A fresh look at the Gauge Coupling Unification and Proton Decay

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In collaboration with:

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◉ Motivation of SUSY

- Fine-tuning Problem

- Dark Matter

- Gauge Coupling Unification

◉ Motivation of SUSY

- Fine-tuning Problem ➡ tension with LHC;
but better than non-SUSY

- Dark Matter ➡ well studied; consistent if its pure Higgsino (~1TeV) or pure Wino (~3TeV)

- Gauge Coupling Unification

◉ Motivation of SUSY

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- Dark Matter → well studied; consistent if its pure Higgsino (~1TeV) or pure Wino (~3TeV)

- Gauge Coupling Unification

↓

not well studied compared to the other two

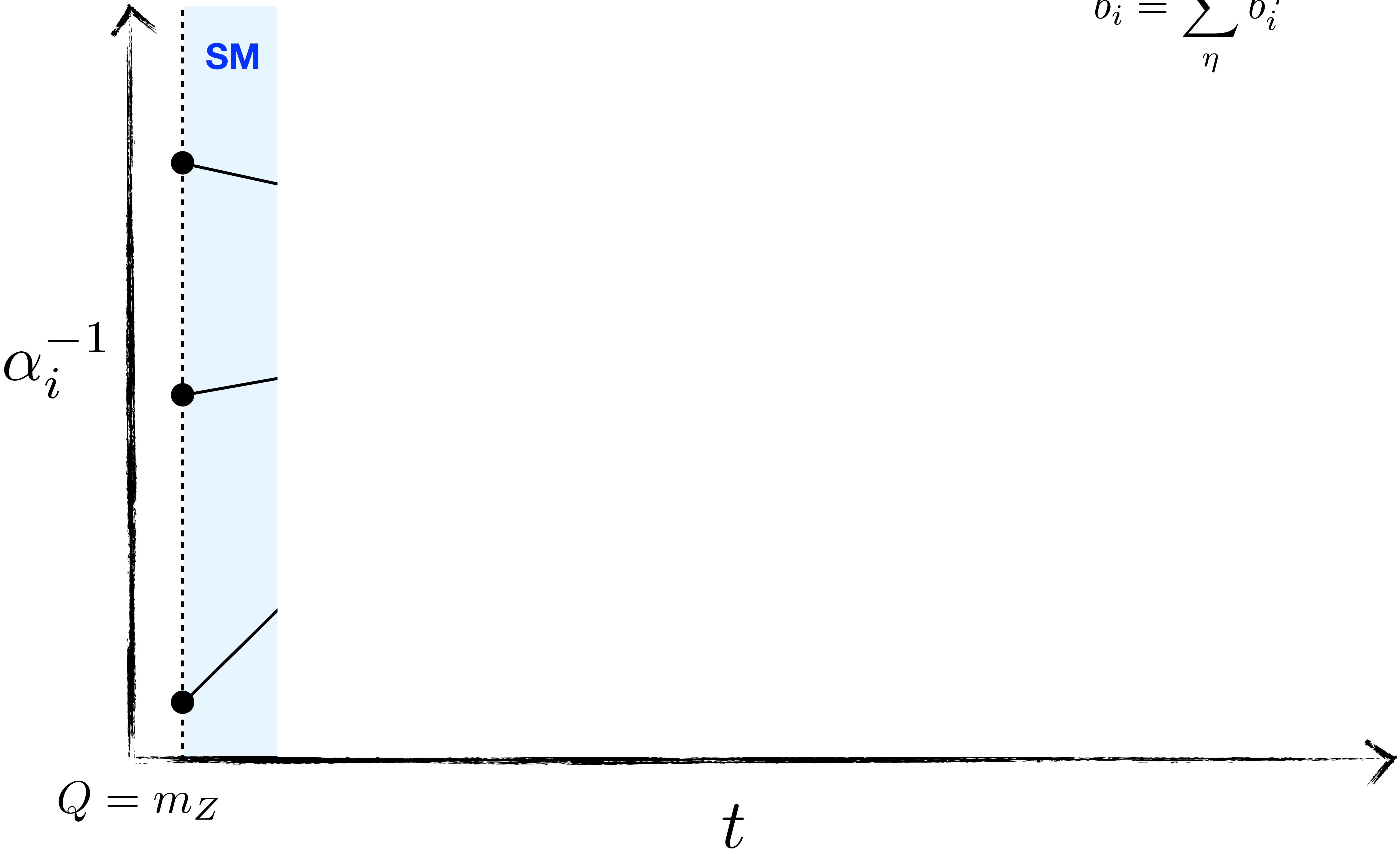
- How is the condition of GCU formulated?
- Is there an upper/lower limit on SUSY masses from GCU?
- Any relation between low energy SUSY and proton decay?

$$\boxed{\frac{d}{dt}\tilde{\alpha}_i^{-1} = b_i}$$

$$\tilde{\alpha}_i^{-1} \equiv 2\pi\alpha_i^{-1}$$

$$t \equiv \ln\left(Q/Q_0\right) > 0$$

$$b_i = \sum_{\eta} b_i^{\eta}$$



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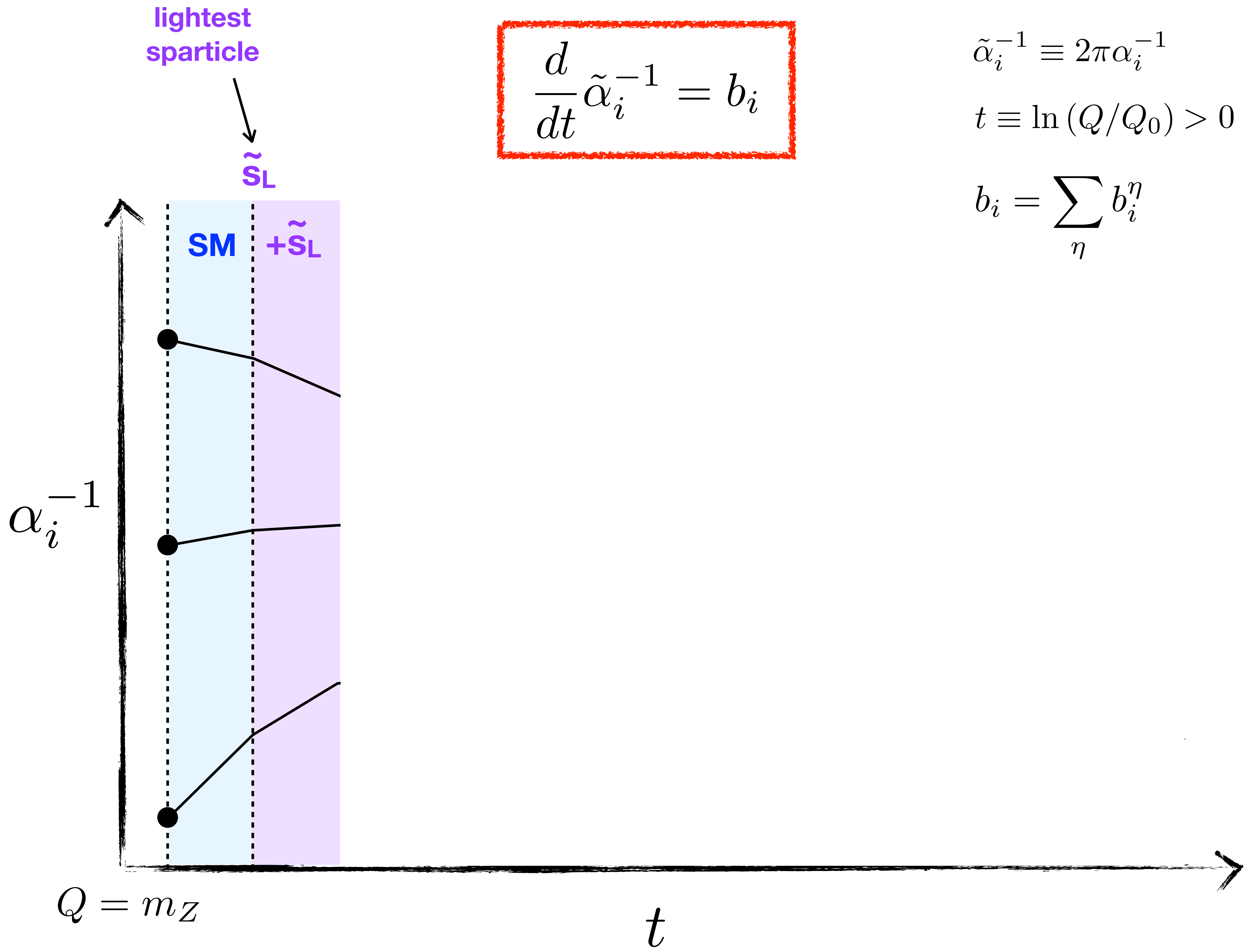

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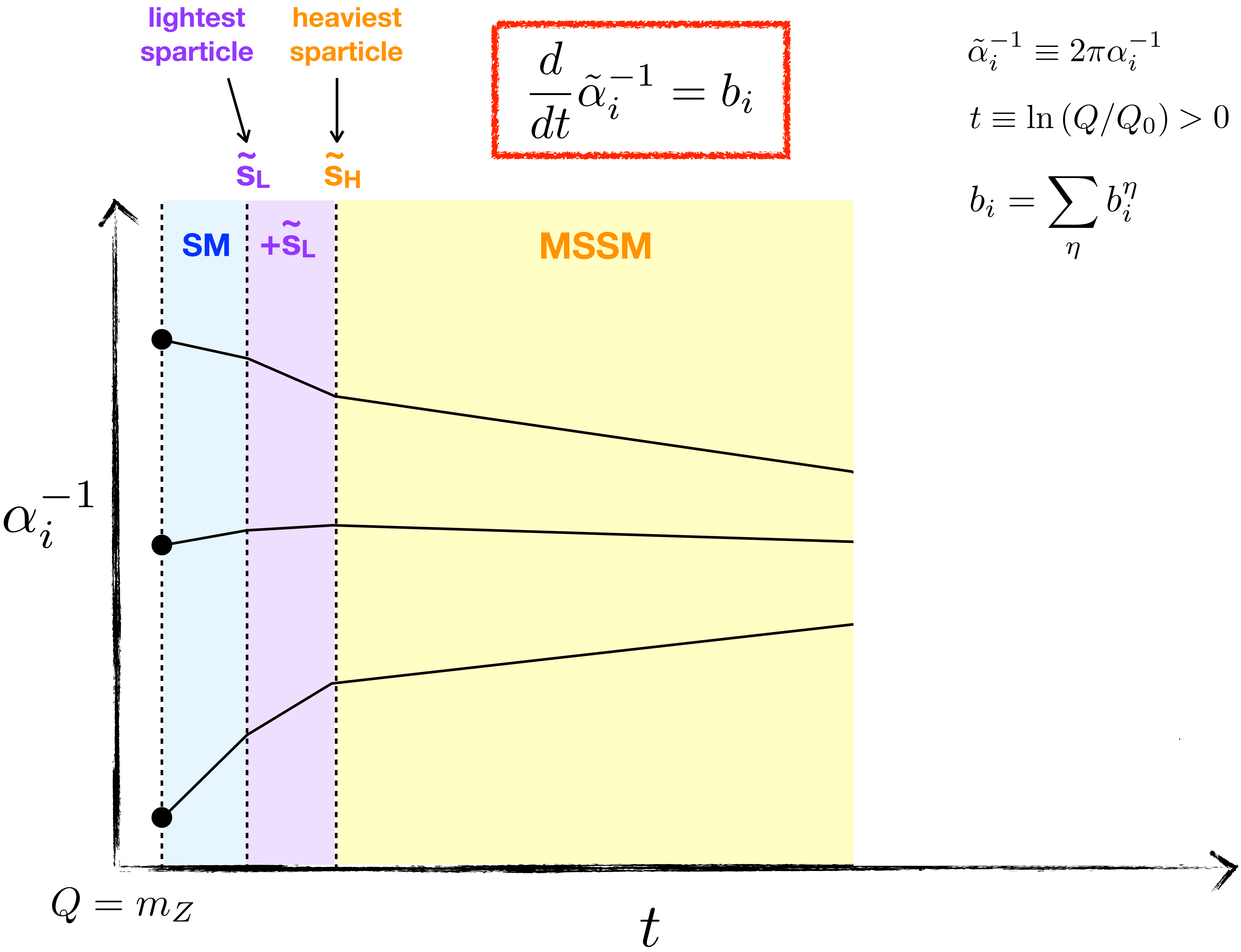

