New perspectives on the emergence of (3+1)D expanding space-time in the Lorentzian type IIB matrix model

Jun Nishimura (KEK, SOKENDAI)

Talk at workshop on "Quantum Geometry, Field Theory and Gravity"
Corfu, Greece, September 18-25, 2019

Ref.) J.N. and Asato Tsuchiya, JHEP 1906 (2019) 077 [arXiv:1904.05919 [hep-th]]
Anagnostopoulos-Aoki-Azuma-Hirasawa-Ito-J.N.-Tsuchiya-Papadoudis, work in progress

type IIB matrix model

Ishibashi, Kawai, Kitazawa, Tsuchiya, 1996

c.f.) Harolod Steinacher's talk yesterday

a conjectured nonperturbative formulation of superstring theory

$$S_{\mathrm{b}} = -\frac{1}{4g^2}\mathrm{tr}([A_\mu, A_\nu][A^\mu, A^\nu])$$

$$S_{\mathrm{f}} = -\frac{1}{2g^2}\mathrm{tr}(\Psi_\alpha(\mathcal{C}\,\Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta])$$

SO(9,1) symmetry

$N \times N$ Hermitian matrices

$A_\mu \quad (\mu = 0, \cdots, 9)$ Lorentz vector

$\Psi_\alpha \quad (\alpha = 1, \cdots, 16)$ Majorana-Weyl spinor

Lorentzian metric $\eta = \mathrm{diag}(-1, 1, \cdots, 1)$
is used to raise and lower indices.

Wick rotation $(A_0 = -iA_{10}\ , \quad \Gamma^0 = i\Gamma_{10})$

⇒ Euclidean matrix model SO(10) symmetry

c.f.) Stratos Papadoudis' talk in the next session

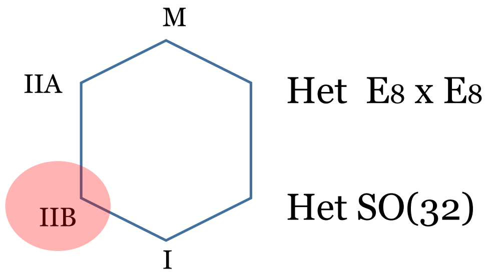
Crucial properties of the type IIB matrix model

as a nonperturbative formulation of superstring theory

- The connection to perturbative formulations can be seen manifestly by considering type IIB superstring theory in 10d.

  worldsheet action, light-cone string field Hamiltonian, etc.

- It is expected to be a nonperturbative formulation of <u>the unique theory</u> underlying the web of string dualities.



- The model has $10D\ \mathcal{N}=2$ SUSY, which cannot be realized in quantum field theories without gravity.
  The low energy effective theory should inevitably include quantum gravity !

  In the SUSY algebra, translation is realized as $A_\mu \mapsto A_\mu + \alpha_\mu \mathbf{1}$,

  which suggests that the space-time is represented as the eigenvalue distribution of A_μ.

Geometry emerges from matrix degrees of freedom dynamically in this approach .

Plan of the talk

0. Introduction
1. Definition of the Lorentzian type IIB matrix model
2. Complex Langevin method
3. Emergence of (3+1)-dim. expanding behavior
4. Emergence of a smooth space-time
5. Summary and discussions

1. Definition of the Lorentzian type IIB matrix model

Regularizing the Lorentzian model

- Unlike the Euclidean model,
  the Lorentzian model is NOT well defined as it is.

$$Z = \int dA \, d\Psi \, e^{i(S_{\mathrm{b}}+S_{\mathrm{f}})} = \int dA \, e^{iS_{\mathrm{b}}} \underbrace{\mathrm{Pf}\mathcal{M}(A)}_{\text{polynomial in } A}$$

pure phase factor

(which is real,
unlike the Euclidean case)

We definitely need some sort of regularization :
IR cutoffs in both temporal and spatial directions

- Difficult to study by Monte Carlo methods due to the sign problem.

We use the complex Langevin method,
which has developed significantly in recent years.

IR cutoffs as a regularization

- Pure imaginary action is hard to deal with numerically.

