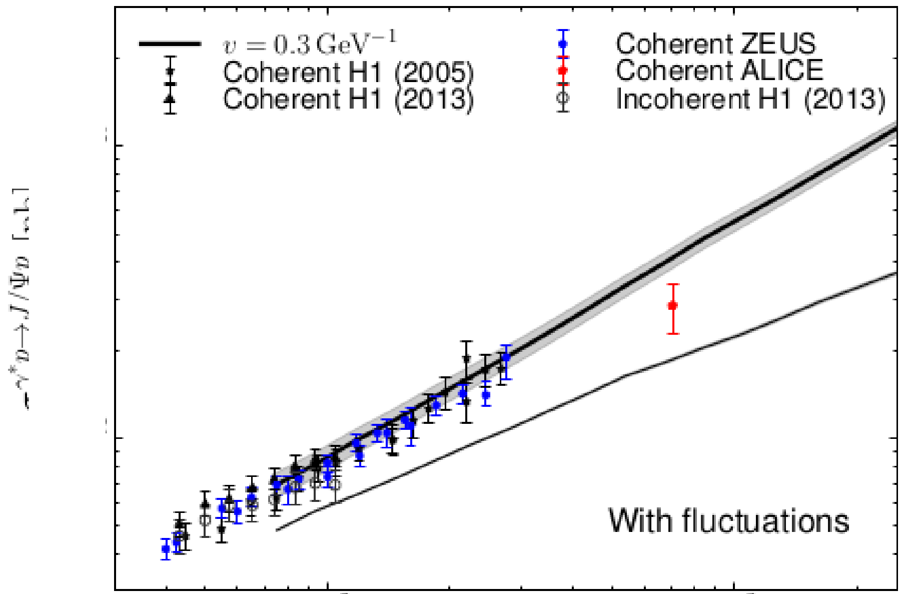
*E. Iancu, J. Madrigal, A. Mueller G. Soyez, D. Triantafyllopoulos*
*Phys. Lett. B744 (2015) 293-302*

# Imaging fluctuating proton and role of fluctuations

*Studies of the proton-lead collisions have revealed a surprisingly similar collective effects,such as large elliptic and triangular flows ($v_2$ and $v_3$ ), what was previously seen in heavy ion collisions.*




H. Mantysaari, B.Schenke, Phys. Rev. D98,034013 (2018))




$$\frac{dN}{d\varphi} = A\left(1 + 2\sum_n v_n \cos n(\varphi - \Psi_n)\right)$$

*The incoherent cross section amount of fluctuations in the event-by-event distribution of the dipole amplitude. The width B qc of the distribution from which the positions for the constituent quark positions are sampled, and the width B q of the Gaussian distribution of gluon density around the sampled 3 quarks. Saturation scale fluctuates too*

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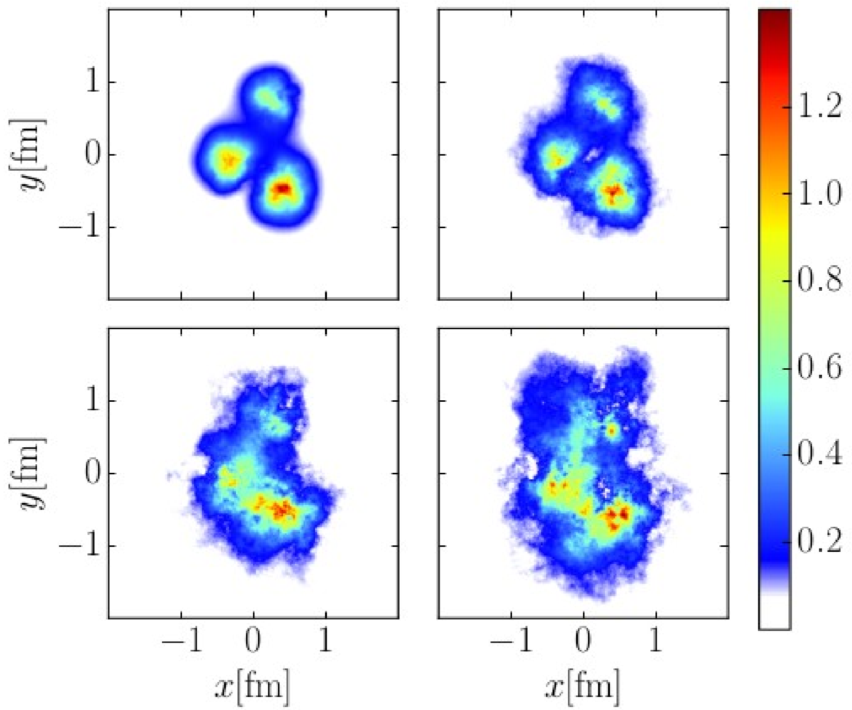