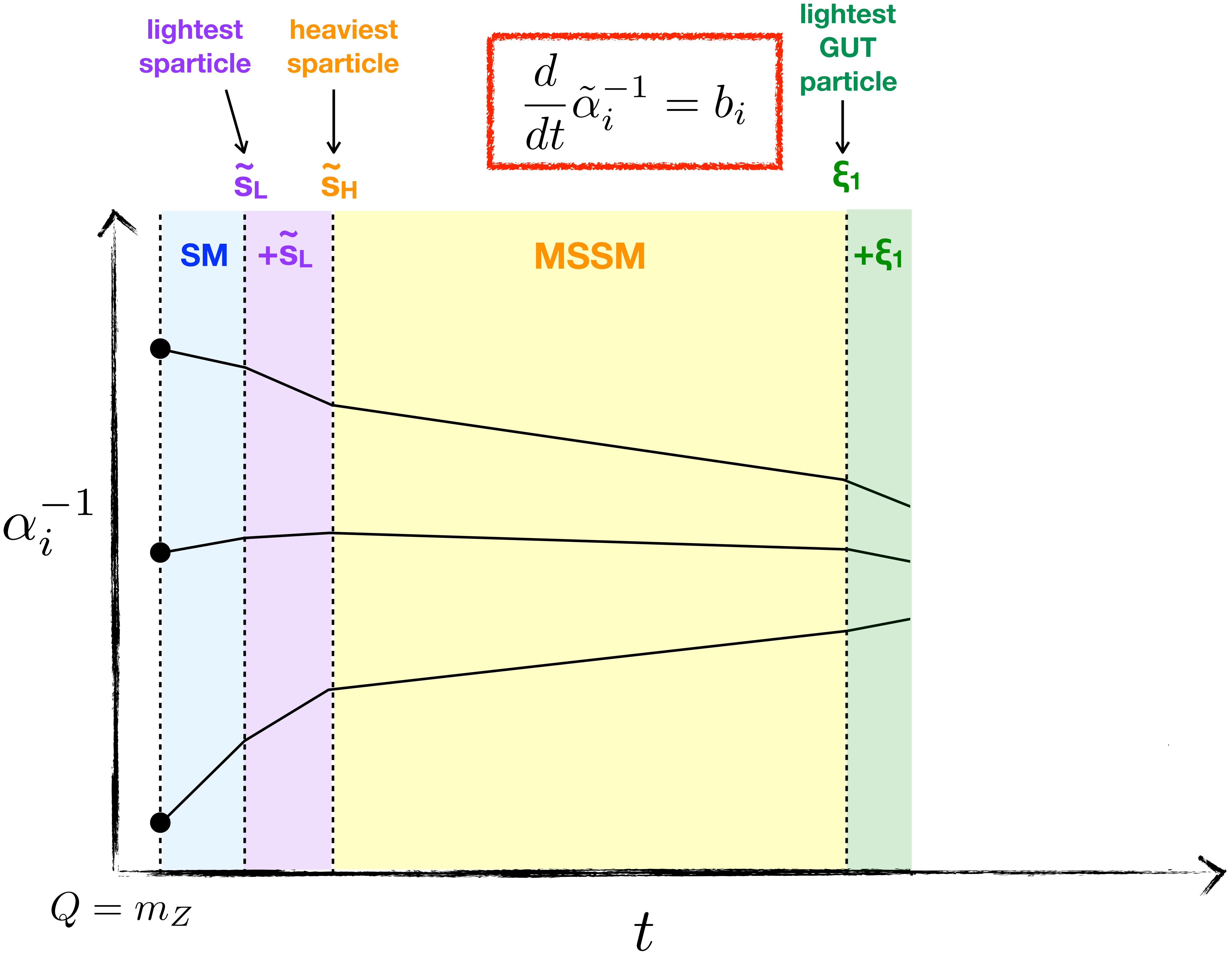

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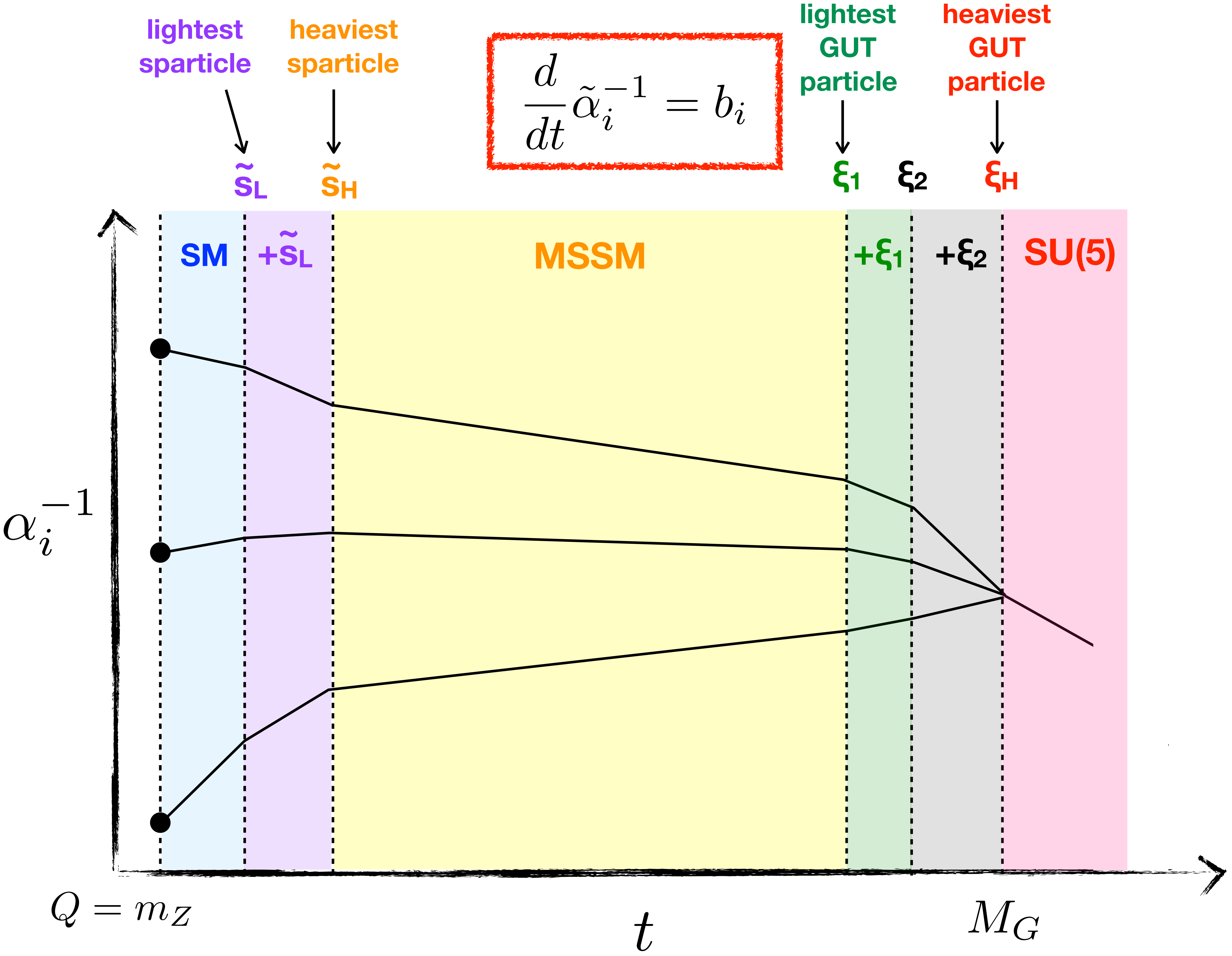
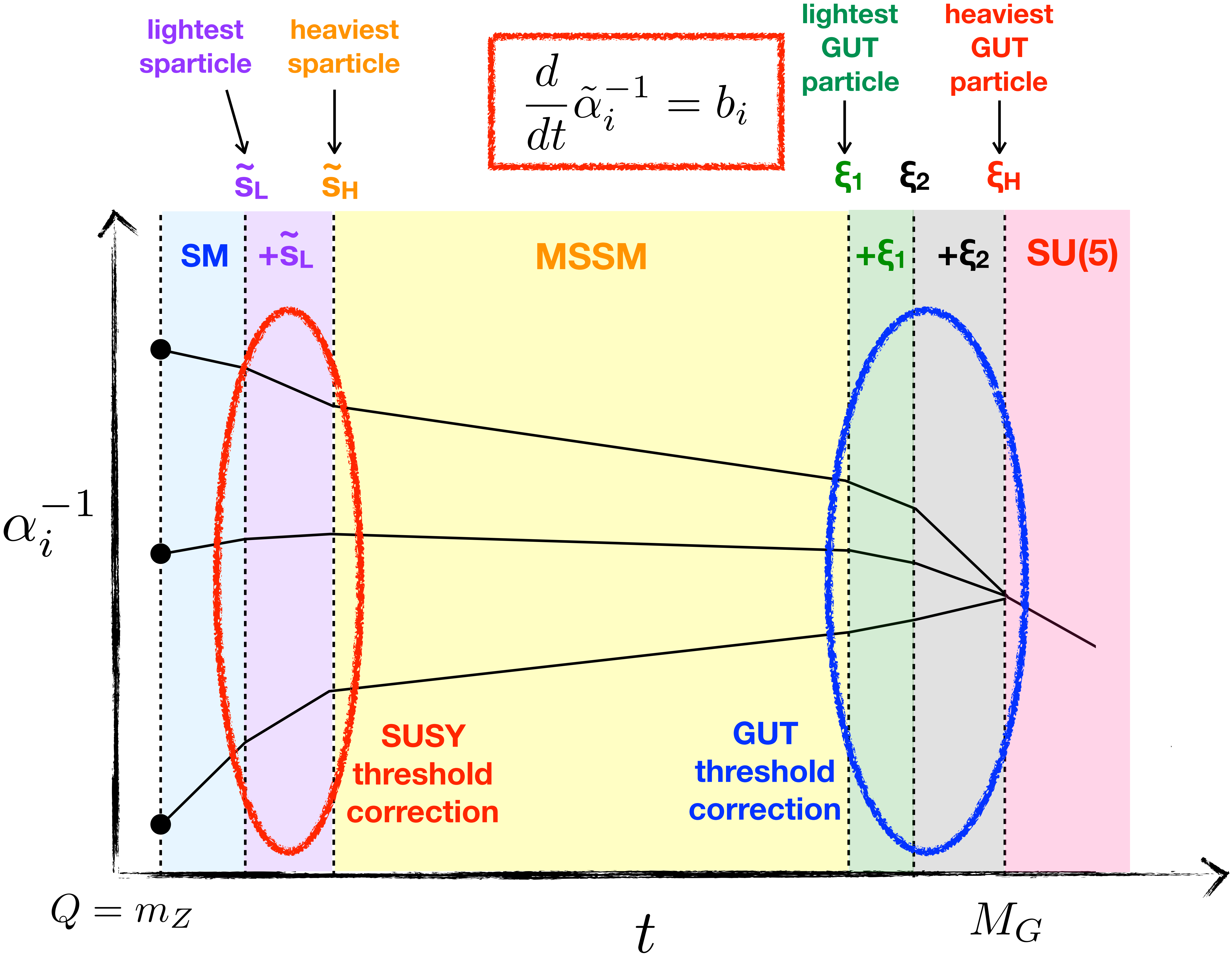

lightest sparticle
heaviest sparticle
$$\frac{d}{dt}\tilde{\alpha}_i^{-1} = b_i$$
lightest GUT particle
heaviest GUT particle
$\tilde{s}_L$
$\tilde{s}_H$
ξ_1
ξ_2
ξ_H
SM
$+\tilde{s}_L$
MSSM
$+\xi_1$
$+\xi_2$
SU(5)
α_i^{-1}
$Q = m_Z$
t
M_G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α_i^{-1}
SUSY threshold correction
GUT threshold correction
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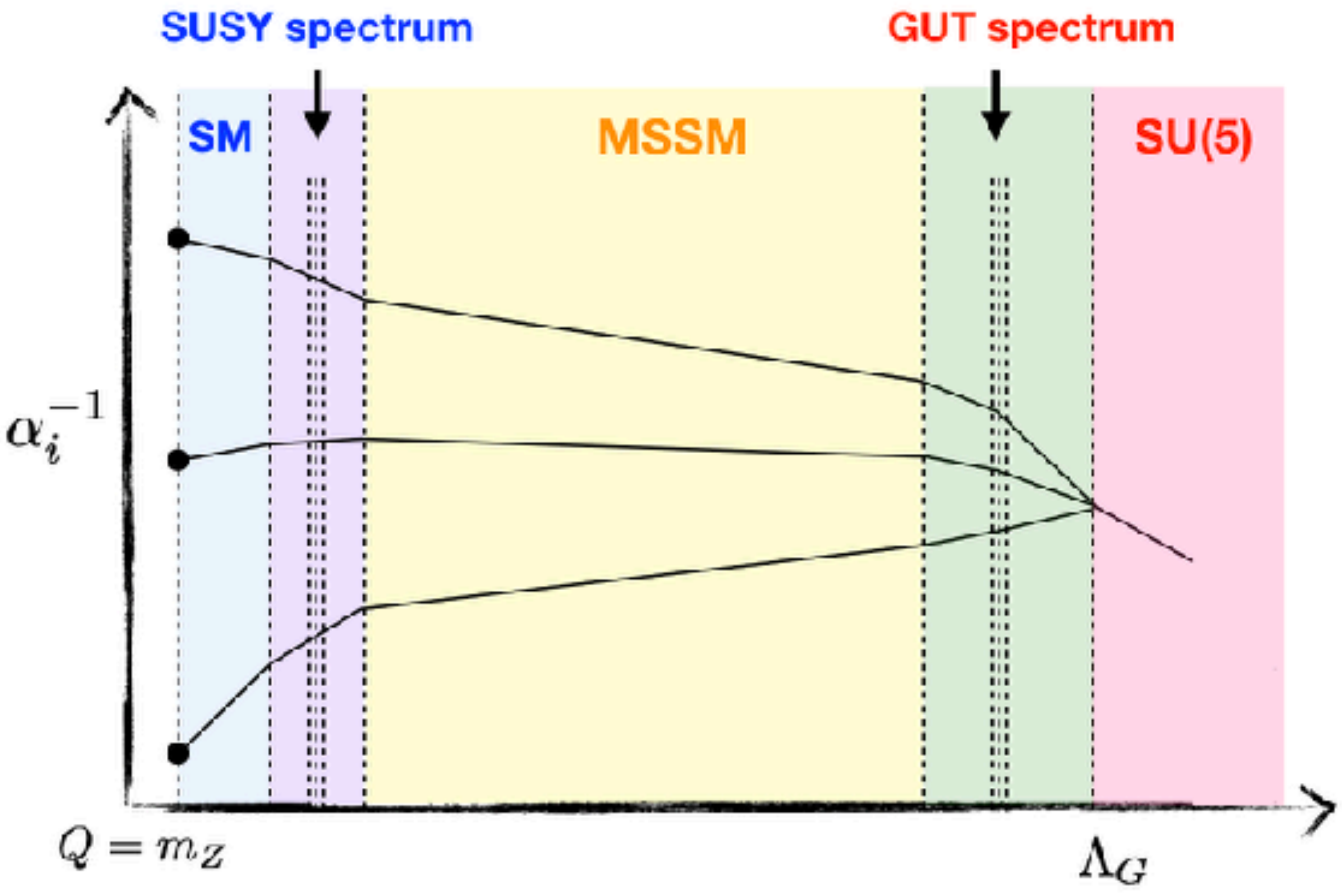
t
M_G



GCU gives constraints on SUSY and GUT spectrum.

Can we formulate such a constraint analytically?

