UNIVERSITY OF Southampton
School of Physics and Astronomy

elusiνes invisiblesPlus
neutrinos, dark matter & dark energy physics

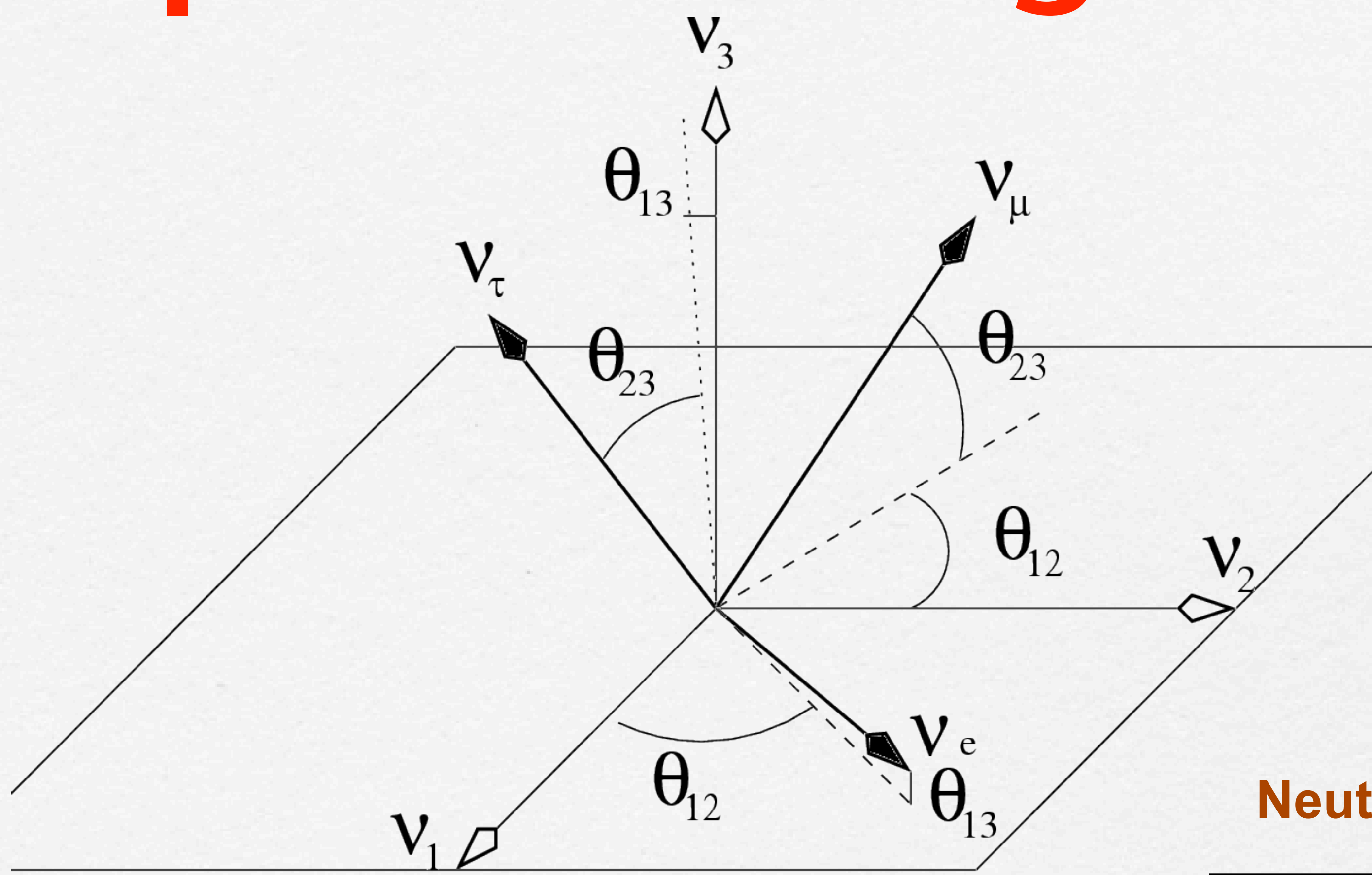
Neutrino Physics - a review

Steve King, 4th September 2019, Corfu

EISA
European Institute for Sciences and Their Applications

PMNS Lepton mixing matrix

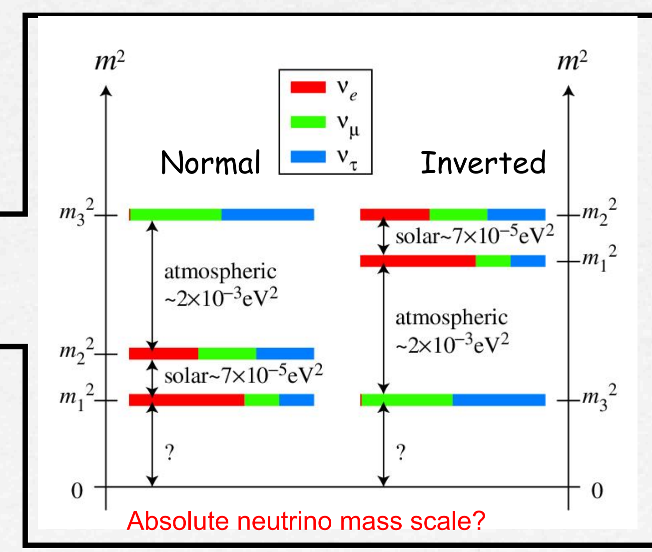
Pontecorvo
Maki
Nakagawa
Sakata



Standard Model states

$$\begin{pmatrix}\nu_e \\ e^-\end{pmatrix}_L \quad \begin{pmatrix}\nu_\mu \\ \mu^-\end{pmatrix}_L \quad \begin{pmatrix}\nu_\tau \\ \tau^-\end{pmatrix}_L$$

Neutrino mass states



$$\begin{pmatrix}\nu_e \\ \nu_\mu \\ \nu_\tau\end{pmatrix} = \begin{pmatrix}U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3}\end{pmatrix} \begin{pmatrix}\nu_1 \\ \nu_2 \\ \nu_3\end{pmatrix}$$

PMNS Lepton mixing matrix

$$U_{PMNS}=\begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & s_{13}e^{-i\delta}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta} & 0 & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0\\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}}\end{pmatrix}$$

Atmospheric Reactor Solar Majorana

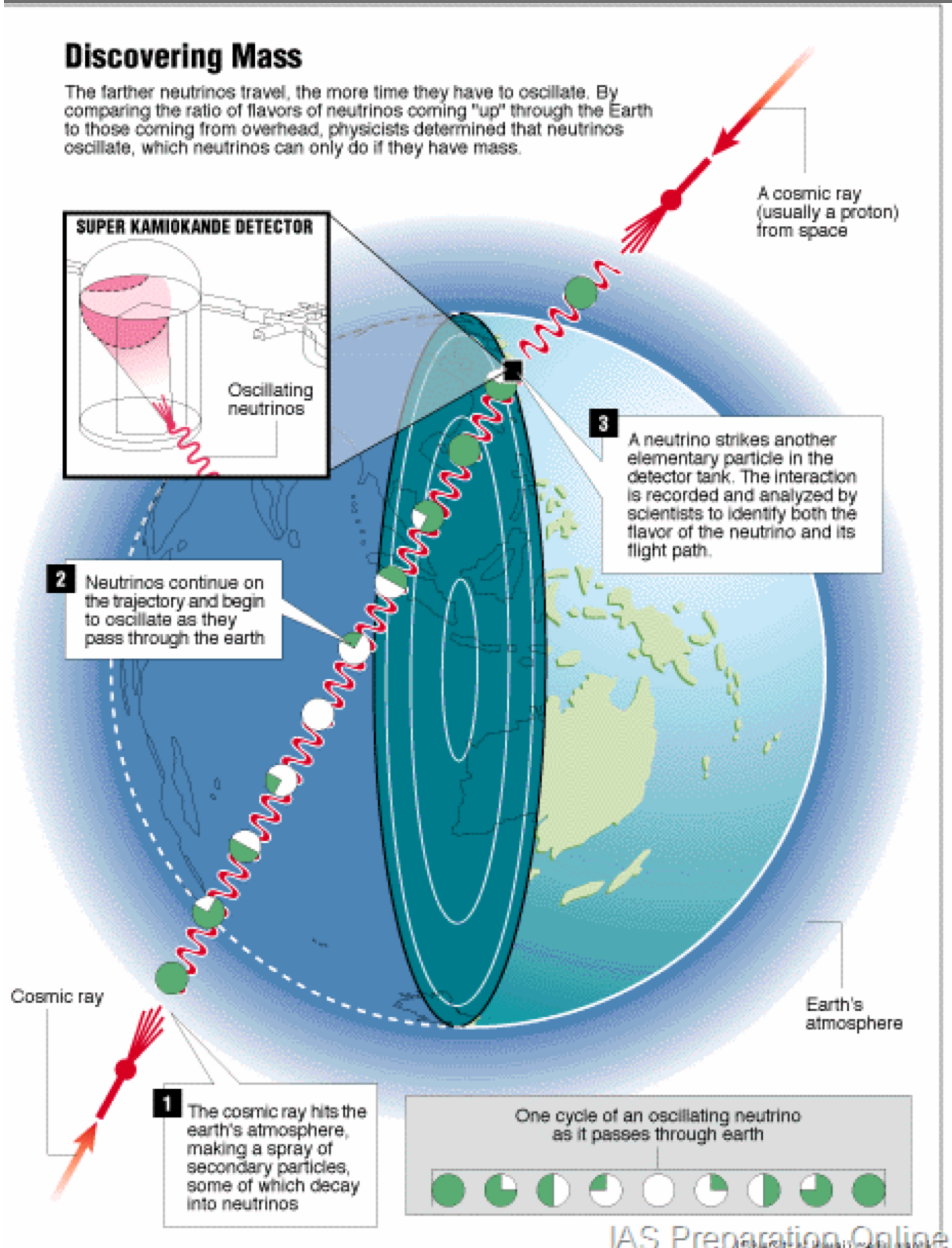
$$=\begin{pmatrix}c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta}\\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13}\\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}\end{pmatrix}$$

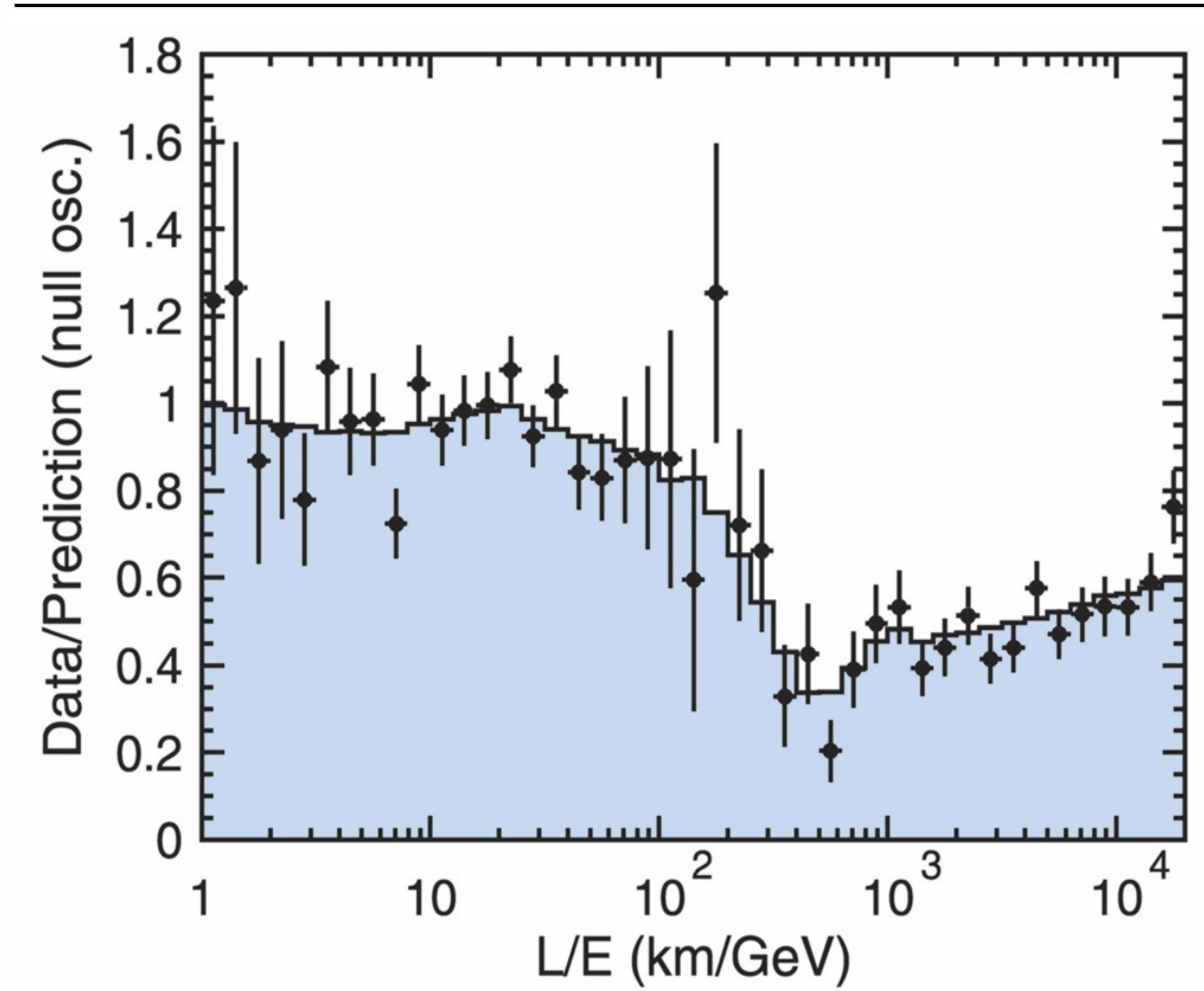
$$\times\,\mathrm{diag}(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2})$$

The 6 parameters measurable in neutrino oscillations (assuming 3 active neutrinos):

- ✳ The atmospheric mass squared difference Δm^2_{31}
- ✳ The solar mass squared difference $\Delta m^2_{21} = m^2_2 - m^2_1$
- ✳ The atmospheric angle θ_{23}
- ✳ The solar angle θ_{12}
- ✳ The reactor angle θ_{13}
- ✳ The CP violating phase δ

Atmospheric Neutrino Oscillations (1998)





$$Prob. = \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{E}$$

Atmospheric neutrino oscillations show characteristic L/E variation

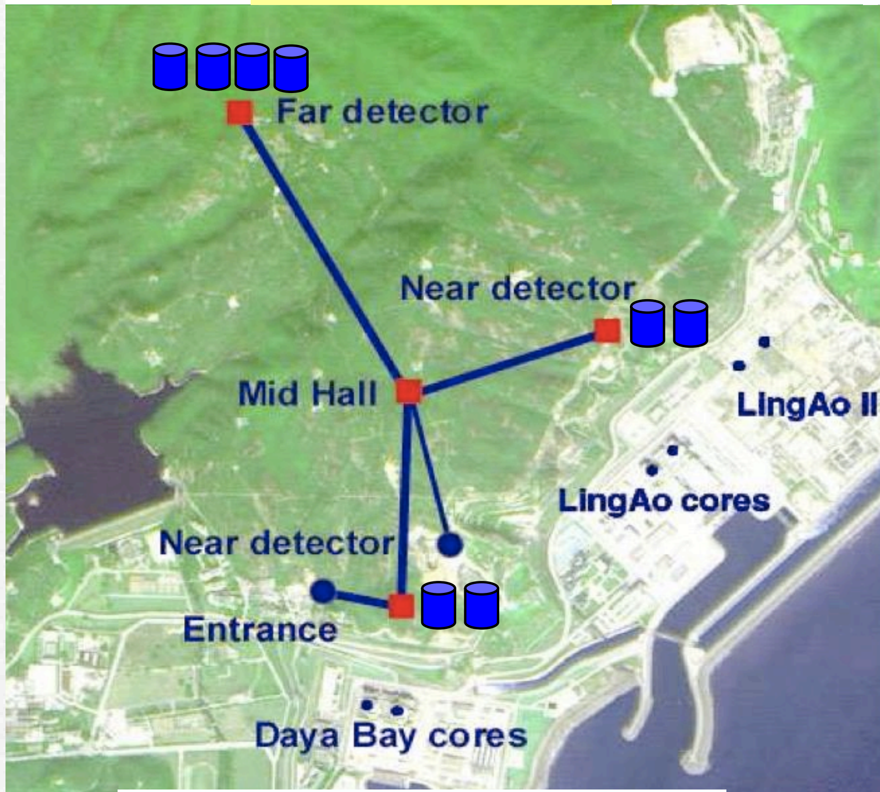
Brief History of Neutrino Physics post 1998

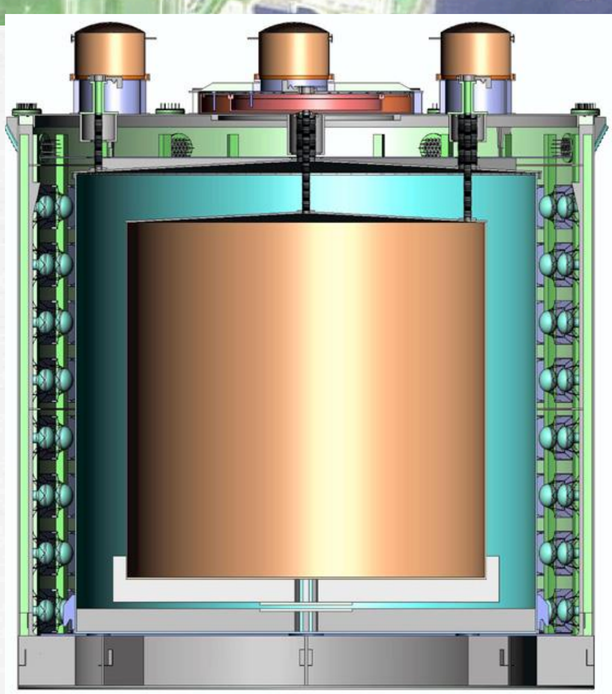
- ☑ Atmospheric ν_μ disappear, large θ_{23} (1998)  SK
- ☑ Solar ν_e disappear, large θ_{12} (2002)  SK, SNO
- ☑ Solar ν_e are converted to ν_μ+ν_τ (2002) SNO
- ☑ Reactor anti-ν_e disappear/reappear (2004) Kamland
- ☑ Accelerator ν_μ disappear (2006) MINOS
- ☑ Accelerator ν_μ converted to ν_τ (2010) OPERA
- ☑ Accelerator ν_μ converted to ν_e , θ_{13} hint (2011) T2K
- ☑ Reactor anti-ν_e disapp θ_{13} meas.(2012) DB, Reno,DC

Daya Bay

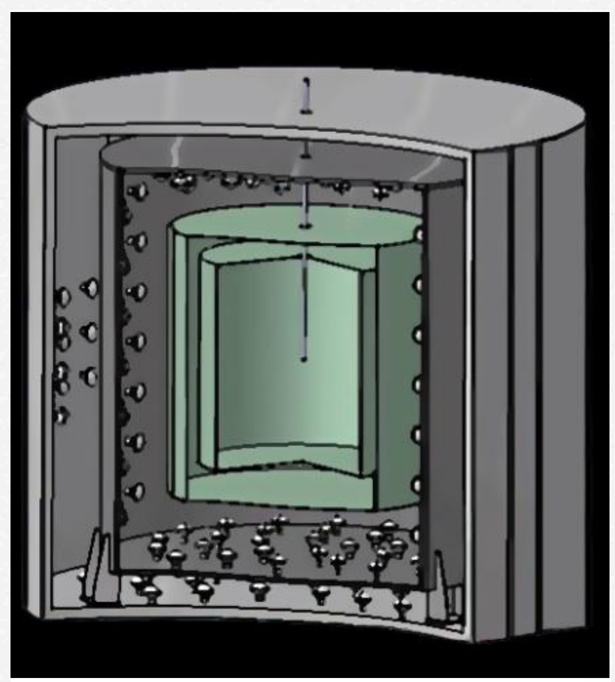
Double Chooz

Reno

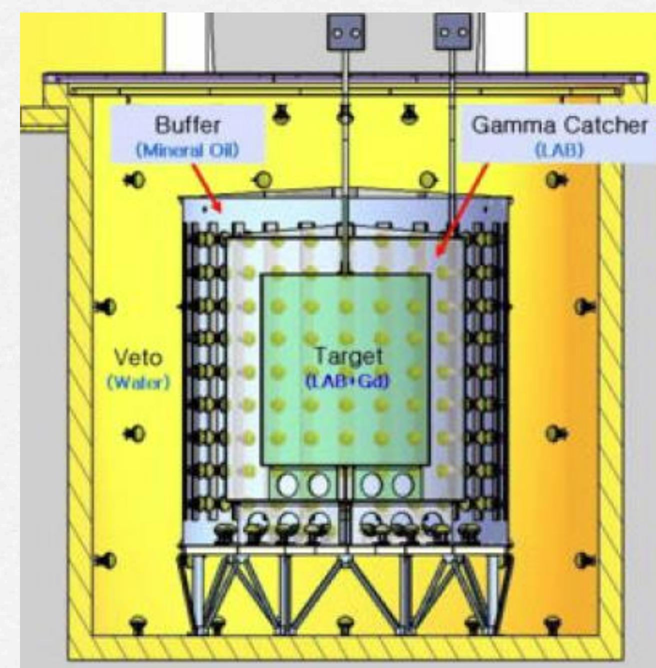




Daya Bay



Double Chooz

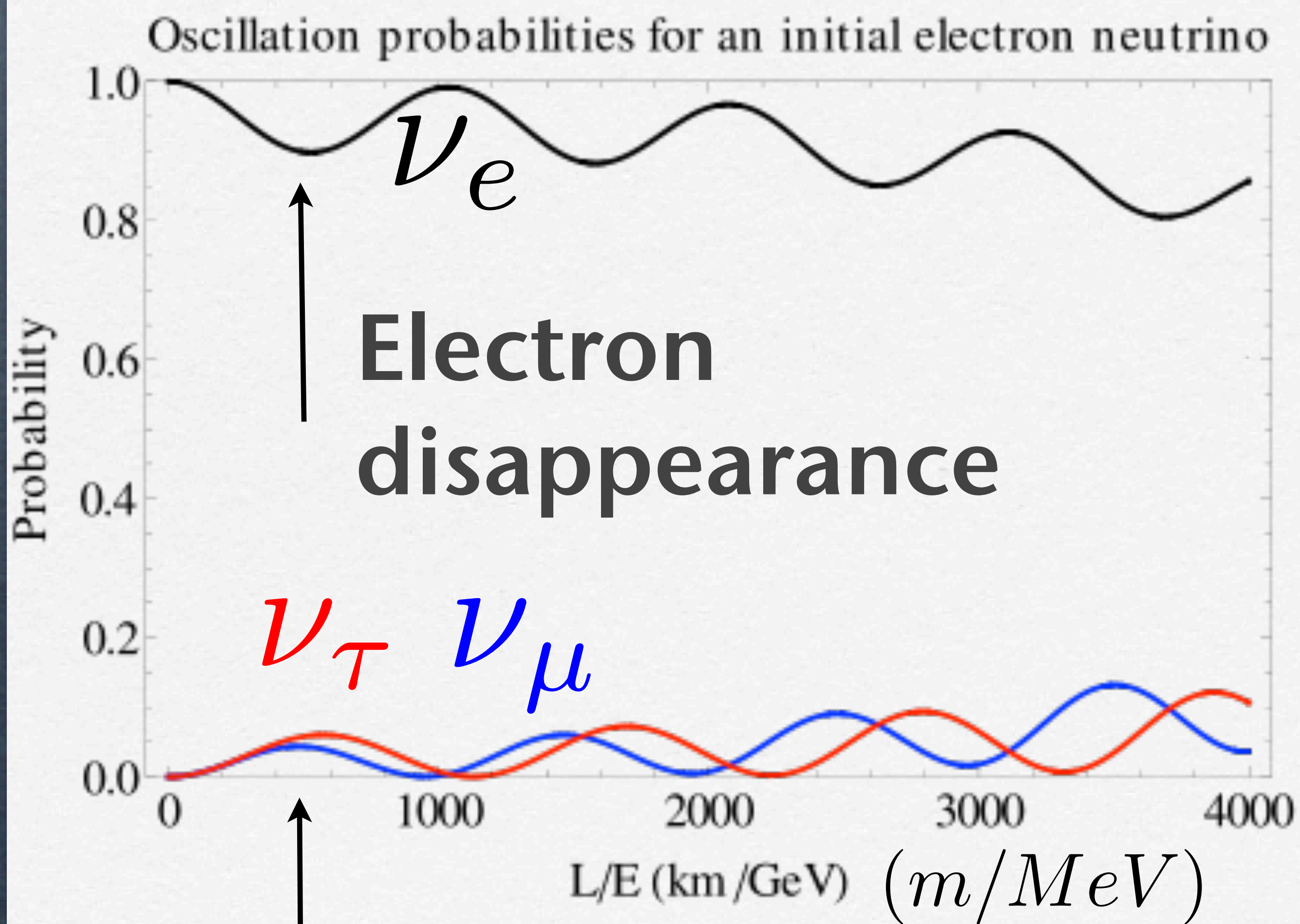


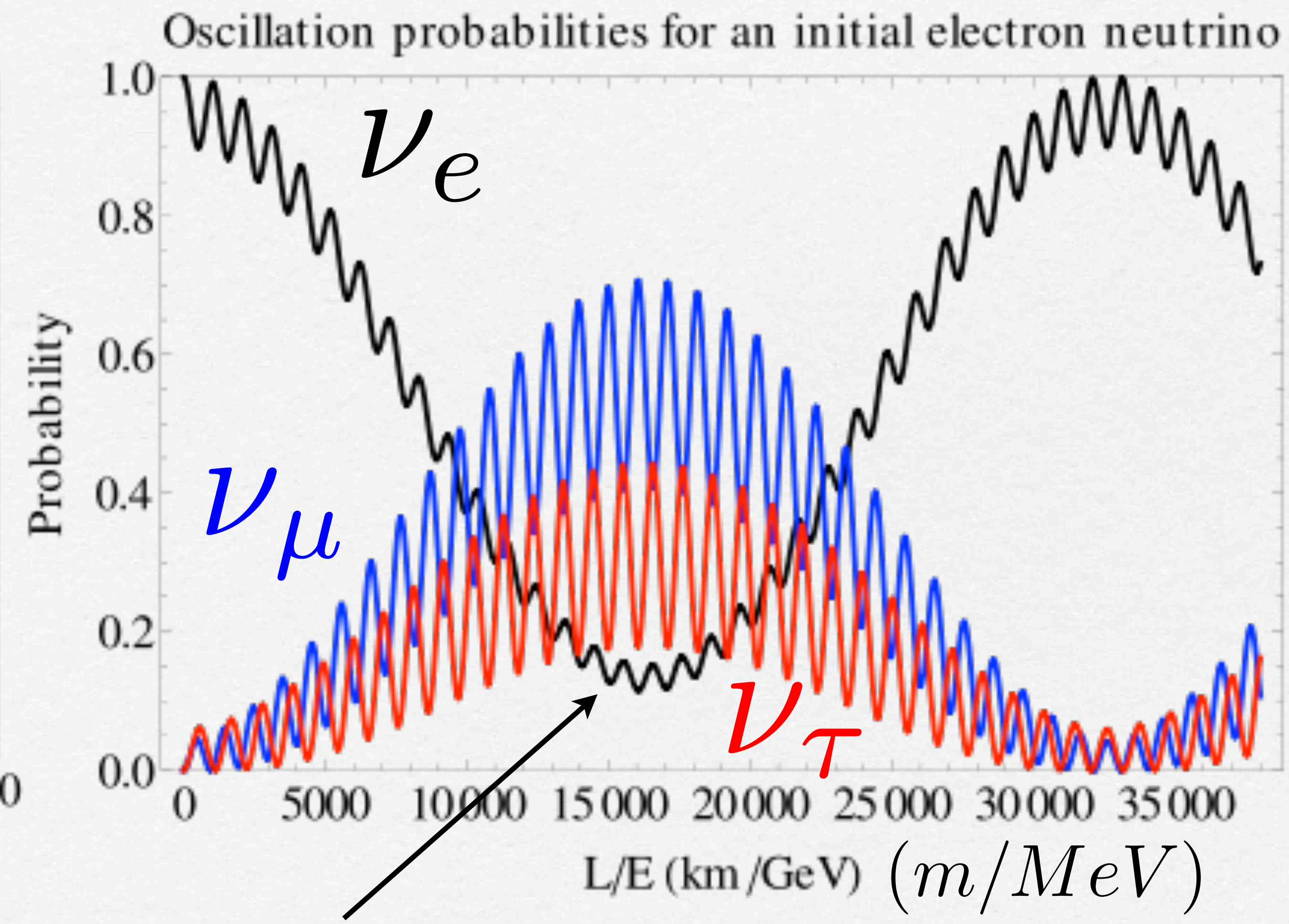
RENO

Electron Neutrino Oscillations

$$P(\bar{\nu}_e \to \bar{\nu}_e; E, L) = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} \qquad \Delta_{ij} = \frac{\Delta m^2_{ij} L}{4E}$$

$$- \sin^2 2\theta_{13}(\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32})$$





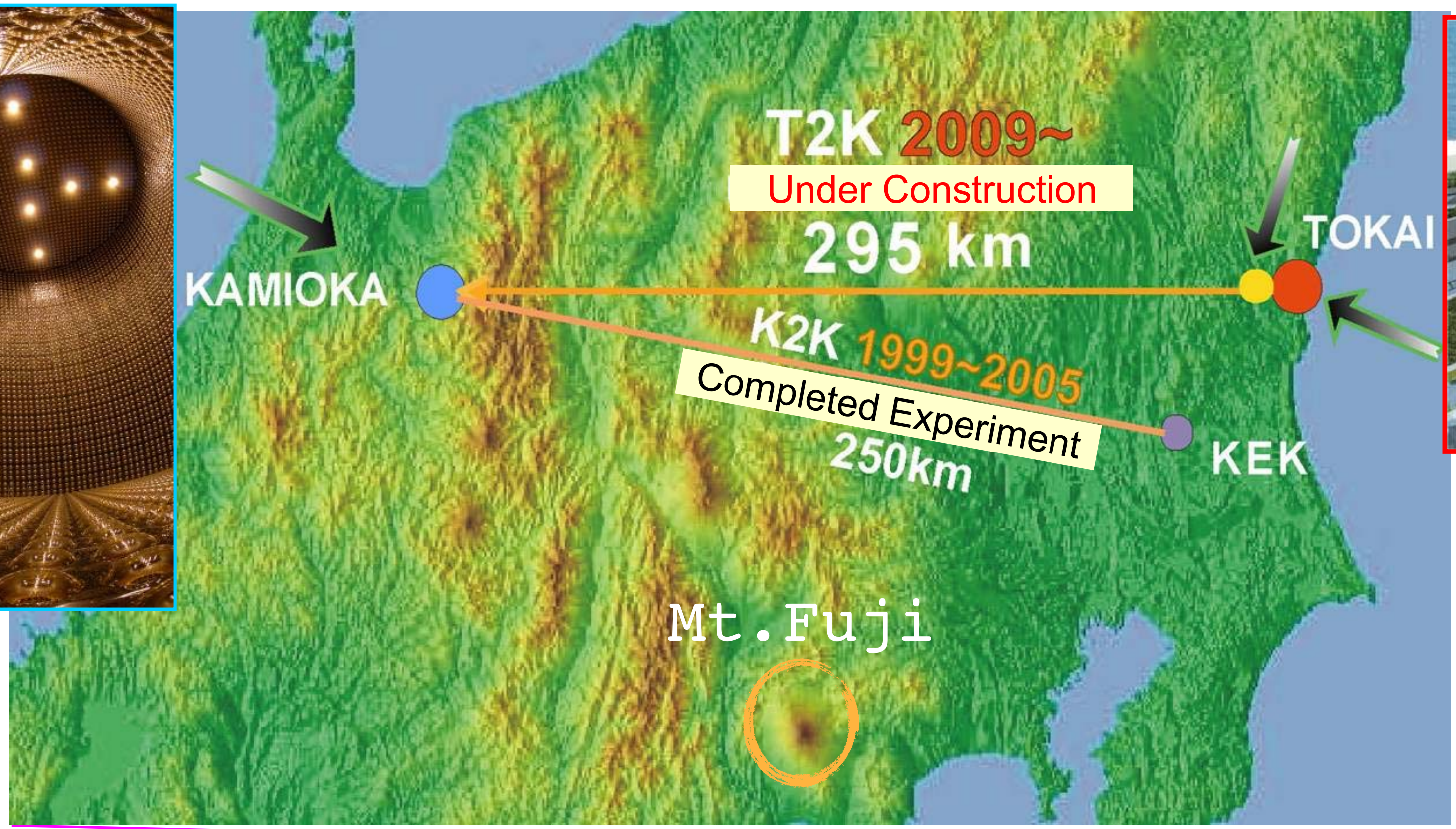
Daya Bay
RENO
(1st atm max)