  ⇒ We deform the model by introducing two parameters *(s,k)*.

$$Z = \int dA\, e^{-S(A)} \mathrm{Pf}\mathcal{M}(A)$$

$$S(A) = N\beta\, e^{-i\frac{\pi}{2}(1-s)} \left\{ \frac{1}{2}\mathrm{tr}\,[A_0, A_i]^2 - \frac{1}{4}\mathrm{tr}\,[A_i, A_j]^2 \right\}$$

$$A_0 \mapsto e^{-ik\pi/2} A_0$$

Hermitian

$(s,k) = (0,0)$ corresponds to the Lorentzian model.

"*s*" : Wick rotation parameter on the worldsheet

"*k*" : Wick rotation parameter in the target space

- Introduce the IR cutoffs so that the extent in temporal and spatial directions become finite.

$$\frac{1}{N}\mathrm{tr}\,(A_0)^2 = \kappa L^2$$

$$\frac{1}{N}\mathrm{tr}\,(A_i)^2 = L^2$$

In what follows, we set $L = 1$ without loss of generality.

In this talk, we focus on the case $k = \frac{1+s}{2}$

$$S = N\beta\, e^{-i\frac{\pi}{2}(1-s)} \left\{ \frac{1}{2} e^{-ik\pi} \mathrm{tr}\,[A_0, A_i]^2 - \frac{1}{4} \mathrm{tr}\,[A_i, A_j]^2 \right\}$$

IR cutoffs

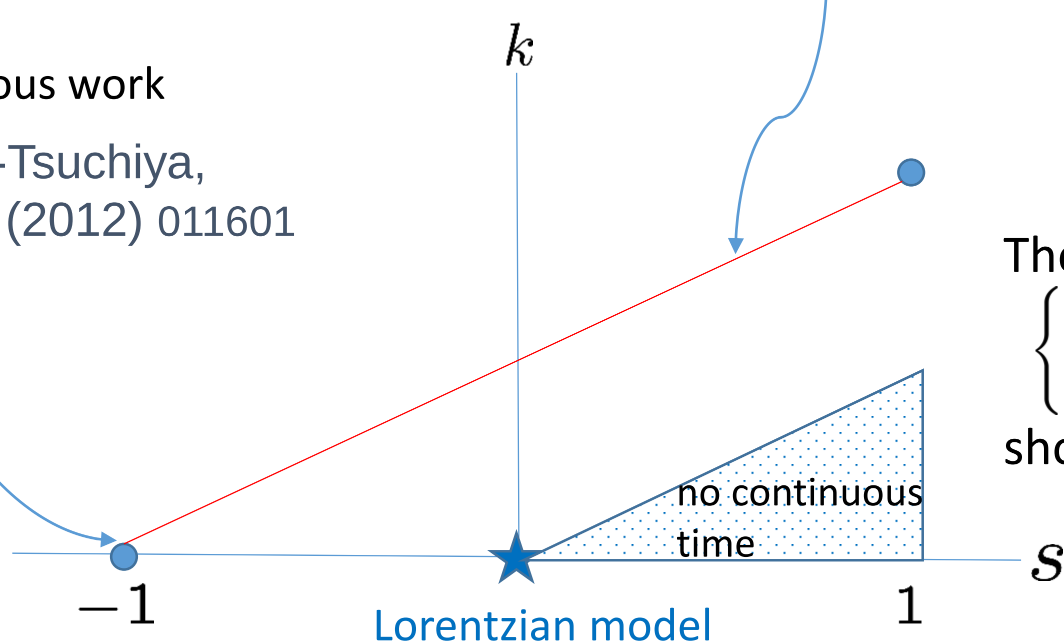
$$\frac{1}{N}\mathrm{tr}\,(A_0)^2 = \kappa$$
$$\frac{1}{N}\mathrm{tr}\,(A_i)^2 = 1$$

The first term can be made real positive by choosing $e^{-i\frac{\pi}{2}(1-s)} e^{-ik\pi} = -1$

$\Rightarrow$ $k = \frac{1+s}{2}$ We focus on this case for the moment.