The condition of GCU (2-loop level)

$$T_S(\mathbf{m}_S) = M_S^*(\alpha_s^{mz})\,\Omega_G(\mathbf{m}_\xi) \quad \bigcap \quad T_G(\mathbf{m}_\xi) = M_G^*(\alpha_s^{mz})\,\Omega_S(\mathbf{m}_S)$$

dim-1 function of
SUSY masses

dim-0 function of
GUT masses

dim-1 function of
GUT masses

dim-0 function of
SUSY masses

$$M_S^* = 2.08\,\mathrm{TeV} + \epsilon\,(\alpha_s^{mz})$$

$$M_G^* = 1.27 \cdot 10^{16}\,\mathrm{GeV} + \epsilon'\,(\alpha_s^{mz})$$

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$$M_G^* = 1.27 \cdot 10^{16}\,\mathrm{GeV} \; + \epsilon'\,(\alpha_s^{mz})$$

gluino **wino** **higgsino** **heavy Higgs** **sfermions**

$$T_S = \left[M_3^{-28} M_2^{32} \mu^{12} m_A^3 X_T\right]^{\frac{1}{19}}$$

$$\Omega_S = \left[M_3^{-100} M_2^{60} \mu^{32} m_A^8 X_\Omega\right]^{\frac{1}{288}}$$

$$X_T \equiv \prod_{i=1\ldots3} \left(\frac{m^3_{\tilde{l}_i}}{m^3_{\tilde{d}_{Ri}}}\right)\left(\frac{m^7_{\tilde{q}_i}}{m^2_{\tilde{e}_{Ri}} m^5_{\tilde{u}_{Ri}}}\right)$$

$$X_\Omega \equiv \prod_{i=1\ldots3} \left(\frac{m^8_{\tilde{l}_i}}{m^8_{\tilde{d}_{Ri}}}\right)\left(\frac{m^6_{\tilde{q}_i} m_{\tilde{e}_{Ri}}}{m^7_{\tilde{u}_{Ri}}}\right)$$

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$$M_G^* = 1.27 \cdot 10^{16}\,\text{GeV} \;+ \epsilon'\,(\alpha_s^{mz})$$

$$\ln\left(\frac{T_G}{\Lambda}\right) = \sum_\xi \left(-\frac{5}{288}b_1^\xi - \frac{15}{76}b_2^\xi + \frac{25}{114}b_3^\xi\right)\ln\left(\frac{m_\xi}{\Lambda}\right)$$

$$\ln\Omega_G = \sum_\xi \left(\frac{10}{19}b_1^\xi - \frac{24}{19}b_2^\xi + \frac{14}{19}b_3^\xi\right)\ln\left(\frac{m_\xi}{\Lambda}\right)$$

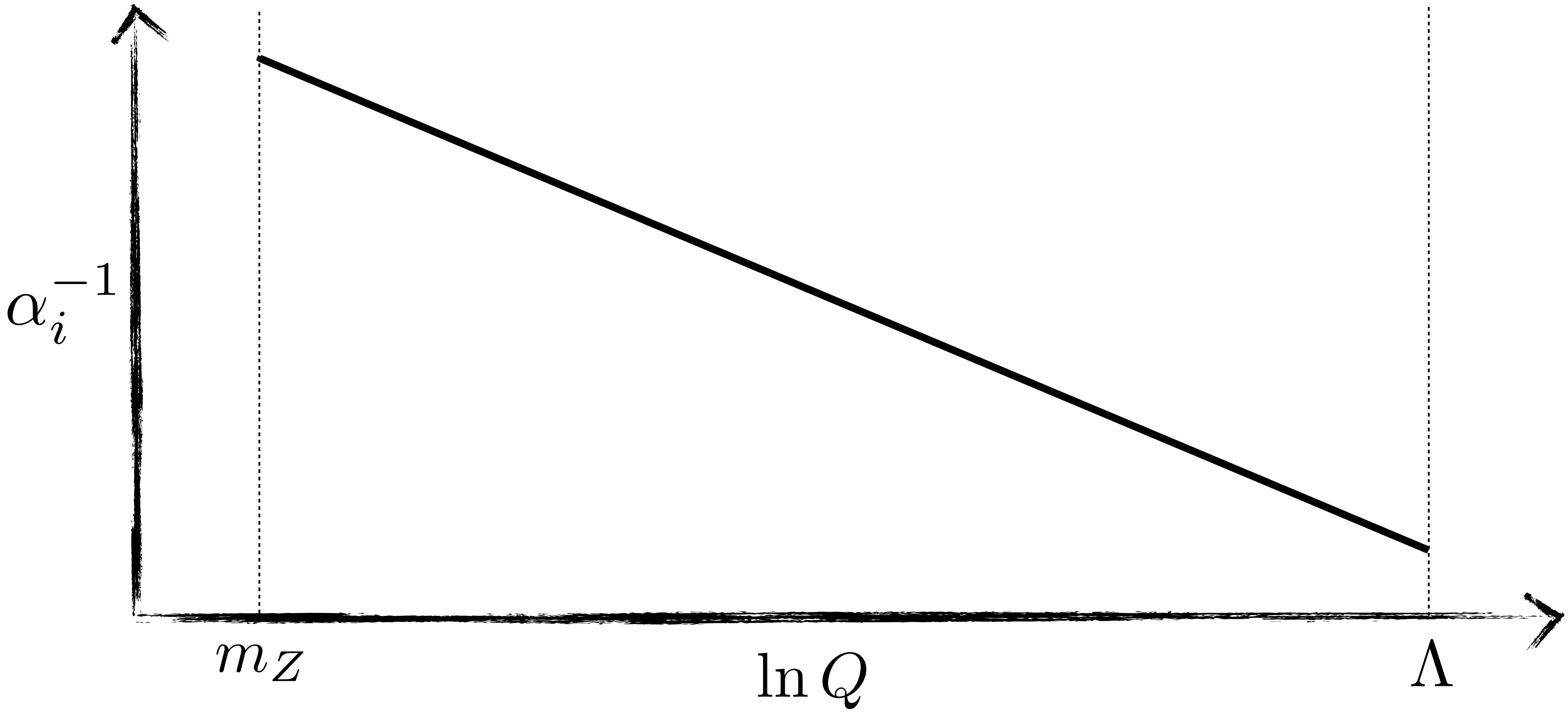
GUT mass spectrum

Unification scale

b_i^ξ **: contribution to the β-coefficient from ξ**

Contents

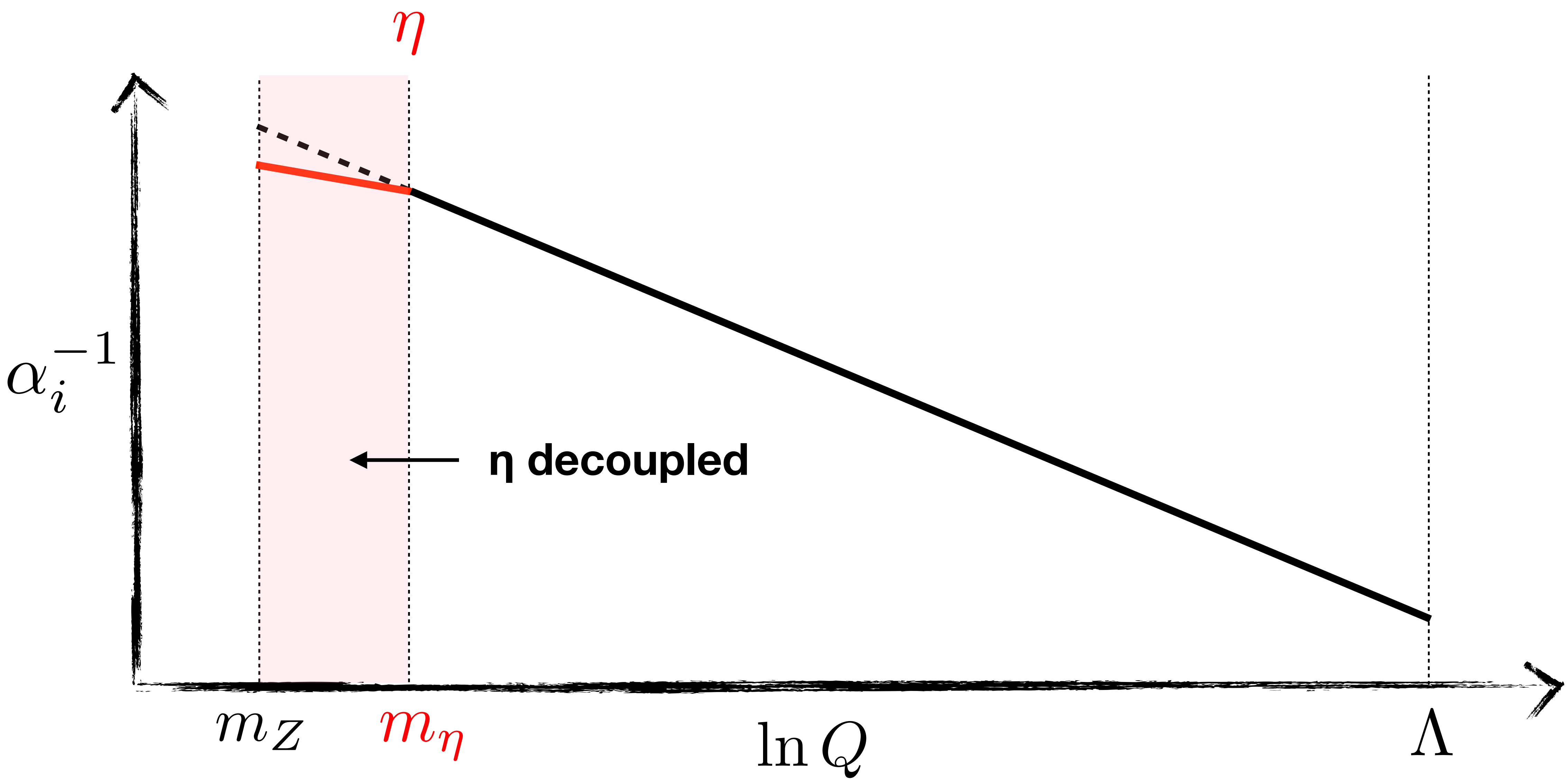
- Derivation of the GCU condition
- Application
  - minimal SUSY SU(5)
  - orbifold SUSY SU(5)



$$\frac{2\pi}{\alpha_i(m_Z)} = \frac{2\pi}{\alpha_i(\Lambda)} + \underset{\Big\uparrow}{b_i} \ln\left(\frac{\Lambda}{m_Z}\right)$$

full MSSM

$$b_i = (\frac{33}{5}, 1, -3)$$



$$\frac{2\pi}{\alpha_i(m_Z)} = \frac{2\pi}{\alpha_i(\Lambda)} + b_i \ln\left(\frac{\Lambda}{m_Z}\right) - b_i^{\eta} \ln\left(\frac{m_\eta}{m_Z}\right)$$

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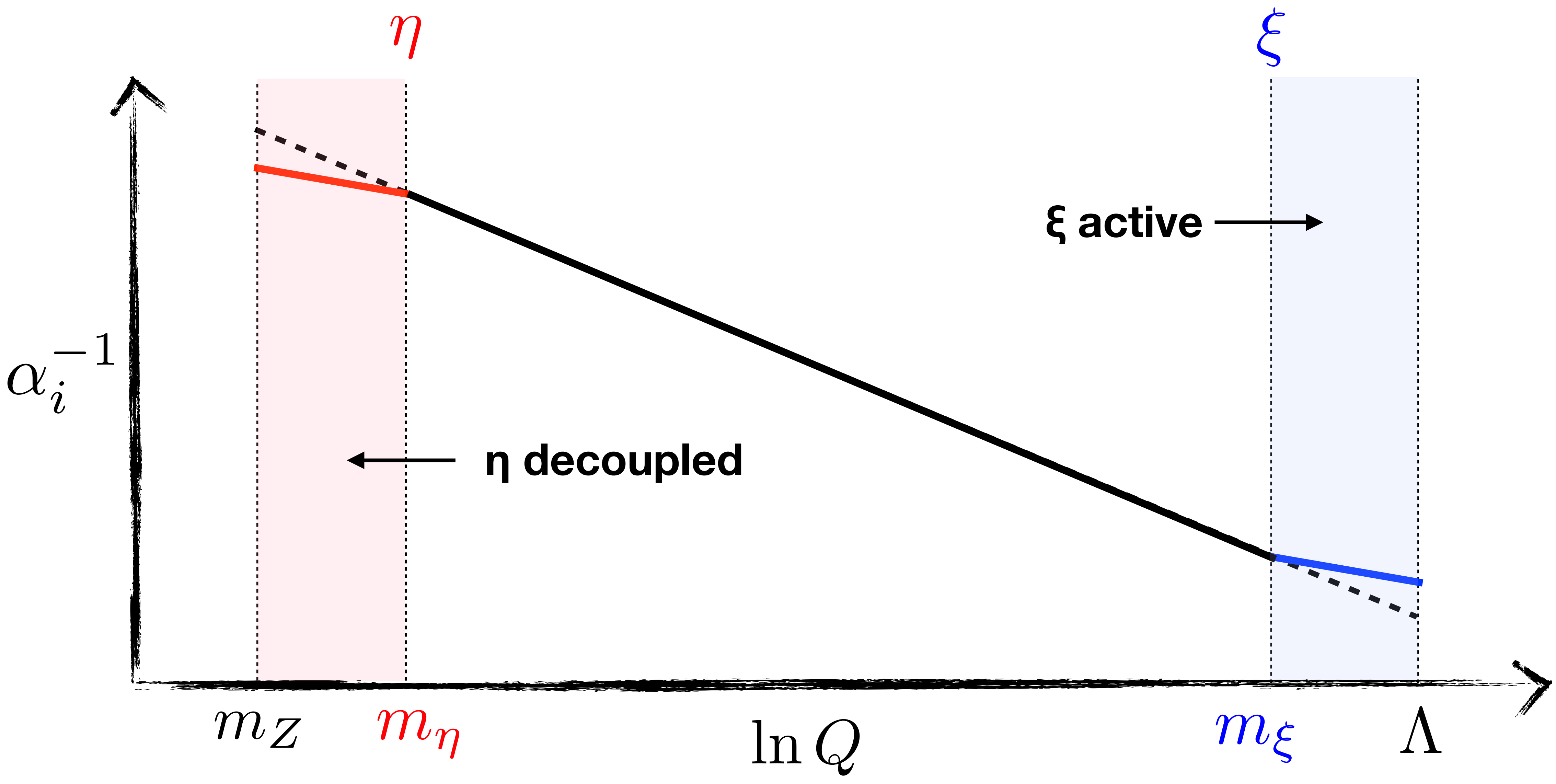
threshold corr.
from η



$$\frac{2\pi}{\alpha_i(m_Z)} = \frac{2\pi}{\alpha_i(\Lambda)} + b_i \ln\left(\frac{\Lambda}{m_Z}\right) - b_i^\eta \ln\left(\frac{m_\eta}{m_Z}\right) + b_i^\xi \ln\left(\frac{\Lambda}{m_\xi}\right)$$

full MSSM

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threshold corr. from η

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General solution to RGE

$$\underset{\underset{\text{unified coupling}}{\uparrow}}{\frac{2\pi}{\alpha(\Lambda)}} = \underset{\underset{\text{experimental input}}{\uparrow}}{\frac{2\pi}{\alpha_i(m_Z)}} - \overset{\overset{\text{full MSSM}}{\downarrow}}{b_i} \ln\left(\frac{\Lambda}{m_Z}\right) + \overset{\overset{\text{SUSY threshold}}{\downarrow}}{s_i} + \underset{\underset{\text{GUT threshold}}{\uparrow}}{r_i}$$

$$s_i = \sum_\eta b_i^\eta \ln\left(\frac{m_\eta}{m_Z}\right)$$

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General solution to RGE

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$$s_i = \sum_\eta b_i^\eta \ln\left(\frac{m_\eta}{m_Z}\right)$$

$$r_i = \sum_\xi b_i^\xi \ln\left(\frac{m_\xi}{\Lambda}\right)$$

[2-loop contribution]

$$b_{ij} = \begin{pmatrix} 199/25 & 27/5 & 88/5 \\ 9/5 & 25 & 24 \\ 11/5 & 9 & 14 \end{pmatrix}$$

$$\begin{aligned} \gamma_i &= -\frac{1}{2}\sum_j \frac{b_{ij}}{b_j} \ln\left(\frac{\alpha_j(\Lambda)}{\alpha_j(m_Z)}\right) \\ &\simeq -\frac{1}{2}\sum_j \frac{b_{ij}}{b_j} \ln\left(1 + \frac{b_j \alpha(\Lambda)}{2\pi} \ln \frac{\Lambda}{m_Z}\right) \end{aligned}$$

one can find γ_i by iteratively updating $\alpha(\Lambda)$ and Λ

General solution to RGE

$$s_i = \sum_\eta b_i^\eta \ln\left(\frac{m_\eta}{m_Z}\right)$$

$$r_i = \sum_\xi b_i^\xi \ln\left(\frac{m_\xi}{\Lambda}\right)$$

$$\underset{\text{unified coupling}}{\frac{2\pi}{\alpha(\Lambda)}} = \underset{\text{experimental input}}{\frac{2\pi}{\alpha_i(m_Z)}} \overset{\text{full MSSM}}{-\, b_i \ln\left(\frac{\Lambda}{m_Z}\right)} + \overset{\text{SUSY threshold}}{s_i} + \underset{\text{GUT threshold}}{r_i} + \overset{\text{2-loop contribution}}{\gamma_i} + \underset{\text{top-quark threshold, MSbar-DRber conversion}}{\Delta_i}$$