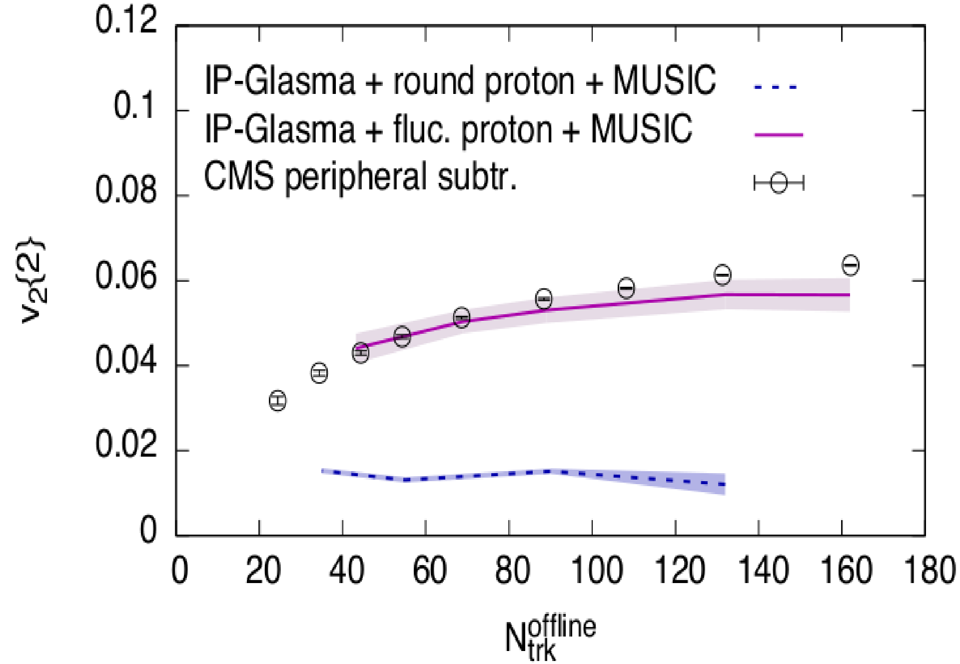


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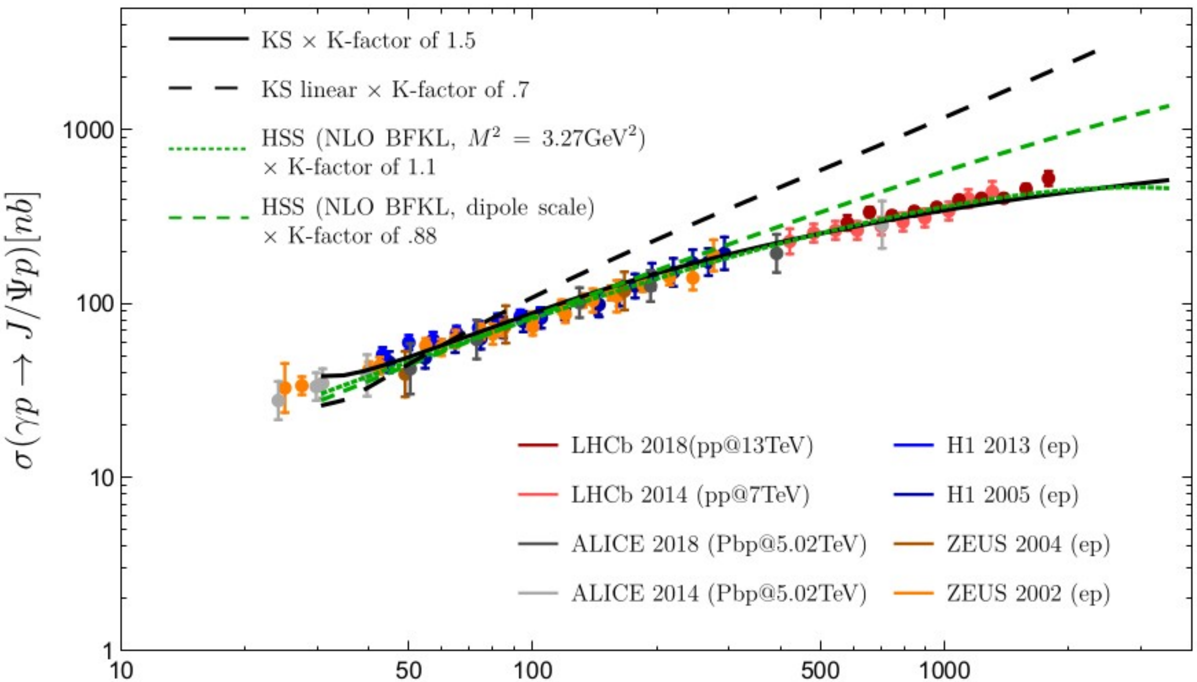
# *Linear vs. nonlinear and J/ψ*