Our previous work

Kim-J.N.-Tsuchiya, PRL 108 (2012) 011601



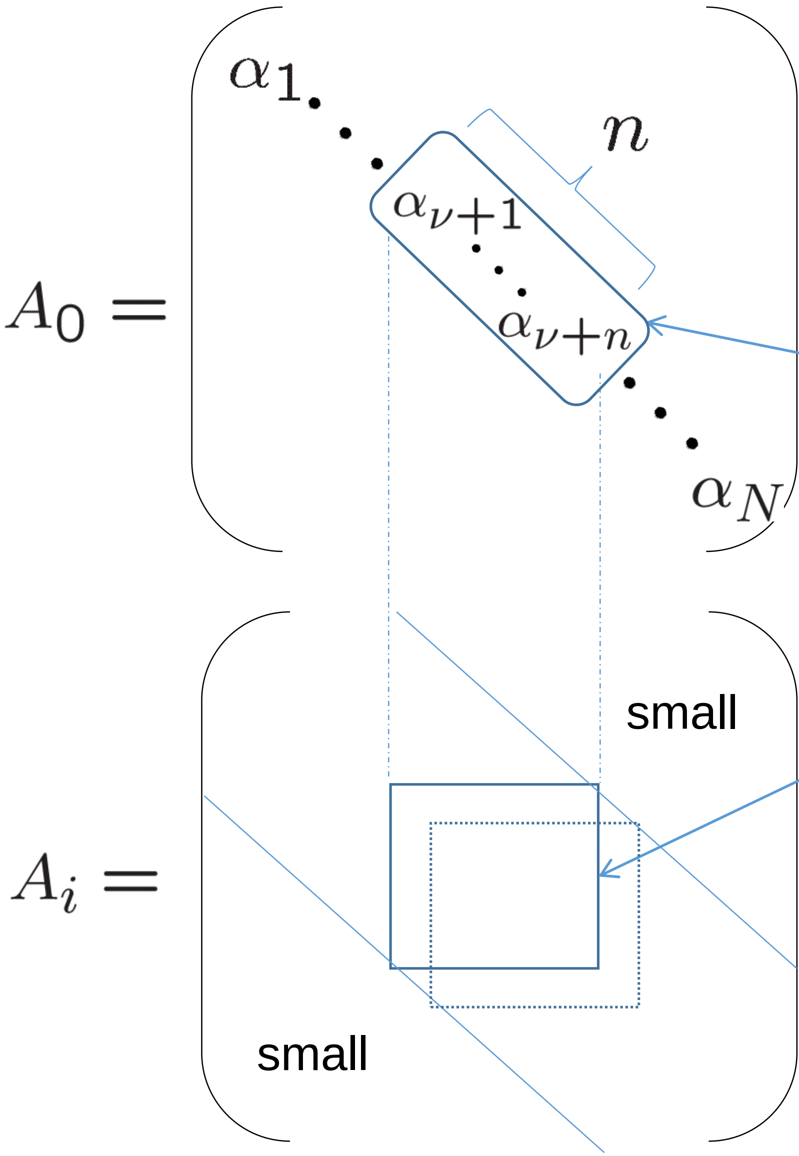
The limits

$$\begin{cases} 1) & N \to \infty \\ 2) & (s, k) \to (0, 0) \end{cases}$$

should be taken eventually.

Extracting time-evolution from the Lorentzian model

Kim-J.N.-Tsuchiya   PRL 108 (2012) 011601   [arXiv:1108.1540]



$\alpha_1 < \cdots < \alpha_N$

definition of time "t"

$$t = \frac{1}{n} \sum_{i=1}^{n} \alpha_{\nu+i}$$

The state of the universe $\bar{A}_i(t)$ at time t

A_i has a band diagonal structure

non-trivial dynamical property

2. Complex Langevin method

The complex Langevin method

Parisi ('83), Klauder ('83)

$$Z = \int dx\, w(x) \qquad\qquad x \in \mathbb{R}$$

complex

MC methods inapplicable due to sign problem !

Complexify the dynamical variables, and consider their (fictitious) time evolution :

$$z^{(\eta)}(t) = x^{(\eta)}(t) + i\, y^{(\eta)}(t)$$

defined by the complex Langevin equation

Gaussian noise (real)

$$\frac{d}{dt} z^{(\eta)}(t) = \boxed{v(z^{(\eta)}(t))} + \eta(t)$$

probability $\propto \mathrm{e}^{-\frac{1}{4}\int dt\, \eta(t)^2}$

$$v(x) \equiv \frac{1}{w(x)} \frac{\partial w(x)}{\partial x}$$

$$\langle \mathcal{O} \rangle \stackrel{?}{=} \lim_{t\to\infty} \langle \boxed{\mathcal{O}(z^{(\eta)}(t))} \rangle_\eta$$

Rem 1 : When w(x) is real positive, it reduces to one of the usual MC methods.