- We trade 3 exp inputs with the ***3 constants***: $[\alpha_1(m_Z), \alpha_2(m_Z), \alpha_3(m_Z)] \rightarrow [M_S^*, M_G^*, \alpha_G^*]$

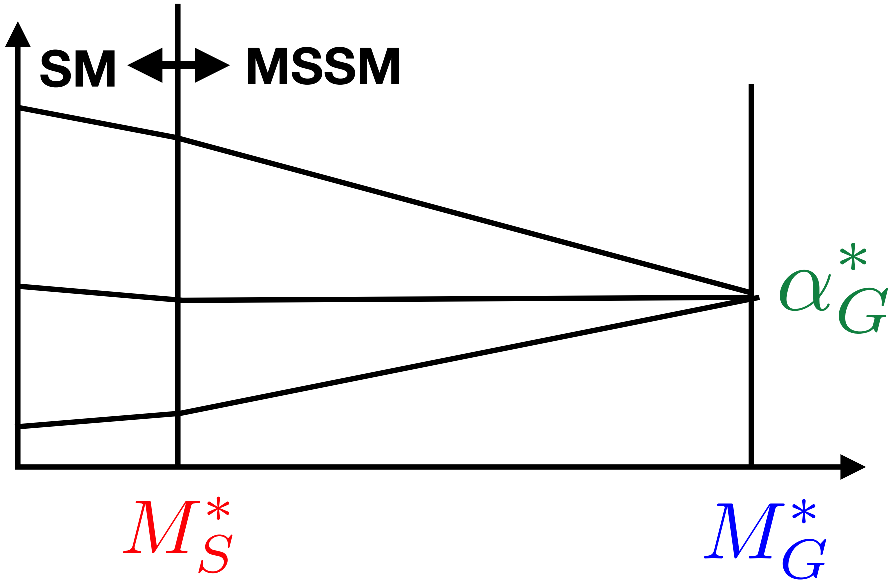
$$\frac{2\pi}{\alpha_i(m_Z)} = \frac{2\pi}{\alpha_G^*} + b_i \ln\left(\frac{M_G^*}{m_Z}\right) - \delta_i \ln\left(\frac{M_S^*}{m_Z}\right) - \gamma_i - \Delta_i$$

degenerate SUSY without GUT thres

$$b_i = (\frac{33}{5}, 1, -3)$$

$$\delta_i \equiv \sum_\eta b_i^\eta = b_i - b_i^{\rm SM}$$

$$= (\frac{2}{5}, \frac{25}{6}, 4)$$



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degenerate SUSY without GUT thres

$$\Big\downarrow$$

$$\frac{2\pi}{\alpha(\Lambda)} = \boxed{\frac{2\pi}{\alpha_i(m_Z)}} - b_i \ln\left(\frac{\Lambda}{m_Z}\right) + s_i + r_i + \gamma_i + \Delta_i$$

general solution

$$= \frac{2\pi}{\alpha_G^*} + b_i \ln\left(\frac{M_G^*}{\Lambda}\right) - \delta_i \ln\left(\frac{M_s^*}{m_Z}\right) + \underset{\substack{\uparrow \\ \text{SUSY threshold}}}{s_i} + \underset{\substack{\uparrow \\ \text{GUT threshold}}}{r_i}$$

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SUSY threshold **GUT threshold**

Any 3D vector can be decomposed into a sum of 3 independent vectors: *1, b_i, δ_i*

$$\vec{1} = (1,1,1) \qquad \vec{b} = (\frac{33}{5}, 1, -3) \qquad \vec{\delta} \equiv \vec{b} - \vec{b}_{\rm SM} = (\frac{2}{5}, \frac{25}{6}, 4)$$

$$s_i = \sum_\eta b_i^\eta \ln\left(\frac{m_\eta}{m_Z}\right) = C_S + b_i \ln \Omega_S + \delta_i \ln\left(\frac{T_S}{m_Z}\right)$$

$$r_i = \sum_\xi b_i^\xi \ln\left(\frac{m_\xi}{\Lambda}\right) = C_G - b_i \ln\left(\frac{T_G}{\Lambda}\right) - \delta_i \ln \Omega_G$$

$$\boxed{\frac{2\pi}{\alpha_i(m_Z)}} = \frac{2\pi}{\alpha_G^*} + b_i \ln\left(\frac{M_G^*}{m_Z}\right) - \delta_i \ln\left(\frac{M_s^*}{m_Z}\right) - \gamma_i - \Delta_i$$

degenerate SUSY without GUT thres

$$\Big\downarrow$$

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general solution

i-independent

$$= \frac{2\pi}{\alpha_G^*} + b_i \ln\left(\frac{M_G^*}{\Lambda}\right) - \delta_i \ln\left(\frac{M_s^*}{m_Z}\right) + s_i + r_i$$

must vanish

$$= \left[\frac{2\pi}{\alpha_G^*} + C_S + C_G\right] + b_i \ln\left(\frac{M_G^* \Omega_S}{T_G}\right) + \delta_i \ln\left(\frac{T_S}{M_s^* \Omega_G}\right)$$

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degenerate SUSY without GUT thres

$$\downarrow$$

$$\frac{2\pi}{\alpha(\Lambda)} = \boxed{\frac{2\pi}{\alpha_i(m_Z)}} - b_i \ln\left(\frac{\Lambda}{m_Z}\right) + s_i + r_i + \gamma_i + \Delta_i$$

general solution

***i*-independent**

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must vanish

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The condition of gauge coupling unification:

$$T_S = M_s^* \Omega_G \quad \cap \quad T_G = M_G^* \Omega_S$$

$$M_s^* = 2.08\,\mathrm{TeV}$$

$$M_G^* = 1.27 \cdot 10^{16}\,\mathrm{GeV}$$

$$\alpha_G^{*-1} = 25.5$$

The unified coupling at Λ

$$\alpha^{-1}(\Lambda) = \alpha_G^{*-1} + \frac{1}{2\pi}\left(C_S + C_G\right)$$

$$s_i = \sum_{\eta} b_i^{\eta} \ln\left(\frac{m_\eta}{m_Z}\right) = C_S + b_i \ln \Omega_S + \delta_i \ln\left(\frac{T_S}{m_Z}\right)$$

$$b_i = (\frac{33}{5}, 1, -3)$$

$$\delta_i = (\frac{2}{5}, \frac{25}{6}, 4)$$

⇔

$$\begin{pmatrix} \ln\left(\prod_{\eta}\left[\frac{m_\eta}{m_Z}\right]^{b_1^\eta}\right) \\ \ln\left(\prod_{\eta}\left[\frac{m_\eta}{m_Z}\right]^{b_2^\eta}\right) \\ \ln\left(\prod_{\eta}\left[\frac{m_\eta}{m_Z}\right]^{b_3^\eta}\right) \end{pmatrix} = \begin{pmatrix} 1 & b_1 & \delta_1 \\ 1 & b_2 & \delta_2 \\ 1 & b_3 & \delta_3 \end{pmatrix} \begin{pmatrix} C_S \\ \ln \Omega_S \\ \ln\left(\frac{T_S}{m_Z}\right) \end{pmatrix}$$

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⇒

$$T_S = \left[M_3^{-28} M_2^{32} \mu^{12} m_A^3 X_T\right]^{\frac{1}{19}}$$

$$\Omega_S = \left[M_3^{-100} M_2^{60} \mu^{32} m_A^8 X_\Omega\right]^{\frac{1}{288}}$$

$$X_T \equiv \prod_{i=1\ldots3} \left(\frac{m_{\tilde{l}_i}^3}{m_{\tilde{d}_{Ri}}^3}\right)\left(\frac{m_{\tilde{q}_i}^7}{m_{\tilde{e}_{Ri}}^2 m_{\tilde{u}_{Ri}}^5}\right)$$

$$X_\Omega \equiv \prod_{i=1\ldots3} \left(\frac{m_{\tilde{l}_i}^8}{m_{\tilde{d}_{Ri}}^8}\right)\left(\frac{m_{\tilde{q}_i}^6 m_{\tilde{e}_{Ri}}}{m_{\tilde{u}_{Ri}}^7}\right)$$

$$C_S = \frac{125}{19} \ln M_3 - \frac{113}{19} \ln M_2 - \frac{40}{19} \ln \mu - \frac{10}{19} \ln m_A + \sum_{i=1\ldots3} \left[\frac{79}{114} \ln m_{\tilde{d}_{Ri}} - \frac{10}{19} \ln m_{\tilde{l}_i} - \frac{121}{114} \ln m_{\tilde{q}_i} + \frac{257}{228} \ln m_{\tilde{u}_{Ri}} + \frac{33}{76} \ln m_{\tilde{e}_{Ri}}\right]$$

$$r_i = \sum_{\xi} b_i^{\xi} \ln\left(\frac{m_\xi}{\Lambda}\right) = C_G - b_i \ln\left(\frac{T_G}{\Lambda}\right) - \delta_i \ln \Omega_G$$

GUT particle masses

$$\ln\left(\frac{T_G}{\Lambda}\right) = \sum_{\xi} \left(- \frac{5}{288} b_1^{\xi} - \frac{15}{76} b_2^{\xi} + \frac{25}{114} b_3^{\xi} \right) \ln\left(\frac{m_\xi}{\Lambda}\right)$$

$$\ln \Omega_G = \sum_{\xi} \left(\frac{10}{19} b_1^{\xi} - \frac{24}{19} b_2^{\xi} + \frac{14}{19} b_3^{\xi} \right) \ln\left(\frac{m_\xi}{\Lambda}\right)$$

$$C_G = \sum_{\xi} \left(\frac{165}{76} b_1^{\xi} - \frac{339}{76} b_2^{\xi} + \frac{125}{38} b_3^{\xi} \right) \ln\left(\frac{m_\xi}{\Lambda}\right)$$

The condition of gauge coupling unification:

$$T_S = M_s^{*} \Omega_G \ \cap \ T_G = M_G^{*} \Omega_S$$

$$M_s^{*} = 2.08 \ \text{TeV}$$

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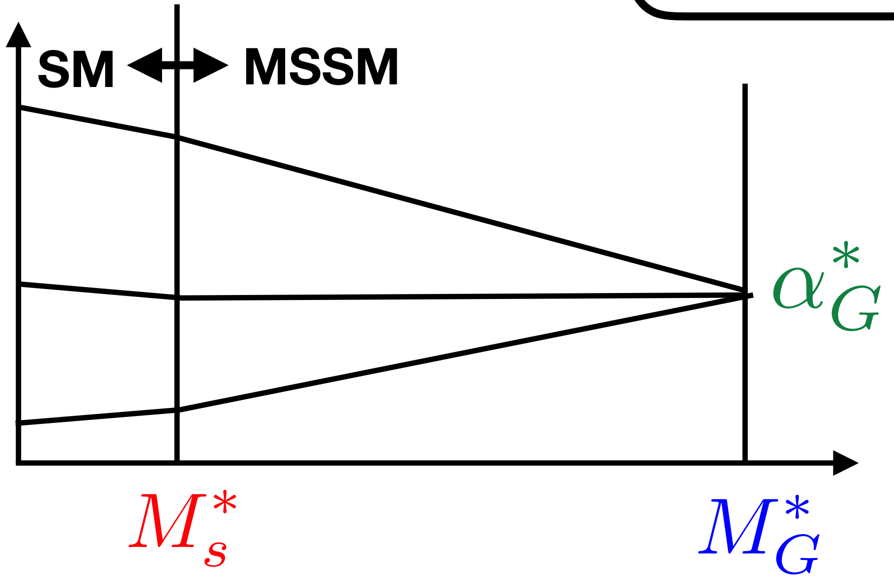
The unified coupling at Λ

$$\alpha^{-1}(\Lambda) = \alpha_G^{*-1} + \frac{1}{2\pi}\left(C_S + C_G\right)$$

Uncertainty of $\alpha_s(m_Z)$

$$\boxed{\alpha_s(m_Z) = \overset{\alpha_s^0}{0.1183} \pm \overset{\Delta\alpha_s}{0.0008}}$$

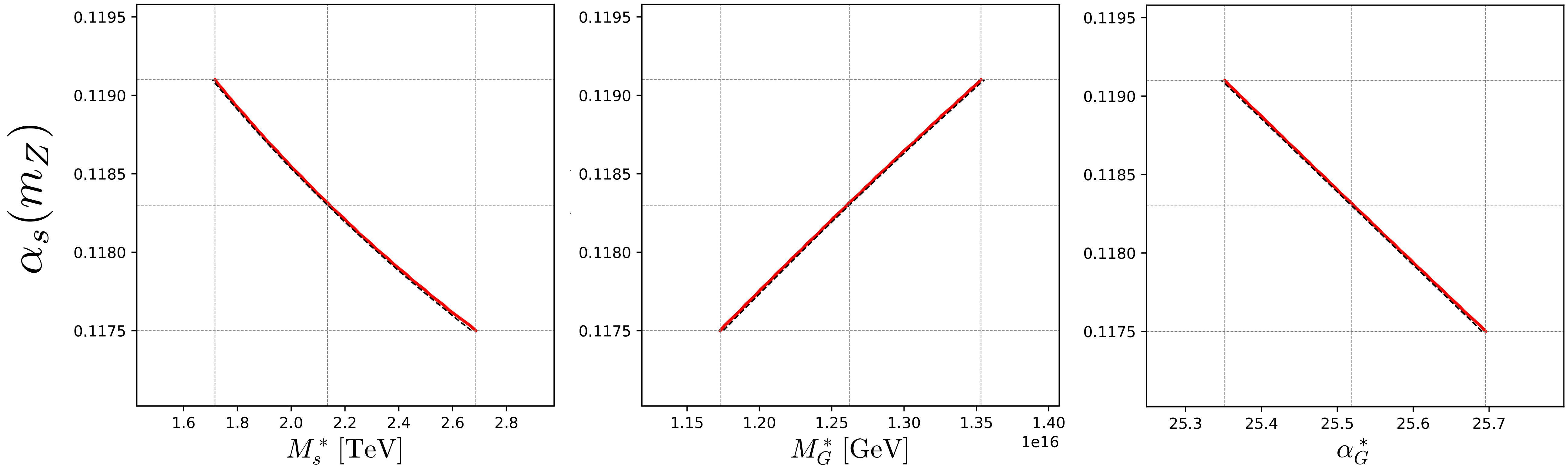
D. d'Enterria [1806.06156]



$$\frac{M_s^*}{\mathrm{TeV}} = \frac{2.08}{\mathrm{TeV}} \cdot \exp\left[-0.224\left(\frac{\alpha_s - \alpha_s^0}{\Delta\alpha_s}\right)\right],$$

$$\frac{M_G^*}{\mathrm{GeV}} = \frac{1.27 \cdot 10^{16}}{\mathrm{GeV}} \cdot \exp\left[0.0715\left(\frac{\alpha_s - \alpha_s^0}{\Delta\alpha_s}\right)\right]$$

$$\alpha_G^{*-1} = 25.5 - 0.172\left(\frac{\alpha_s - \alpha_s^0}{\Delta\alpha_s}\right).$$