*However, one can also arrive at good description of the J/ψ using BK with kinematical corrections and more complete splitting function in the kernel…*



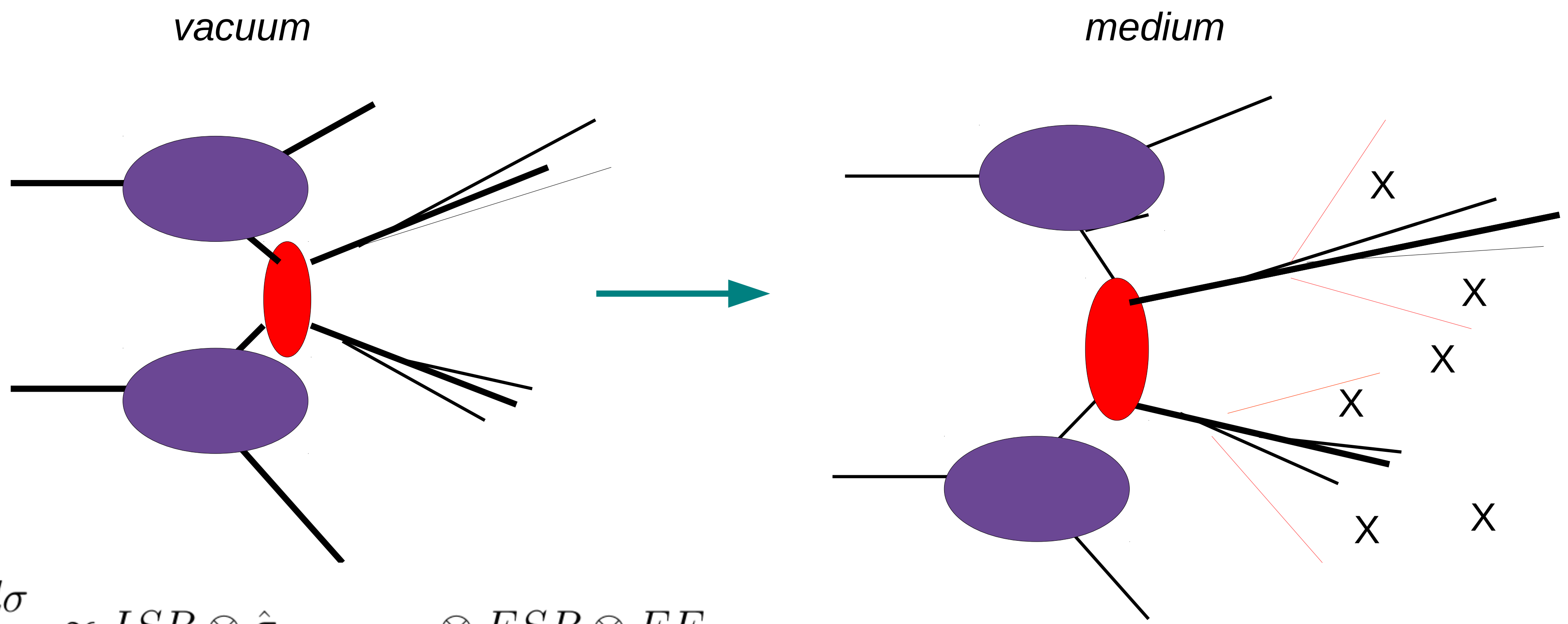

$$\frac{d\sigma}{dt}(\gamma p \to Vp) = \frac{1}{16\pi}\left|\mathcal{A}_{T,L}^{\gamma p \to Vp}(W^2, t)\right|^2$$

*A. Arroyo Garcia, M. Hentschinski, K. Kutak*
*Phys.Lett. B795 (2019) 569-575*

$$\Im\mathrm{m}\mathcal{A}_T^{\gamma p \to Vp}(W, t=0) = 2\int d^2\boldsymbol{r}\int d^2\boldsymbol{b}\int_0^1 \frac{dz}{4\pi}\,(\Psi_V^*\Psi)_T \cdot N(x, r, b)$$