Rem 2 : The drift term $v(x) \equiv \frac{1}{w(x)} \frac{\partial w(x)}{\partial x}$ and the observables $\mathcal{O}(x)$ should be evaluated for complexified variables by analytic continuation.

Complex Langevin equation

The effective action

$$\begin{aligned} S_{\text{eff}} &= N\beta\, e^{-i\frac{\pi}{2}(1-s)} \left\{ \frac{1}{2} e^{-ik\pi} \text{tr}\,[X_0, X_i]^2 - \frac{1}{4} \text{tr}\,[X_i, X_j]^2 \right\} \\ &\quad + \frac{1}{2} N \text{tr}\,(A_i)^2 + \frac{1}{2} N \text{tr}\,(A_0)^2 \\ &\quad - \log \Delta(\alpha) - \sum_{a=1}^{N-1} \tau_a \end{aligned}$$

Complex Langevin equation

$$\left\{ \begin{aligned} \frac{d\tau_a}{dt} &= -\frac{\partial S_{\text{eff}}}{\partial \tau_a} + \eta_a \\ \frac{d(A_i)_{ab}}{dt} &= -\frac{\partial S_{\text{eff}}}{\partial (A_i)_{ba}} + (\eta_i)_{ab} \end{aligned} \right.$$

τ_a :complex variables, A_i : general complex matrices.

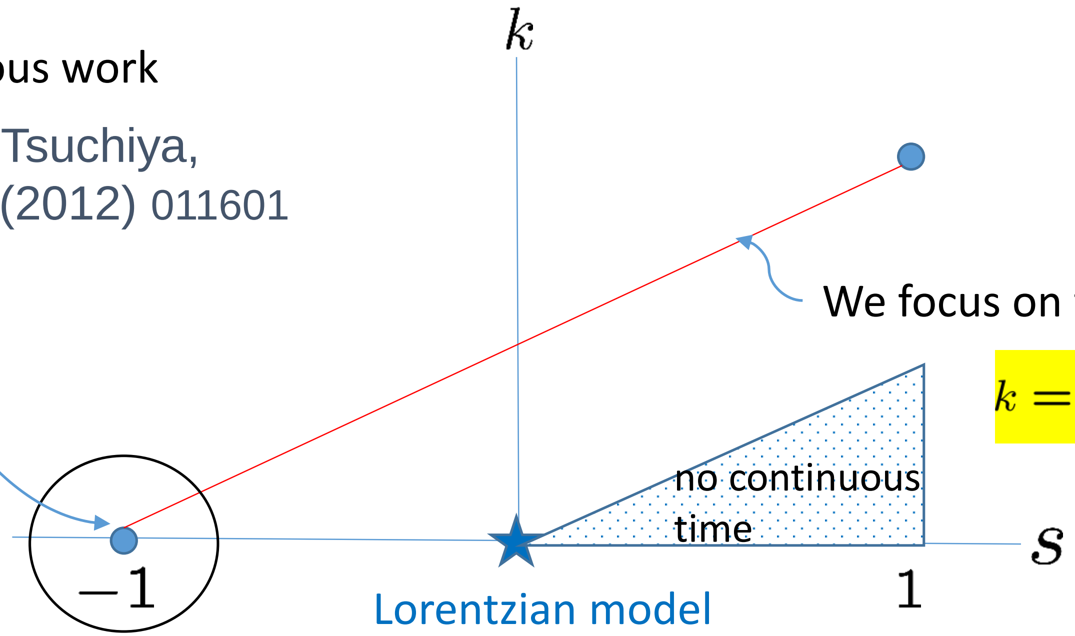
In this work, we omit the fermionic matrices, and consider 6d version instead of 10d to reduce computation time.

3. Emergence of (3+1)-dimensional expanding behavior

Results at (s,k)=(-1,0) in the 6D bosonic model

Our previous work

Kim-J.N.-Tsuchiya, PRL 108 (2012) 011601



We focus on this line for the moment.

$$k = \frac{1+s}{2}$$

k = 0 → no tilt in the time direction (real time).