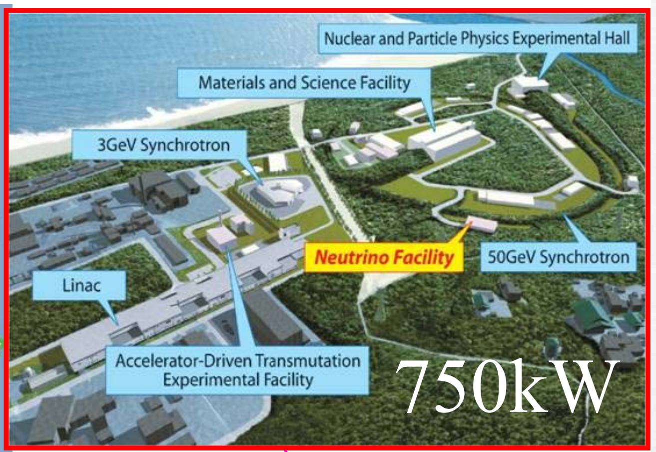
$$\frac{\Delta m^2_{31} L}{4E} = \frac{\pi}{2}$$

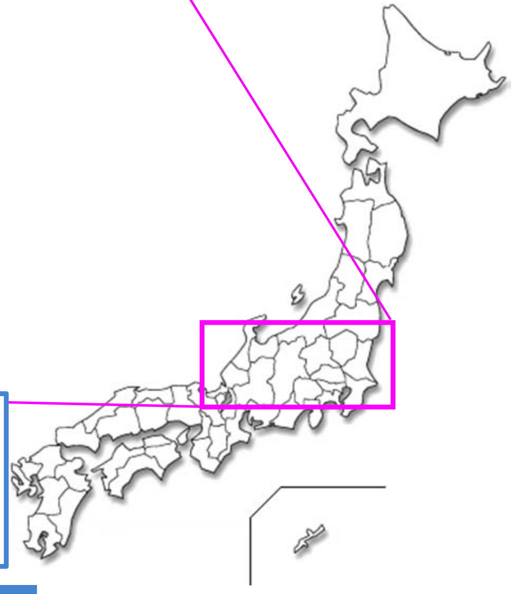
JUNO
RENO50km
(1st sol max)

$$\frac{\Delta m^2_{21} L}{4E} = \frac{\pi}{2}$$

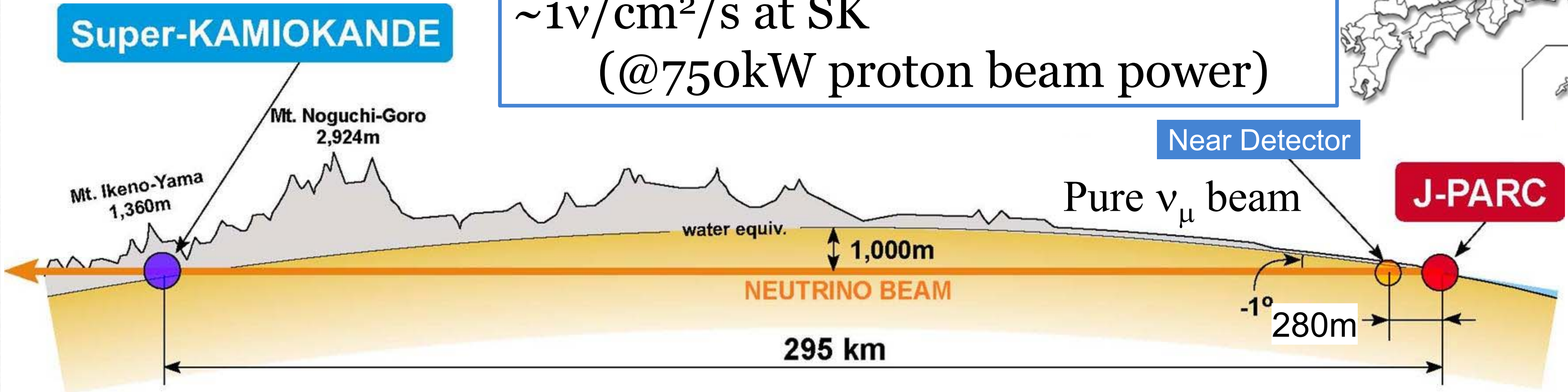
T2K (Tokai to Kamioka)
Long Baseline ν experiment







~1ν/cm²/s at SK
(@750kW proton beam power)

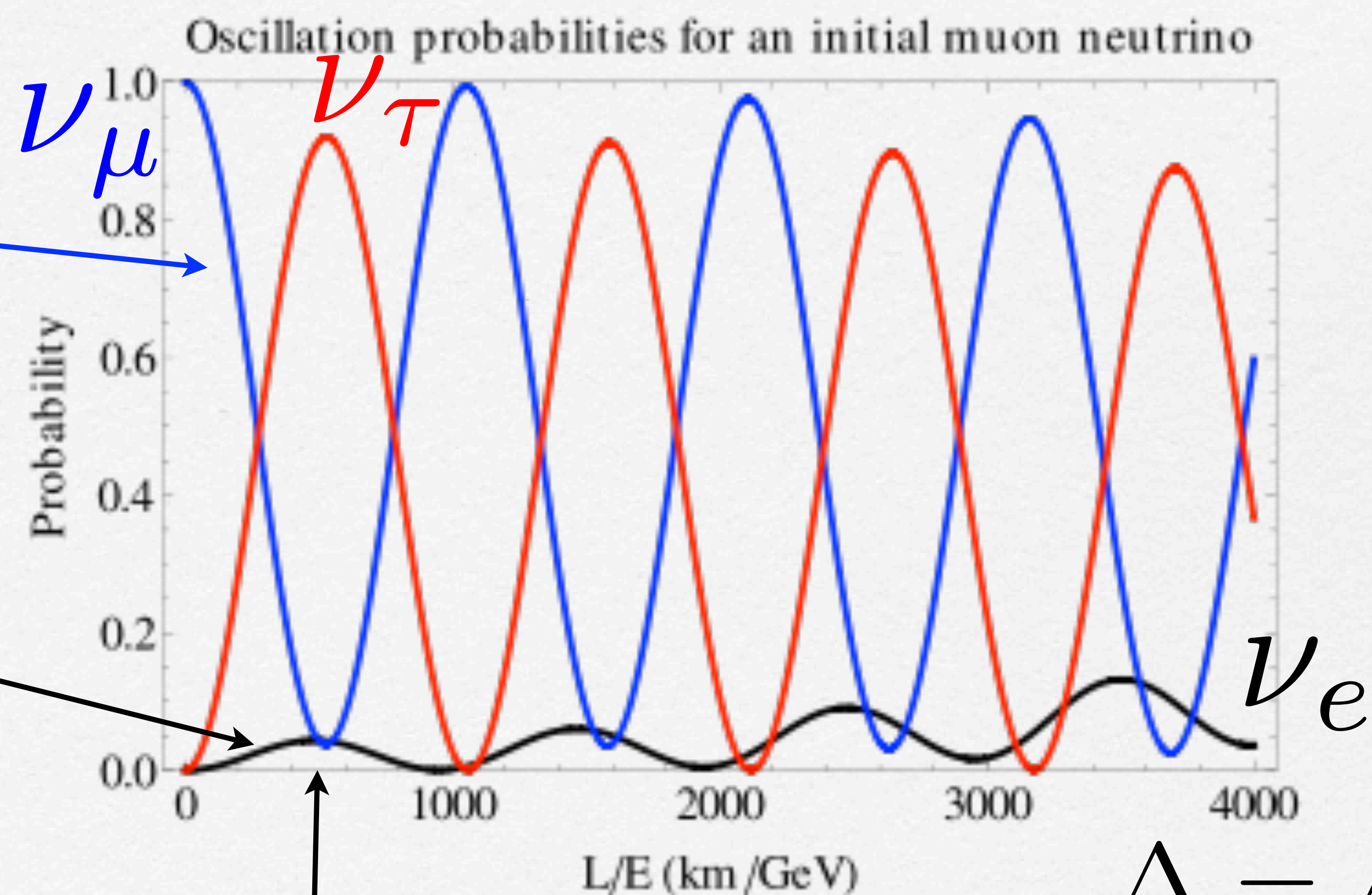


Muon Neutrino Oscillations

$$P(\nu_\mu \to \nu_\mu; E, L) = 1 - \sin^2(2\theta_{23}) \sin^2\left(\frac{\Delta L}{2}\right) + \mathcal{O}(\epsilon)$$

Muon disappearance

Electron appearance



$$\frac{\Delta m^2_{31} L}{4E} = \frac{\pi}{2}$$

$$\Delta = \Delta m^2_{31}/2E$$

$$\epsilon \equiv \Delta m^2_{21}/\Delta m^2_{31} \approx 0.03$$

Accelerator LBL (1st atm max)

Electron Neutrino Appearance

$$P(\nu_\mu \to \nu_e; E, L) \equiv P_1 + P_{\frac{3}{2}} + \mathcal{O}\left(\epsilon^2\right)$$

$$P_1 = \frac{4}{(1-r_A)^2} \sin^2\theta_{23} \sin^2\theta_{13} \sin^2\left(\frac{(1-r_A)\Delta L}{2}\right),$$

$$P_{\frac{3}{2}} = 8J_r \frac{\epsilon}{r_A(1-r_A)} \cos\left(\delta + \frac{\Delta L}{2}\right) \sin\left(\frac{r_A \Delta L}{2}\right) \sin\left(\frac{(1-r_A)\Delta L}{2}\right)$$

CP phase **Matter effect**

Electron appearance depends on CP phase

$$\Delta = \Delta m^2_{31}/2E$$

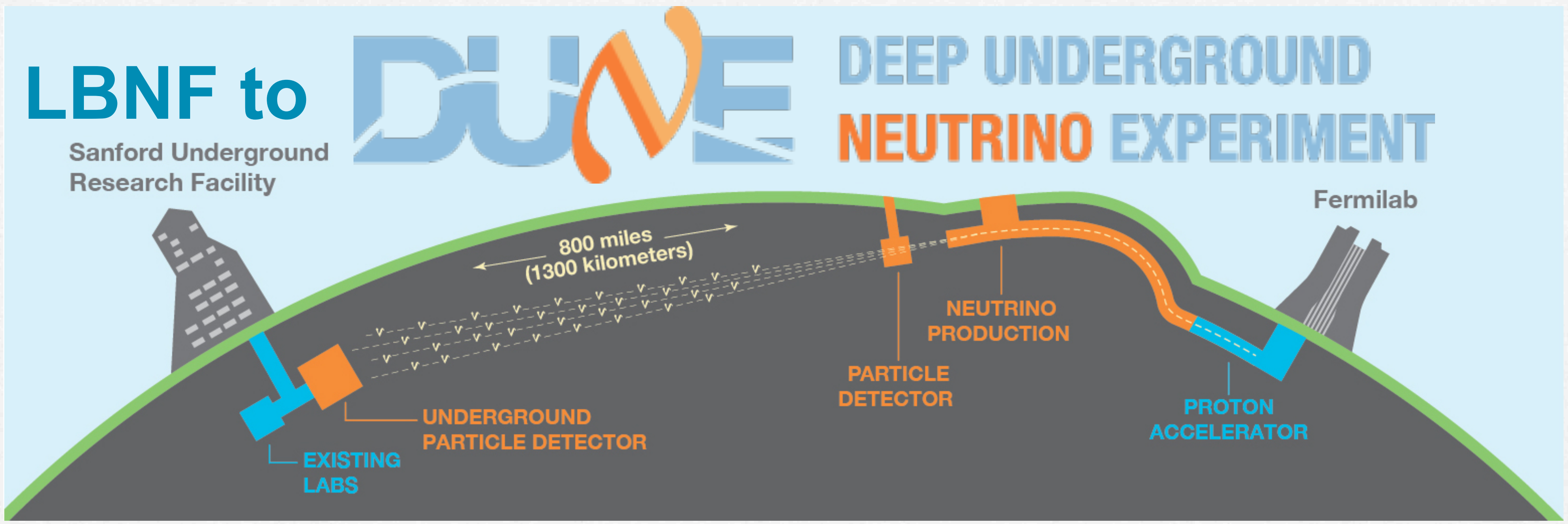
$$\epsilon \equiv \Delta m^2_{21}/\Delta m^2_{31} \approx 0.03$$

r_A, δ change sign for antineutrinos

$$r_A = 2\sqrt{2} G_{\mathrm{F}} N_e E/\Delta m^2_{31}$$

$$J_r = \cos\theta_{12} \sin\theta_{12} \cos\theta_{23} \sin\theta_{23} \sin\theta_{13}$$

Future LBL experiments

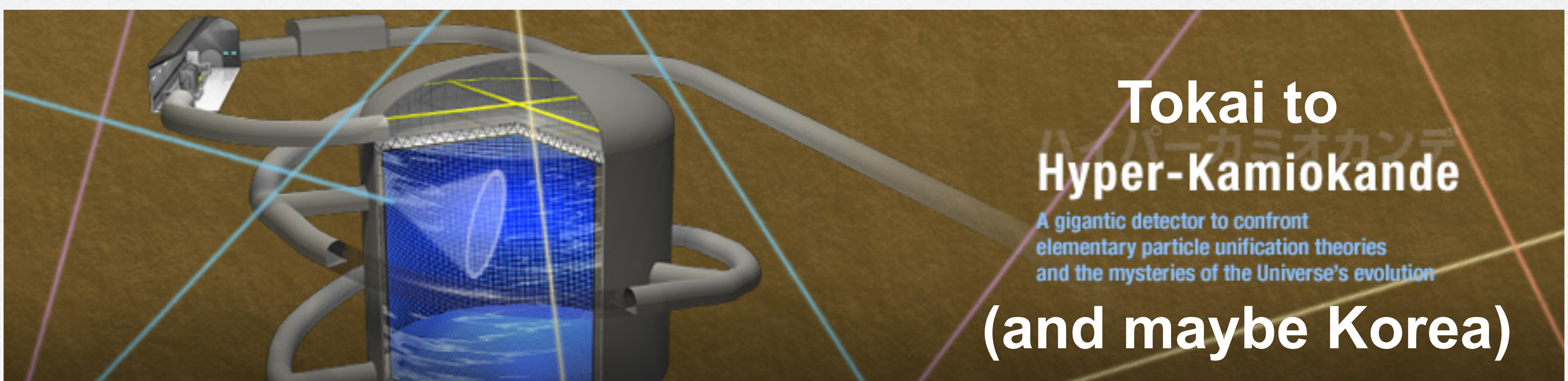


Beams of

$$\nu_\mu \qquad \bar{\nu}_\mu$$

from

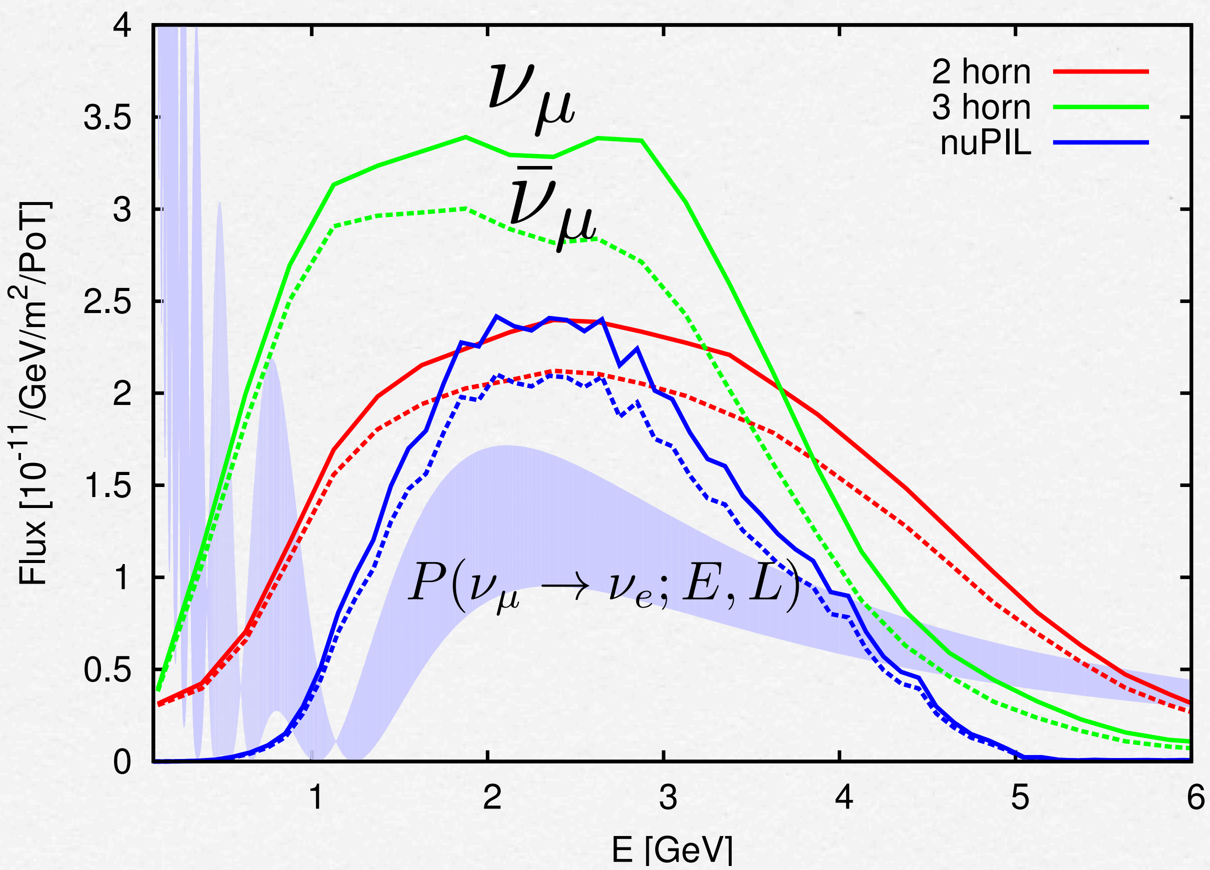
$$\pi^+ \to \mu^+ + \nu_\mu$$
$$\pi^- \to \mu^- + \bar{\nu}_\mu$$



Highly complementary experiments:

DUNE

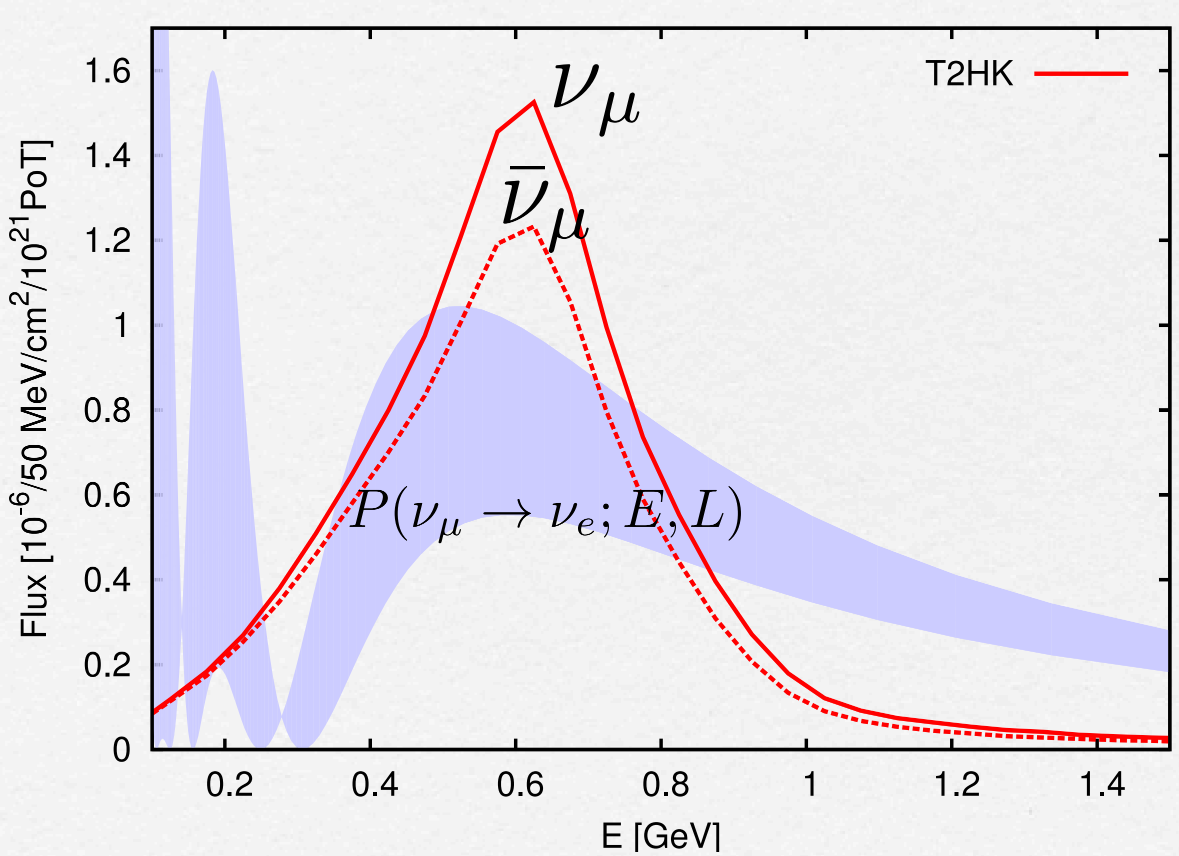
$L = 1300km$



Wide Band Beam
LAr detector

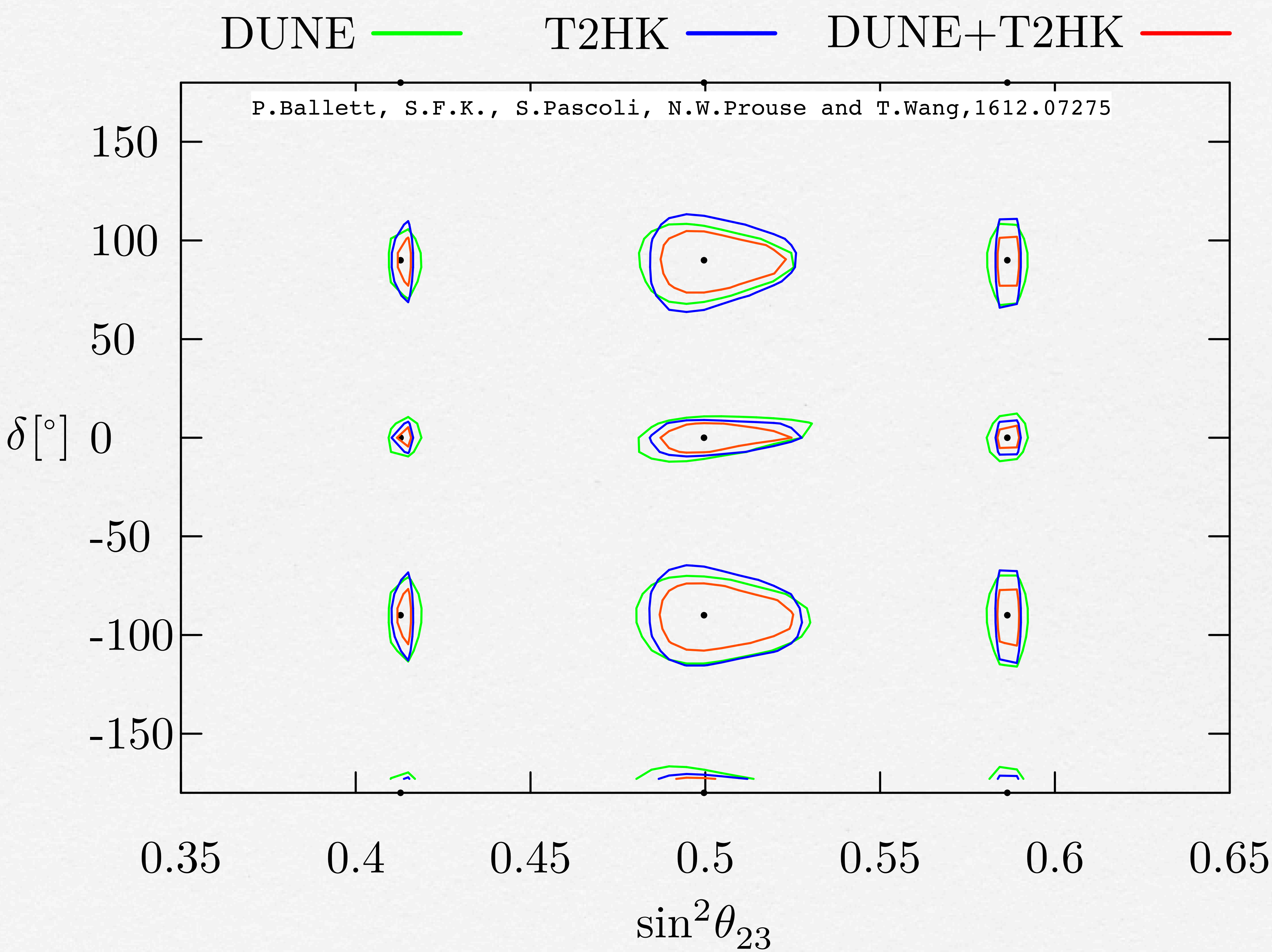
T2HK

$L = 295km$



Narrow Band Beam (off-axis)
Water detector

Precision measurements



1 sigma contours in future

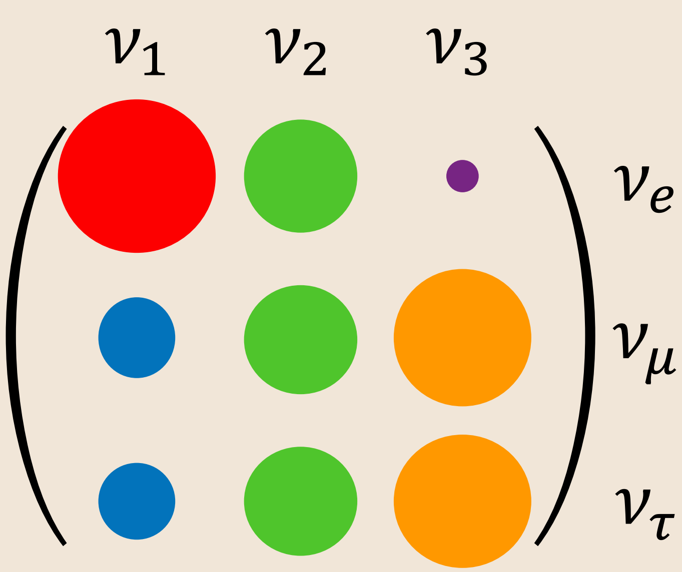
Parameters	Neutrino Oscillation Experiments
Δm^2_{21}	KamLAND ($\overline{\nu}_e \to \overline{\nu}_e$)[21]
Δm^2_{31}	T2K ($\nu_\mu \to \nu_\mu$)[22] MINOS ($\overline{\nu}_\mu \to \overline{\nu}_\mu$, $\nu_\mu \to \nu_\mu$)[23]
θ_{12}	solar neutrinos ($\nu_e \to \nu_e$) Borexino[24], SNO[25,26], Super-Kamionkande I-IV[27]
θ_{13}	Daya Bay ($\overline{\nu}_e \to \overline{\nu}_e$)[28] RENO ($\overline{\nu}_e \to \overline{\nu}_e$)[29]
θ_{23}	atmospheric neutrinos ($\overline{\nu}_\mu \to \overline{\nu}_\mu$, $\nu_\mu \to \nu_\mu$) Super-Kamiokande I-IV[30]
δ	–

NuFIT 4.1 (2019)		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 6.2$)
		bfp $\pm 1\sigma$	3σ range	
without SK atmospheric data	$\sin^2\theta_{12}$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$	
	$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	
	$\sin^2\theta_{23}$	$0.558^{+0.020}_{-0.033}$	$0.427 \to 0.609$	
	$\theta_{23}/^\circ$	$48.3^{+1.1}_{-1.9}$	$40.8 \to 51.3$	
	$\sin^2\theta_{13}$	$0.02241^{+0.00066}_{-0.00065}$	$0.02046 \to 0.02440$	
	$\theta_{13}/^\circ$	$8.61^{+0.13}_{-0.13}$	$8.22 \to 8.99$	
	$\delta_{\mathrm{CP}}/^\circ$	222^{+38}_{-28}	$141 \to 370$	
	$\dfrac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	
	$\dfrac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.523^{+0.032}_{-0.030}$	$+2.432 \to +2.618$	

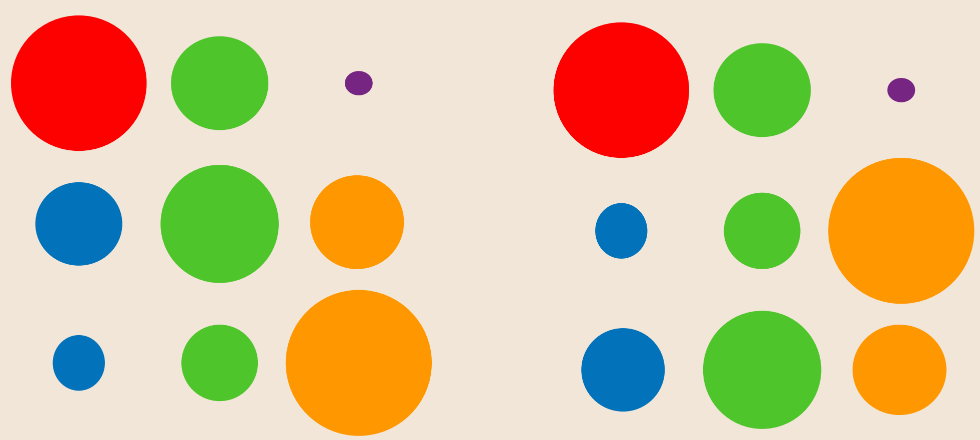
Open questions for neutrino mixing

Phill Litchfield

$$|U_{\mathrm{PMNS}}|^2 \simeq$$

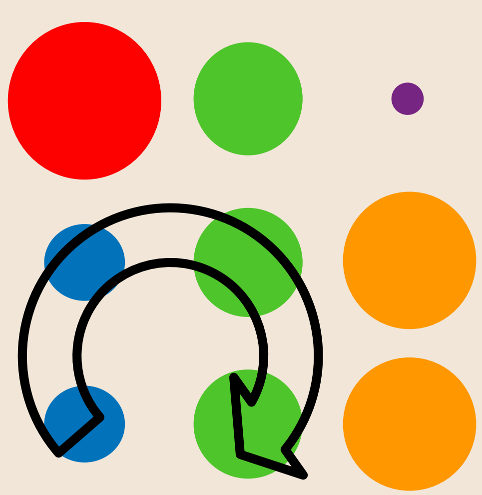


Octant degeneracy



Lower ($\theta_{23} < 45°$) Upper ($\theta_{23} > 45°$)