*Jet quenching*

# From vacuum to medium

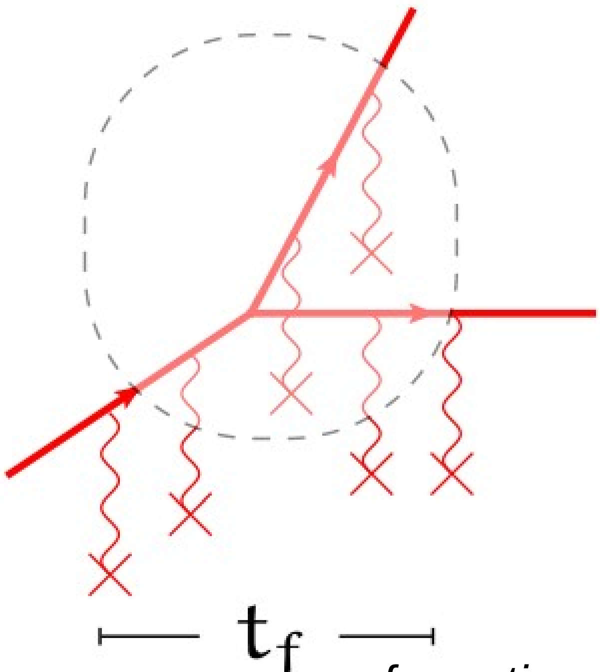



$$\frac{d\sigma}{dPS} \propto ISR \otimes \hat{\sigma}_{ab \to c_1 \ldots . c_n} \otimes FSR \otimes FF$$

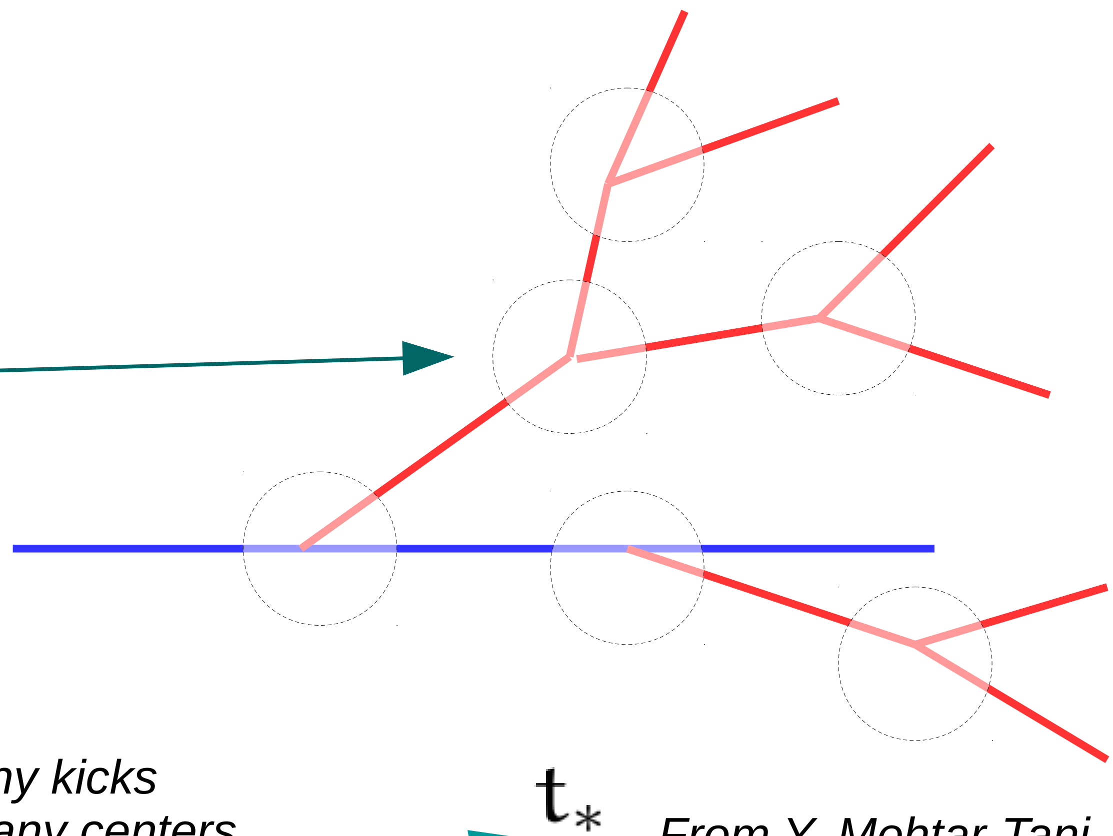
$$\frac{d\sigma_{medium}}{dPS} \propto ISR \otimes \hat{\sigma}_{ab \to c_1 \ldots . c_n} \otimes FSR \otimes FF \otimes D$$