$M_s^* \in [2.69,\ 1.72]$ TeV $\quad$ $M_G^* \in [1.17,\ 1.35] \cdot 10^{16}$ GeV $\quad$ $\alpha_G^{*-1} \in [25.7,\ 25.4]$

Application to Minimal SU(5)

Minimal SUSY SU(5)

$$H(\mathbf{5}) = (\boxed{H_C}, H_u) \qquad \Sigma(\mathbf{24}) = (\boxed{\Sigma_8, \Sigma_3}, \Sigma_1, \Sigma_{(2,3)}, \Sigma_{(2,3^*)})$$

$$\overline{H}(\overline{\mathbf{5}}) = (\boxed{\overline{H}_C}, H_d) \qquad \mathcal{V}(\mathbf{24}) = (G, W, B, \boxed{(X,Y),(X,Y)^\dagger})$$

$$\boxed{(H_C, \overline{H}_C)} \rightarrow M_{H_C} = 5\lambda_H V$$

$$\boxed{(X,Y),(X,Y)^\dagger} \rightarrow M_V = 5\sqrt{2} g_5 V$$

$$\boxed{(\Sigma_8, \Sigma_3)} \rightarrow M_\Sigma = \frac{2}{5}\lambda_\Sigma V$$

mass	$(U(1)\times \mathrm{SU}(2)\times \mathrm{SU}(3))$	(b_1, b_2, b_3)
M_{H_C}	$(-\frac{1}{3}, \mathbf{1}, \mathbf{3}),\ (\frac{1}{3}, \mathbf{1}, \overline{\mathbf{3}})$	$(\frac{2}{5}, 0, 1)$
M_V	$(-\frac{5}{6}, \mathbf{2}, \mathbf{3}),\ (\frac{5}{6}, \mathbf{2}, \overline{\mathbf{3}})$	$(-10, -6, -4)$
M_Σ	$(0, \mathbf{3}, \mathbf{1}),\ (0, \mathbf{1}, \mathbf{8})$	$(0, 2, 3)$

Minimal SUSY SU(5)

$$H(\mathbf{5}) = (\boxed{H_C}, H_u) \qquad \Sigma(\mathbf{24}) = (\boxed{\Sigma_8, \Sigma_3}, \Sigma_1, \Sigma_{(2,3)}, \Sigma_{(2,3^*)})$$

$$\overline{H}(\overline{\mathbf{5}}) = (\boxed{\overline{H}_C}, H_d) \qquad \mathcal{V}(\mathbf{24}) = (G, W, B, \boxed{(X,Y), (X,Y)^\dagger})$$

$$\boxed{(H_C, \overline{H}_C)} \rightarrow M_{H_C} = 5\lambda_H V$$

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mass	$(U(1)\times\mathrm{SU}(2)\times\mathrm{SU}(3))$	(b_1, b_2, b_3)
M_{H_C}	$(-\frac{1}{3}, \mathbf{1}, \mathbf{3})$, $(\frac{1}{3}, \mathbf{1}, \overline{\mathbf{3}})$	$(\frac{2}{5}, 0, 1)$
M_V	$(-\frac{5}{6}, \mathbf{2}, \mathbf{3})$, $(\frac{5}{6}, \mathbf{2}, \overline{\mathbf{3}})$	$(-10, -6, -4)$
M_Σ	$(0, \mathbf{3}, \mathbf{1})$, $(0, \mathbf{1}, \mathbf{8})$	$(0, 2, 3)$

$$\ln\left(\frac{T_G}{\Lambda}\right) = \sum_\xi \left(-\frac{5}{288}b_1^\xi - \frac{15}{76}b_2^\xi + \frac{25}{114}b_3^\xi\right) \ln\left(\frac{m_\xi}{\Lambda}\right)$$

$$\ln \Omega_G = \sum_\xi \left(\frac{10}{19}b_1^\xi - \frac{24}{19}b_2^\xi + \frac{14}{19}b_3^\xi\right) \ln\left(\frac{m_\xi}{\Lambda}\right)$$

$$T_G = M_{H_C}^{\frac{4}{19}} \left(M_V^2 M_\Sigma\right)^{\frac{5}{19}}$$

$$\Omega_G = M_{H_C}^{\frac{18}{19}} \left(M_V^2 M_\Sigma\right)^{-\frac{6}{19}}$$

Minimal SUSY SU(5)

$$H(\mathbf{5}) = (\boxed{H_C}, H_u) \qquad \Sigma(\mathbf{24}) = (\boxed{\Sigma_8, \Sigma_3}, \Sigma_1, \Sigma_{(2,3)}, \Sigma_{(2,3^*)})$$

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$$\boxed{(H_C, \overline{H}_C)} \rightarrow M_{H_C} = 5\lambda_H V$$

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M_{H_C}	$(-\frac{1}{3}, \mathbf{1}, \mathbf{3}),\ (\frac{1}{3}, \mathbf{1}, \overline{\mathbf{3}})$	$(\frac{2}{5}, 0, 1)$
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$$\ln\left(\frac{T_G}{\Lambda}\right) = \sum_\xi \left(-\frac{5}{288} b_1^\xi - \frac{15}{76} b_2^\xi + \frac{25}{114} b_3^\xi\right) \ln\left(\frac{m_\xi}{\Lambda}\right)$$

$$\ln \Omega_G = \sum_\xi \left(\frac{10}{19} b_1^\xi - \frac{24}{19} b_2^\xi + \frac{14}{19} b_3^\xi\right) \ln\left(\frac{m_\xi}{\Lambda}\right)$$

$$\Longrightarrow$$

$$T_G = M_{H_C}^{\frac{4}{19}} \left(M_V^2 M_\Sigma\right)^{\frac{5}{19}}$$

$$\Omega_G = M_{H_C}^{\frac{18}{19}} \left(M_V^2 M_\Sigma\right)^{-\frac{6}{19}}$$

GCU condition

$$T_S = M_s^* \Omega_G \ \cap \ T_G = M_G^* \Omega_S$$

$$\Longrightarrow$$

$$M_{H_C} = M_G^* \Omega_S \left(\frac{T_S}{M_s^*}\right)^{\frac{5}{6}}$$

$$(M_V^2 M_\Sigma)^{\frac{1}{3}} = M_G^* \Omega_S \left(\frac{T_S}{M_s^*}\right)^{-\frac{2}{9}}$$

Minimal SUSY SU(5)

D=5 proton decay rate can be calculated from the SUSY masses!

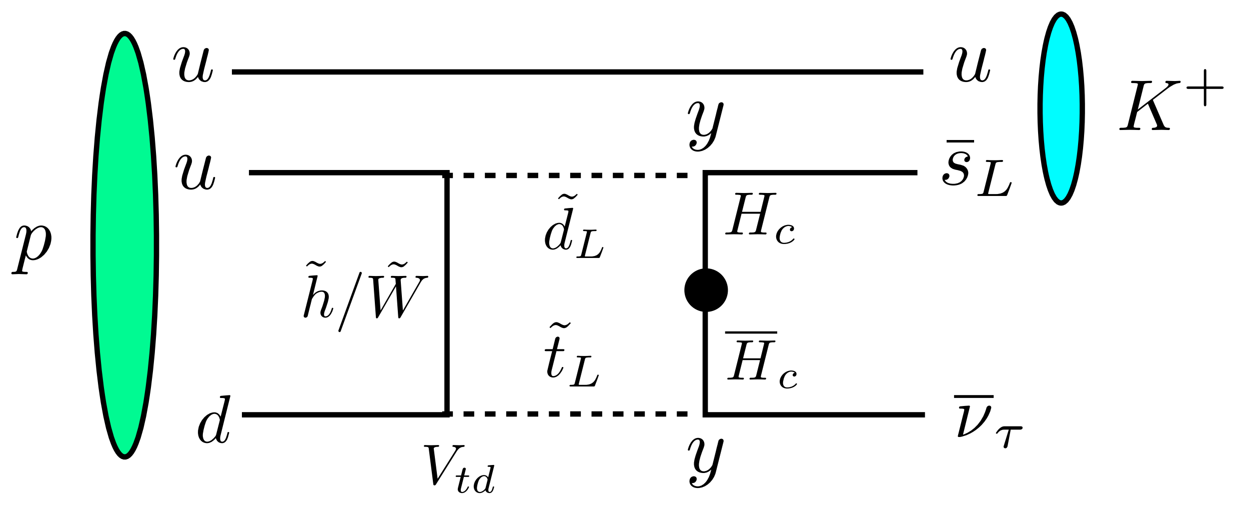
$$T_S = \left[M_3^{-28} M_2^{32} \mu^{12} m_A^3 X_T\right]^{\frac{1}{19}}$$

$$\Omega_S = \left[M_3^{-100} M_2^{60} \mu^{32} m_A^8 X_\Omega\right]^{\frac{1}{288}}$$

$$M_s^* = 2.08\,\mathrm{TeV}$$

$$M_G^* = 1.27 \cdot 10^{16}\,\mathrm{GeV}$$

D=5 proton decay



GCU condition

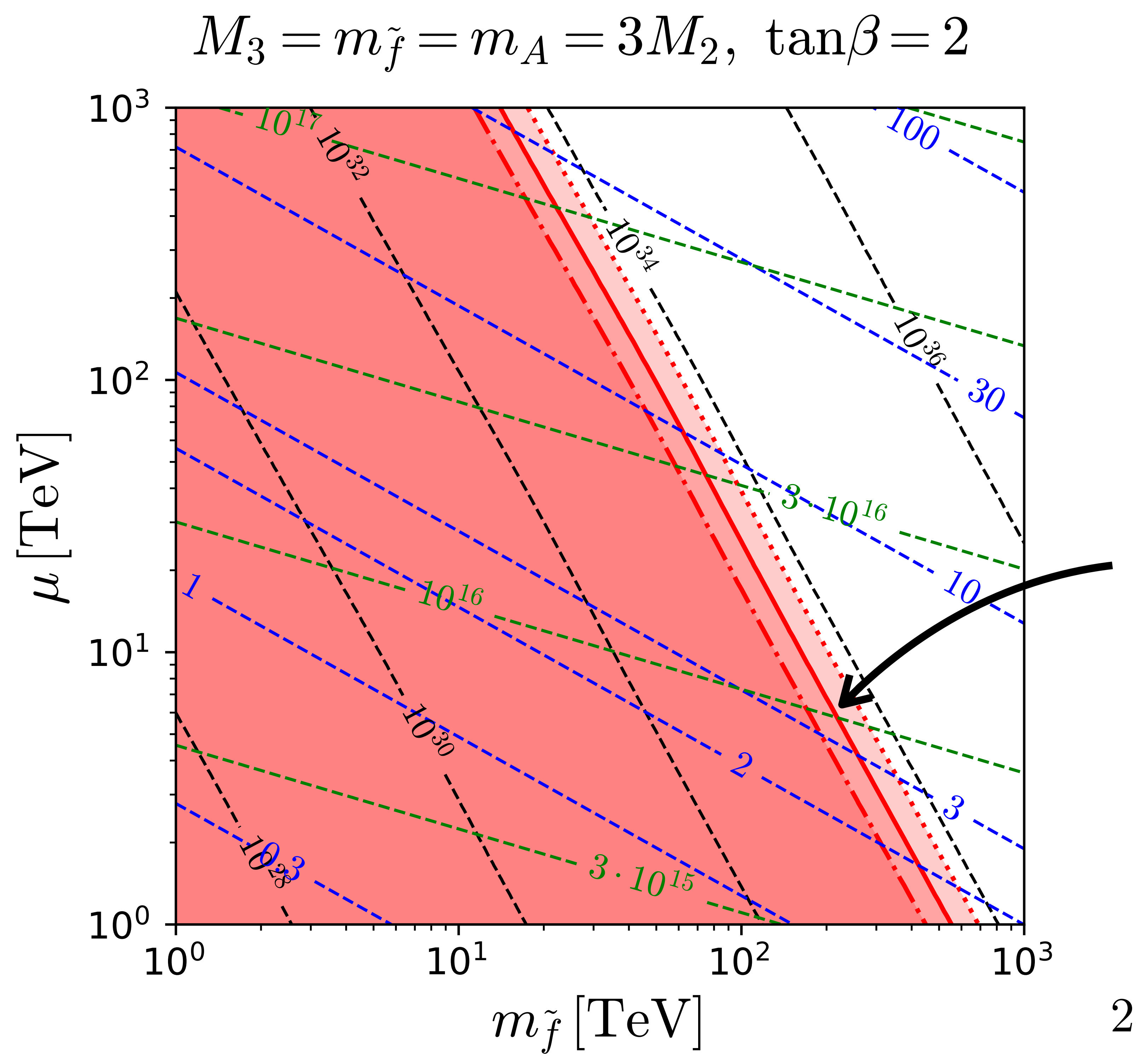
$$T_S = M_s^* \Omega_G \;\cap\; T_G = M_G^* \Omega_S$$

$$\boxed{M_{H_C} = M_G^* \Omega_S \left(\frac{T_S}{M_s^*}\right)^{\frac{5}{6}}}$$

$$(M_V^2 M_\Sigma)^{\frac{1}{3}} = M_G^* \Omega_S \left(\frac{T_S}{M_s^*}\right)^{-\frac{2}{9}}$$