CP Violation

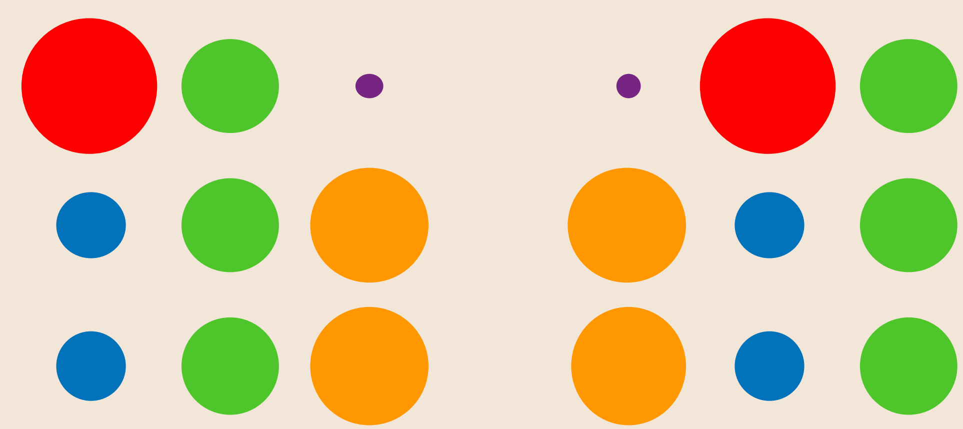


Complex mixing of these 4 elements causes

$$P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

Key parameter: $\boldsymbol{\delta_{CP}}$

Mass Ordering (Hierarchy)



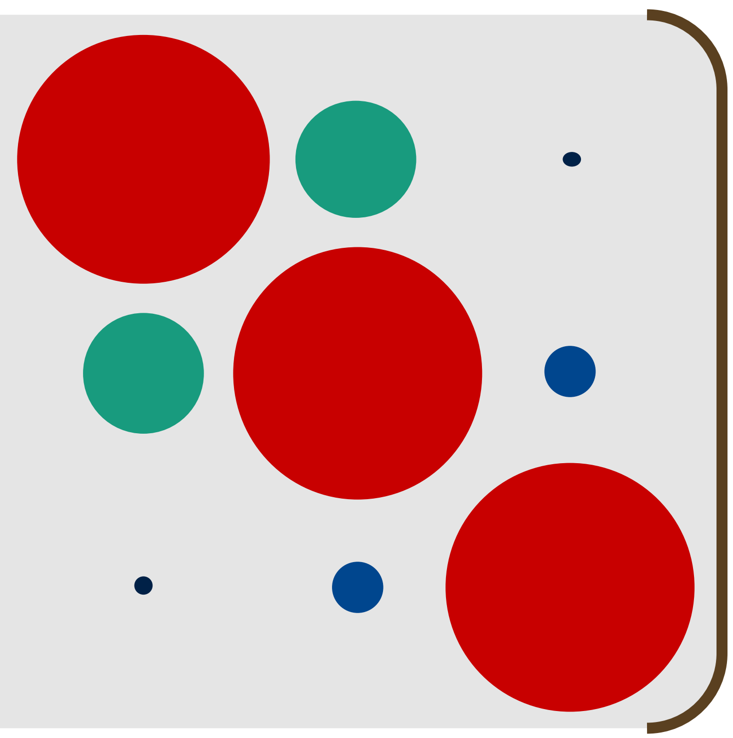
Normal (NO) Inverted (IO)

CKM vs Tri-bimaximal Mixing

Phill Litchfield

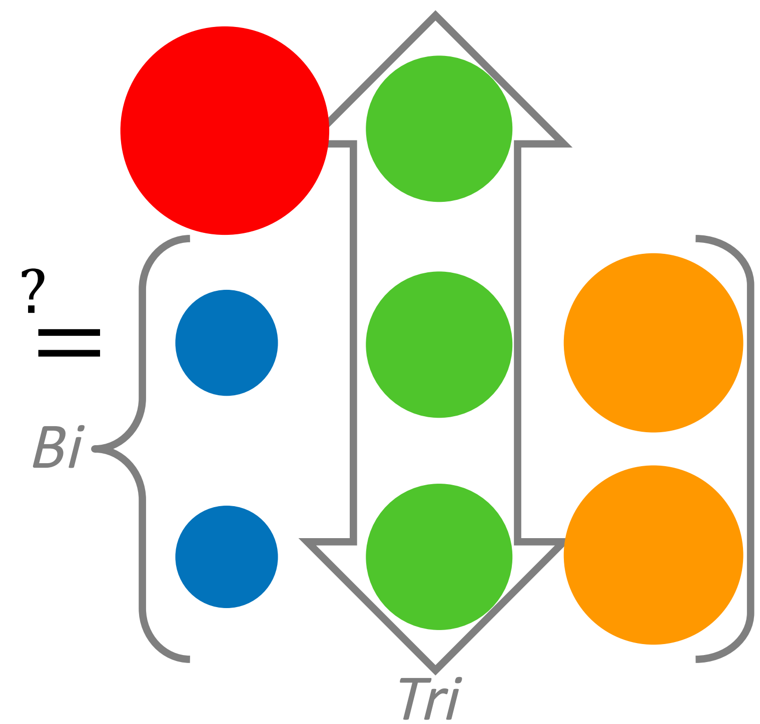
CKM Matrix

$$|U_{CKM}|^2 =$$



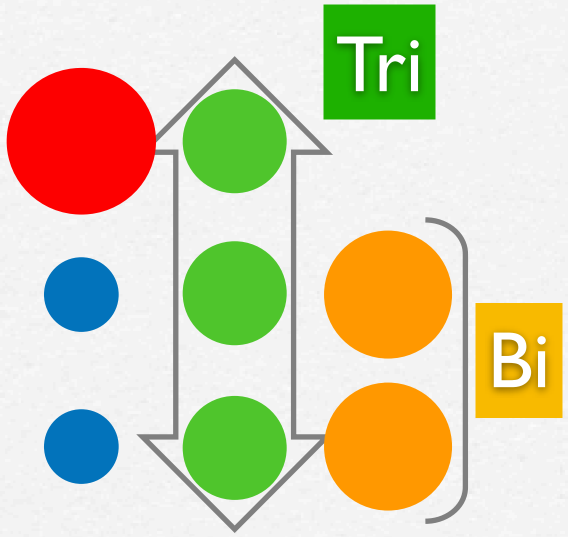
Tri-bimaximal Mixing

$$|U_{PMNS}|^2 \stackrel{?}{=}$$



P.F.Harrison, D.H.Perkins and W.G.Scott, hep-ph/0202074

Tri-Bimaximal Mixing



$$\begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sin\theta_{12} = \frac{1}{\sqrt{3}}$$

Allowed at
3 sigma

$$\sin\theta_{23} = \frac{1}{\sqrt{2}}$$

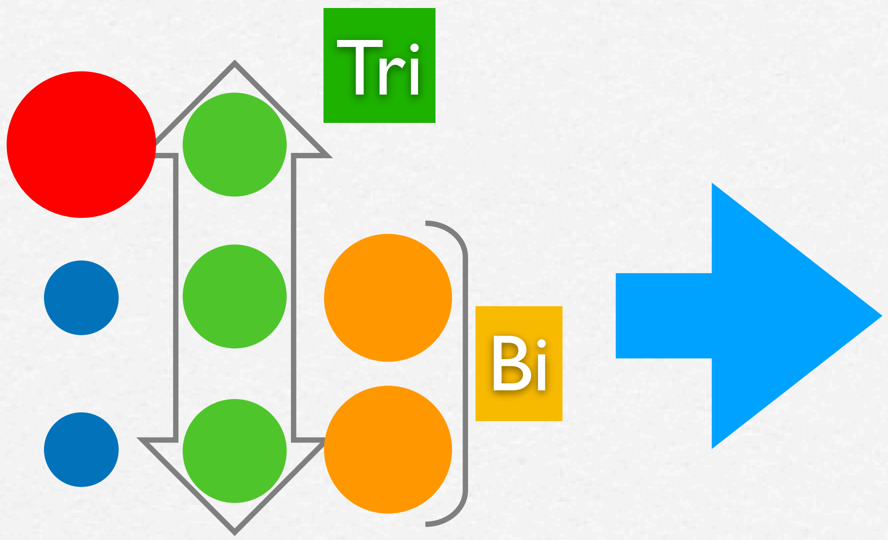
Allowed at
3 sigma

$$\sin\theta_{13} = 0$$

Excluded
at many sigma

P.F.Harrison, D.H.Perkins and W.G.Scott, hep-ph/0202074

Tri-Bimaximal Mixing



$$\begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sin\theta_{12} = \frac{1}{\sqrt{3}}$$

Allowed at 3 sigma

$$\sin\theta_{23} = \frac{1}{\sqrt{2}}$$

Allowed at 3 sigma

$$\sin\theta_{13} = 0$$

Excluded at many sigma

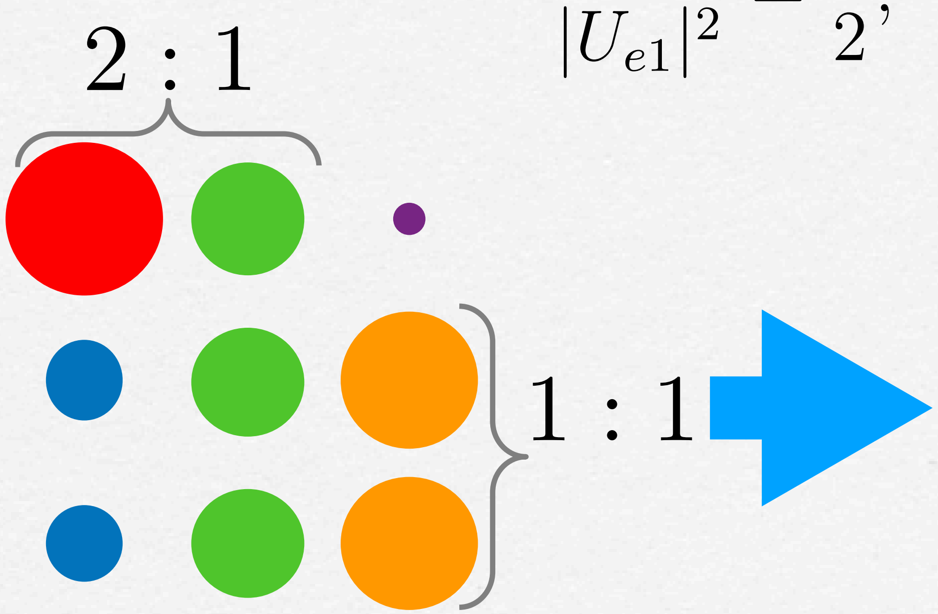
NuFIT 4.1 (2019) Best Fit Preferences:

$$s^2_{12} < \frac{1}{3} \qquad s^2_{23} > \frac{1}{2} \qquad s^2_{13} = 0.02241 \pm 0.00065$$

SFK 0903.3199, 1205.0506

Tri-Bimaximal-Reactor

$$\frac{|U_{e2}|^2}{|U_{e1}|^2} = \frac{1}{2}, \quad \frac{|U_{\mu 3}|^2}{|U_{\tau 3}|^2} = 1.$$



$$\begin{pmatrix} \sqrt{\frac{2}{3}}(1-\frac{1}{4}\lambda^2) & \frac{1}{\sqrt{3}}(1-\frac{1}{4}\lambda^2) & \frac{1}{\sqrt{2}}\lambda e^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+\lambda e^{i\delta}) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}\lambda e^{i\delta}) & \frac{1}{\sqrt{2}}(1-\frac{1}{4}\lambda^2) \\ \frac{1}{\sqrt{6}}(1-\lambda e^{i\delta}) & -\frac{1}{\sqrt{3}}(1+\frac{1}{2}\lambda e^{i\delta}) & \frac{1}{\sqrt{2}}(1-\frac{1}{4}\lambda^2) \end{pmatrix}$$

$$\sin\theta_{12} = \frac{1}{\sqrt{3}}$$

Allowed at 3 sigma

$$\sin\theta_{23} = \frac{1}{\sqrt{2}}$$

Allowed at 3 sigma

$$\sin\theta_{13} = \frac{\lambda}{\sqrt{2}}$$

Allowed ✓

Charged lepton corrections

Charged lepton rotation

Tri-bimaximal

$$U_{\rm PMNS} = \begin{pmatrix} c^e_{12} & s^e_{12}e^{-i\delta^e_{12}} & 0 \\ -s^e_{12}e^{i\delta^e_{12}} & c^e_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$= \begin{pmatrix} \cdots & \cdots & \frac{s^e_{12}}{\sqrt{2}}e^{-i\delta^e_{12}} \\ \cdots & \cdots & \frac{c^e_{12}}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

- Reactor angle generated
- Third row unchanged implies sum rules

Sum rules first derived and studied in: SFK hep-ph/0506297; S.Antusch, SFK hep-ph/0508044; S.Antusch, P.Huber, S.F.K and T.Schwetz, hep-ph/0702286; S.Antusch, S.F.K., M.Malinsky,0711.4727
More recent detailed phenomenological analyses:
D.Marzocca, S.T.Petcov, A.Romanino and M.C.Sevilla,1302.0423; S.T.Petcov 1405.6006;
P.Ballett, S.F.King, C.Luhn, S.Pascoli and M.A.Schmidt, 1410.7573
I.Girardi, S.T.Petcov and A.V.Titov,1410.8056,1504.00658,1504.02402,1605.04172,…
For asymmetric texture without sum rule see: M.H.Rahat, P.Ramond, B.Xu, 1805.10684

Charged lepton corrections

Charged lepton rotation

Tri-bimaximal

$$U_{\rm PMNS} = \begin{pmatrix} c^e_{12} & s^e_{12}e^{-i\delta^e_{12}} & 0 \\ -s^e_{12}e^{i\delta^e_{12}} & c^e_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$= \begin{pmatrix} \cdots & \cdots & \frac{s^e_{12}}{\sqrt{2}}e^{-i\delta^e_{12}} \\ \cdots & \cdots & \frac{c^e_{12}}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$s_{13} = \frac{s^e_{12}}{\sqrt{2}}$$

Suggests $\theta^e_{12} \approx \theta_C$

$$c_{23}c_{13} = \frac{1}{\sqrt{2}} \rightarrow s^2_{23} < \frac{1}{2}$$

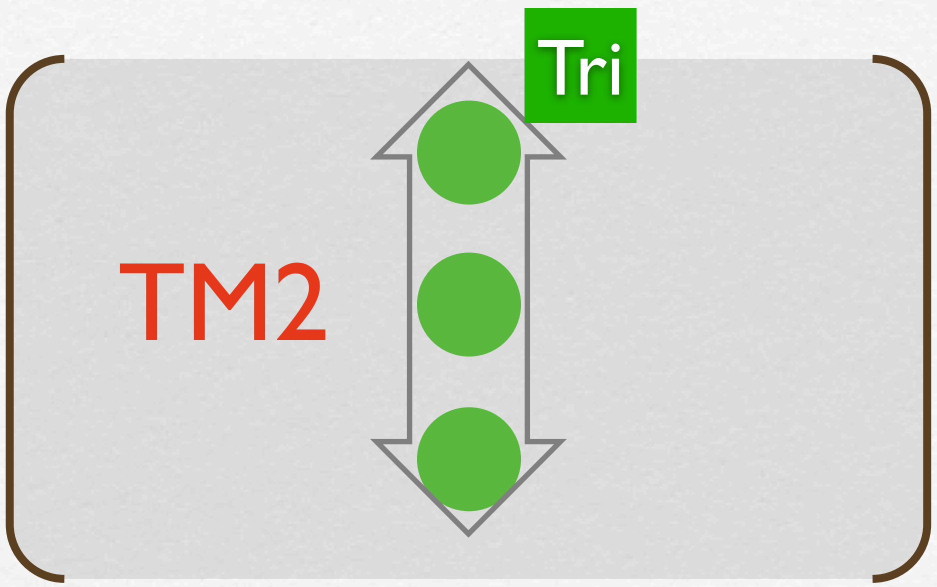
Not best fit

$$\frac{|U_{\tau 1}|}{|U_{\tau 2}|} = \frac{|s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta}|}{|-c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta}|} = \frac{1}{\sqrt{2}} \rightarrow \cos\delta = \frac{t_{23}s^2_{12} + s^2_{13}c^2_{12}/t_{23} - \frac{1}{3}(t_{23} + s^2_{13}/t_{23})}{\sin 2\theta_{12}s_{13}}$$

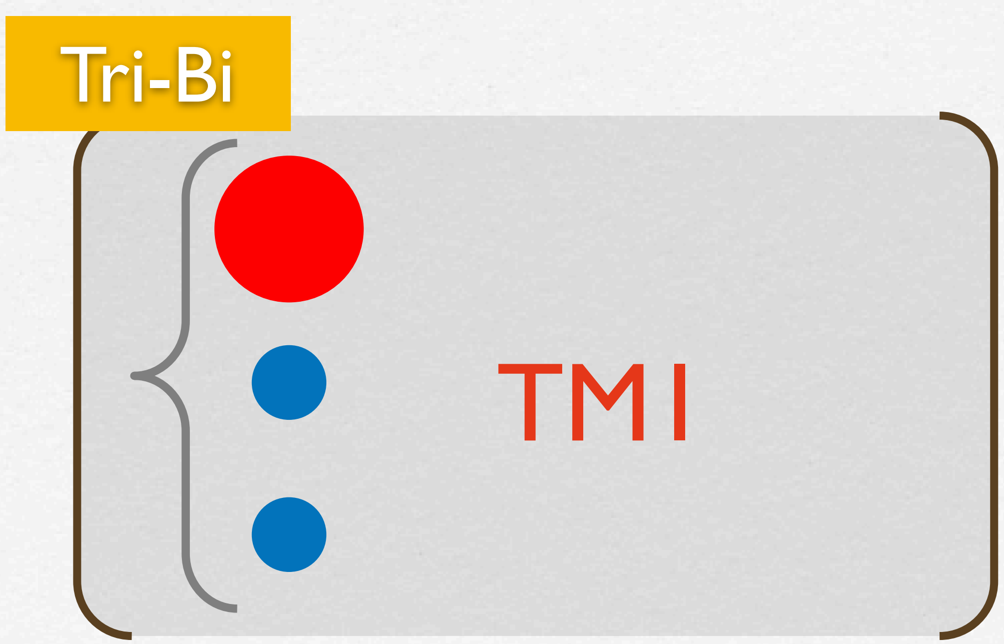
This derivation: P.Ballett, S.F.King, C.Luhn, S.Pascoli and M.A.Schmidt, 1410.7573

Tri-maximal Mixing

C.H.Albright and W.Rodejohann, 0812.0436; C.H.Albright, A.Dueck and W.Rodejohann, 1004.2798



$$U_{\text{TM2}} \approx \begin{pmatrix} - & \frac{1}{\sqrt{3}} & - \\ - & \frac{1}{\sqrt{3}} & - \\ - & -\frac{1}{\sqrt{3}} & - \end{pmatrix}$$



$$U_{\text{TM1}} \approx \begin{pmatrix} \sqrt{\frac{2}{3}} & - & - \\ -\frac{1}{\sqrt{6}} & - & - \\ \frac{1}{\sqrt{6}} & - & - \end{pmatrix}$$

Tri-maximal Mixing

C.H.Albright and W.Rodejohann, 0812.0436; C.H.Albright, A.Dueck and W.Rodejohann, 1004.2798

Not best fit

$$U_{\rm TM2} \approx \begin{pmatrix} - & \frac{1}{\sqrt{3}} & - \\ - & \frac{1}{\sqrt{3}} & - \\ - & -\frac{1}{\sqrt{3}} & - \end{pmatrix}$$

$$\rightarrow |U_{e2}| = s_{12}c_{13} = \sqrt{\frac{1}{3}} \rightarrow s_{12}^2 > \frac{1}{3}$$

$$\rightarrow |U_{\mu 2}| = |c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta}| = \sqrt{\frac{1}{3}}$$

$$\rightarrow |U_{\tau 2}| = |-c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta}| = \sqrt{\frac{1}{3}}$$

$$\rightarrow \cos\delta = \frac{2c_{13}\cot 2\theta_{23}\cot 2\theta_{13}}{\sqrt{2-3s_{13}^2}}$$

Tri-maximal Mixing

C.H.Albright and W.Rodejohann, 0812.0436; C.H.Albright, A.Dueck and W.Rodejohann, 1004.2798

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$$\rightarrow |U_{\tau 2}| = |-c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta}| = \sqrt{\frac{1}{3}}$$

$$\rightarrow \cos\delta = \frac{2c_{13}\cot 2\theta_{23}\cot 2\theta_{13}}{\sqrt{2-3s_{13}^2}}$$

Best fit

$$U_{\mathrm{TM1}} \approx \begin{pmatrix} \sqrt{\frac{2}{3}} & - & - \\ -\frac{1}{\sqrt{6}} & - & - \\ \frac{1}{\sqrt{6}} & - & - \end{pmatrix}$$

$$\rightarrow |U_{e1}| = c_{12}c_{13} = \sqrt{\frac{2}{3}} \rightarrow s_{12}^2 < \frac{1}{3}$$

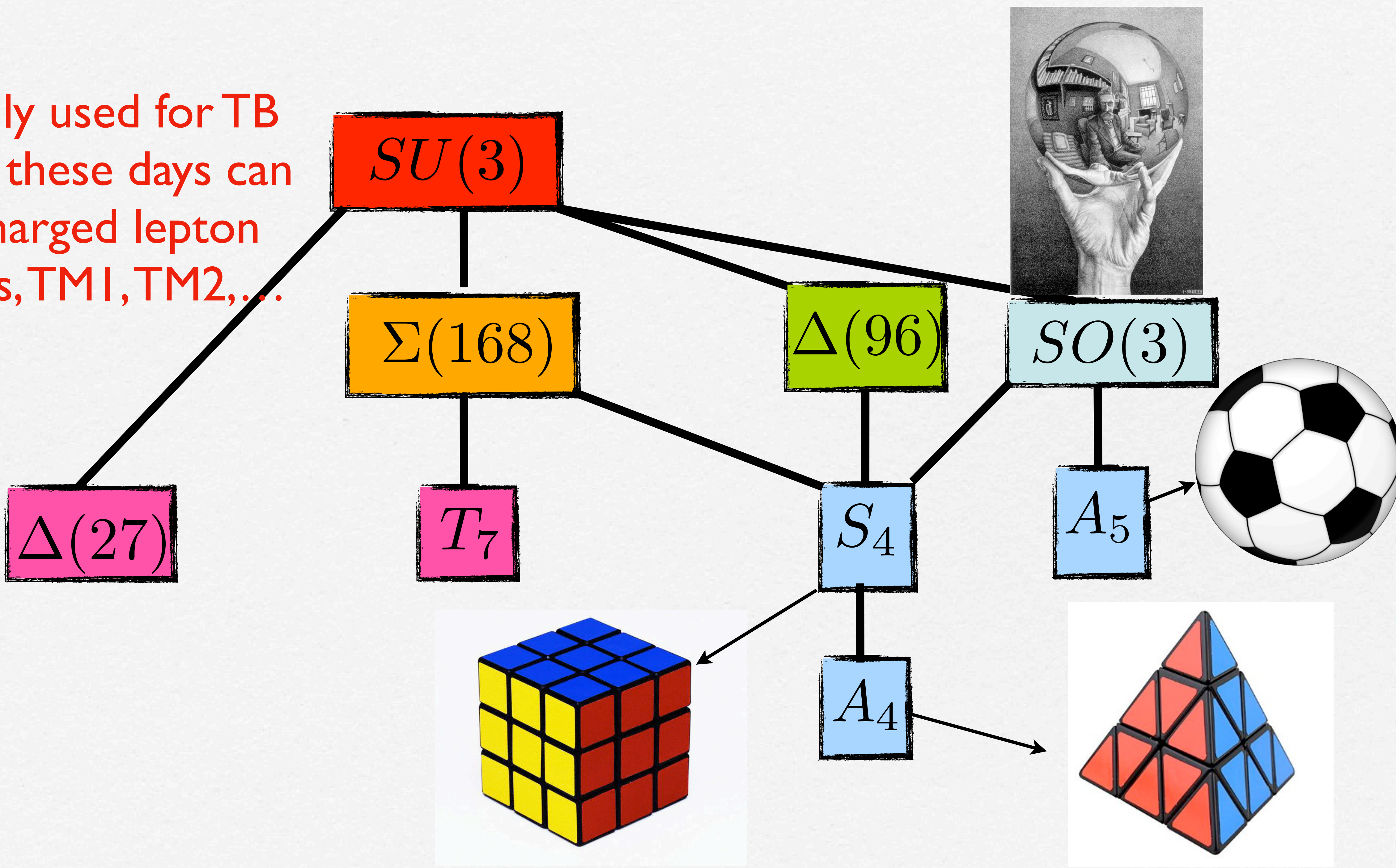
$$\rightarrow |U_{\mu 1}| = |-s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta}| = \sqrt{\frac{1}{6}}$$

$$\rightarrow |U_{\tau 1}| = |s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta}| = \sqrt{\frac{1}{6}}$$

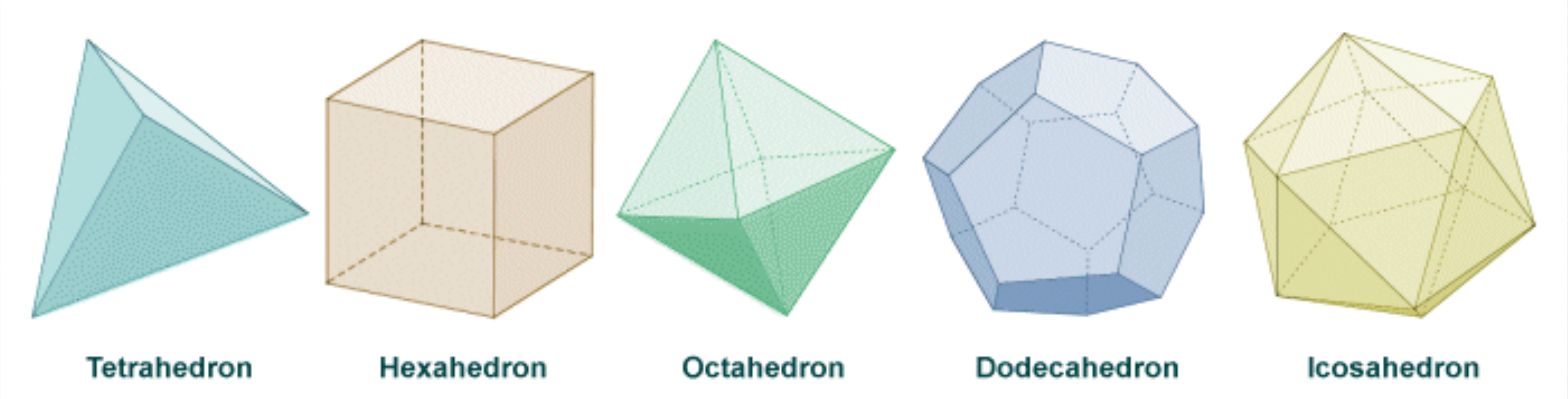
$$\rightarrow \cos\delta = -\frac{\cot 2\theta_{23}(1-5s_{13}^2)}{2\sqrt{2}s_{13}\sqrt{1-3s_{13}^2}}$$

Family Symmetry

Traditionally used for TB mixing, but these days can explain charged lepton corrections, TM1, TM2,….