# Multiple branching

*Beyond energy lost by the leading particle, the LHC data call for a more thorough analysis of the jet shape for which the effects of multiple branching at large angles are important….*



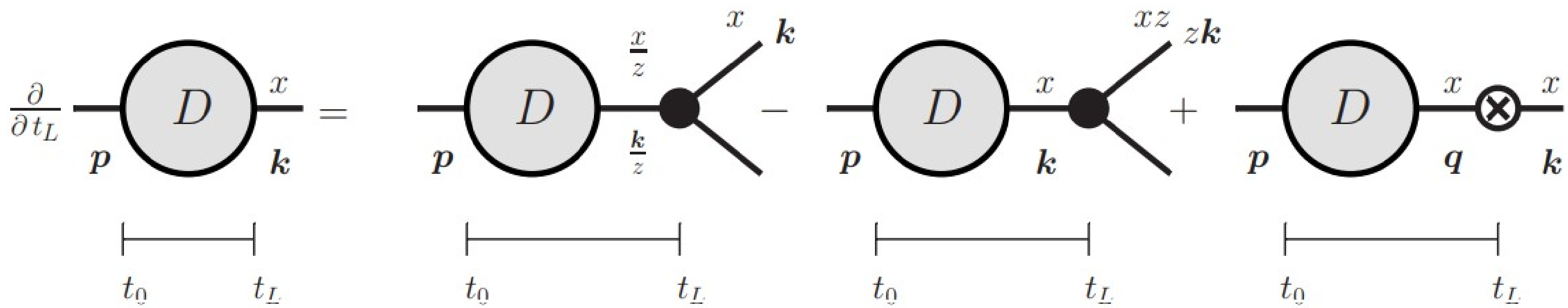

*formation time; many kicks before radiation; many centers act as a single source*




*From Y. Mehtar-Tani*

*stopping time → time at which energy has been emitted in form of soft gluons*

# *The BDIM equation*




*J. Blaizot, F. Dominguez, E. Iancu, Y. Mehtar-TaniJHEP 1406 (2014) 075*

$$\frac{\partial}{\partial t} D(x,\mathbf{k},t) = \frac{1}{t^*}\int_0^1 dz\, \mathcal{K}(z) \left[ \frac{1}{z^2}\sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \frac{\mathbf{k}}{z}, t\right) \theta(z-x) - \frac{z}{\sqrt{x}} D(x,\mathbf{k},t) \right]$$

*Inclusive gluon distribution as produced by hard jet*

$$+ \int \frac{d^2\mathbf{q}}{(2\pi)^2} C(\mathbf{q}) D(x, \mathbf{k}-\mathbf{q}, t)$$

$$\frac{1}{t^*} = \frac{\bar{\alpha}}{\tau_{\rm br}(E)} = \bar{\alpha}\sqrt{\frac{\hat{q}}{E}}$$

$$C(\mathbf{q}) = w(\mathbf{q}) - \delta(\mathbf{q}) \int d^2\mathbf{q}'\, w(\mathbf{q}')$$

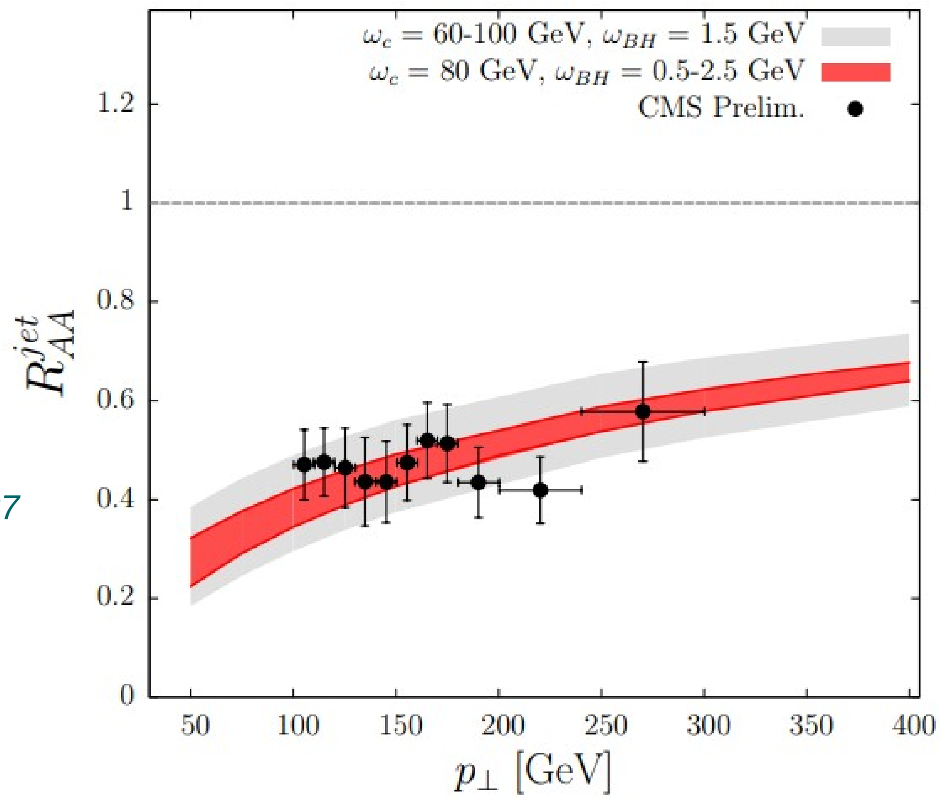
$$\mathcal{K}(z) = \frac{[f(z)]^{5/2}}{[z(1-z)]^{3/2}} \qquad f(z) = 1 - z + z^2$$