Vanilla SUSY

$(M_V = M_\Sigma)$



$\tau(p \to K^+\bar\nu)/\mathrm{yrs}$

M_{H_C}/GeV

T_S/TeV

$\tau(p \to K^+\bar\nu) > 4.0 \times 10^{33}\,\mathrm{yrs}$

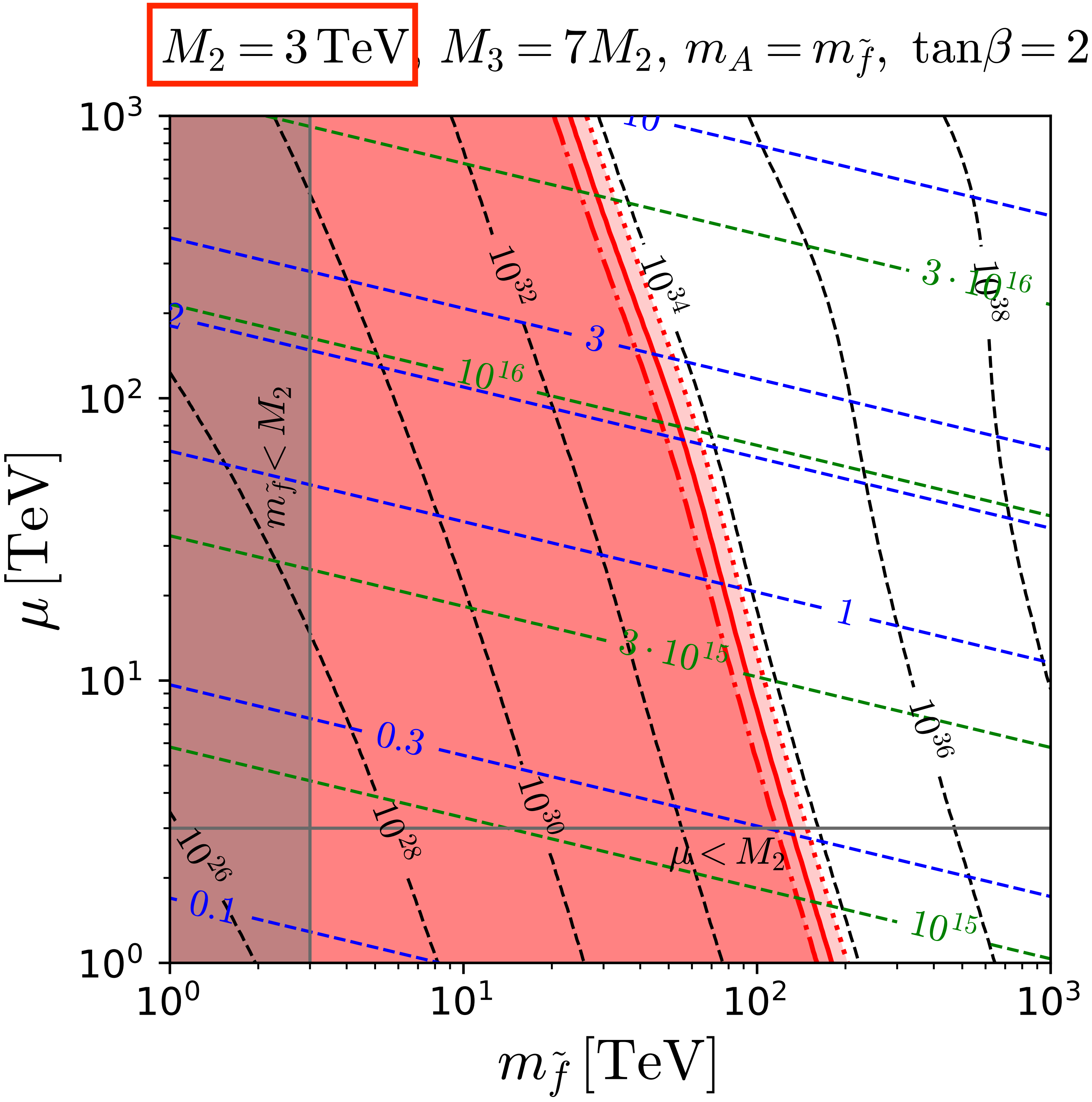
$\alpha_s(m_Z) = 0.1183 \pm 0.0008$

$\Lambda \equiv \max\{M_{H_C}, M_V\}$

$24.7 < \alpha^{-1}(\Lambda) < 31.8 \ \ (M_V = M_\Sigma)$

$3.77 \cdot 10^{15}\,\mathrm{GeV} < (M_V^2 M_\Sigma)^{\frac{1}{3}} < 1.83 \cdot 10^{16}\,\mathrm{GeV}$

AMSB with Wino DM



$$\tau(p \to K^+\bar\nu) > 4.0 \times 10^{33}\,\mathrm{yrs}$$

$$\alpha_s(m_Z) = 0.1183 \pm 0.0008$$

$$\Lambda \equiv \max\{M_{H_C}, M_V\}$$

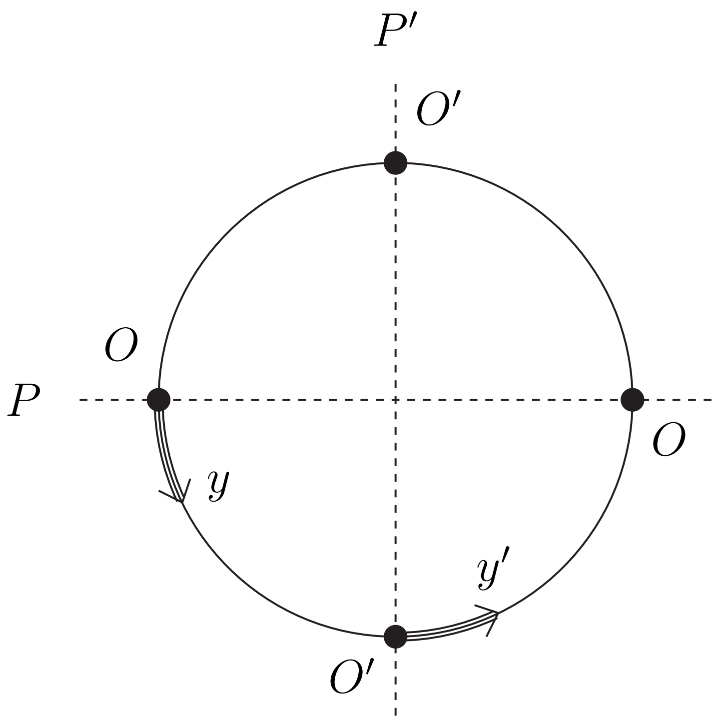
$$25.3 < \alpha^{-1}(\Lambda) < 29.5 \ \ (M_V = M_\Sigma)$$

$$8.90 \cdot 10^{15}\,\mathrm{GeV} < (M_V^2 M_\Sigma)^{\frac{1}{3}} < 1.19 \cdot 10^{16}\,\mathrm{GeV}$$

Orbifold SUSY SU(5) GUT

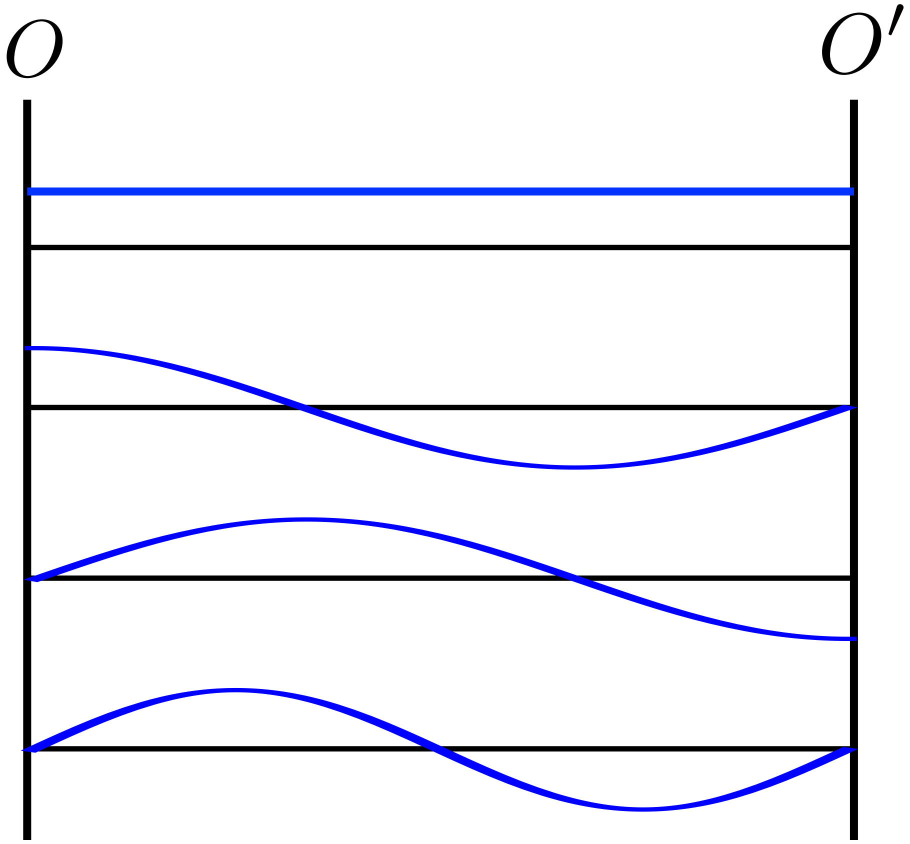
$S^1/(Z_2 \times Z_2')$ orbifold in the fifth dimension

[Hall, Nomura '01]



two orbifold parities (P, P')

$$\begin{aligned} \phi(x^\mu, y) \rightarrow \phi(x^\mu, -y) &= P\phi(x^\mu, y), \\ \phi(x^\mu, y') \rightarrow \phi(x^\mu, -y') &= P'\phi(x^\mu, y'), \end{aligned}$$



(P, P')

$(+,+) \longrightarrow 2n/R$ ← **zero mode only here**

$(+,-) \longrightarrow (2n+1)/R$

$(-,+) \longrightarrow (2n+1)/R$

$(-,-) \longrightarrow (2n+2)/R$

$$n = 0, 1, 2, \cdots$$

a = unbroken generators $\hat{a}$ = broken generators $n = 0, 1, 2, \cdots$

KK mode	m_ξ	(P, P')	4d fields	$\sum(b_1, b_2, b_3)$
zero	0	$(+,+)$	$V^a,\ H_F,\ H_{\overline{F}}$	
even	$(2n+2)/R$	$(+,+)$	$V^a,\ H_F,\ H_{\overline{F}}$	$(\frac{6}{5}, -2, -6)$
		$(-,-)$	$\Sigma^a,\ H_F^c,\ H_{\overline{F}}^c$	
odd	$(2n+1)/R$	$(+,-)$	$V^{\hat{a}},\ H_C,\ H_{\overline{C}}$	$(-\frac{46}{5}, -6, -2)$
		$(-,+)$	$\Sigma^{\hat{a}},\ H_C^c,\ H_{\overline{C}}^c$	

a = unbroken generators $\quad \hat{a}$ = broken generators $\quad n = 0, 1, 2, \cdots$

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zero	0	$(+,+)$	$V^a,\ H_F,\ H_{\overline{F}}$	
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		$(-,-)$	$\Sigma^a,\ H_F^c,\ H_{\overline{F}}^c$	
odd	$(2n+1)/R$	$(+,-)$	$V^{\hat{a}},\ H_C,\ H_{\overline{C}}$	$(-\frac{46}{5}, -6, -2)$
		$(-,+)$	$\Sigma^{\hat{a}},\ H_C^c,\ H_{\overline{C}}^c$	

$(r \equiv R\Lambda)$

$$\ln\left(\frac{T_G}{\Lambda}\right) = \sum_\xi \left(-\frac{5}{288}b_1^\xi - \frac{15}{76}b_2^\xi + \frac{25}{114}b_3^\xi\right)\ln\left(\frac{m_\xi}{\Lambda}\right)$$

$$\ln \Omega_G = \sum_\xi \left(\frac{10}{19}b_1^\xi - \frac{24}{19}b_2^\xi + \frac{14}{19}b_3^\xi\right)\ln\left(\frac{m_\xi}{\Lambda}\right)$$

$r \in$	$(1,2]$	$(2,3]$	$(3,4]$	$(4,5]$	$\cdots$
Ω_G	$\left[\frac{1}{r}\right]^{\frac{24}{19}}$	$\left[\frac{1}{2}\right]^{\frac{24}{19}}$	$\left[\frac{1\cdot3}{2}\frac{1}{r}\right]^{\frac{24}{19}}$	$\left[\frac{1\cdot3}{2\cdot4}\right]^{\frac{24}{19}}$	$\cdots$
T_G/Λ	$\left[\frac{1}{r}\right]^{\frac{18}{19}}$	$\left[\frac{1}{2}\right]^{\frac{18}{19}}$	$\left[\frac{1\cdot3}{2}\frac{1}{r}\right]^{\frac{18}{19}}$	$\left[\frac{1\cdot3}{2\cdot4}\right]^{\frac{18}{19}}$	$\cdots$

a = unbroken generators $\quad \hat{a}$ = broken generators $\quad n = 0, 1, 2, \cdots$

KK mode	m_ξ	(P, P')	4d fields	$\sum(b_1, b_2, b_3)$
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		$(-,-)$	$\Sigma^a,\ H^c_F,\ H^c_{\overline{F}}$	
odd	$(2n+1)/R$	$(+,-)$	$V^{\hat{a}},\ H_C,\ H_{\overline{C}}$	$(-\frac{46}{5}, -6, -2)$
		$(-,+)$	$\Sigma^{\hat{a}},\ H^c_C,\ H^c_{\overline{C}}$	

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$r \in$	$(1,2]$	$(2,3]$	$(3,4]$	$(4,5]$	$\cdots$
Ω_G	$\left[\frac{1}{r}\right]^{\frac{24}{19}}$	$\left[\frac{1}{2}\right]^{\frac{24}{19}}$	$\left[\frac{1\cdot 3}{2}\frac{1}{r}\right]^{\frac{24}{19}}$	$\left[\frac{1\cdot 3}{2\cdot 4}\right]^{\frac{24}{19}}$	$\cdots$
T_G/Λ	$\left[\frac{1}{r}\right]^{\frac{18}{19}}$	$\left[\frac{1}{2}\right]^{\frac{18}{19}}$	$\left[\frac{1\cdot 3}{2}\frac{1}{r}\right]^{\frac{18}{19}}$	$\left[\frac{1\cdot 3}{2\cdot 4}\right]^{\frac{18}{19}}$	$\cdots$

GCU condition

$$T_S = M_s^* \Omega_G$$

$$T_G = M_G^* \Omega_S$$

$$T_S = \left[M_3^{-28} M_2^{32} \mu^{12} m_A^3 X_T\right]^{\frac{1}{19}} = M_s^* \Omega_G$$

non-trivial constraint on SUSY masses

a = unbroken generators $\hat{a}$ = broken generators $n = 0, 1, 2, \cdots$

KK mode	m_ξ	(P, P')	4d fields	$\sum(b_1, b_2, b_3)$
zero	0	$(+,+)$	$V^a,\ H_F,\ H_{\overline{F}}$	
even	$(2n+2)/R$	$(+,+)$	$V^a,\ H_F,\ H_{\overline{F}}$	$(\frac{6}{5}, -2, -6)$
		$(-,-)$	$\Sigma^a,\ H^c_F,\ H^c_{\overline{F}}$	
odd	$(2n+1)/R$	$(+,-)$	$V^{\hat{a}},\ H_C,\ H_{\overline{C}}$	$(-\frac{46}{5}, -6, -2)$
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$(r \equiv R\Lambda)$

$r \in$	$(1, 2]$	$(2, 3]$	$(3, 4]$	$(4, 5]$	$\cdots$
Ω_G	$\left[\frac{1}{r}\right]^{\frac{24}{19}}$	$\left[\frac{1}{2}\right]^{\frac{24}{19}}$	$\left[\frac{1\cdot 3}{2}\frac{1}{r}\right]^{\frac{24}{19}}$	$\left[\frac{1\cdot 3}{2\cdot 4}\right]^{\frac{24}{19}}$	$\cdots$
T_G/Λ	$\left[\frac{1}{r}\right]^{\frac{18}{19}}$	$\left[\frac{1}{2}\right]^{\frac{18}{19}}$	$\left[\frac{1\cdot 3}{2}\frac{1}{r}\right]^{\frac{18}{19}}$	$\left[\frac{1\cdot 3}{2\cdot 4}\right]^{\frac{18}{19}}$	$\cdots$

GCU condition

$$T_S = M_s^* \Omega_G$$

$$T_G = M_G^* \Omega_S$$

$$T_S = \left[M_3^{-28} M_2^{32} \mu^{12} m_A^3 X_T\right]^{\frac{1}{19}} = M_s^* \Omega_G$$