Platonic Solids



solid	faces	vertices	Plato	Group
tetrahedron	4	4	fire	A_4
octahedron	8	6	air	S_4
icosahedron	20	12	water	A_5
hexahedron	6	8	earth	S_4
dodecahedron	12	20	?	A_5

Plato's fire
A4 can explain
Tri-bimaximal
Mixing

E.Ma and G.Rajasekaran, hep-ph/0106291;
K.S.Babu, E.Ma, J.W.F.Valle, hep-ph/0206292;
G.Altarelli and F.Feruglio, hep-ph/0504165,hep-ph/0512103

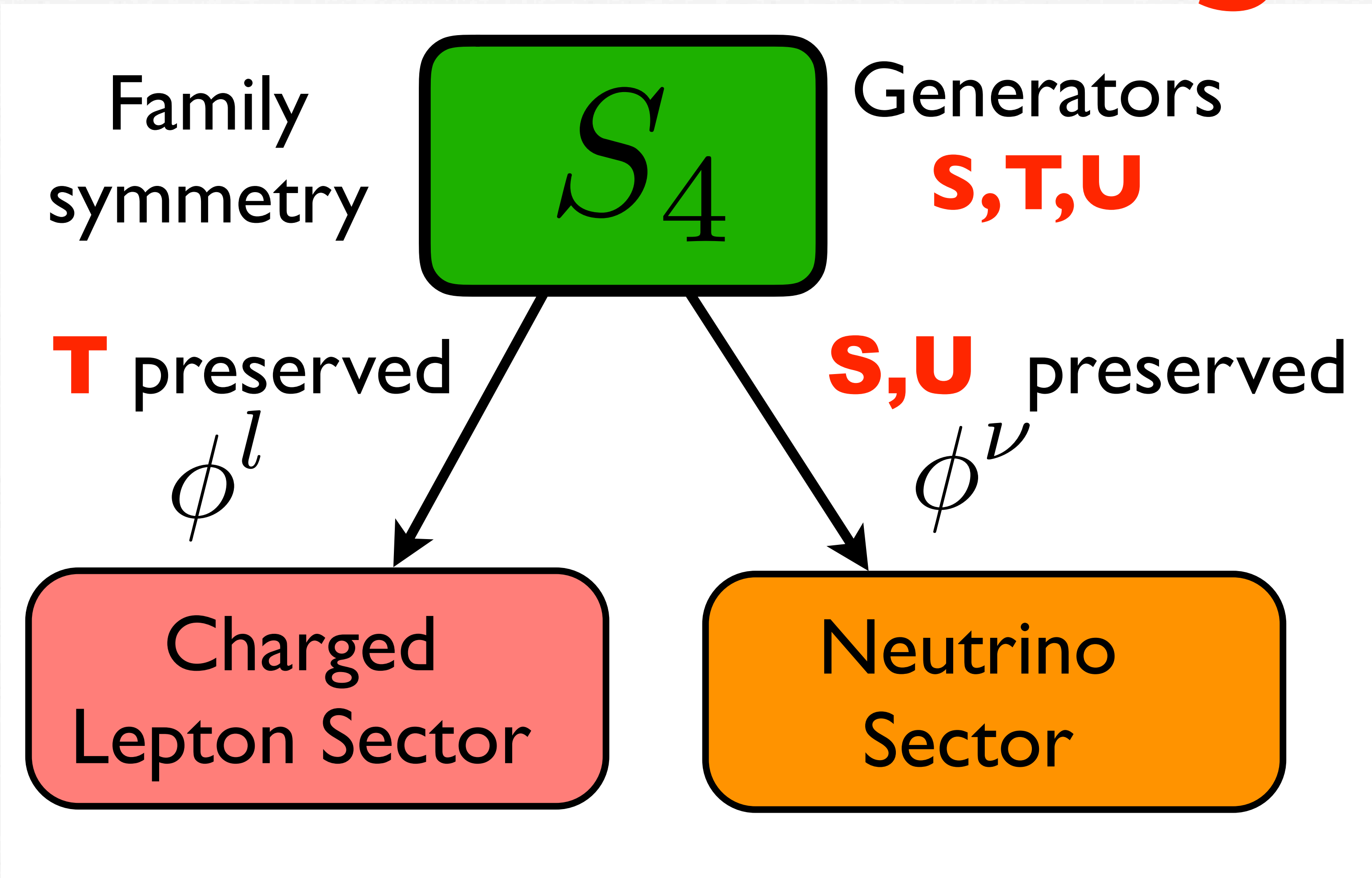
S.F.K., C.Luhn, 1301.1340

A_4 and S_4 Group Theory

S_4	A_4	S	T	U
$\mathbf{1}, \mathbf{1}'$	$\mathbf{1}$	1	1	± 1
$\mathbf{2}$	$\begin{pmatrix} \mathbf{1}'' \\ \mathbf{1}' \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\mathbf{3}, \mathbf{3}'$	$\mathbf{3}$	$\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$	$\mp \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

Diagonalised by TB matrix

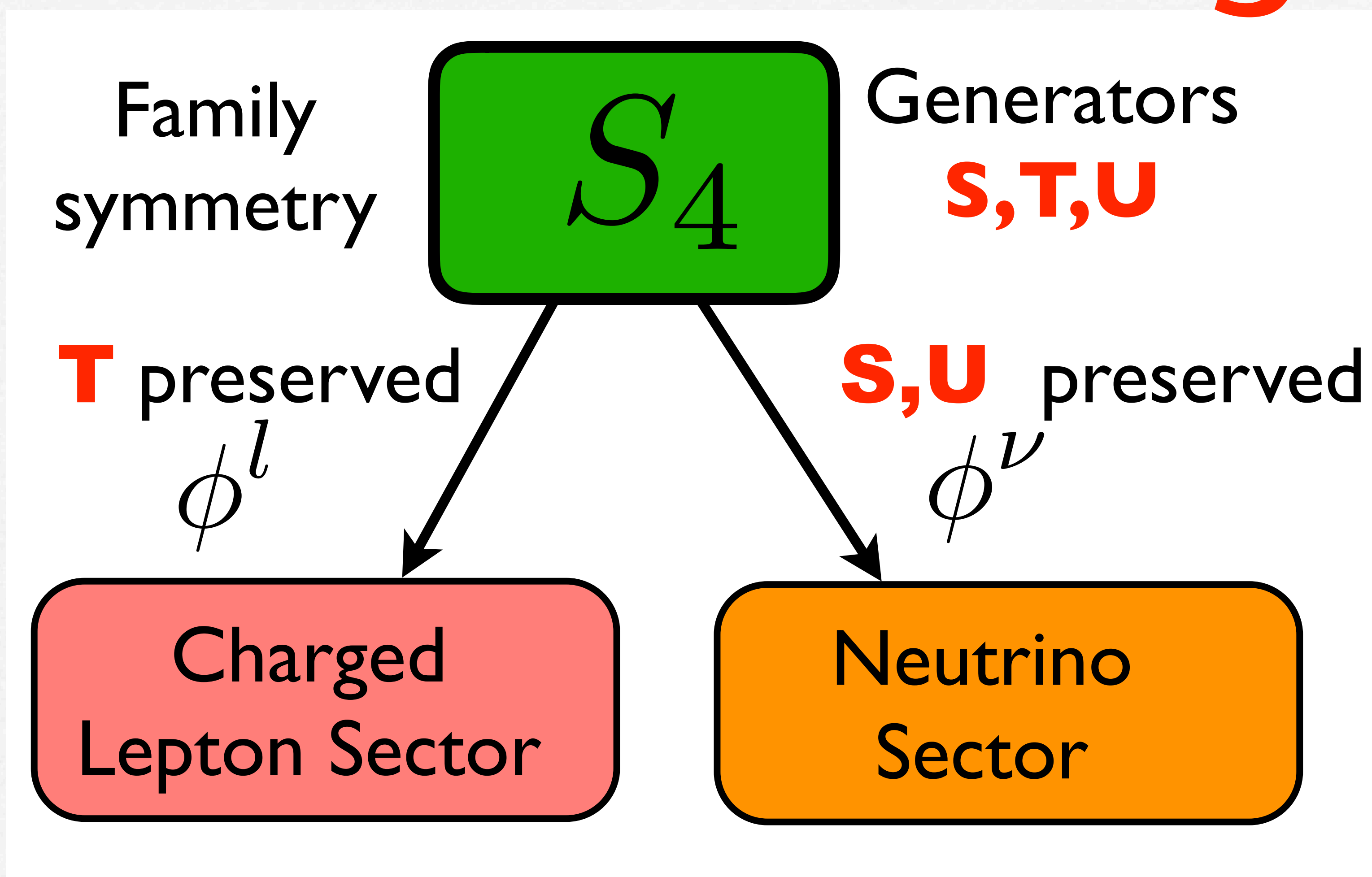
Tri-bimaximal mixing from S_4



S.F.K., C.Luhn,
1301.1340

Tri-bimaximal mixing from S_4

S.F.K., C.Luhn, 1301.1340

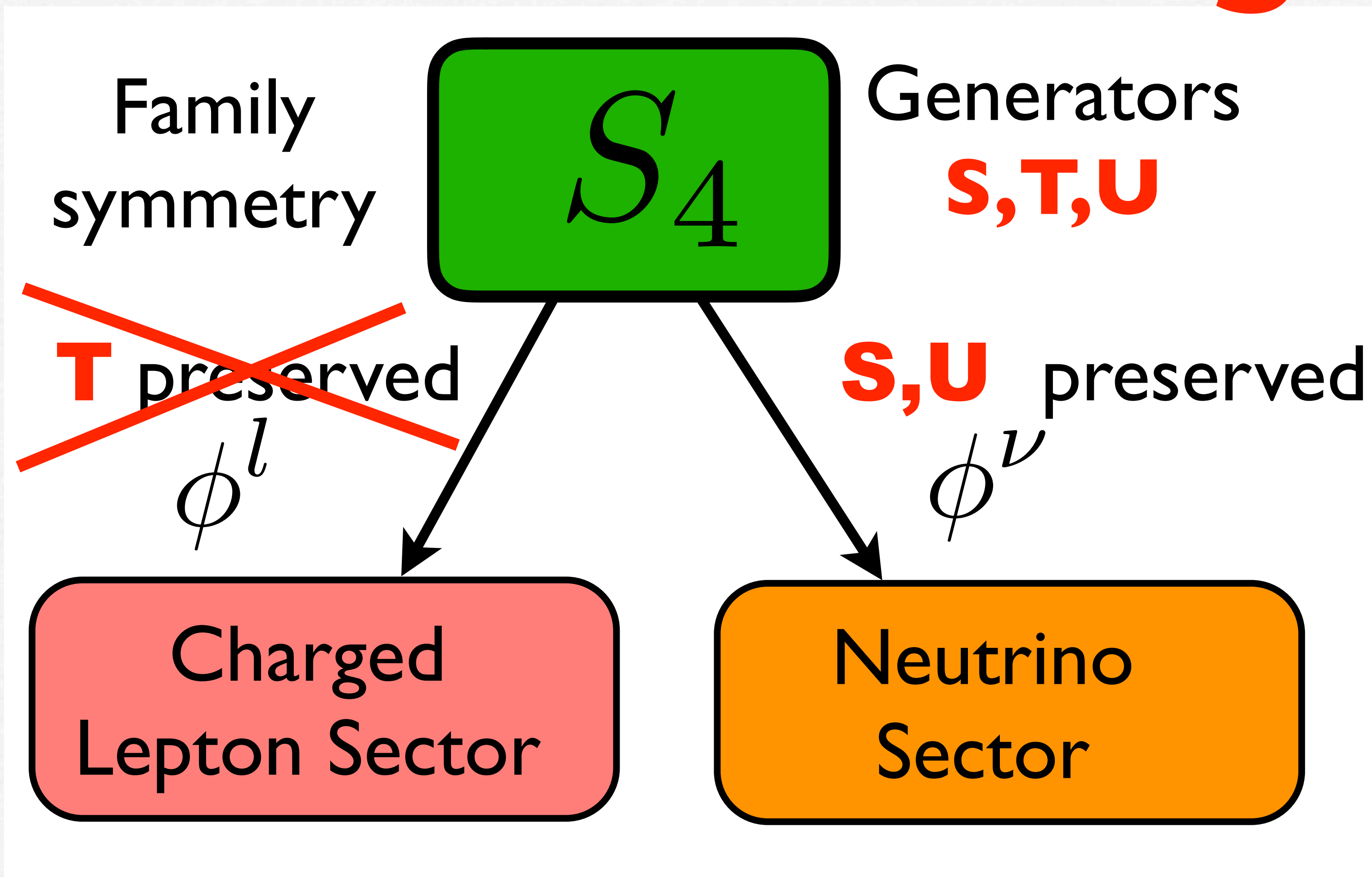


$$\begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

TB mixing excluded so need to break S,T,U

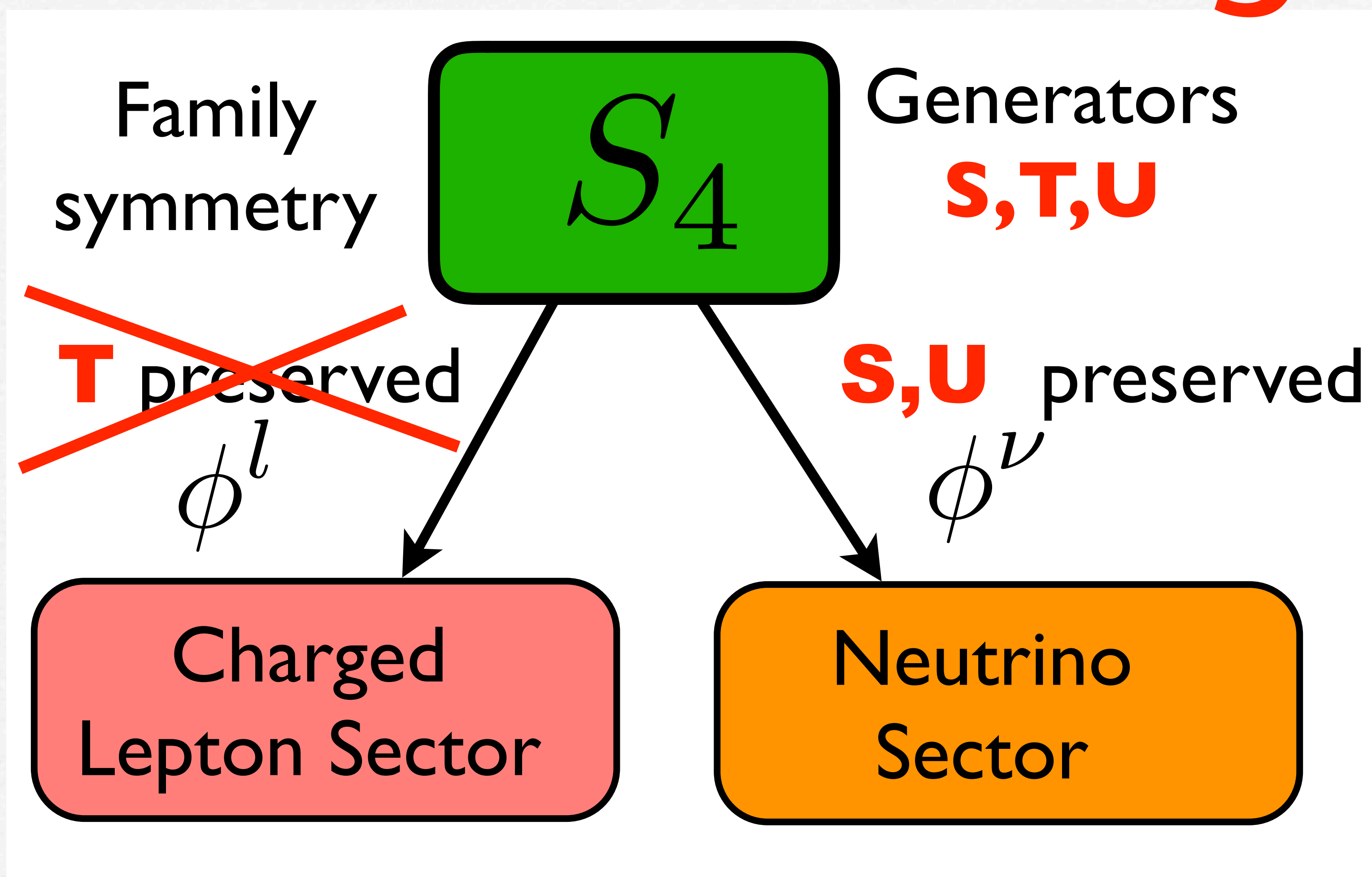
Tri-bimaximal mixing from S_4

break T



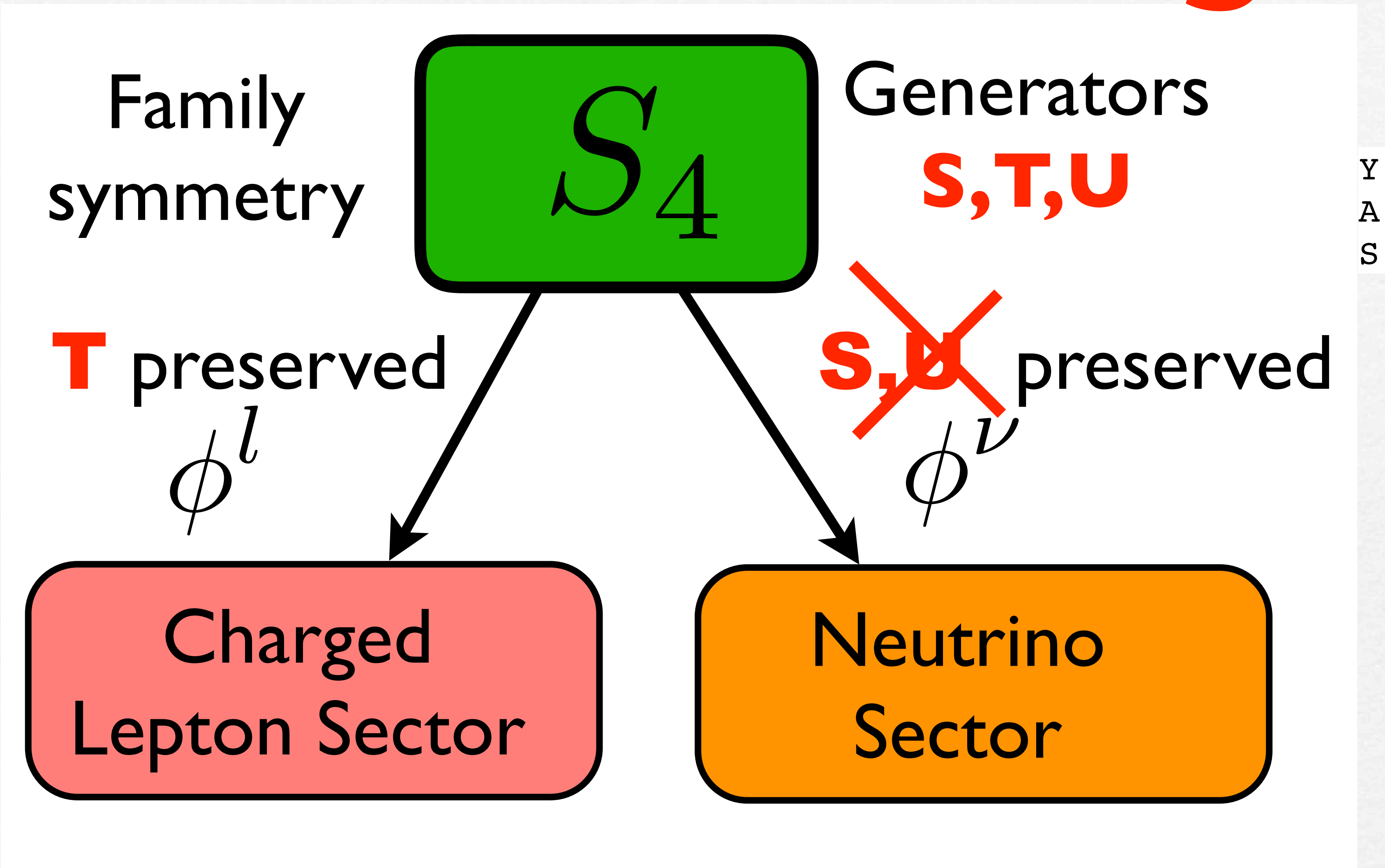
Tri-bimaximal mixing from S_4

break T



$$U_{\mathrm{PMNS}} = \underbrace{\begin{pmatrix} c^e_{12} & s^e_{12}e^{-i\delta^e_{12}} & 0 \\ -s^e_{12}e^{i\delta^e_{12}} & c^e_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Charged lepton rotation}} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Tri-bimaximal mixing from S_4

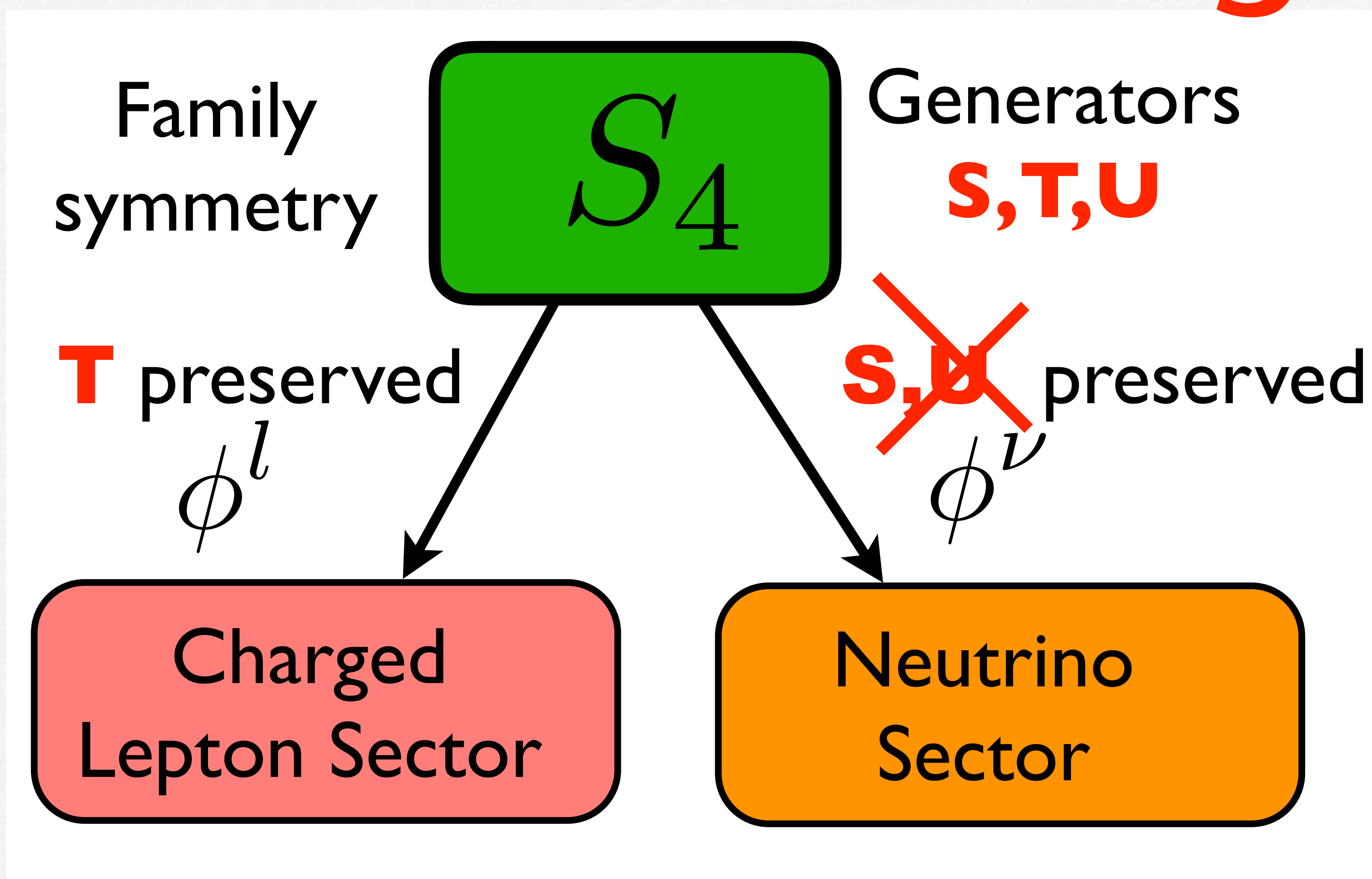


S.F.K., C.Luhn, 1301.1340

Y.Shimizu, M.Tanimoto, A.Watanabe, 1105.2929; S.F.K.,C.Luhn,1107.5332

break U

Tri-bimaximal mixing from S_4



S.F.K., C.Luhn, 1301.1340

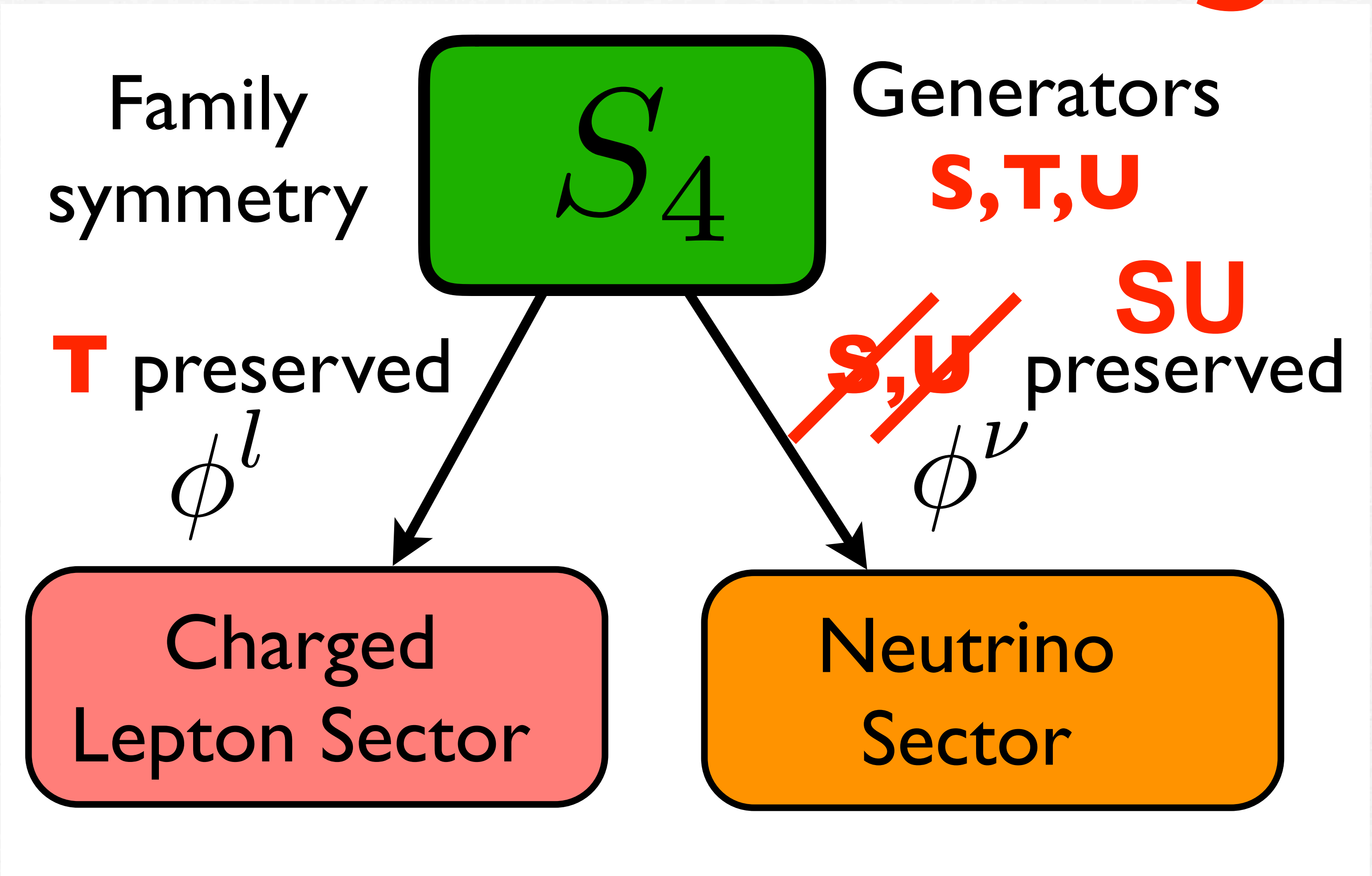
Y.Shimizu, M.Tanimoto, A.Watanabe, 1105.2929; S.F.K.,C.Luhn,1107.5332

break U

$$U_{\mathrm{TM2}} \approx \begin{pmatrix} - & \frac{1}{\sqrt{3}} & - \\ - & \frac{1}{\sqrt{3}} & - \\ - & -\frac{1}{\sqrt{3}} & - \end{pmatrix}$$

TM2 as A4 with just S and T

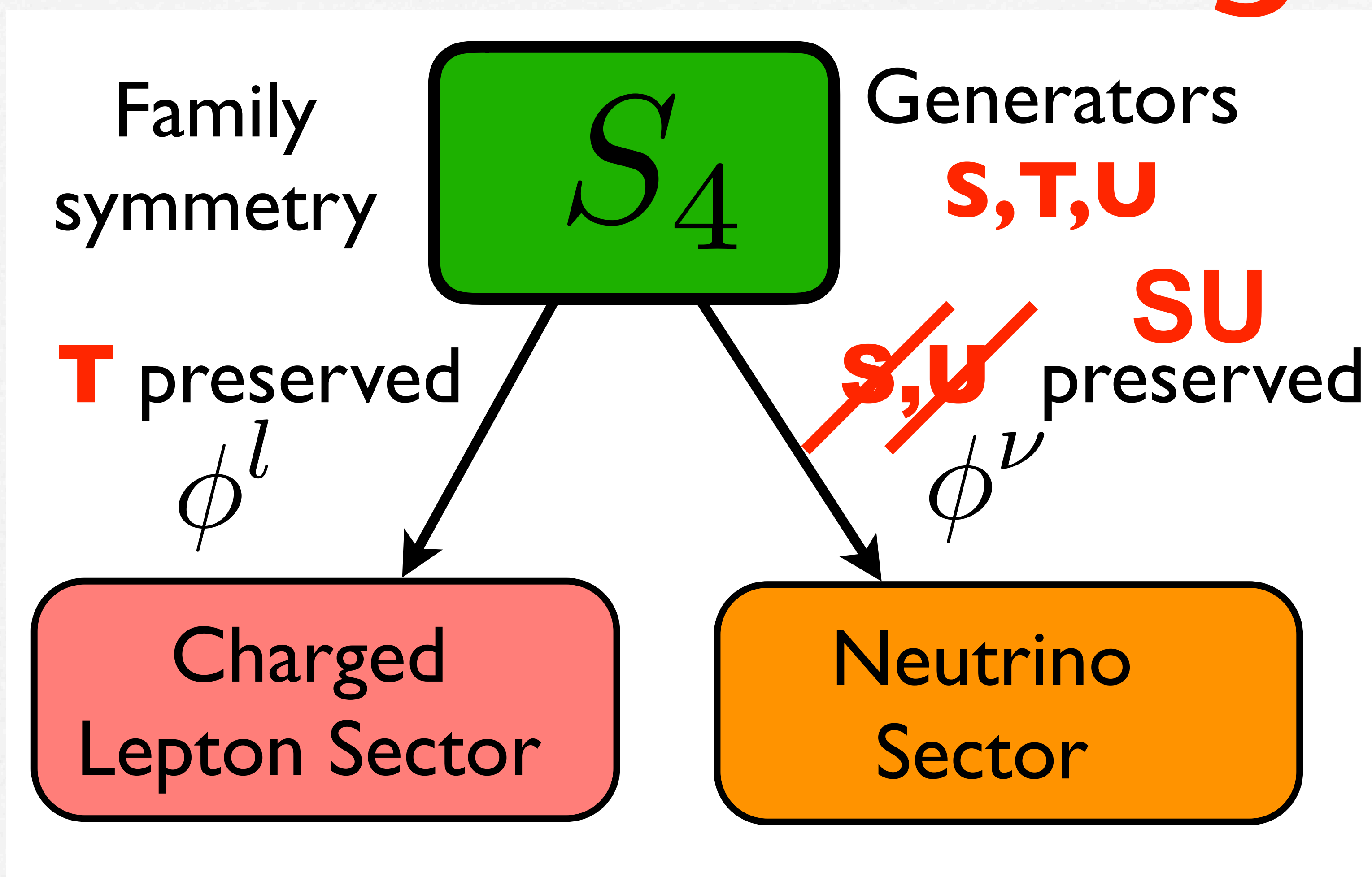
Tri-bimaximal mixing from S_4



S.F.K., C.Luhn, 1301.1340

break S,U separately preserve SU

Tri-bimaximal mixing from S_4



S.F.K., C.Luhn, 1301.1340

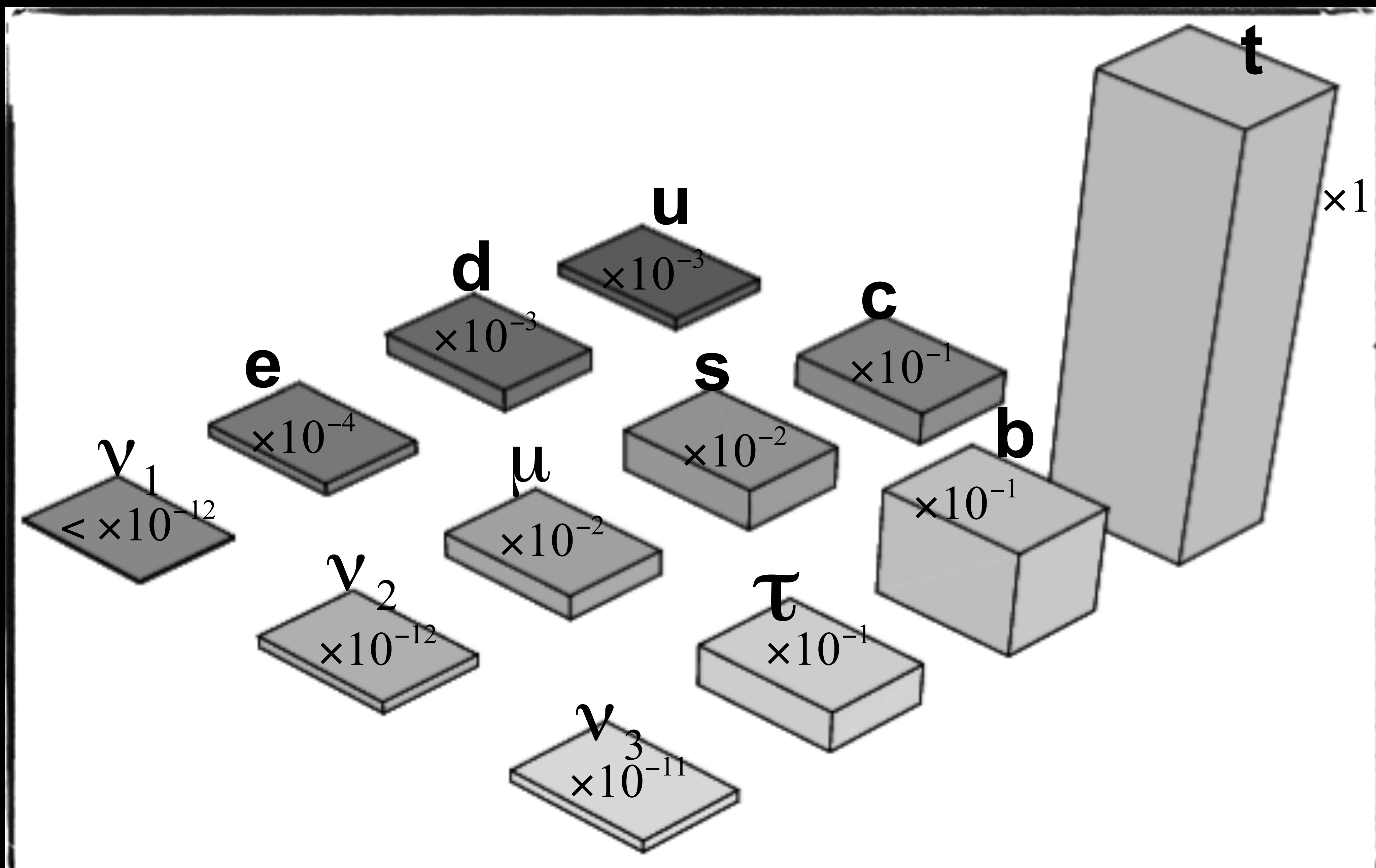
break S,U separately preserve SU

$$U_{\rm TM1} \approx \begin{pmatrix} \sqrt{\frac{2}{3}} & - & - \\ -\frac{1}{\sqrt{6}} & - & - \\ \frac{1}{\sqrt{6}} & - & - \end{pmatrix}$$

TM1 with SU and T

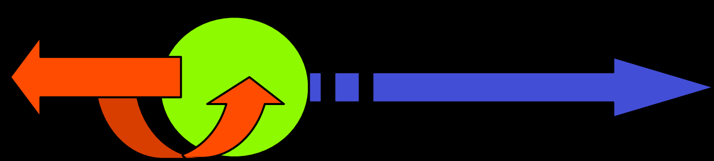
D.Hernandez and A.Y.Smirnov 1204.0445,1212.2149,1304.7738; C.Luhn, 1306.2358 S.F.K.,C.Luhn,1607.05276

Why nu mass small?