*Equation describes interplay of rescatterings and branching. This particular equation has $k_t$ independent kernel. This is an approximation. The whole broadening comes from rescattering. Energy of emitted gluon is much larger than its transverse momentum*

# *Phenomenological applications – nuclear modification ratio*

*Modification of the kernel of equation for energy to account for transversal momentum dependence.*
*Misses broadening contribution*

*Y. Mehtar-Tani, K. Tywoniuk Phys.Lett. B744 (2015) 284-287*




$$\frac{\mathrm{d}^2 N^{\mathrm{jet}}_{\mathrm{Pb\text{-}Pb}}(p_\perp)}{T_{\mathrm{AA}}\,\mathrm{d}^2 p_\perp} \simeq \int_0^1 \frac{\mathrm{d}x}{x} D_q^{\mathrm{med}}\left(x, \frac{p_\perp}{x}, L\right) \frac{\mathrm{d}^2 \sigma^{\mathrm{jet}}_{\mathrm{p\text{-}p}}\left(\frac{p_\perp}{x}\right)}{\mathrm{d}^2 p_\perp}$$

# Solution of the equation for gluon density in QGP

*procedure almost the same as for energy distribution*

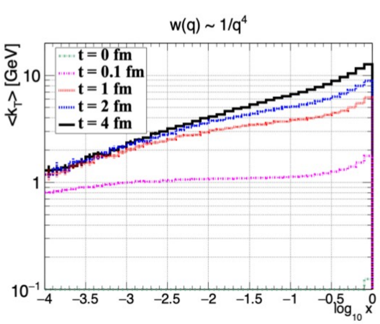
*K. Kutak, W. Płaczek, R. Straka, Eur.Phys.J. C79 (2019)*

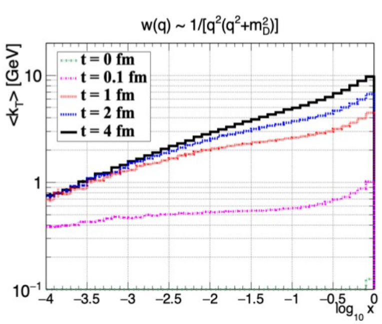
$$\frac{\partial}{\partial t}D(x,\mathbf{k},t) = \frac{1}{t^*}\int_0^1 dz\,\mathcal{K}(z)\left[\frac{1}{z^2}\sqrt{\frac{z}{x}}\,D\left(\frac{x}{z},\frac{\mathbf{k}}{z},t\right)\theta(z-x) - \frac{z}{\sqrt{x}}D(x,\mathbf{k},t)\right]$$
$$+\int\frac{d^2\mathbf{q}}{(2\pi)^2}\,C(\mathbf{q})\,D(x,\mathbf{k}-\mathbf{q},t),$$

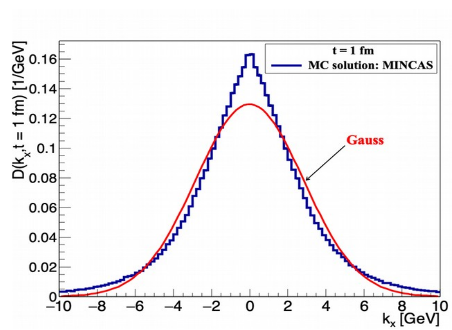
*Resummation of virtual and unresolved real contributions*

$$D(x,\mathbf{k},\tau) = e^{-\Psi(x)(\tau-\tau_0)}\,D(x,\mathbf{k},\tau_0)$$
$$+\int_{\tau_0}^{\tau} d\tau'\int_0^1 dz\int_0^1 dy\int d^2\mathbf{k}'\int d^2\mathbf{q}\;\mathcal{G}(z,\mathbf{q})$$
$$\times\,\delta(x-zy)\,\delta(\mathbf{k}-\mathbf{q}-z\mathbf{k}')\,e^{-\Psi(x)(\tau-\tau')}\,D(y,\mathbf{k}',\tau')$$