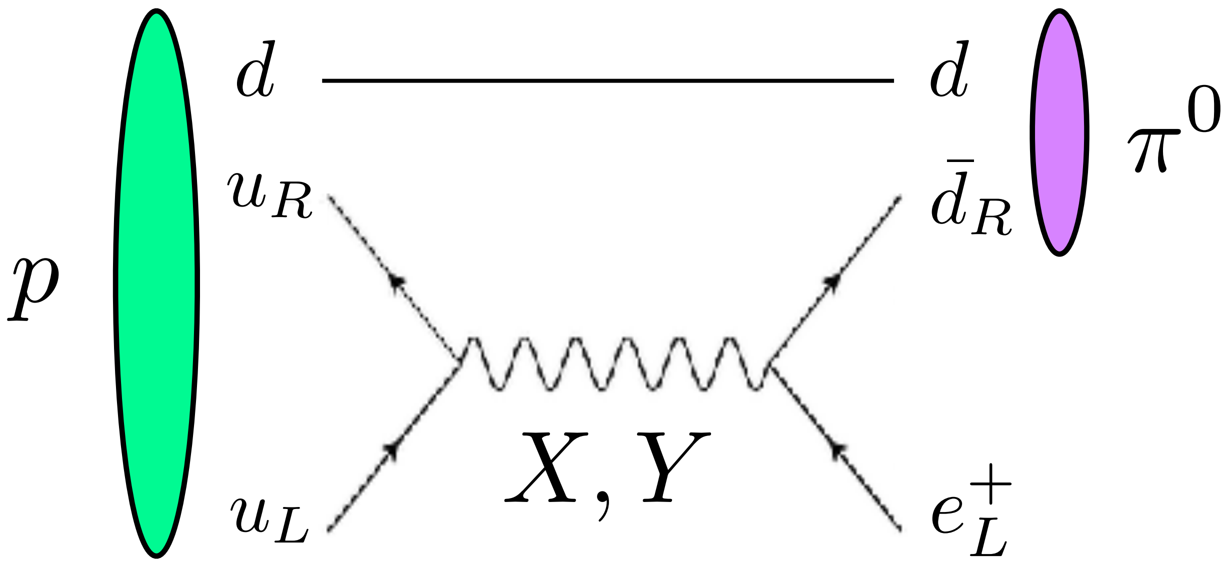
non-trivial constraint on SUSY masses

$$\frac{1}{R} = M_{(X,Y)_1} = M_G^* \Omega_G{}^{-\frac{3}{4}} \Omega_S / r$$

$$= M_G^* M_s^*{}^{\frac{19}{108}} \Omega_G{}^{-\frac{31}{54}} M_3{}^{-\frac{19}{216}} M_2{}^{-\frac{19}{216}} X_T{}^{-\frac{1}{108}} X_\Omega{}^{-\frac{1}{288}} / r$$

M_c = 1/R, (i.e. X,Y boson mass) can be predicted from SUSY spectrum, allowing to predict D=6 proton decay

D=6 proton decay



GCU condition

$$T_S = M_s^* \Omega_G$$

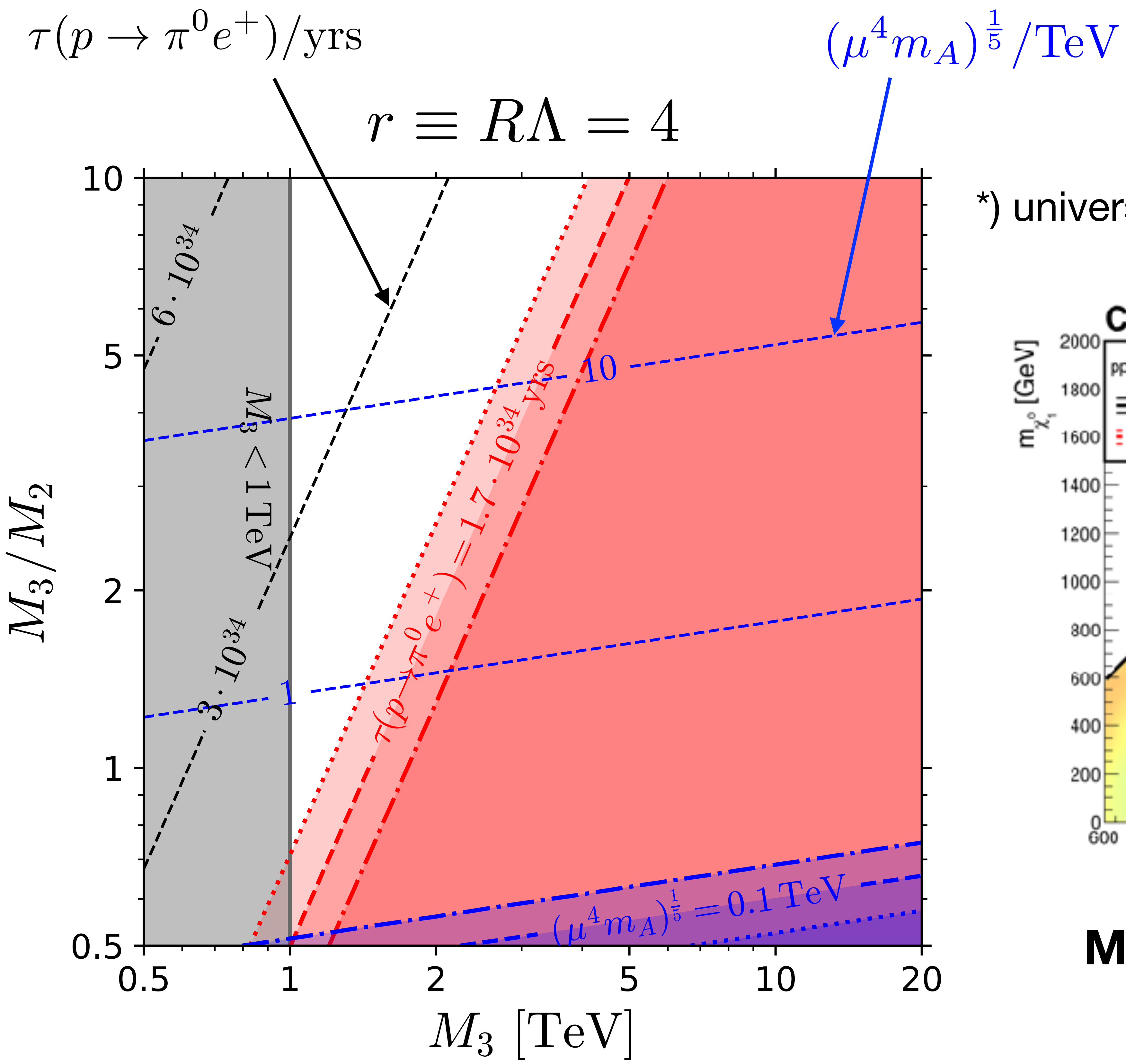
$$T_G = M_G^* \Omega_S$$

$$\frac{1}{R} = M_{(X,Y)_1} = M_G^* \Omega_G{}^{-\frac{3}{4}} \Omega_S \, /r$$

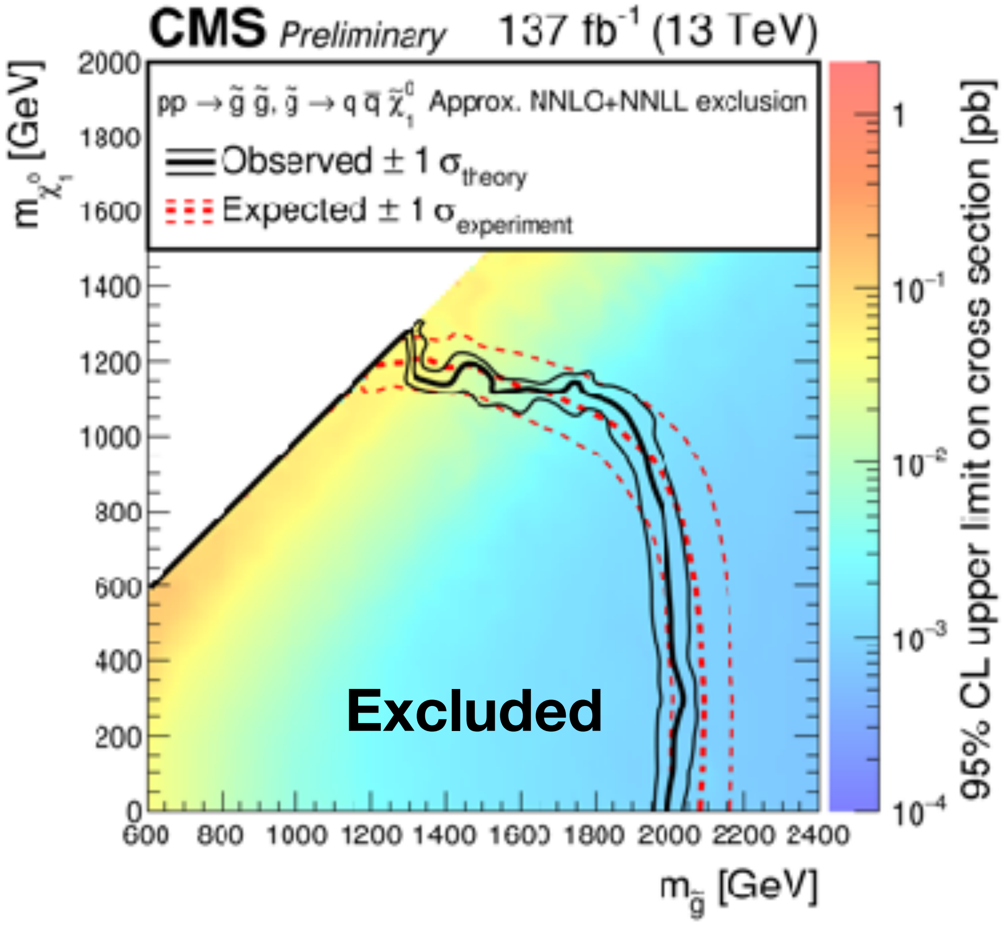
$$= M_G^* M_s^*{}^{\frac{19}{108}} \Omega_G{}^{-\frac{31}{54}} M_3{}^{-\frac{19}{216}} M_2{}^{-\frac{19}{216}} X_T{}^{-\frac{1}{108}} X_\Omega{}^{-\frac{1}{288}} \, /r$$

M_c = 1/R, (i.e. X,Y boson mass) can be predicted from SUSY spectrum, allowing to predict D=6 proton decay