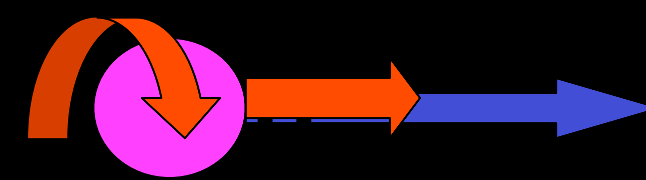


Dirac or Majorana?

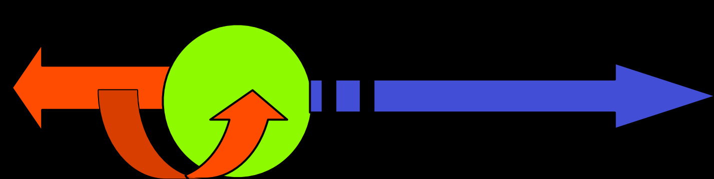
ν_L **Dirac** ν_R



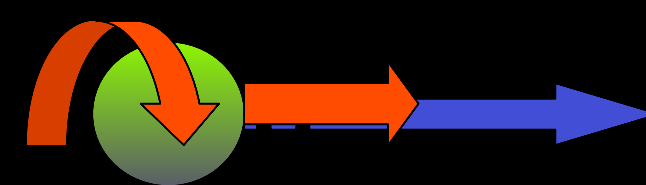
$$m_{\mathrm{D}}\bar{\nu}_L\nu_R$$



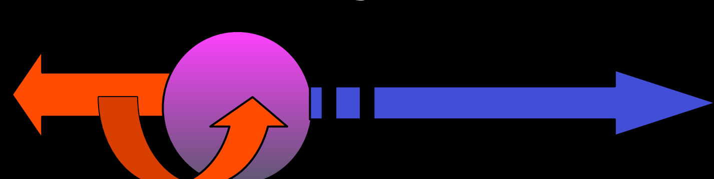
ν_L **Majorana** ν_L^c



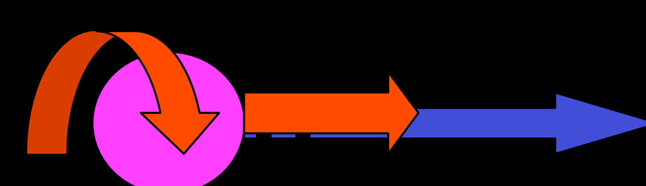
$$m_{\mathrm{L}}\bar{\nu}_L\nu_L^c$$



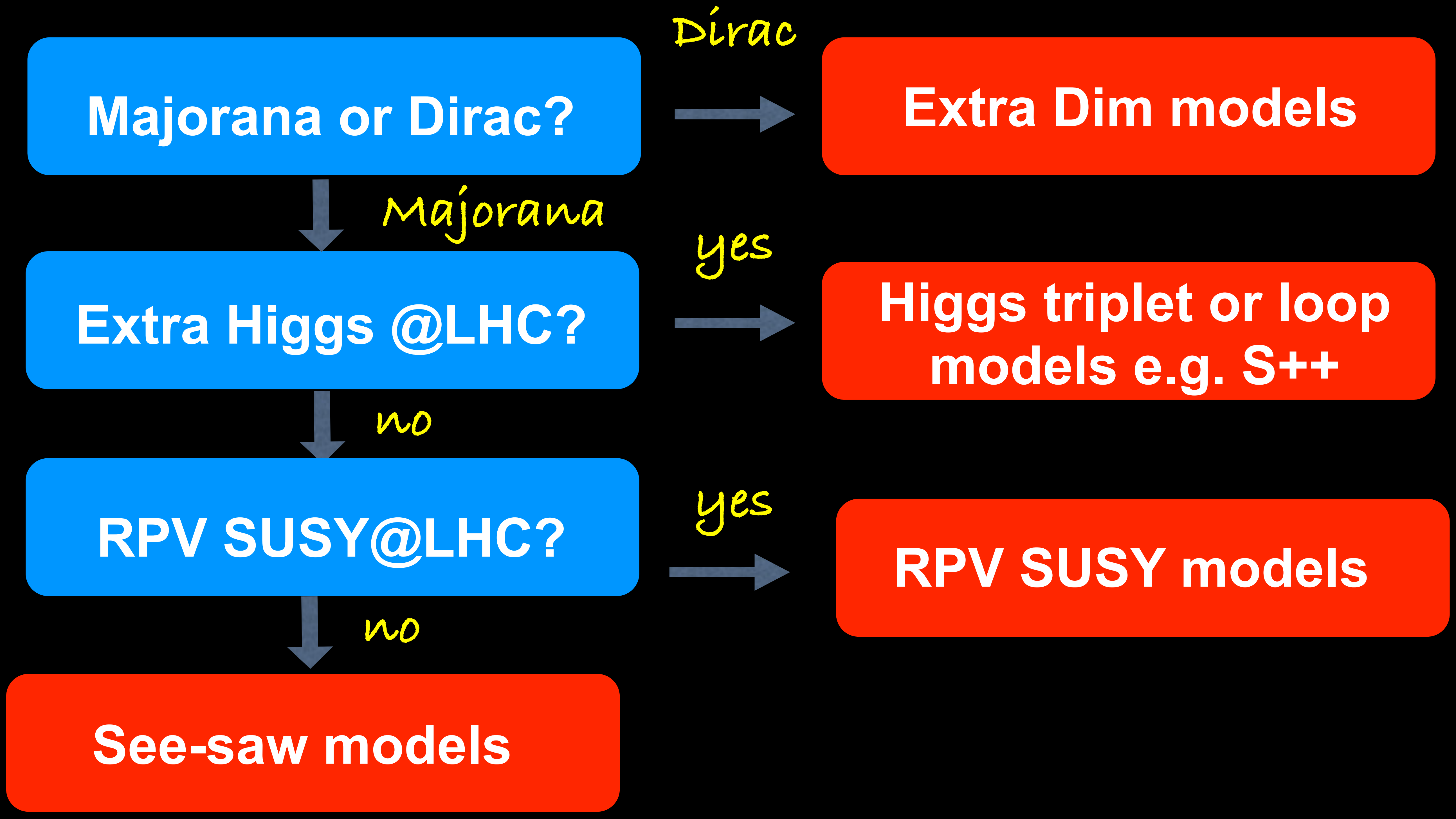
ν_R^c **Majorana** ν_R



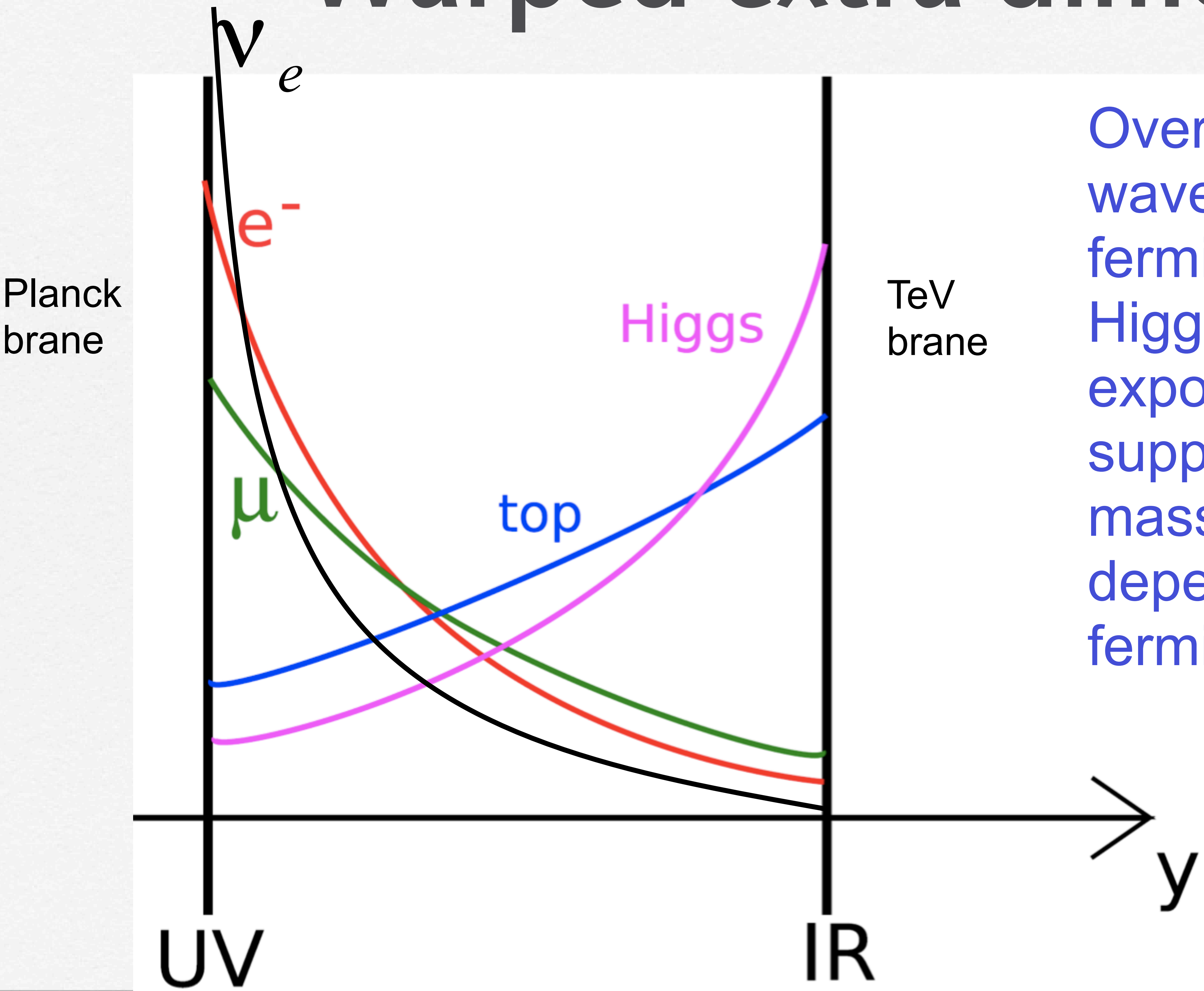
$$M_{\mathrm{R}}\bar{\nu}_R^c\nu_R$$



Roadmap of neutrino mass

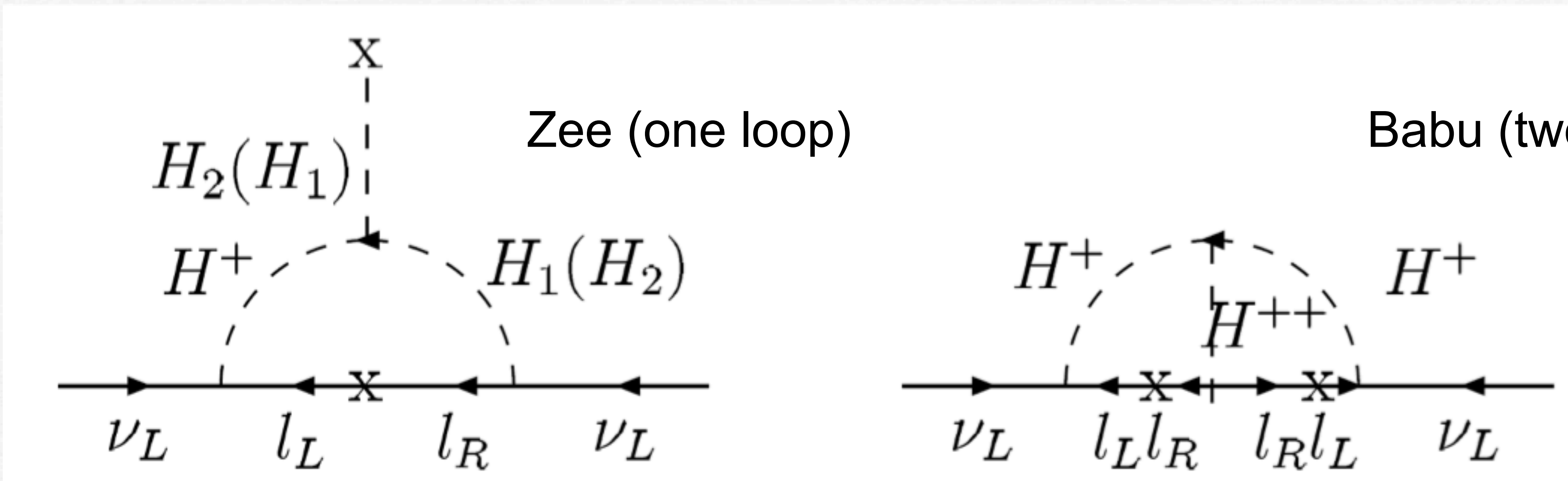


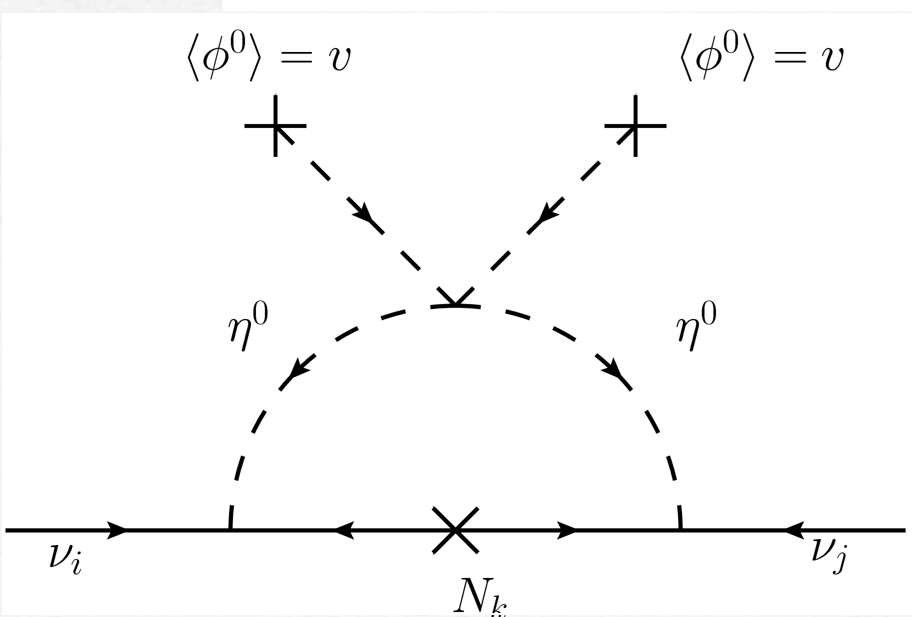
Warped extra dimensions

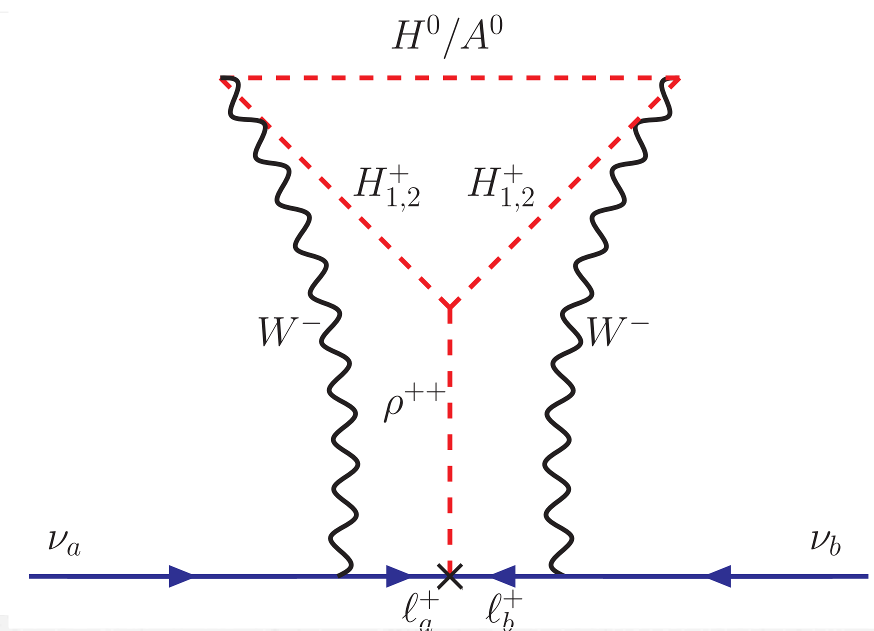


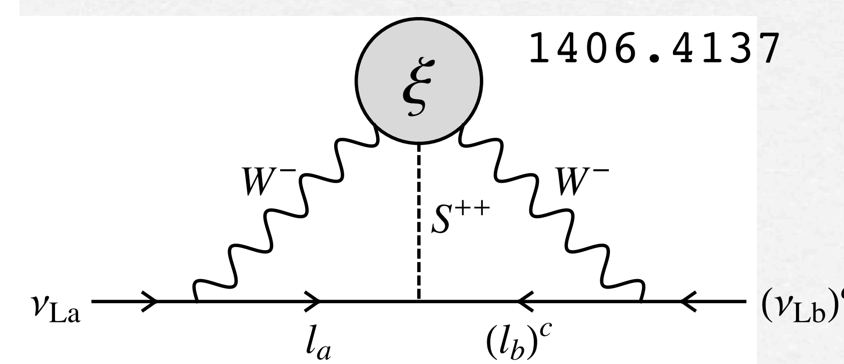
Overlap wavefunction of fermions with Higgs gives exponentially suppressed Dirac masses, depending on the fermion profiles

Loop Models of Neutrino Mass









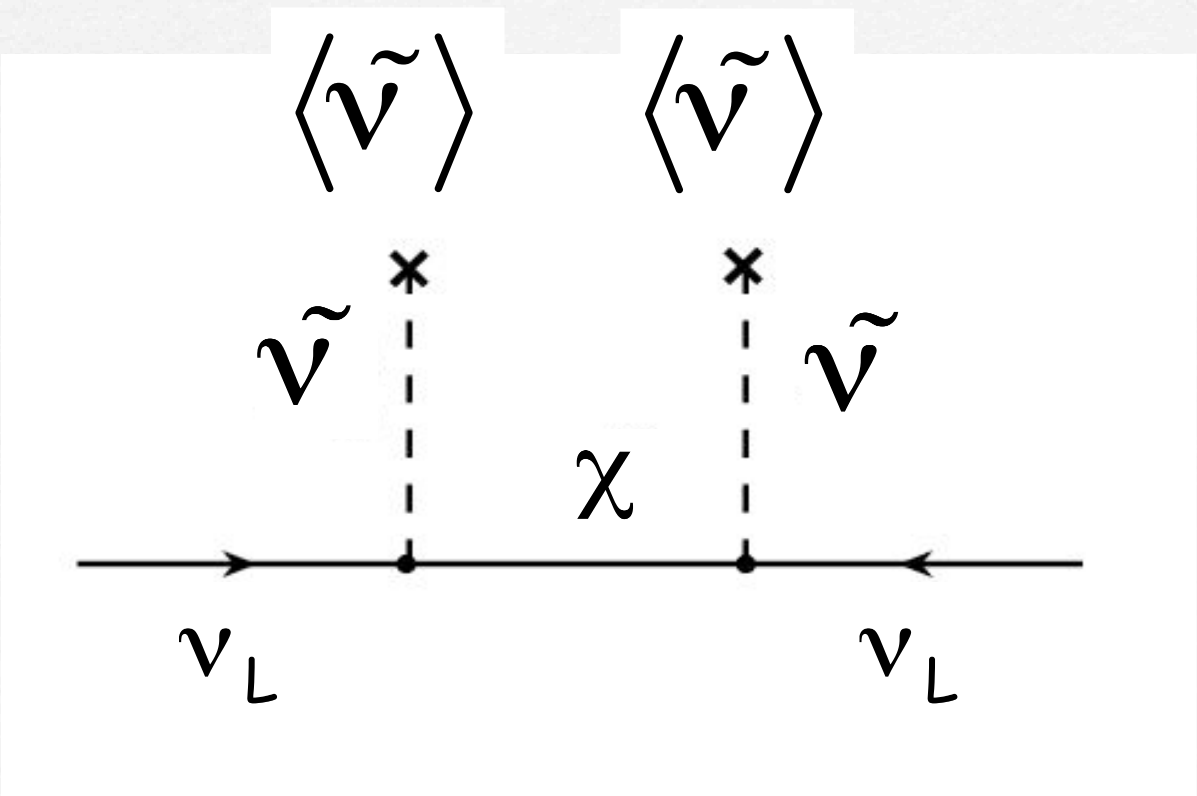
Scotogenic model

Cocktail model

Effective theory

R-Parity Violating SUSY

- Majorana masses can be generated via RPV SUSY
- Scalar partners of lepton doublets (slepton doublets) have same quantum numbers as Higgs doublets
- If R-parity is violated then sneutrinos may get (small) VEVs inducing a mixing between neutrinos and neutralinos χ

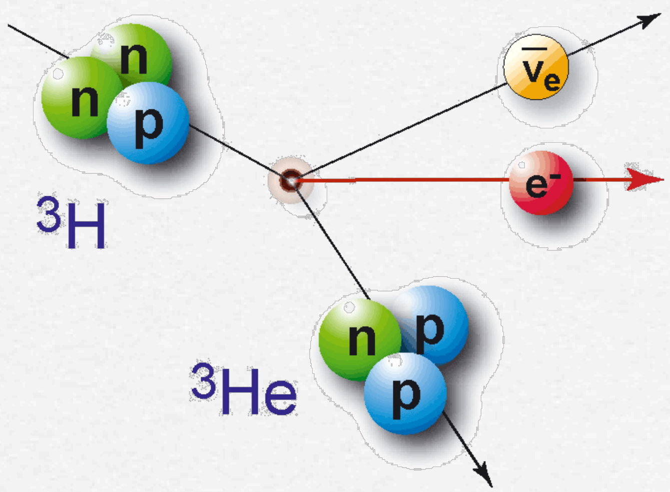


$$m_{LL}^{\nu} \approx \frac{\langle\tilde{\nu}\rangle^2}{M_\chi} \approx \frac{MeV^2}{TeV} \approx eV$$

Experimental determination of neutrino mass

Majorana only
(no signal if Dirac)

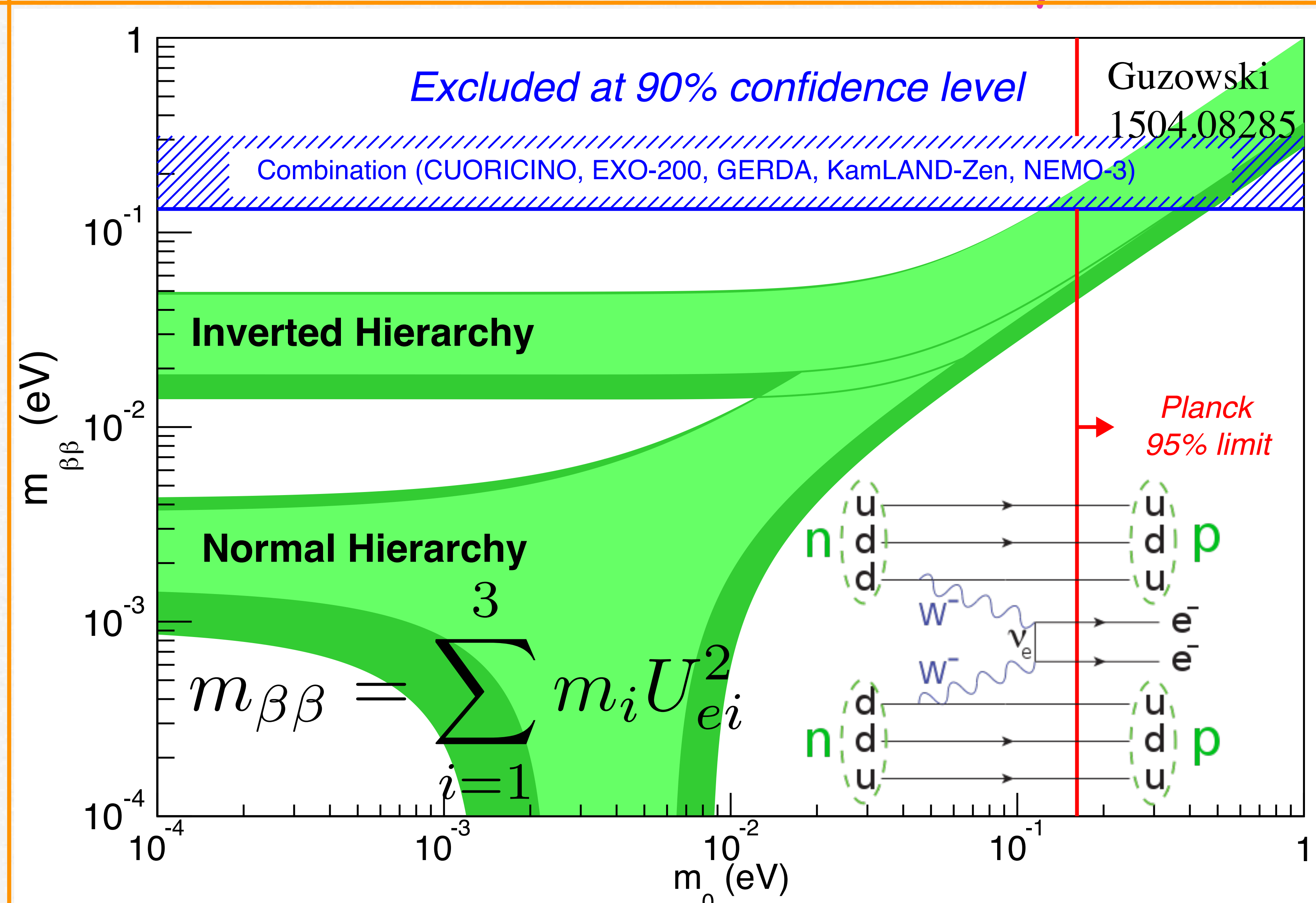
Tritium beta decay



$$m_{\nu_e}^2 = \sum_{i=1}^{3} |U_{ei}|^2 m_i^2$$

Present Mainz	< 2.2 eV
KATRIN	~0.35eV

Neutrinoless double beta decay



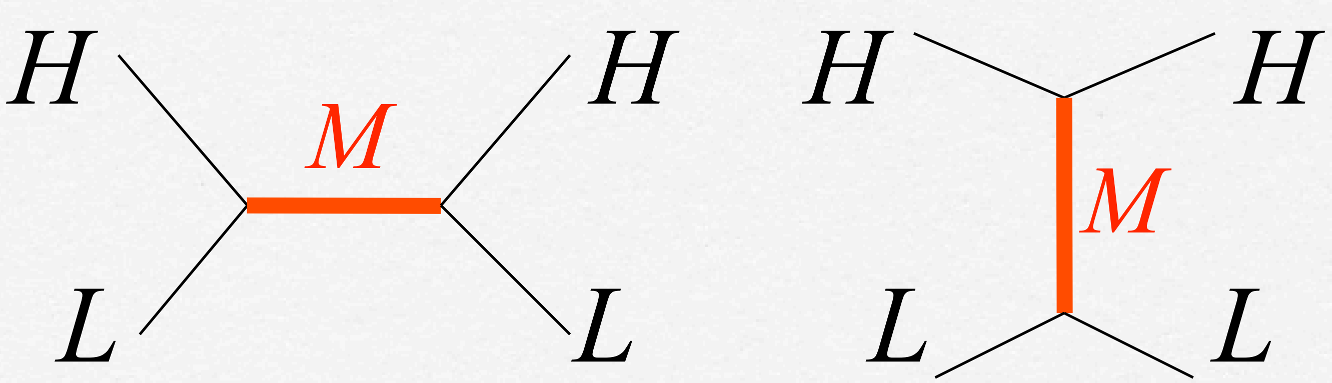
Is Majorana mass renormalisable?

Renormalisable $\Delta L = 2$ operator $\lambda_\nu LL\Delta$ where Δ is light Higgs triplet with VEV < 8GeV from ρ parameter

Non-renormalisable $\Delta L = 2$ operator $$\frac{\lambda_\nu}{M} LLHH = \frac{\lambda_\nu}{M} \langle H^0 \rangle^2 \overline{\nu}_{eL} \nu^c_{eL}$$ Weinberg

This is nice because it gives naturally small Majorana neutrino masses $m_{LL} \sim \langle H^0 \rangle^2 / M$ where M is some high energy scale

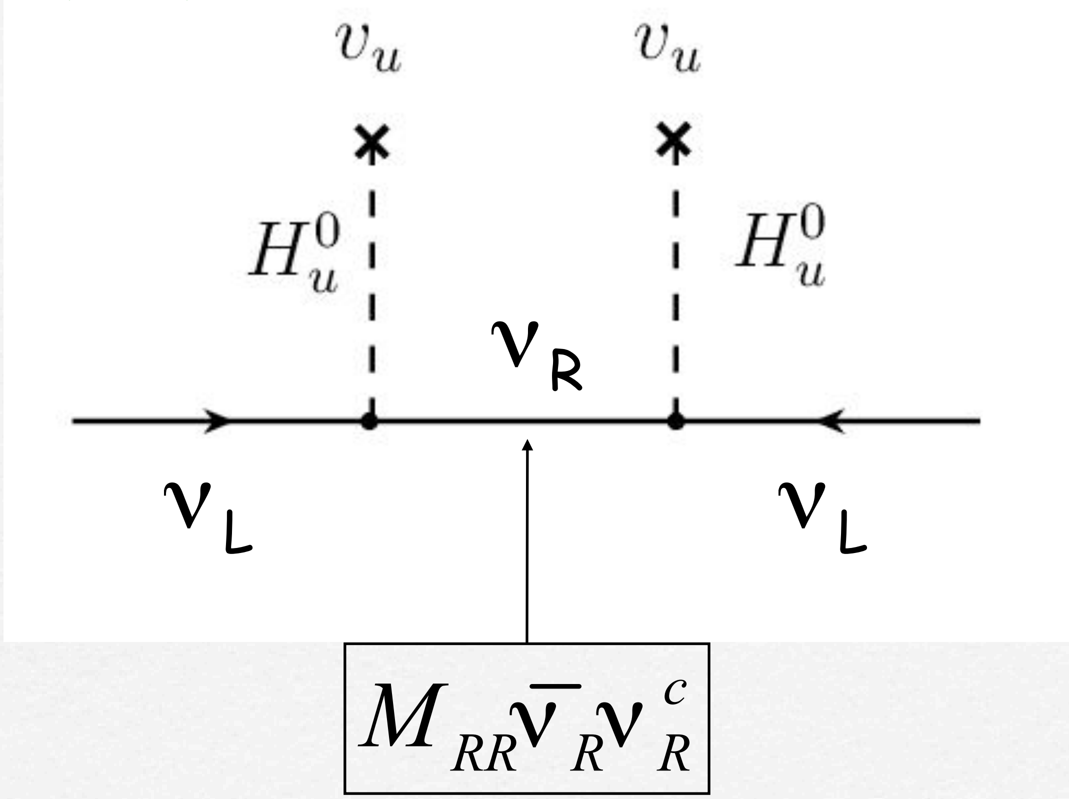
The high mass scale can be associated with some heavy particle of mass M being exchanged (can be singlet or triplet)



See-saw mechanisms

Type Ia see-saw mechanism

P. Minkowski (1977), Gell-Mann, Glashow, Mohapatra, Ramond, Senjanovic, Slanski, Yanagida (1979/1980), Schechter and Valle (1980)...