*can be solved by Monte Carlo methods!*



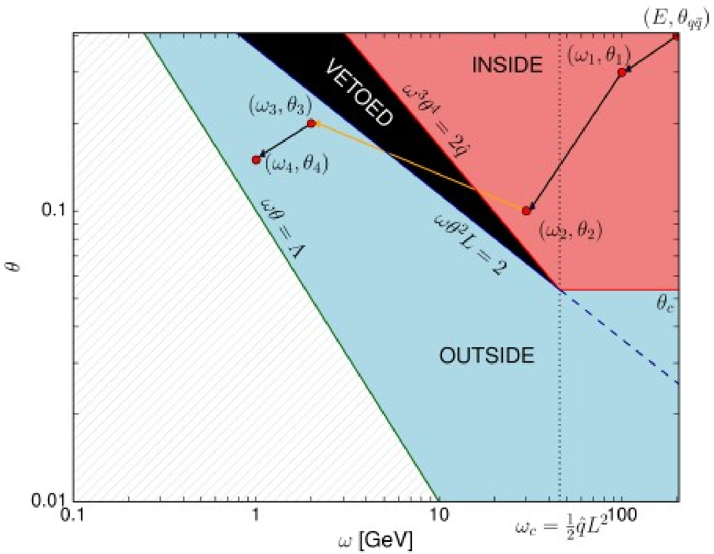



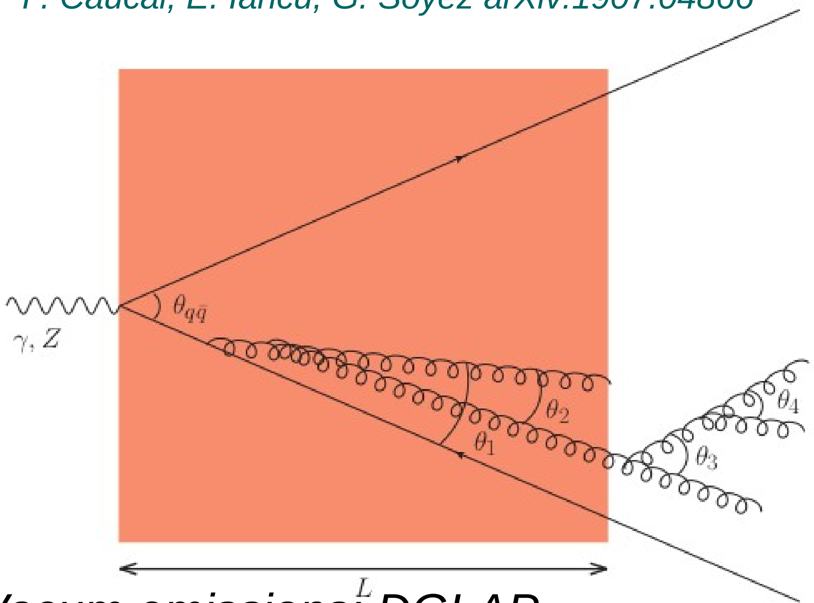

*Thermalized medium suppresses jets stronger. Universal behavior at larger times The jet gets delocalized in transverse in transverse plane and lower and lower "x". Sum of many gaussians with different width. This is a result of the exact treatment of the gluon transverse momentum broadening due to an arbitrary number of the collisions with the medium together with its shrinking due an arbitrary number of the emission branching.*

# Factorized shower

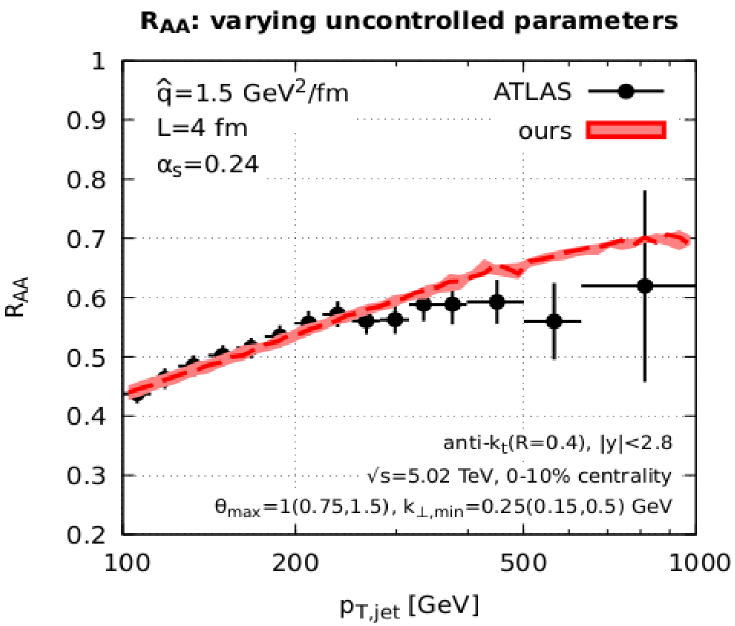
*Formalism from slides before for in medium + vacumlike shower in medium + vacum shower*