A SUSY plane with GCU

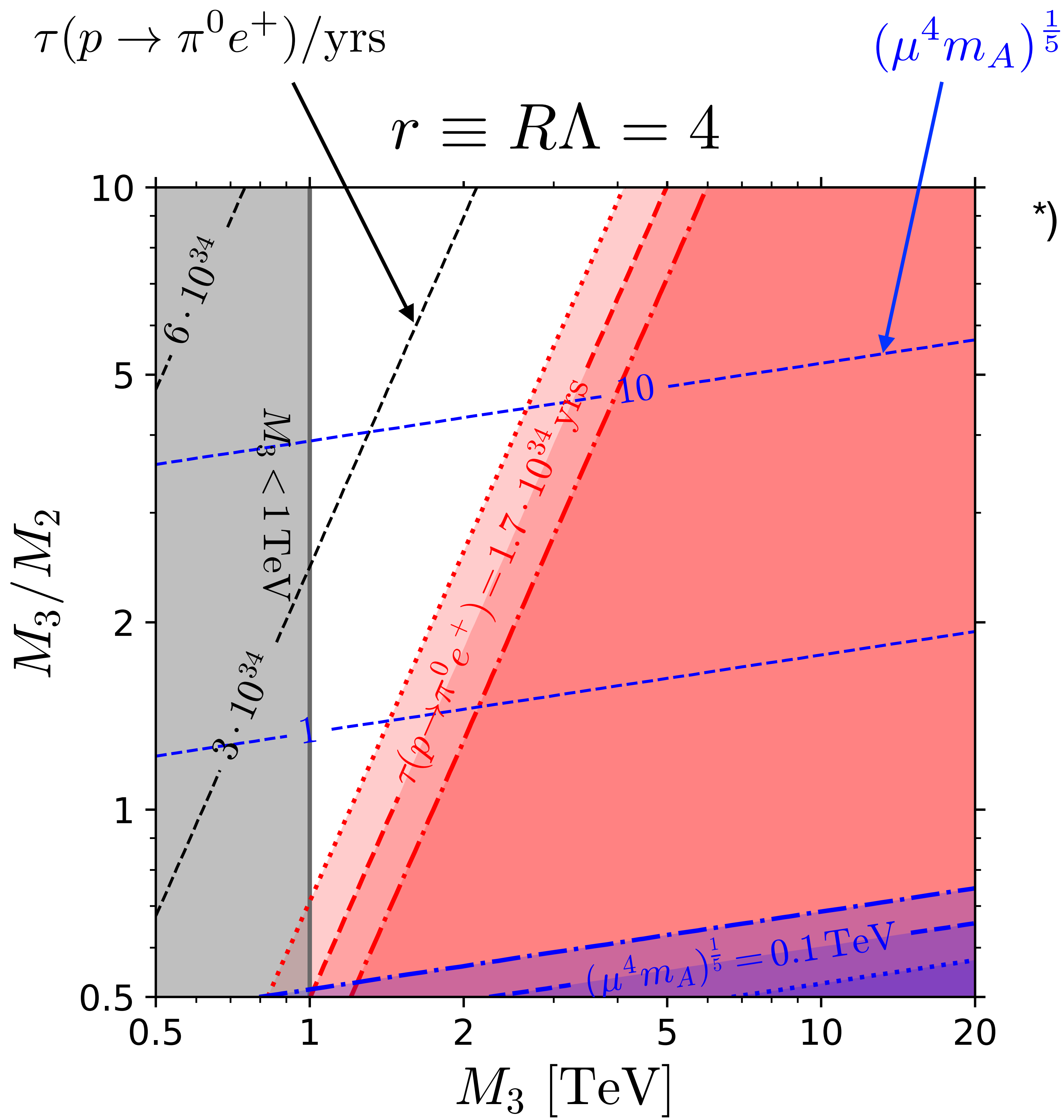


*) universal sfermion mass is assumed



M_3 > 1 - 2 TeV from LHC

A SUSY plane with GCU



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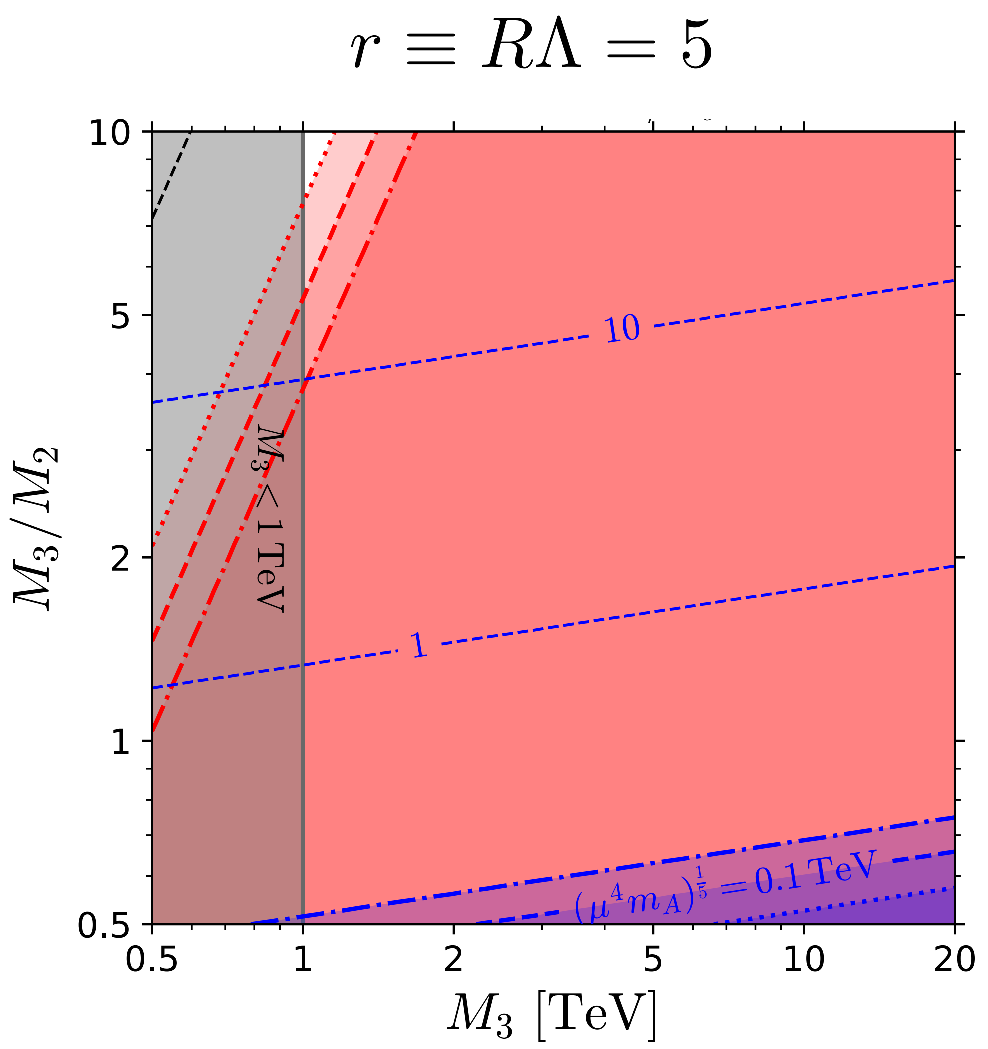
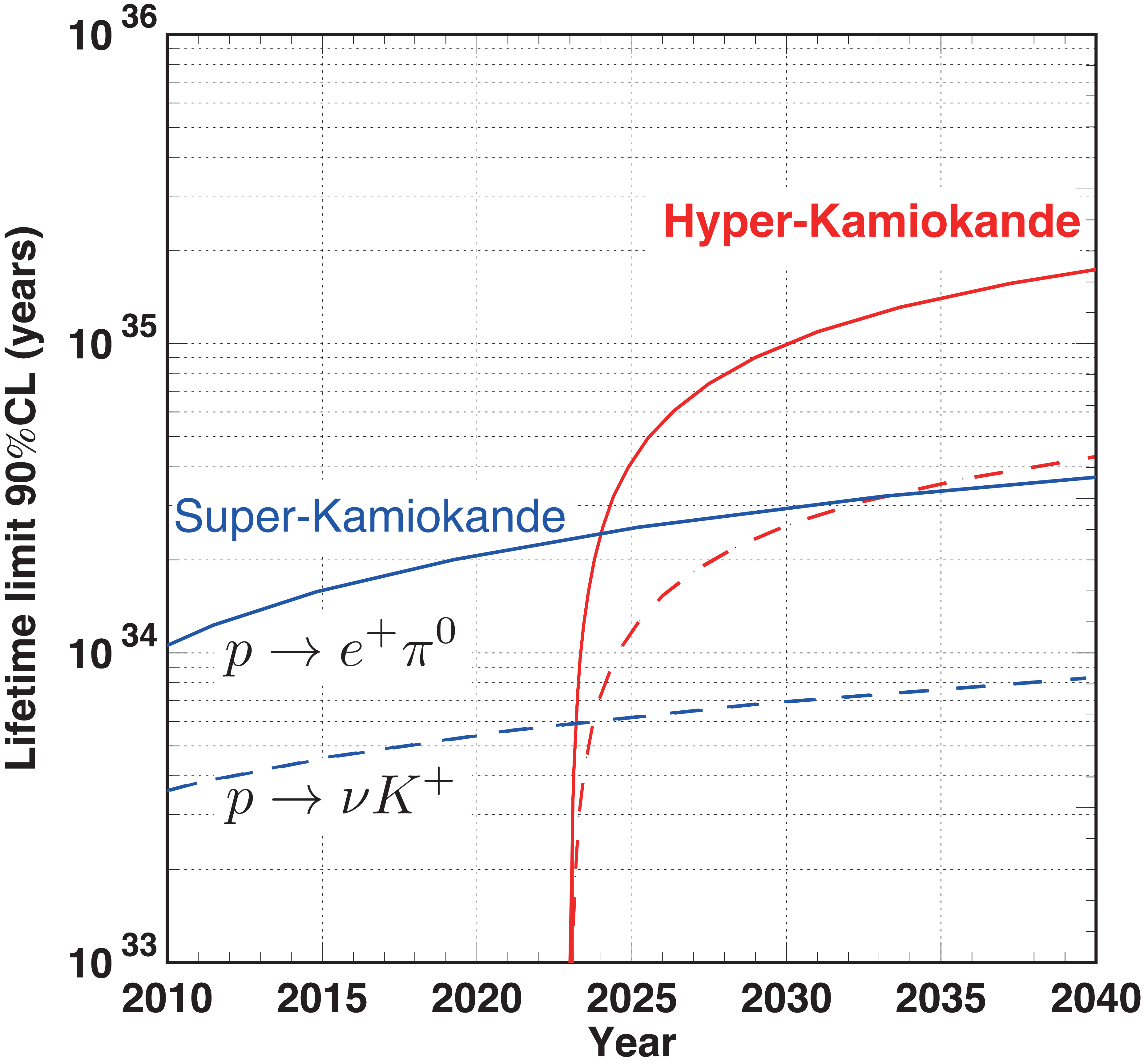
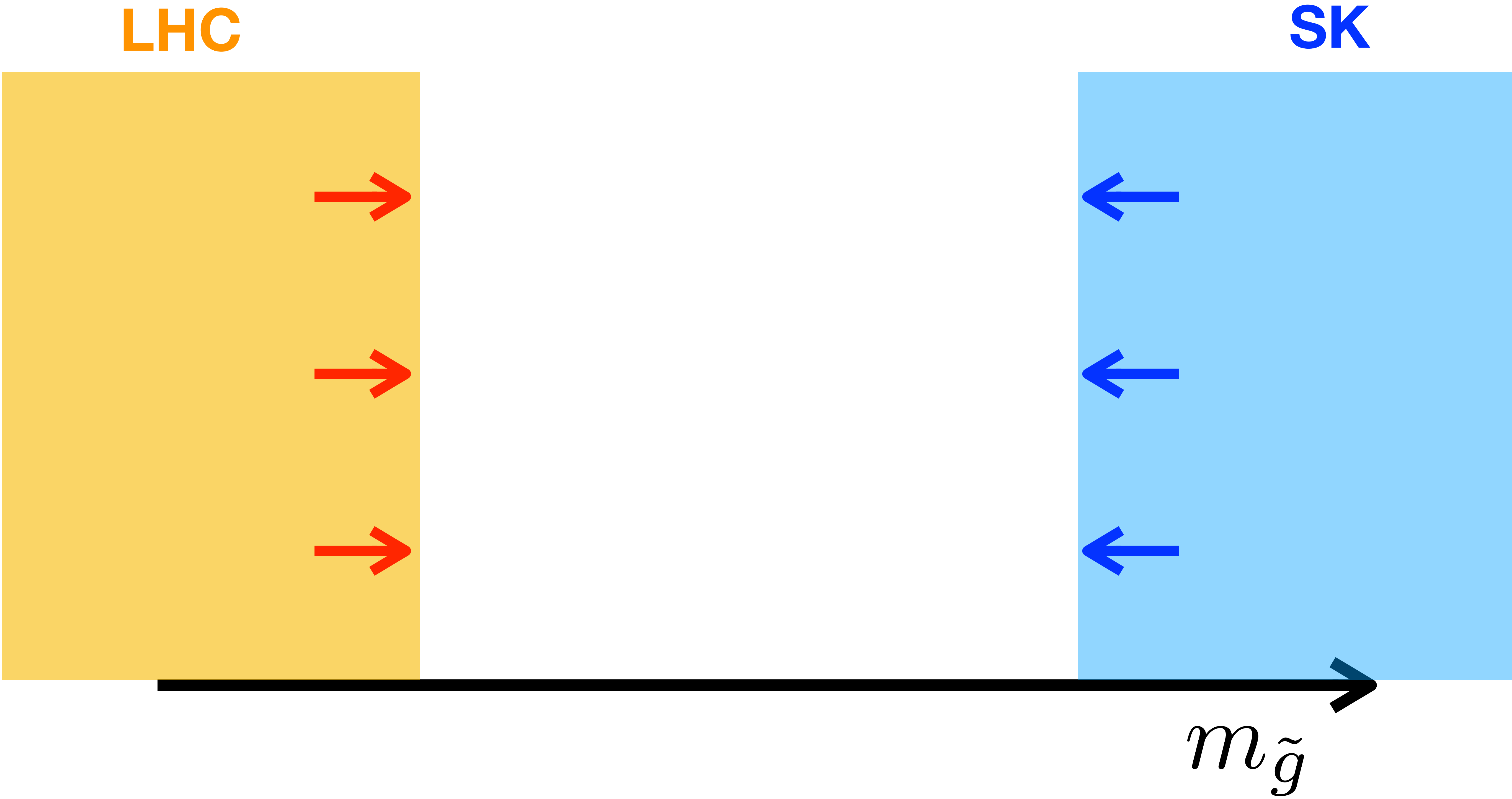
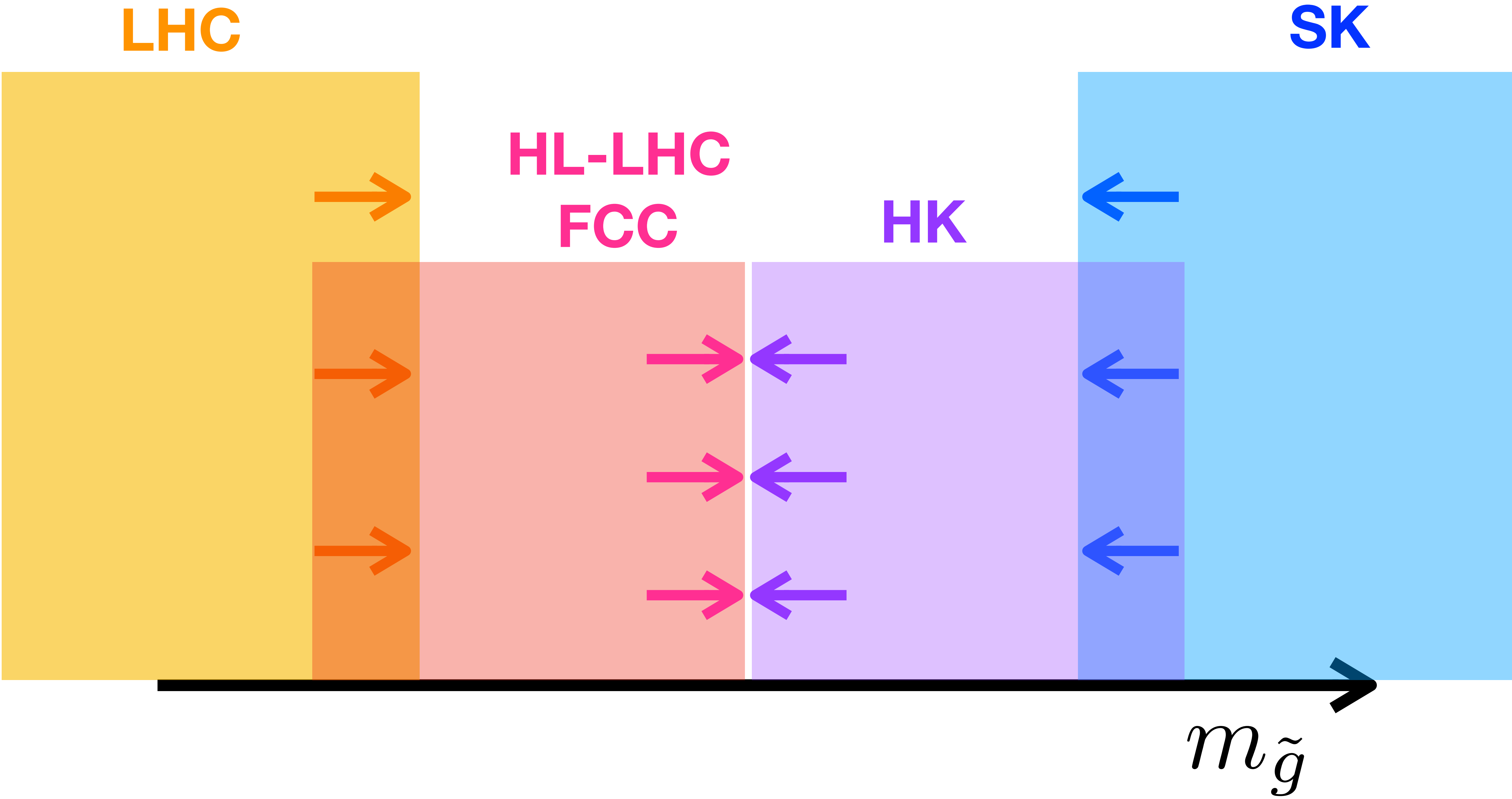



Hyper-Kamiokande
Super-Kamiokande
$p \rightarrow e^+\pi^0$
$p \rightarrow \nu K^+$
Lifetime limit 90%CL (years)
10^{36}
10^{35}
10^{34}
10^{33}
2010
2015
2020
2025
2030
2035
2040
Year


LHC
SK
$m_{\tilde{g}}$


LHC
SK
HL-LHC
FCC
HK
$m_{\tilde{g}}$

Conclusions

- We have derived an analytic formula for the condition of GCU including the 2-loop effect and $\alpha_s(m_Z)$ uncertainty.

$$\boxed{\begin{aligned} T_S(\mathbf{m}_S) &= M_S^*(\alpha_s^{m_Z})\,\Omega_G(\mathbf{m}_\xi) \\ T_G(\mathbf{m}_\xi) &= M_G^*(\alpha_s^{m_Z})\,\Omega_S(\mathbf{m}_S) \end{aligned}}$$

- Minimal SU(5):

The coloured Higgs mass is given as a function of low energy SUSY masses: D=5 proton decay can be predicted by the SUSY spectrum.

- Orbifold SUSY SU(5):

There is a non-trivial constraint on the SUSY spectrum. The X,Y boson mass is given as a function of low energy SUSY masses: D=6 proton decay can be predicted by the SUSY spectrum.