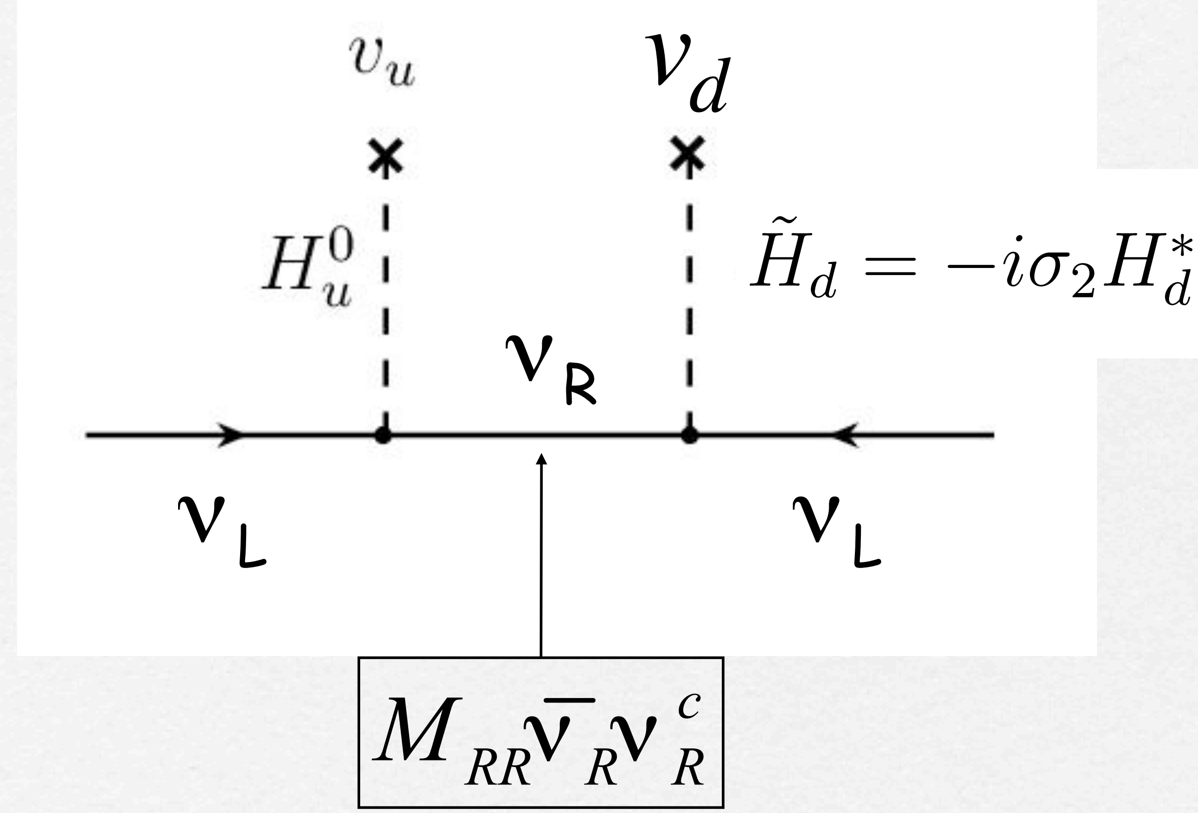
$$m_{LL}^{I} \approx -m_{LR} M_{RR}^{-1} m_{LR}^{T}$$

Type Ia

Type Ib see-saw mechanism

Hernandez-Garcia and SFK 1903.01474

More details - see backup

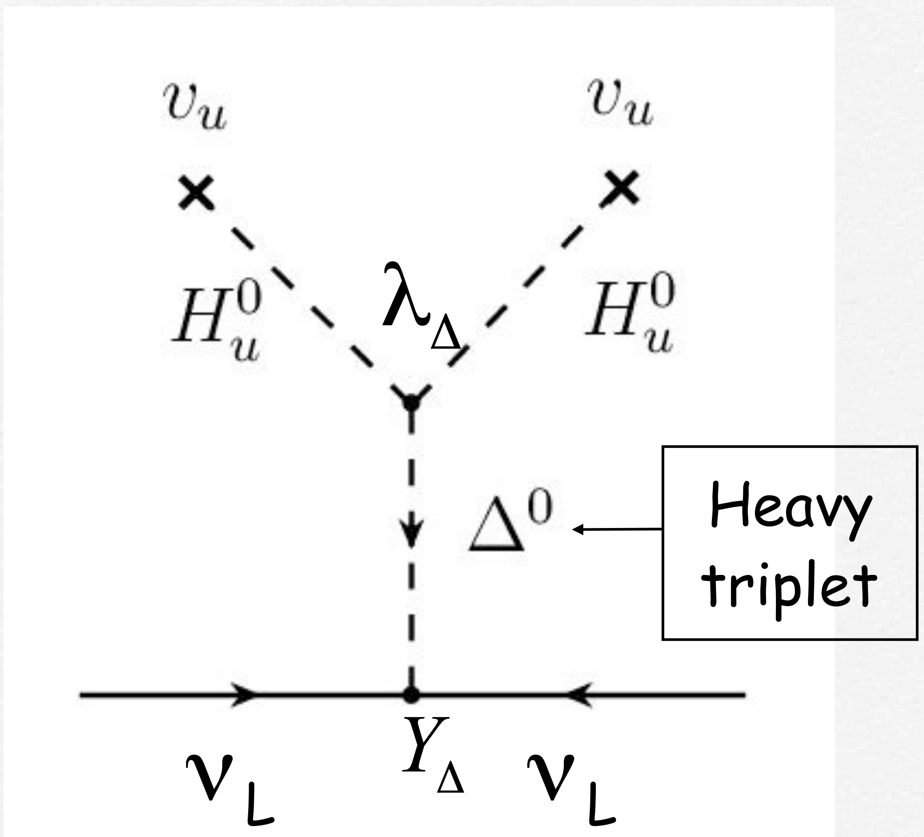


$$m_{LL}^{Ib} = -m_{LR1} M_{RR}^{-1} m_{LR2}^{T}$$

Type Ib

Type II see-saw mechanism (SUSY)

Lazarides, Magg, Mohapatra, Senjanovic,
Shafi, Wetterich, Schechter and Valle...



$$m_{LL}^{II} \approx \lambda_\Delta Y_\Delta \frac{v_u^2}{M_\Delta}$$

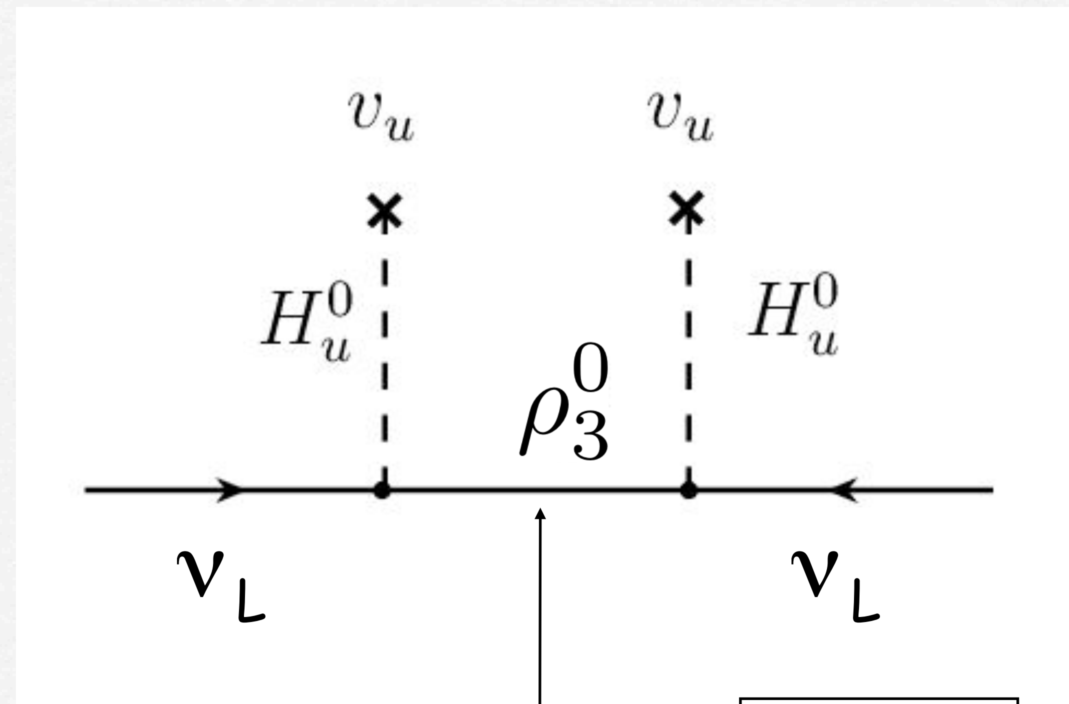
Type II

Type III see-saw mechanism

Foot, Lew, He, Joshi; Ma...

Supersymmetric adjoint SU(5)

Perez et al; Cooper, SFK, Luhn,...



$SU(2)_L$ fermion triplet

$M_\rho \rho\rho$

$$m_{LL}^{III} \approx -m_{LR} M_\rho^{-1} m_{LR}^T$$

Type III

See-saw w/extra singlets S

Inverse see-saw

Wyler, Wolferstein; Mohapatra, Valle

$$\begin{pmatrix} 0 & M_D & 0 \\ M_D^T & 0 & M \\ 0 & M^T & \mu \end{pmatrix}$$

M ≈ TeV➔LHC

$$M_\nu = M_D {M^T}^{-1} \mu M^{-1} M_D^T$$

Minimal example - see backup

Linear see-saw

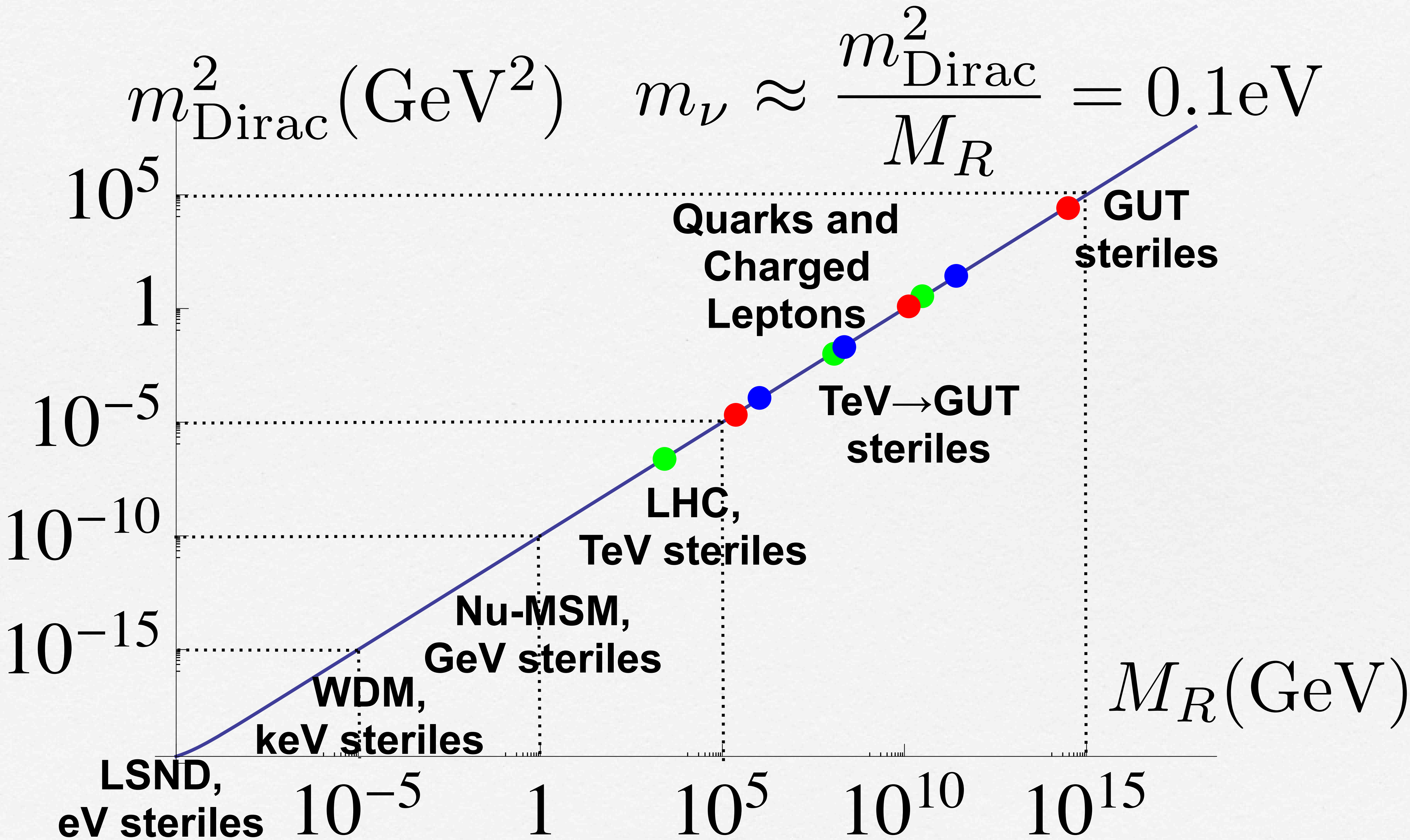
$$\begin{pmatrix} 0 & M_D & M_L \\ M_D^T & 0 & M \\ M_L^T & M^T & 0 \end{pmatrix}$$

Malinsky, Romao, Valle

$$M_\nu = M_D (M_L M^{-1})^T + (M_L M^{-1}) {M_D}^T$$

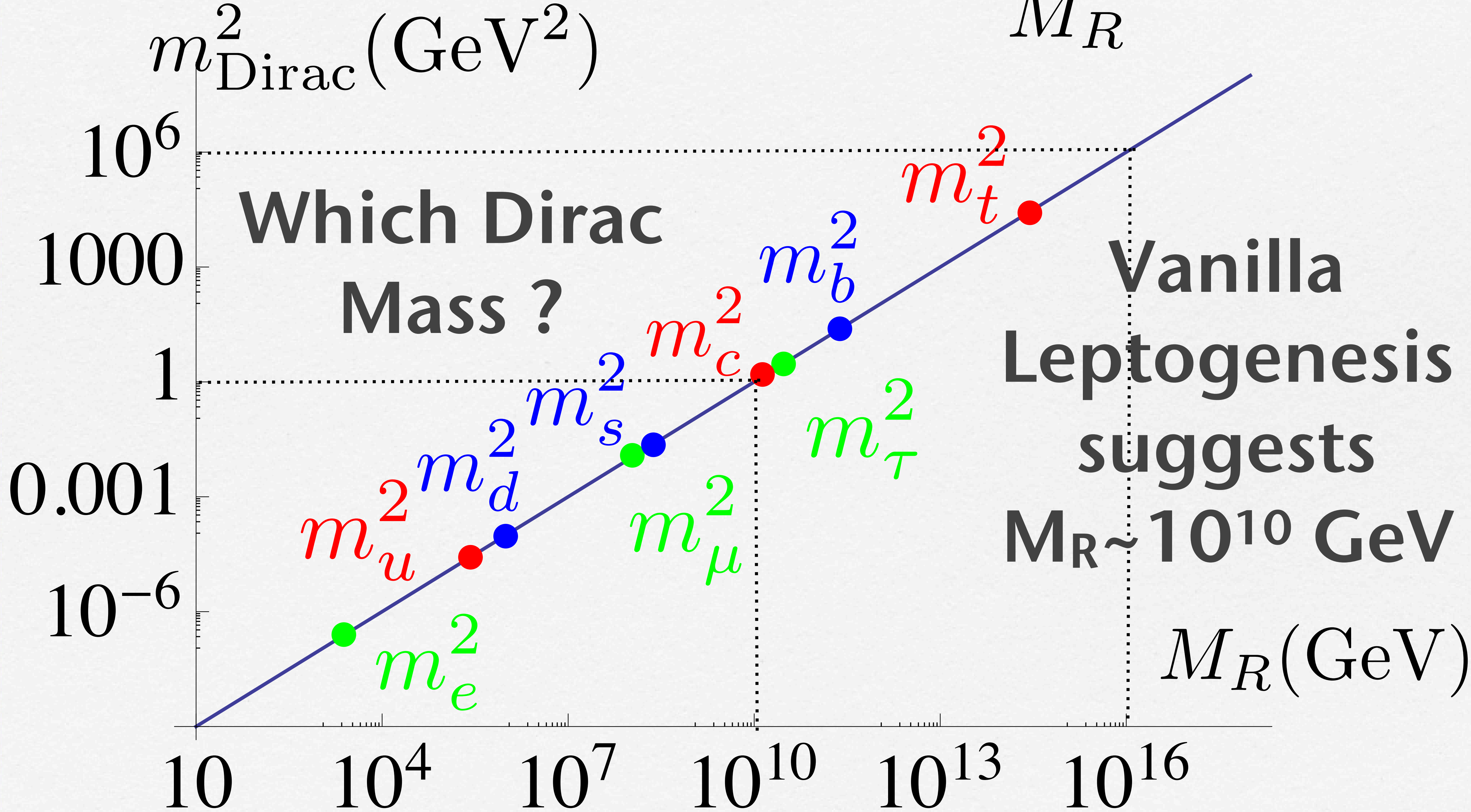
LFV predictions

RHN masses in Type Ia Seesaw

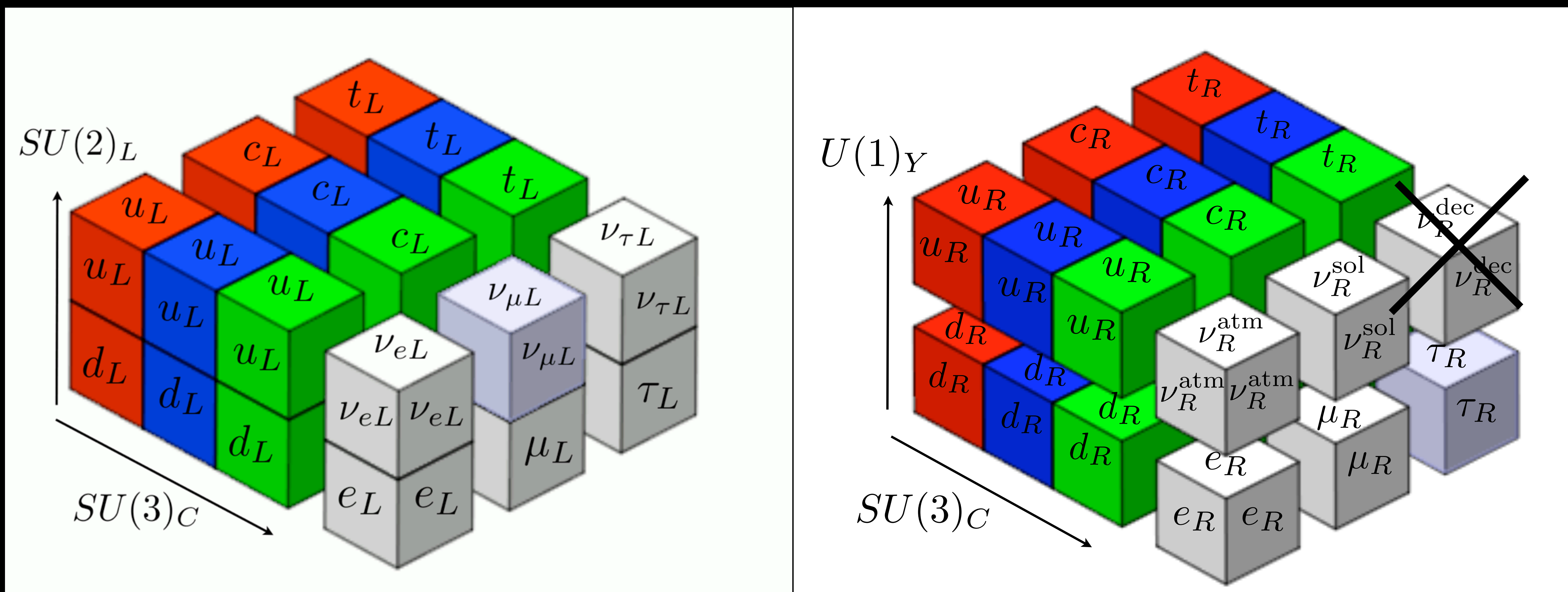


Classic seesaw:

$$m_\nu \approx \frac{m^2_{\rm Dirac}}{M_R} = 0.1\text{eV}$$



Minimal Type Ia seesaw



Type Ia seesaw with two RHNs — S.F.K, hep-ph/9912492

Either one Dirac texture zero (NO) — S.F.K, hep-ph/0204360

Or two Dirac texture zeros (IO) — Frampton, Glashow, Yanagida, hep-ph/0208157

Littlest Seesaw

SFK 1304.6264; 1512.07531

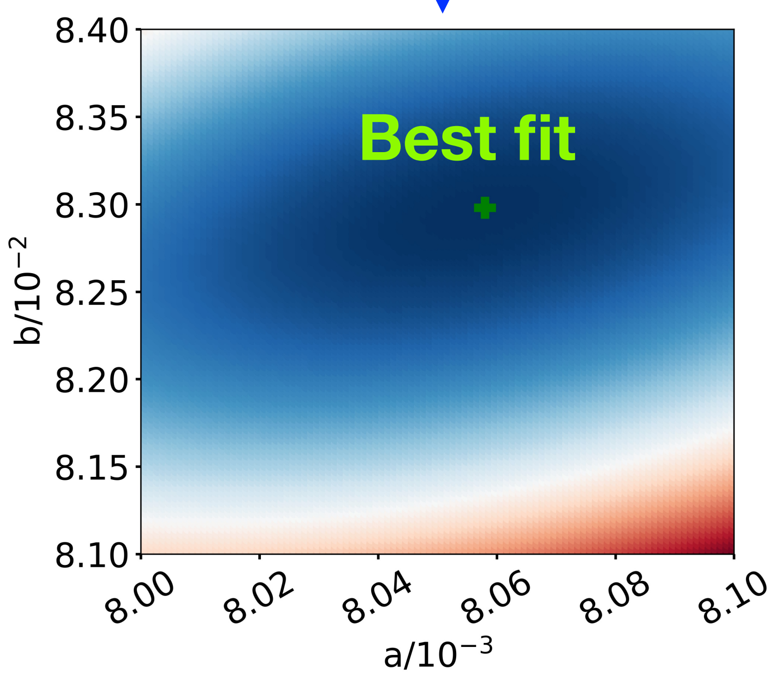
SFK, Molina Sedgwick, Rowley, 1808.01005

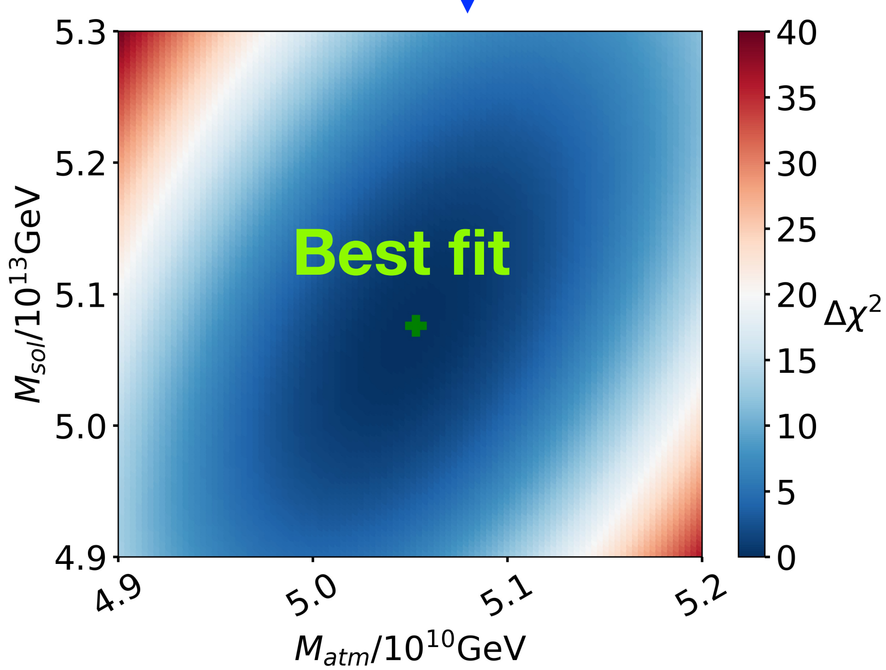
Dirac texture zero

From Type Ia with S4 (backup)

$$Y^\nu = \begin{pmatrix} 0 & be^{i\pi/3} \\ a & 3be^{i\pi/3} \\ a & be^{i\pi/3} \end{pmatrix} \qquad M_R = \begin{pmatrix} M_{\rm atm} & 0 \\ 0 & M_{\rm sol} \end{pmatrix}$$

4 real input parameters





- **Fit includes effects of RG corrections**
- **Determines the RHN masses!**

Describes:

3 neutrino masses (m_1=0),
3 mixing angles,
1 Dirac CP phase,
2 Majorana phases (1 zero)
1 BAU parameter Y_B
= 10 observables
of which 7 are constrained

Predictions	1σ range
$\theta_{12}/^\circ$	$34.254 \to 34.350$
$\theta_{13}/^\circ$	$8.370 \to 8.803$
$\theta_{23}/^\circ$	$45.405 \to 45.834$
$\Delta m_{12}{}^2/10^{-5}\mathrm{eV}^2$	$7.030 \to 7.673$
$\Delta m_{31}{}^2/10^{-3}\mathrm{eV}^2$	$2.434 \to 2.561$
$\delta/^\circ$	$-88.284 \to -86.568$
$Y_B/10^{-10}$	$0.839 \to 0.881$

Also predicts NO and m_l=0

Littlest Seesaw

SFK 1304.6264; 1512.07531

SFK, Molina Sedgwick, Rowley, 1808.01005

Seesaw formula $M_\nu = m_D M_R^{-1} m_D^T \longrightarrow (M_\nu)_{ij}\nu^c_{iL}\nu^c_{jL} = (M^*_\nu)_{ij}\nu_{iL}\nu_{jL}$

$$\text{Case I}: \quad \mathrm{M}_\nu^{\mathrm{I}} = \omega \mathrm{m_a}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + \mathrm{m_s}\begin{pmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{pmatrix}$$

$$\text{Case II}: \quad \mathrm{M}_\nu^{\mathrm{II}} = \omega^2 \mathrm{m_a}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + \mathrm{m_s}\begin{pmatrix} 1 & 1 & 3 \\ 1 & 1 & 3 \\ 3 & 3 & 9 \end{pmatrix}$$

Fits neutrino data with $m_a/m_s=10$

$$\omega = e^{i2\pi/3}$$

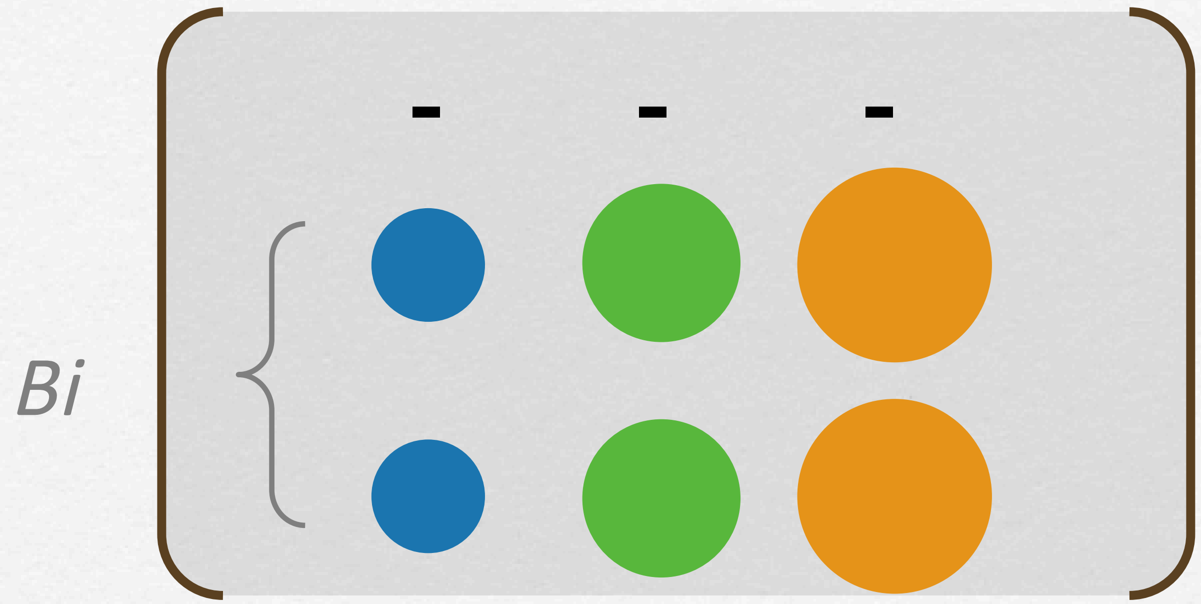
Special case $m_a/m_s=11$ gives **Littlest mu-tau seesaw**

$$\text{Case I:} \quad M_\nu = m_s\begin{pmatrix} 1 & 3 & 1 \\ 3 & 9+11\omega & 3+11\omega \\ 1 & 3+11\omega & 1+11\omega \end{pmatrix},$$

$$\text{Case II:} \quad M_\nu = m_s\begin{pmatrix} 1 & 1 & 3 \\ 1 & 1+11\omega^2 & 3+11\omega^2 \\ 3 & 3+11\omega^2 & 9+11\omega^2 \end{pmatrix}.$$

How can this be since it looks nothing like mu-tau symmetry ?