P. Caucal, E. Iancu, G. Soyez arXiv:1907.04866




*Vacum emissions: DGLAP*




*Vacum like emissions in medium: DGLAP (with constrained phase space) Proceed fast enough for their formation not to be influenced by the medium*

*Medium like emissions: generalized BDMPS*

$$k^2_{\perp,\text{vac}} = \omega^2\theta^2$$

$$k^2_{\perp,\text{med}} = \hat{q}t_f = \frac{2\hat{q}}{\omega\theta^2}$$

$$k^2_{\perp,\text{med}} = k^2_{\perp,vac} \text{ i.e. } \omega^3\theta^4 = 2\hat{q}$$

# Summary

*New QCD factorization formula for dilute-dense collision has been proposed and proved.*

*New evidence for saturation has been found*

*New developments in Monte Carlo → one can get full set of TMD dependent parton densities which are unmodified by shower.*

*New proposals for imaging of hadrons using EIC and Wigner distributions.*

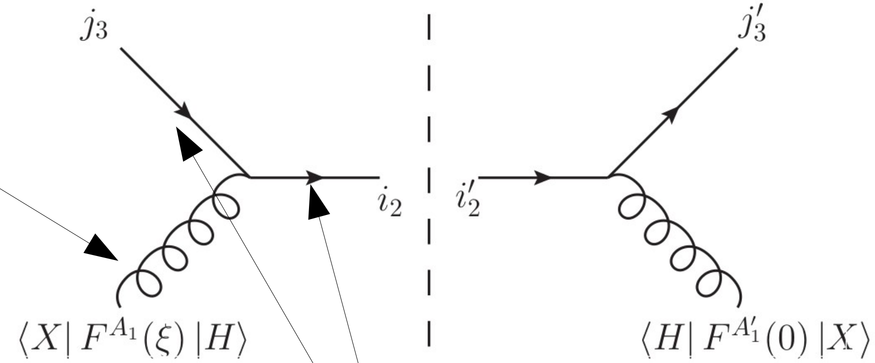
*There is new evidence for strong fluctuations in the shape of proton.*
*Perhaps this can explain similarities between p-p, p-Pb, PB-PB.*

*Emerging new understanding of dynamics of quenching of jets in Pb-Pb collision.*

# *BACKUP*

# Example $qg \to q$

*We want to get TMD distribution of*




Resummation

replacement of deltas with operators

*We need to resum all collinear emissions from*

$$\mathcal{M} = \left(t^{A_1}\right)^{i_2}_{j_3} \mathcal{A}(2,1,3)$$

$$\mathcal{M}^*\mathcal{M}\, \delta^{i_2 i'_2}\, \delta_{j_3 j'_3} = \left(t^{A_1}\right)^{i_2}_{j_3} \left(t^{A'_1}\right)^{j'_3}_{i'_2} \delta^{A_1 A'_1} \delta^{i_2}_{i'_2}\, \delta^{j'_3}_{j_3}\, \mathcal{A}^*(2,1,3)\mathcal{A}(2,1,3)$$

$$\left(t^{A_1}\right)^{i_2}_{j_3} \left(t^{A'_1}\right)^{j'_3}_{i'_2} \left(\mathcal{U}^{[+]}\right)^{i_2}_{i'_2} \left(\mathcal{U}^{[-]\dagger}\right)^{j'_3}_{j_3} F^{A'_1}(0) F^{A_1}(\xi) =$$

$$= \left(F(\xi)\right)^{i_2}_{j_3} \left(\mathcal{U}^{[-]\dagger}\right)^{j'_3}_{j_3} \left(F(0)\right)^{j'_3}_{i'_2} \left(\mathcal{U}^{[+]}\right)^{i_2}_{i'_2} = \mathrm{Tr}\left[F(\xi)\mathcal{U}^{[-]\dagger}F(0)\mathcal{U}^{[+]}\right]$$

$$\mathcal{F}^{(1)}_{qg}(x, k_T) = 2\int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+}\, e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T} \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$