Mu-Tau Symmetry $\nu_\mu \leftrightarrow \nu_\tau^*$



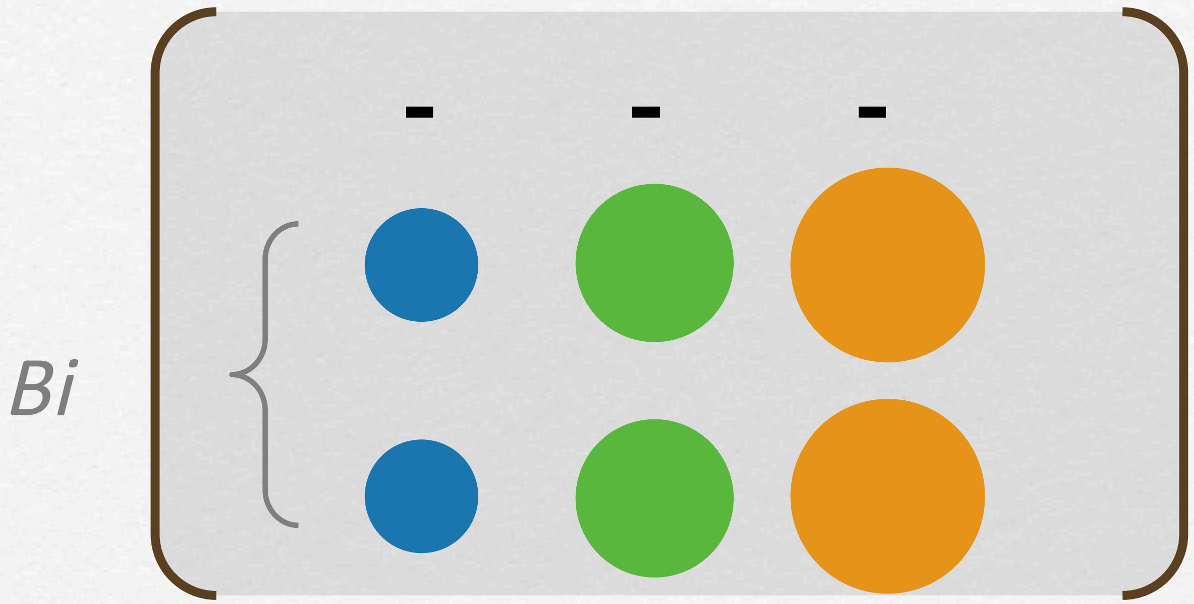
Basic Idea:

Two rows have equal magnitudes

Z.z.Xing and S.Zhou, 0804.3512

$$\theta_{13} \neq 0\,, \quad \theta_{23} = 45^\circ, \quad \delta_{\rm CP} = \pm 90^\circ$$

Mu-Tau Symmetry $\nu_\mu \leftrightarrow \nu_\tau^*$



Basic Idea:

→ Two rows have
→ equal magnitudes

Z.z.Xing and S.Zhou, 0804.3512

$$\rightarrow \quad \theta_{13} \neq 0\,, \quad \theta_{23} = 45^\circ, \quad \delta_{\text{CP}} = \pm 90^\circ$$

$$V_0 = \begin{pmatrix} |V_{e1}| & |V_{e2}| & |V_{e3}| \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\mu 1}^* & V_{\mu 2}^* & V_{\mu 3}^* \end{pmatrix}$$

Generalisation of:

Mu-tau reflection symmetry

P.F.Harrison and W.G.Scott, hep-ph/0210197

Mu-Tau Symmetry $\nu_\mu \leftrightarrow \nu_\tau^*$

Mu-tau reflection symmetric Majorana mass matrix:

$$H_\nu = M_\nu^\dagger M_\nu = \begin{pmatrix} A & D & D^* \\ D^* & B & C^* \\ D & C & B \end{pmatrix}$$

P.F.Harrison and W.G.Scott, hep-ph/0210197

Mu-Tau Symmetry $\nu_\mu \leftrightarrow \nu_\tau^*$

Mu-tau reflection symmetric Majorana mass matrix:

$$H_\nu = M_\nu^\dagger M_\nu = \begin{pmatrix} A & D & D^* \\ D^* & B & C^* \\ D & C & B \end{pmatrix}$$

P.F.Harrison and W.G.Scott, hep-ph/0210197

Can arise from:

$$M_\nu = \begin{pmatrix} a & d & d^* \\ d & c & b \\ d^* & b & c^* \end{pmatrix}$$

W.Grimus and L.Lavoura,hep-ph/0305309

More general examples:

H.J.He, W.Rodejohann and X.J.Xu, 1507.03541

A.S.Joshipura and K.M.Patel,1507.01235

Littlest Mu-Tau Seesaw $\nu_\mu \leftrightarrow \nu_\tau^*$

S.F.K. and C.C.Nishi,1807.00023

$$M_\nu = m_s \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1+11\omega^2 & 3+11\omega^2 \\ 3 & 3+11\omega^2 & 9+11\omega^2 \end{pmatrix}$$

$$\omega = e^{i2\pi/3}$$

unequal

Littlest Mu-Tau Seesaw $\nu_\mu \leftrightarrow \nu_\tau^*$

S.F.K. and C.C.Nishi,1807.00023

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$$\omega = e^{i2\pi/3}$$

unequal

$$H_\nu = M_\nu^\dagger M_\nu = 11\,|m_{\rm s}|^2 \begin{pmatrix} 1 & -1-2i\sqrt{3} & 1-2i\sqrt{3} \\ -1+2i\sqrt{3} & 19 & 17+4i\sqrt{3} \\ 1+2i\sqrt{3} & 17-4i\sqrt{3} & 19 \end{pmatrix}$$

equal

Littlest Mu-Tau Seesaw $\nu_\mu \leftrightarrow \nu_\tau^*$

S.F.K. and C.C.Nishi,1807.00023

$$M_\nu = m_s \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1+11\omega^2 & 3+11\omega^2 \\ 3 & 3+11\omega^2 & 9+11\omega^2 \end{pmatrix}$$

$$\omega = e^{i2\pi/3}$$

unequal

$$H_\nu = M_\nu^\dagger M_\nu = 11\,|m_\mathrm{s}|^2 \begin{pmatrix} 1 & -1-2i\sqrt{3} & 1-2i\sqrt{3} \\ -1+2i\sqrt{3} & 19 & 17+4i\sqrt{3} \\ 1+2i\sqrt{3} & 17-4i\sqrt{3} & 19 \end{pmatrix}$$

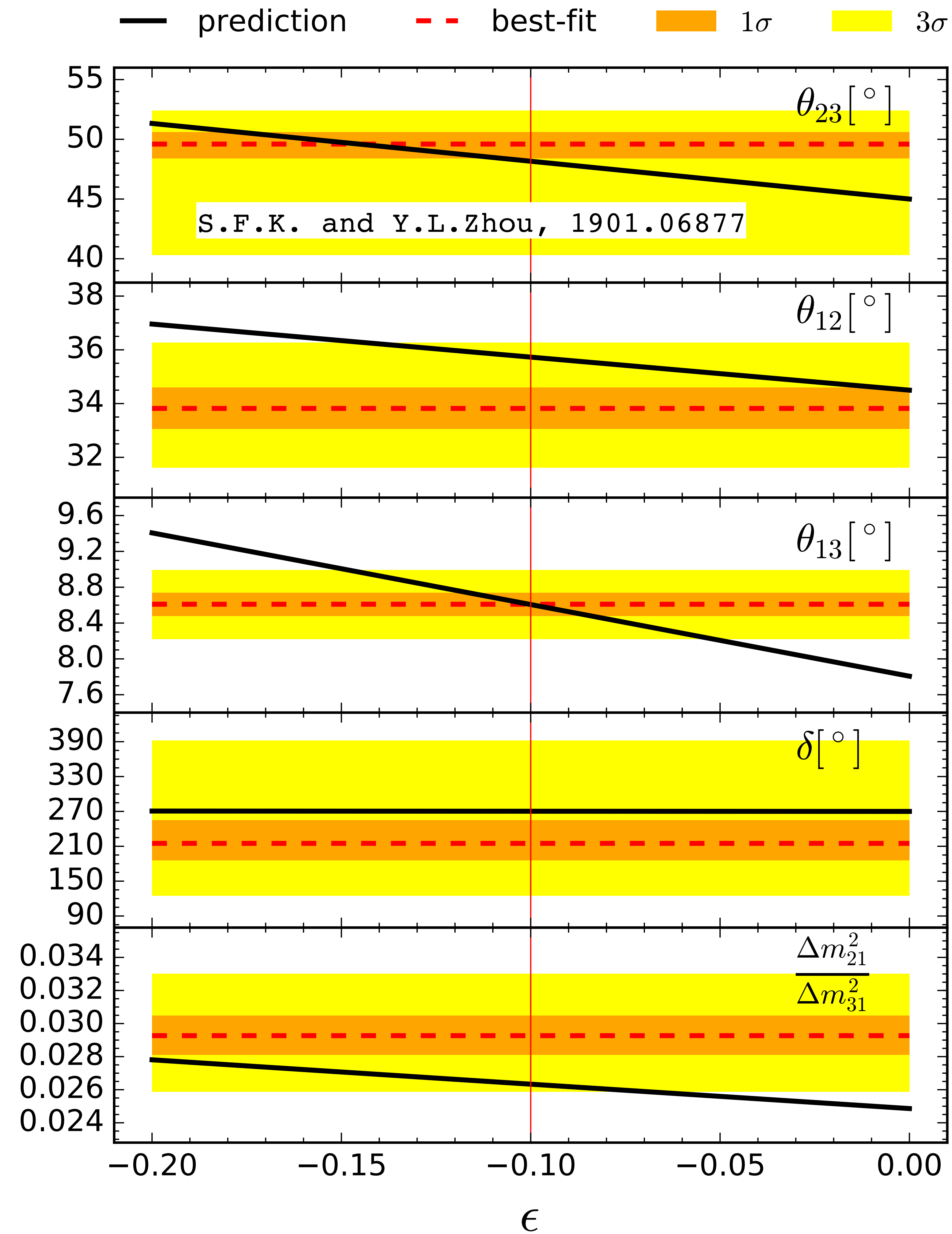
equal

$$U = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{c_+}{\sqrt{6}} & \frac{c_-}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} & -\frac{c_+}{\sqrt{6}} - i\frac{c_-}{2} & -\frac{c_-}{\sqrt{6}} + i\frac{c_+}{2} \\ \frac{1}{\sqrt{6}} & -\frac{c_+}{\sqrt{6}} + i\frac{c_-}{2} & -\frac{c_-}{\sqrt{6}} - i\frac{c_+}{2} \end{pmatrix}$$

TM1

Mu-tau reflection symmetry

$$c_\pm = \sqrt{1 \pm \frac{11}{3\sqrt{17}}}$$



Littlest mu-tau seesaw

$$m_1 = 0$$

$$U = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{c_+}{\sqrt{6}} & \frac{c_-}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} & -\frac{c_+}{\sqrt{6}} - i\frac{c_-}{2} & -\frac{c_-}{\sqrt{6}} + i\frac{c_+}{2} \\ \frac{1}{\sqrt{6}} & -\frac{c_+}{\sqrt{6}} + i\frac{c_-}{2} & -\frac{c_-}{\sqrt{6}} - i\frac{c_+}{2} \end{pmatrix}$$

$$c_\pm = \sqrt{1 \pm \frac{11}{3\sqrt{17}}}$$

Renormalisation Group Corrections

$$\theta_{13} \approx 7.807^\circ - 8.000^\circ\epsilon\,,$$

$$\theta_{12} \approx 34.50^\circ - 12.30^\circ\epsilon\,,$$

$$\theta_{23} \approx 45.00^\circ - 31.64^\circ\epsilon\,,$$

$$\delta \approx 270.00^\circ + 3.23^\circ\epsilon\,,$$

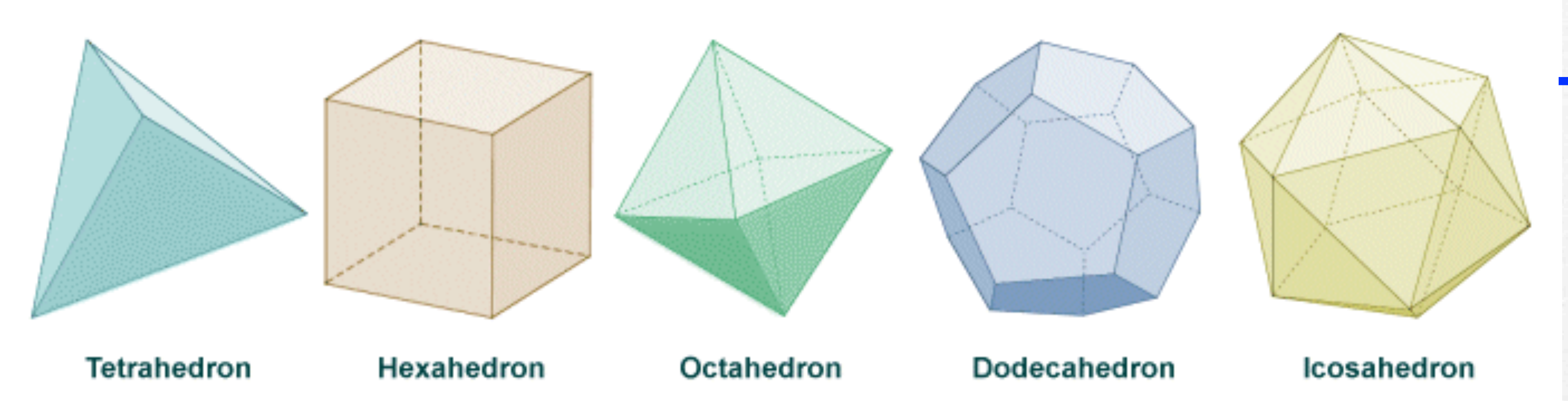
$$\Delta m^2_{21}/\Delta m^2_{31} \approx 0.0247 - 0.0147\epsilon$$

Conclusions

- Most parameters well measured in oscillation experiments…but…CP phase, octant, ordering? Also: Dirac or Majorana? Absolute masses?
- TB mixing explained by S_4…excluded by reactor angle…but…S_4 violations allow: charged lepton corrections, or TM1,TM2, with testable sum rules
- Mu-tau symmetry predicts $\theta_{23} = 45^\circ, \delta = -90^\circ$ Littlest mu-tau seesaw…one parameter…wow!
- Origin of Plato's symmetry? - see backup slides

Backup slides

Origin of Plato's symmetry?



solid	faces	vertices	Plato	Group
tetrahedron	4	4	fire	A_4
octahedron	8	6	air	S_4
icosahedron	20	12	water	A_5
hexahedron	6	8	earth	S_4
dodecahedron	12	20	?	A_5

Two possibilities:

1. From gauge group e.g. SU(3) or SO(3)
2. From extra dimensions e.g. string theory

Origin of Plato's symmetry?

Possibility I:

Y.Koide,0705.2275; T.Banks and N.Seiberg,1011.5120;
Y.L.Wu,1203.2382; A.Merle and R.Zwicky,1110.4891;
B.L.Rachlin and T.W.Kephart,1702.08073; C. Luhn, 1101.2417;
S.F.K. and Ye-Ling Zhou, 1809.10292

Break SO(3) using large Higgs reps

irrep	$\underline{1}$	$\underline{3}$	$\underline{5}$	$\underline{7}$
subgroups	$SO(3)$	$SO(2)$	$Z_2 \times Z_2$	$\mathbf{1}$
		$SO(3)$	$SO(2)$	A_4
			$SO(3)$	Z_3
				D_4
				$SO(2)$
				$SO(3)$

Origin of Plato's symmetry?

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Y.Koide,0705.2275; T.Banks and N.Seiberg,1011.5120;
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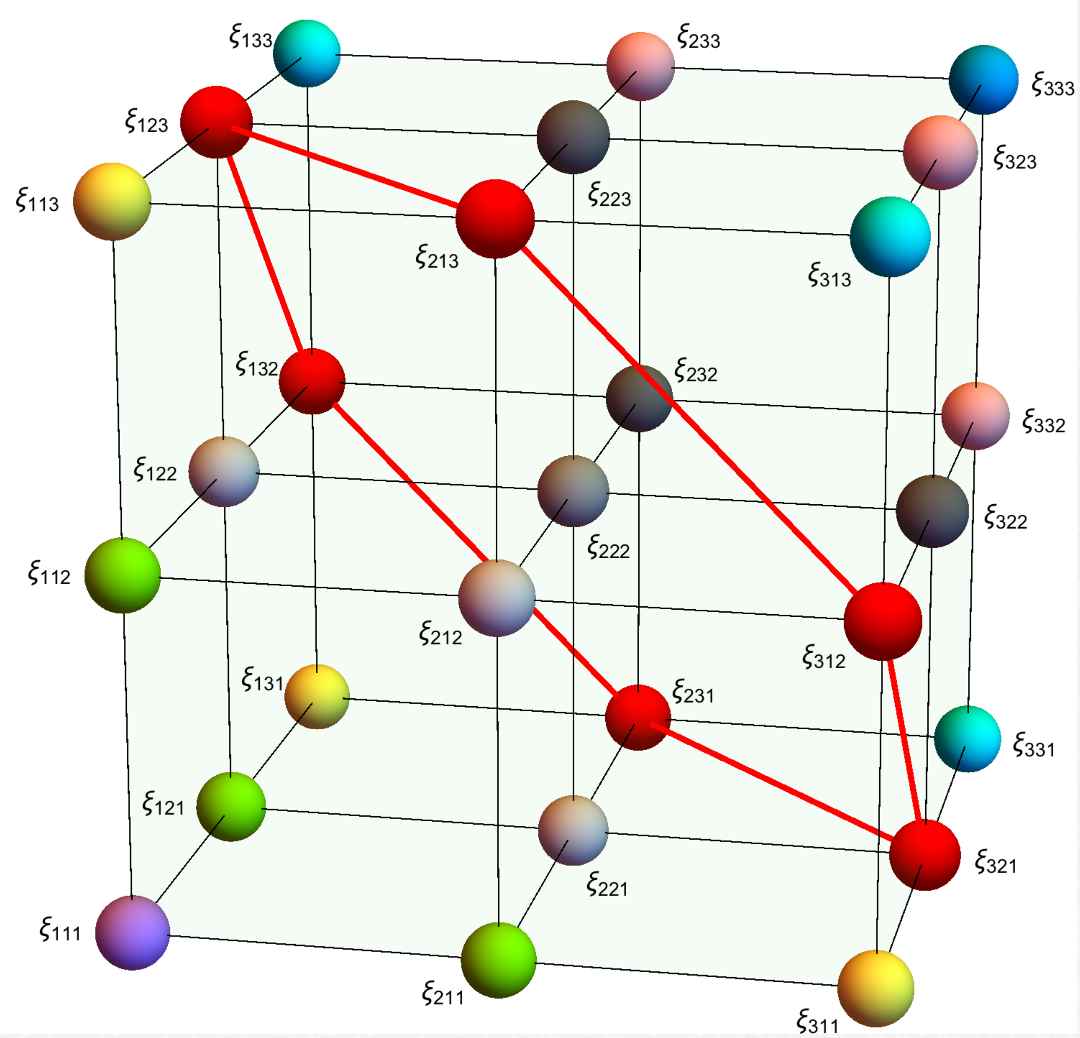
S.F.K. and Ye-Ling Zhou, 1809.10292

Break SO(3) using large Higgs reps E.g. 7-plet

irrep	$\underline{1}$	$\underline{3}$	$\underline{5}$	$\underline{7}$
subgroups	$SO(3)$	$SO(2)$	$Z_2 \times Z_2$	$\mathbf{1}$
		$SO(3)$	$SO(2)$	A_4
			$SO(3)$	Z_3
				D_4
				$SO(2)$
				$SO(3)$



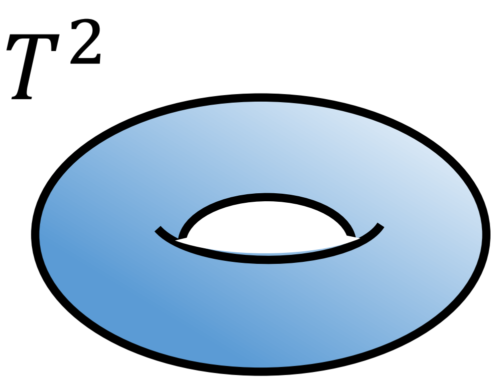
A4 preserving direction of 7-plet VEV

$$\langle\xi_{123}\rangle \equiv \frac{v_\xi}{\sqrt{6}}, \quad \langle\xi_{111}\rangle = \langle\xi_{112}\rangle = \langle\xi_{113}\rangle = \langle\xi_{133}\rangle = \langle\xi_{233}\rangle = \langle\xi_{333}\rangle = 0$$

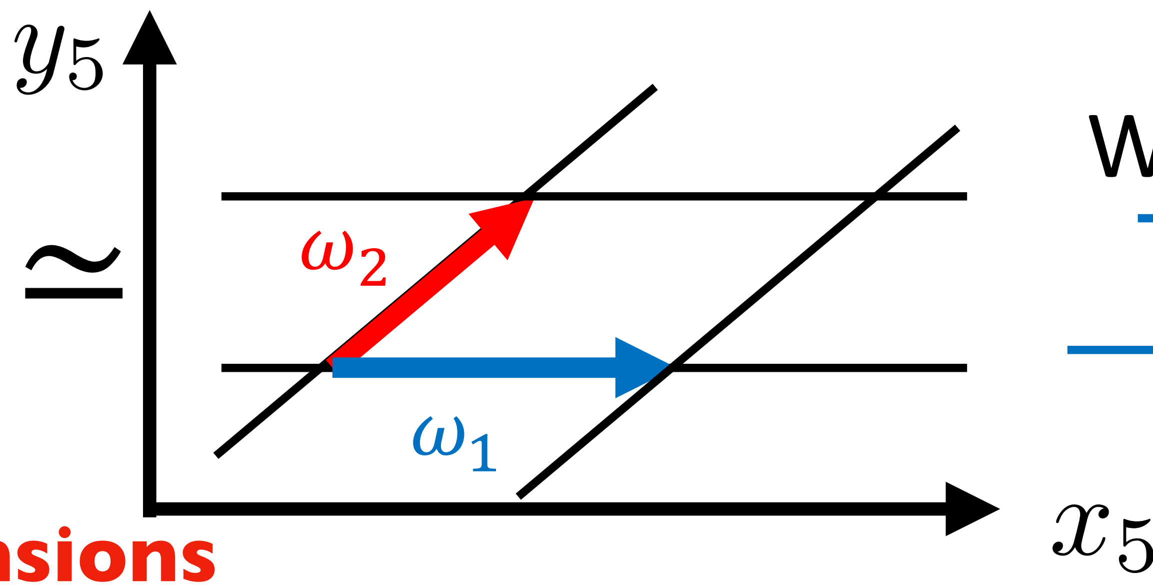
Possibility 2: Extra dimensions (string theory)

G.Altarelli and F.Feruglio, hep-ph/0512103
R.de Adelhart Toorop, F.Feruglio and C.Hagedorn, 1112.1340
F.Feruglio, 1706.08749; J.C.Criado and F.Feruglio, 1807.01125; J.T.Penedo and S.T.Petcov 1806.11040;
P.P.Novichkov, J.T.Penedo, S.T.Petcov and A.V.Titov, 1811.04933, 1812.02158;
T.Kobayashi, K.Tanaka and T.H.Tatsuishi,1803.10391; F.de Anda,S.F.K.,E.Perdomo,1812.05620
T.Kobayashi, N.Omoto, Y.Shimizu, K.Takagi, M.Tanimoto and T.H.Tatsuishi,1808.03012;
G.J.Ding, S.F.King and X.G.Liu, 1903.12588

The structure of a torus $T^2 \simeq$ The structure of a lattice on $\mathbb{C}$-plane



$\simeq$



With identification

T.H.Tatsuishi

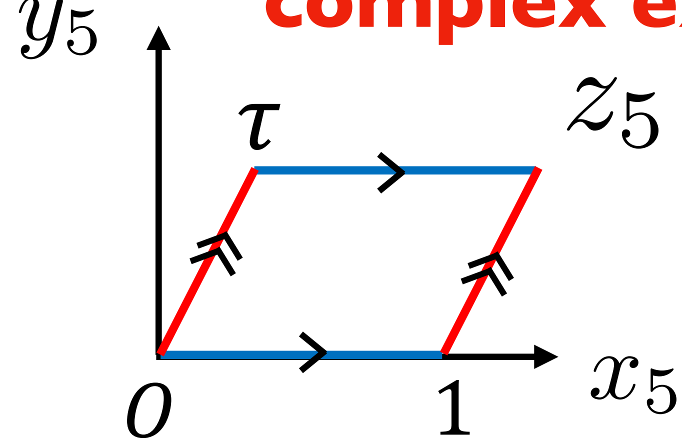
two extra dimensions compactified on torus

Without loss of generality,

$$(\omega_1, \omega_2) \to \left(1, \frac{\omega_2}{\omega_1}\right) \equiv (1, \tau)$$

modulus

complex extra dimension

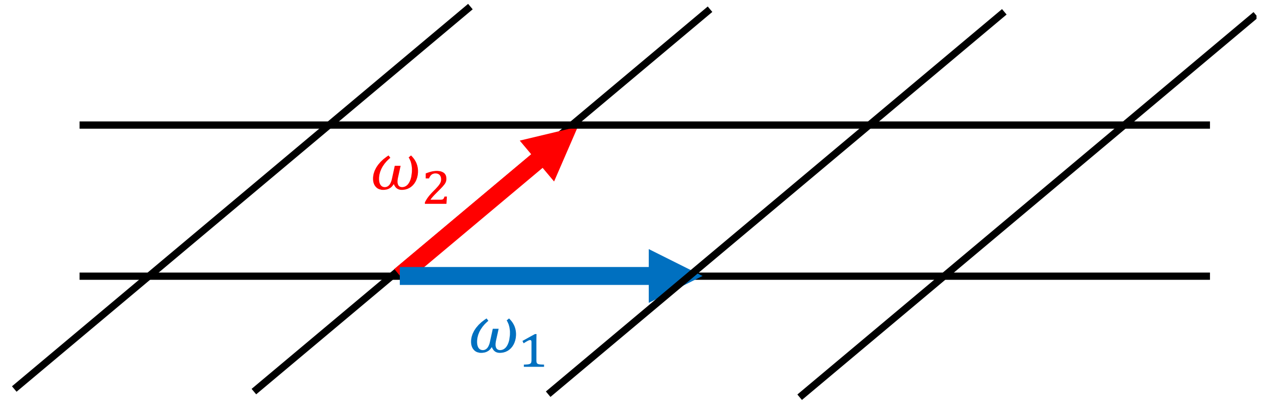


$$z_5 = x_5 + i y_5$$

$$(\mathrm{Im}(\tau) > 0)$$

Possibility 2: Extra dimensions (string theory)

There are two independent lattice invariant transformations.

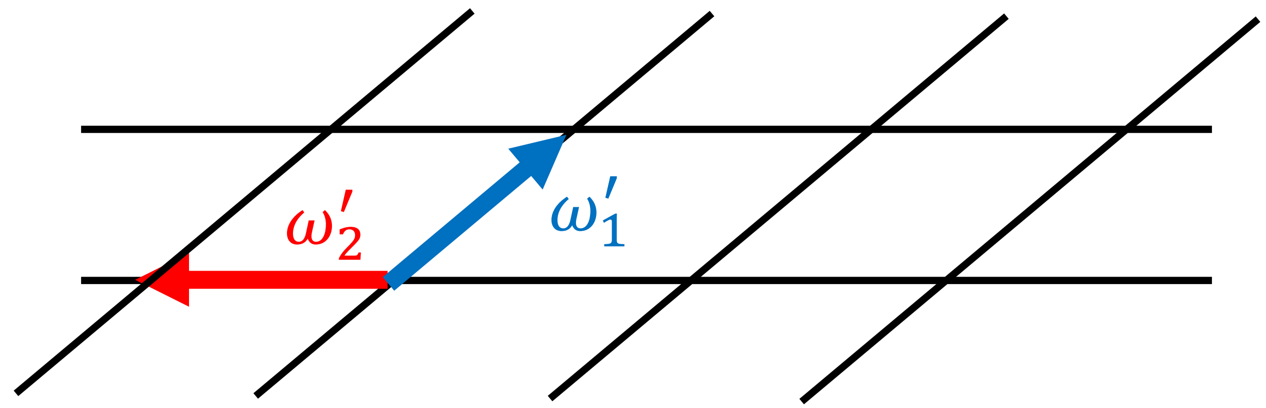


Modular Symmetry SL(2,Z):

$$\tau \rightarrow (a\tau + b)/(c\tau + d)$$

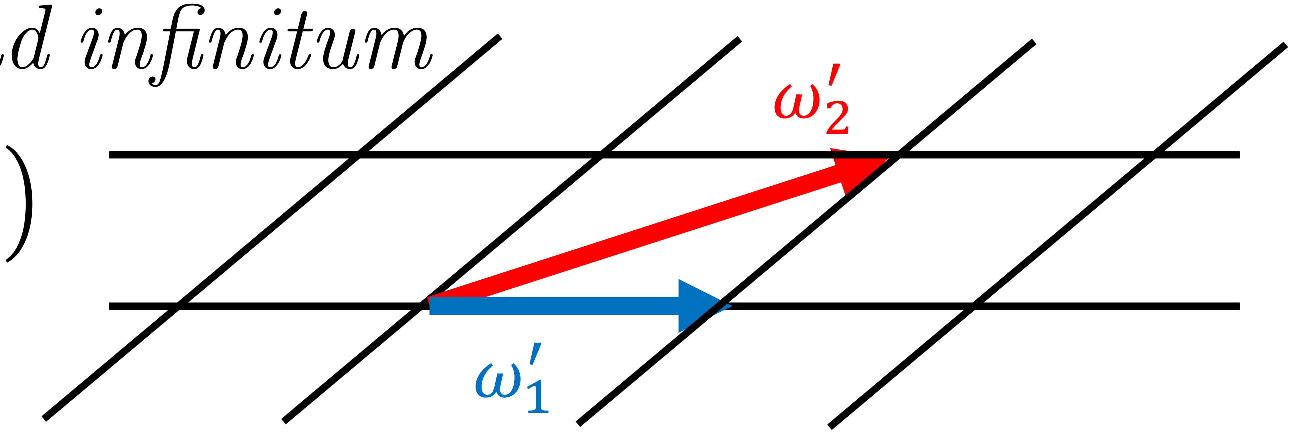
S-transformation $\quad \tau \rightarrow -1/\tau$

$$\begin{pmatrix} \omega_1' \\ \omega_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix} = \begin{pmatrix} \omega_2 \\ -\omega_1 \end{pmatrix}$$



T-transformation $\quad \tau \rightarrow \tau + 1 \quad$ *ad infinitum*

$$\begin{pmatrix} \omega_1' \\ \omega_2' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix} = \begin{pmatrix} \omega_1 \\ \omega_2 + \omega_1 \end{pmatrix}$$



where S^2, $(ST)^3$=I (infinite)

S, T are generators of A_4 if T^3=I, S_4 if T^4=I, A_5 if T^5=I, ...

Flavour symmetries may be identified as finite subgroups of the infinite modular group

F.Feruglio, 1706.08749

Modular Forms

Yukawa couplings involving twisted states whose modular weights do not add up to zero are modular forms

Level 3 Weight 2 acts as A4 triplet:

$$Y = \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix} = \begin{pmatrix} 1 + 12q + 36q^2 + 12q^3 + 84q^4 + 72q^5 + \dots \\ -6q^{1/3}(1 + 7q + 8q^2 + 18q^3 + 14q^4 + \dots) \\ -18q^{2/3}(1 + 2q + 5q^2 + 4q^3 + 8q^4 + \dots) \end{pmatrix}$$

$$q \equiv e^{i2\pi\tau}$$ **free modulus** $$\tau = \frac{\omega_2}{\omega_1}$$

Weinberg operator $$\frac{1}{\Lambda}(H_u H_u \; LL \; Y) \longrightarrow m_\nu = \begin{pmatrix} 2Y_1 & -Y_3 & -Y_2 \\ -Y_3 & 2Y_2 & -Y_1 \\ -Y_2 & -Y_1 & 2Y_3 \end{pmatrix} \frac{v_u^2}{\Lambda}$$

A_4: 3 3 3

G.J.Ding, S.F.K. and X.-G.Liu, 1907.11714

A4 Modular Symmetry

Models		mass matrices	assignment	weight		
			$\rho_{E^c_{1,2,3}}$	$k_{E^c_{1,2,3}}$	k_L	k_{N^c}
Weinberg operator	$\mathcal{A}1$	$W1,\ C1$	$\mathbf{1}, \mathbf{1}, \mathbf{1}$	$1, 3, 5$	1	$-$
	$\mathcal{A}2$	$W1,\ C2$	$\mathbf{1}', \mathbf{1}', \mathbf{1}'$	$1, 3, 5$	1	$-$
	$\mathcal{A}3$	$W1,\ C3$	$\mathbf{1}'', \mathbf{1}'', \mathbf{1}''$	$1, 3, 5$	1	$-$
	$\mathcal{A}4$	$W1,\ C4$	$\mathbf{1}, \mathbf{1}, \mathbf{1}'$	$1, 3, 1$	1	$-$
	$\mathcal{A}5$	$W1,\ C5$	$\mathbf{1}, \mathbf{1}, \mathbf{1}''$	$1, 3, 1$	1	$-$
	$\mathcal{A}6$	$W1,\ C6$	$\mathbf{1}', \mathbf{1}', \mathbf{1}$	$1, 3, 1$	1	$-$
	$\mathcal{A}7$	$W1,\ C7$	$\mathbf{1}'', \mathbf{1}'', \mathbf{1};$	$1, 3, 1$	1	$-$
	$\mathcal{A}8$	$W1,\ C8$	$\mathbf{1}'', \mathbf{1}'', \mathbf{1}'$	$1, 3, 1$	1	$-$
	$\mathcal{A}9$	$W1,\ C9$	$\mathbf{1}', \mathbf{1}', \mathbf{1}''$	$1, 3, 1$	1	$-$
	$\mathcal{A}10$	$W1,\ C10$	$\mathbf{1}, \mathbf{1}'', \mathbf{1}'$	$1, 1, 1$	1	$-$
Type I see-saw	$\mathcal{B}1(\mathcal{C}1)[\mathcal{D}1]$	$S1(S2)[S3],\ C1$	$\mathbf{1}, \mathbf{1}, \mathbf{1}$	$0(3)[1], 2(5)[3], 4(7)[5]$	$2(-1)[1]$	$0(1)[1]$
	$\mathcal{B}2(\mathcal{C}2)[\mathcal{D}2]$	$S1(S2)[S3],\ C2$	$\mathbf{1}', \mathbf{1}', \mathbf{1}'$	$0(3)[1], 2(5)[3], 4(7)[5]$	$2(-1)[1]$	$0(1)[1]$
	$\mathcal{B}3(\mathcal{C}3)[\mathcal{D}3]$	$S1(S2)[S3],\ C3$	$\mathbf{1}'', \mathbf{1}'', \mathbf{1}''$	$0(3)[1], 2(5)[3], 4(7)[5]$	$2(-1)[1]$	$0(1)[1]$
	$\mathcal{B}4(\mathcal{C}4)[\mathcal{D}4]$	$S1(S2)[S3],\ C4$	$\mathbf{1}, \mathbf{1}, \mathbf{1}'$	$0(3)[1], 2(5)[3], 0(3)[1]$	$2(-1)[1]$	$0(1)[1]$
	$\mathcal{B}5(\mathcal{C}5)[\mathcal{D}5]$	$S1(S2)[S3],\ C5$	$\mathbf{1}, \mathbf{1}, \mathbf{1}''$	$0(3)[1], 2(5)[3], 0(3)[1]$	$2(-1)[1]$	$0(1)[1]$
	$\mathcal{B}6(\mathcal{C}6)[\mathcal{D}6]$	$S1(S2)[S3],\ C6$	$\mathbf{1}', \mathbf{1}', \mathbf{1}$	$0(3)[1], 2(5)[3], 0(3)[1]$	$2(-1)[1]$	$0(1)[1]$
	$\mathcal{B}7(\mathcal{C}7)[\mathcal{D}7]$	$S1(S2)[S3],\ C7$	$\mathbf{1}', \mathbf{1}', \mathbf{1}''$	$0(3)[1], 2(5)[3], 0(3)[1]$	$2(-1)[1]$	$0(1)[1]$
	$\mathcal{B}8(\mathcal{C}8)[\mathcal{D}8]$	$S1(S2)[S3],\ C8$	$\mathbf{1}'', \mathbf{1}'', \mathbf{1}$	$0(3)[1], 2(5)[3], 0(3)[1]$	$2(-1)[1]$	$0(1)[1]$
	$\mathcal{B}9(\mathcal{C}9)[\mathcal{D}9]$	$S1(S2)[S3],\ C9$	$\mathbf{1}'', \mathbf{1}'', \mathbf{1}'$	$0(3)[1], 2(5)[3], 0(3)[1]$	$2(-1)[1]$	$0(1)[1]$
	$\mathcal{B}10(\mathcal{C}10)[\mathcal{D}10]$	$S1(S2)[S3],\ C10$	$\mathbf{1}, \mathbf{1}'', \mathbf{1}'$	$0(3)[1], 0(3)[1], 0(3)[1]$	$2(-1)[1]$	$0(1)[1]$

Comprehensive study of 40 simplest cases without flavons

Models	Ordering		Models	Ordering		Models	Ordering		Models	Ordering	
	NO	IO		NO	IO		NO	IO		NO	IO
$\mathcal{A}1$	✘	✘	$\mathcal{B}1$	✔	✔	$\mathcal{C}1$	✘	✘	$\mathcal{D}1$	✔	✔
$\mathcal{A}2$	✘	✘	$\mathcal{B}2$	✔	✔	$\mathcal{C}2$	✘	✘	$\mathcal{D}2$	✔	✔
$\mathcal{A}3$	✘	✘	$\mathcal{B}3$	✔	✔	$\mathcal{C}3$	✘	✘	$\mathcal{D}3$	✔	✔
$\mathcal{A}4$	✘	✘	$\mathcal{B}4$	✘	✘	$\mathcal{C}4$	✘	✘	$\mathcal{D}4$	✘	✔
$\mathcal{A}5$	✘	✘	$\mathcal{B}5$	✘	✘	$\mathcal{C}5$	✘	✘	$\mathcal{D}5$	✔	✘
$\mathcal{A}6$	✘	✘	$\mathcal{B}6$	✘	✔	$\mathcal{C}6$	✘	✘	$\mathcal{D}6$	✔	✘
$\mathcal{A}7$	✘	✘	$\mathcal{B}7$	✘	✘	$\mathcal{C}7$	✘	✘	$\mathcal{D}7$	✔	✔
$\mathcal{A}8$	✘	✘	$\mathcal{B}8$	✘	✘	$\mathcal{C}8$	✘	✘	$\mathcal{D}8$	✔	✔
$\mathcal{A}9$	✘	✘	$\mathcal{B}9$	✔	✔	$\mathcal{C}9$	✘	✘	$\mathcal{D}9$	✔	✔
$\mathcal{A}10$	✘	✘	$\mathcal{B}10$	✔	✔	$\mathcal{C}10$	✘	✘	$\mathcal{D}10$	✔	✔

Minimal Models:

$$\mathcal{B}_9,\ \mathcal{B}_{10},\ \mathcal{D}_5 \sim \mathcal{D}_{10}$$

8 inputs for 12 observables

(6 lepton masses, 6 PMNS)

Large nu mass, deltaCP

G.J.Ding, S.F.K. and X.-G.Liu, 1903.12588

A5 Modular Symmetry

Models		mass matrices	assignment $(\rho_{E^c}, \rho_L, \rho_{N^c})$	weight $k_{E_{1,2,3}}$	k_L	k_{N^c}
With flavons	$\mathcal{A}1$	$W1$	$(\mathbf{1}, \mathbf{3}, -)$	$-$	1	$-$
	$\mathcal{A}2$	$W2$	$(\mathbf{1}, \mathbf{3}', -)$	$-$	1	$-$
	$\mathcal{A}3$	$S1$	$(\mathbf{1}, \mathbf{3}, \mathbf{3})$	$-$	2	0
	$\mathcal{A}4$	$S2$	$(\mathbf{1}, \mathbf{3}, \mathbf{3})$	$-$	-1	1
	$\mathcal{A}5$	$S3$	$(\mathbf{1}, \mathbf{3}', \mathbf{3})$	$-$	2	0
	$\mathcal{A}6$	$S4$	$(\mathbf{1}, \mathbf{3}, \mathbf{3}')$	$-$	2	0
	$\mathcal{A}7$	$S5$	$(\mathbf{1}, \mathbf{3}', \mathbf{3}')$	$-$	2	0
	$\mathcal{A}8$	$S6$	$(\mathbf{1}, \mathbf{3}', \mathbf{3}')$	$-$	-1	1
Without flavons	$\mathcal{B}1$	$C1$, $W1$	$(\mathbf{1}, \mathbf{3}, -)$	1 , 3 , 5	1	$-$
	$\mathcal{B}2$	$C2$, $W2$	$(\mathbf{1}, \mathbf{3}', -)$	1 , 3 , 5	1	$-$
	$\mathcal{B}3$	$C1$, $S1$	$(\mathbf{1}, \mathbf{3}, \mathbf{3})$	0 , 2 , 4	2	0
	$\mathcal{B}4$	$C1$, $S2$	$(\mathbf{1}, \mathbf{3}, \mathbf{3})$	3 , 5 , 7	-1	1
	$\mathcal{B}5$	$C2$, $S3$	$(\mathbf{1}, \mathbf{3}', \mathbf{3})$	0 , 2 , 4	2	0
	$\mathcal{B}6$	$C1$, $S4$	$(\mathbf{1}, \mathbf{3}, \mathbf{3}')$	0 , 2 , 4	2	0
	$\mathcal{B}7$	$C2$, $S5$	$(\mathbf{1}, \mathbf{3}', \mathbf{3}')$	0 , 2 , 4	2	0
	$\mathcal{B}8$	$C2$, $S6$	$(\mathbf{1}, \mathbf{3}', \mathbf{3}')$	3 , 5 , 7	-1	1

Comprehensive study of simplest cases with and without flavons

Results very dependent on **free modulus**

Models		free input parameters p_i	overall factors
With flavons	$\mathcal{A}1$, $\mathcal{A}2$	$\{\mathrm{Re}\,\tau,\ \mathrm{Im}\,\tau\}$	v_u^2/Λ
	$\mathcal{A}4$, $\mathcal{A}5$, $\mathcal{A}6$, $\mathcal{A}8$	$\{\mathrm{Re}\,\tau,\ \mathrm{Im}\,\tau\}$	$g^2 v_u^2/\Lambda$
	$\mathcal{A}3$, $\mathcal{A}7$	$\{\mathrm{Re}\,\tau,\ \mathrm{Im}\,\tau,\ \lvert g_1/g_2\rvert,\ \mathrm{Arg}(g_1/g_2)\}$	$g_2^2 v_u^2/\Lambda$
Without flavons	$\mathcal{B}1$, $\mathcal{B}2$	$\{\mathrm{Re}\,\tau,\ \mathrm{Im}\,\tau,\ \beta/\alpha,\ \gamma_1/\alpha,\ \lvert\gamma_2/\alpha\rvert,\ \mathrm{Arg}(\gamma_2/\alpha)\}$	$\alpha v_d,\ v_u^2/\Lambda$
	$\mathcal{B}4$, $\mathcal{B}5$, $\mathcal{B}6$, $\mathcal{B}8$	$\{\mathrm{Re}\,\tau,\ \mathrm{Im}\,\tau,\ \beta/\alpha,\ \gamma_1/\alpha,\ \lvert\gamma_2/\alpha\rvert,\ \mathrm{Arg}(\gamma_2/\alpha)\}$	$\alpha v_d,\ g^2 v_u^2/\Lambda$
	$\mathcal{B}3$, $\mathcal{B}7$	$\{\mathrm{Re}\,\tau,\ \mathrm{Im}\,\tau, \beta/\alpha,\ \gamma_1/\alpha,\ \lvert\gamma_2/\alpha\rvert,$ $\mathrm{Arg}(\gamma_2/\alpha),\ \lvert g_1/g_2\rvert,\ \mathrm{Arg}(g_1/g_2)\}$	$\alpha v_d,\ g_2^2 v_u^2/\Lambda$

$$\tau = \frac{\omega_2}{\omega_1}$$

F. De Anda, S.F.K., E.Perdomo, 1812.05620

Modular Symmetry and orbifolds

Consider a **finite** modular symmetry

$$\bar{\Gamma}_M \simeq \{S,T|S^2=(ST)^3=T^M=\mathbb{I}\}/\{\pm 1\}$$

Represented by the modular transformations (level M>2)

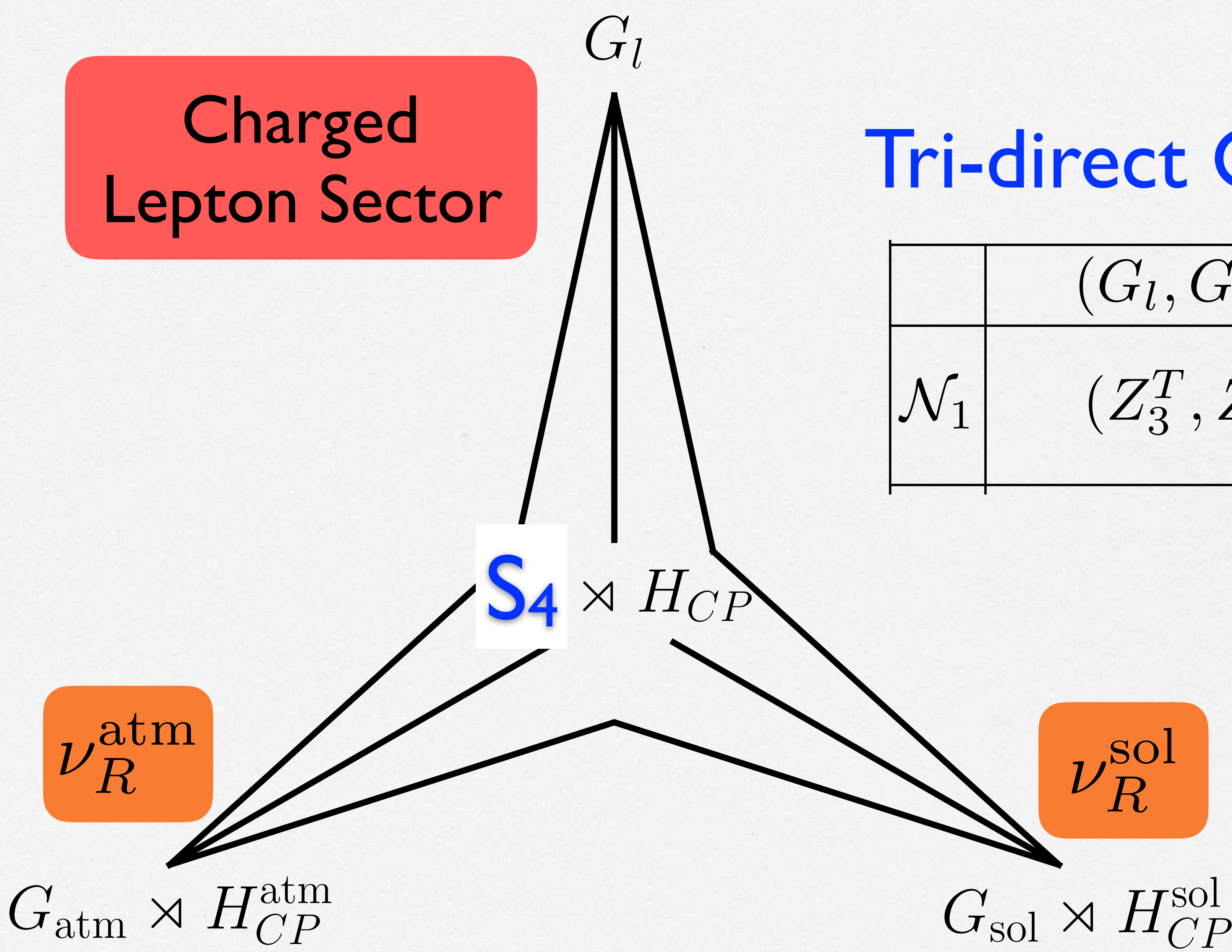
$$S=\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad T_{(M)}=\begin{pmatrix} e^{-2i\pi/M} & 0 \\ 1 & e^{2i\pi/M} \end{pmatrix}$$

We show that for the orbifold $T^2/\mathbb{Z}_2$
the **fixed points** are only invariant for a particular
level M=3 and **fixed modulus** $\omega=e^{i2\pi/3}$

$$\bar{\Gamma}_3=A_4 \text{ with } \tau=\omega \text{ or } \tau=\omega+1.$$

G.J.Ding, S.F.K. and C.C.Li, 1807.07538, 1811.12340

Littlest Seesaw from S$_4$



Tri-direct CP with S$_4$

	$(G_l, G_{\mathrm{atm}}, G_{\mathrm{sol}})$
$\mathcal{N}_1$	(Z_3^T, Z_2^U, Z_2^{SU})

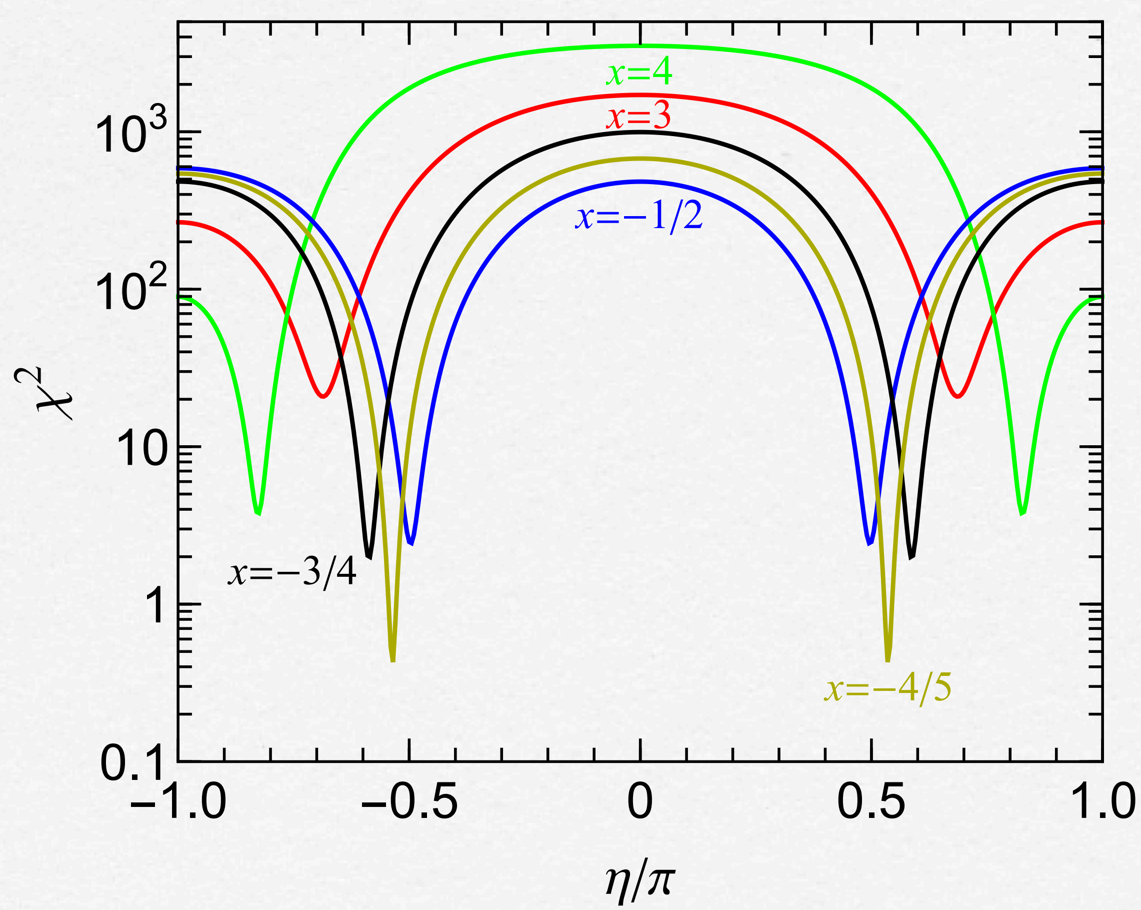
P.T.Chen, G.J.Ding, S.F.K. and C.C.Li, 1906.11414

Littlest Seesaw from S_4

Tri-direct CP with S_4 gives the structure

$$m_\nu = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} + m_s e^{i\eta} \begin{pmatrix} 1 & 2-x & x \\ 2-x & (x-2)^2 & (2-x)x \\ x & (2-x)x & x^2 \end{pmatrix}$$

TM1
NO
$m_1=0$



Original Littlest Seesaw

$$(x,\eta) = (3, 2\pi/3),\ (-1, -2\pi/3)$$

$$\sin^2\theta_{23} \approx 0.5 \quad \delta_{CP} \approx -\pi/2$$

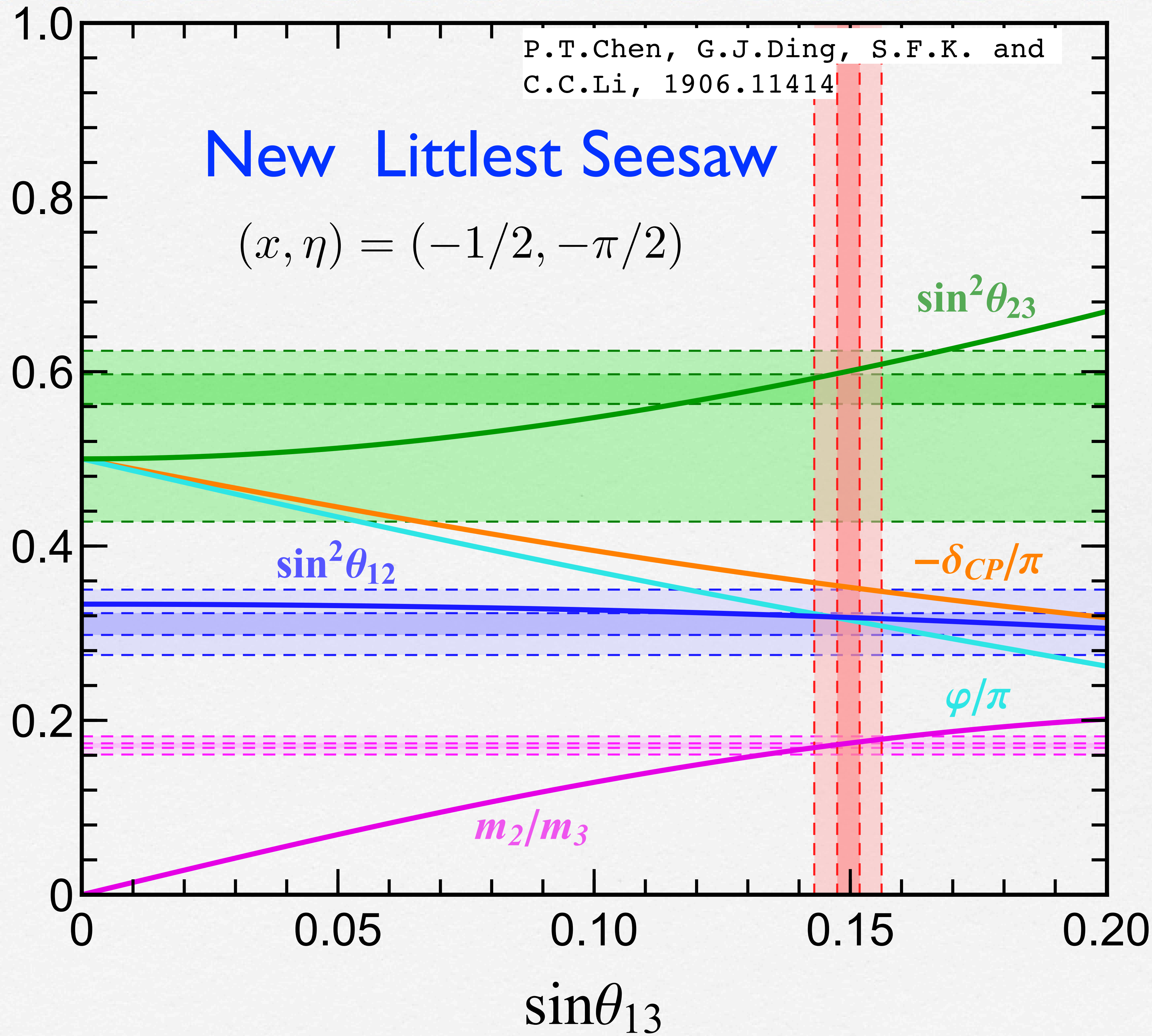
New Littlest Seesaw

$$(x,\eta) = (-1/2, -\pi/2)$$

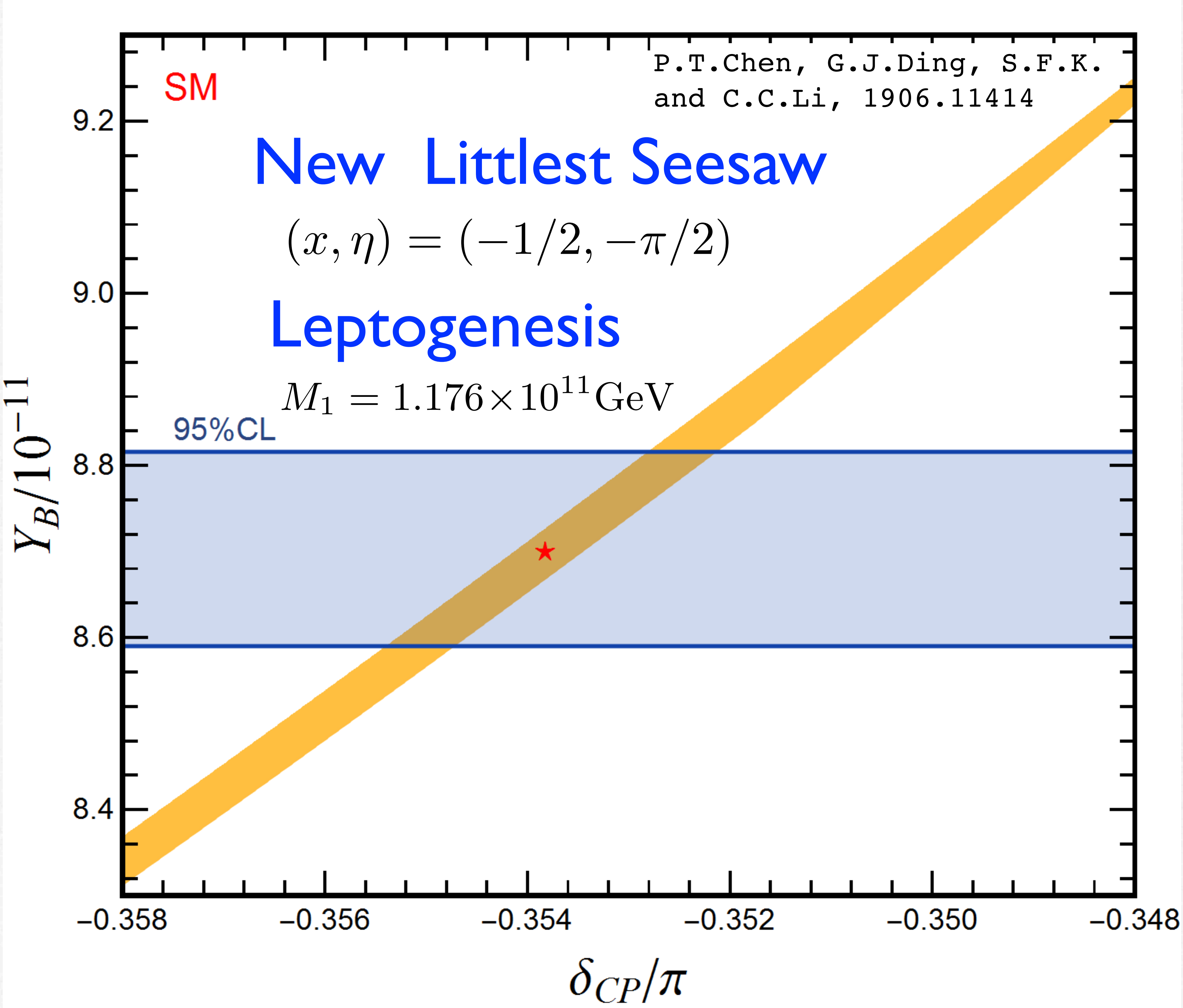
$$0.593 \le \sin^2\theta_{23} \le 0.609$$

$$-0.358 \le \delta_{CP}/\pi \le -0.348$$

UO



P.T.Chen, G.J.Ding, S.F.K. and C.C.Li, 1906.11414
New Littlest Seesaw
$(x, \eta) = (-1/2, -\pi/2)$
$\sin^2\theta_{23}$
$-\delta_{CP}/\pi$
$\sin^2\theta_{12}$
φ/π
m_2/m_3
1.0
0.8
0.6
0.4
0.2
0
0
0.05
0.10
0.15
0.20
$\sin\theta_{13}$


SM
P.T.Chen, G.J.Ding, S.F.K. and C.C.Li, 1906.11414
New Littlest Seesaw
$(x,\eta)=(-1/2,-\pi/2)$
Leptogenesis
$M_1 = 1.176\times 10^{11}\mathrm{GeV}$
95%CL
9.2
9.0
8.8
8.6
8.4
$Y_B/10^{-11}$
−0.358
−0.356
−0.354
−0.352
−0.350
−0.348
δ_{CP}/π

A.E.Cárcamo Hernández and S.F.King,1903.02565

Littlest Inverse Seesaw

Another Possibility

$$M_\nu = \begin{pmatrix} 0_{3\times3} & m_D & 0_{3\times2} \\ m_D^T & 0_{2\times2} & M \\ 0_{2\times3} & M^T & \mu \end{pmatrix}$$

cLFV, collider…

Talk by Antusch

$$m_D \sim \begin{pmatrix} 0 & b \\ a & 3b \\ a & b \end{pmatrix}, \qquad M \sim \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \mu \sim \begin{pmatrix} 1 & 0 \\ 0 & \omega \end{pmatrix}, \qquad \omega = e^{\frac{2\pi i}{3}}.$$

$$m_\nu = -m_D (M^T)^{-1} \mu M^{-1} m_D^T$$

Talk by Valle

Same low energy matrix

$$m_\nu = m_{\nu a} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_{\nu b}\omega \begin{pmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{pmatrix}$$

Hernandez-Garcia and SFK 1903.01474

Minimal Type Ib seesaw

Z’

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)'$
Q_i	$\mathbf{3}$	$\mathbf{2}$	$1/6$	0
u_i^c	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	0
d_i^c	$\overline{\mathbf{3}}$	$\mathbf{1}$	$1/3$	0
L_i	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	0
e_i^c	$\mathbf{1}$	$\mathbf{1}$	1	0
ν^c	$\mathbf{1}$	$\mathbf{1}$	0	1
$\overline{\nu^c}$	$\mathbf{1}$	$\mathbf{1}$	0	-1
ϕ	$\mathbf{1}$	$\mathbf{1}$	0	1
H_u	$\mathbf{1}$	$\mathbf{2}$	$1/2$	-1
H_d	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	-1

$$y_i^\nu H_u L_i \nu^c + \epsilon_1 y_i^{\nu\prime} \tilde{H}_d L_i \overline{\nu^c}$$

Assume Hd couplings small

$$M^\nu = \begin{array}{c} \\ \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu^c \\ \overline{\nu^c} \end{array} \begin{array}{c} \begin{array}{ccccc} \nu_1 & \nu_2 & \nu_3 & \nu^c & \overline{\nu^c} \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & y_1^\nu v & \epsilon_1 y_1^{\nu\prime} v' \\ 0 & 0 & 0 & y_2^\nu v & \epsilon_1 y_2^{\nu\prime} v' \\ 0 & 0 & 0 & y_3^\nu v & \epsilon_1 y_3^{\nu\prime} v' \\ y_1^\nu v & y_2^\nu v & y_3^\nu v & 0 & M^\nu \\ \epsilon_1 y_1^{\nu\prime} v' & \epsilon_1 y_2^{\nu\prime} v' & \epsilon_1 y_3^{\nu\prime} v' & M^\nu & 0 \end{pmatrix} \end{array}$$

Light effective neutrino matrix

$$\hat{m}_{ij} = \frac{\epsilon_1 v v'}{M^\nu} \left(y_i^\nu y_j^{\nu\prime} + y_i^{\nu\prime} y_j^\nu \right)$$

Unitarity violation due to large y

$$\eta_{ij} = \frac{1}{2 M^{\nu 2}} \left(v^2 y_i^{\nu *} y_j^\nu + \epsilon_1^2 v'^2 y_i^{\nu\prime *} y_j^{\nu\prime} \right) \simeq \frac{v^2}{2 M^{\nu 2}} y_i^{\nu *} y_j^\nu$$

Hernandez-Garcia and SFK 1903.01474

Minimal Type Ib seesaw