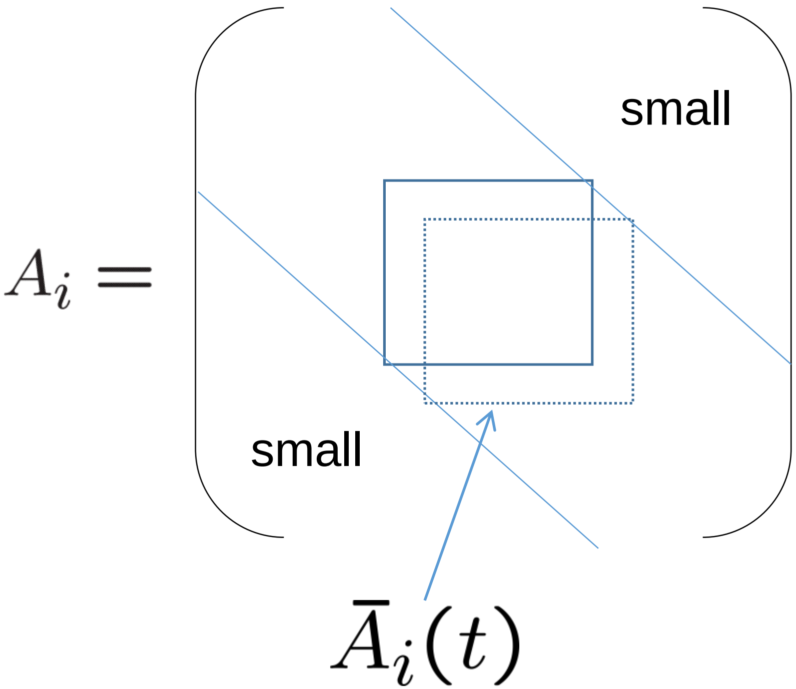
$$S = N\beta \left\{ -\frac{1}{2} \mathrm{tr}\, [A_0, A_i]^2 + \frac{1}{4} \mathrm{tr}\, [A_i, A_j]^2 \right\}$$

Boltzmann weight = e^{-S}

→ no sign problem in this case !

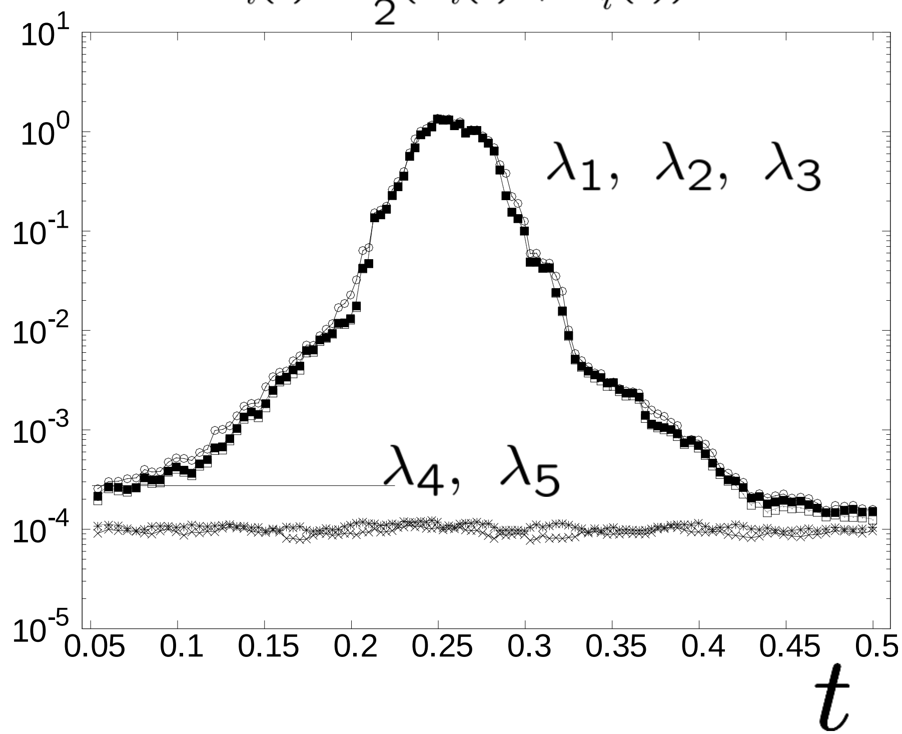
Emergence of (3+1)-dim. expanding behavior

$$N = 128\ ,\quad \kappa = 0.02\ ,\quad \beta = 8\ ,\quad (s,k) = (-1,0)\ ,\quad n = 16$$



eigenvalues of $T_{ij}(t) = \frac{1}{n}\mathrm{tr}\left\{X_i(t)X_j(t)\right\}$

$$X_i(t) = \frac{1}{2}(\bar{A}_i(t) + \bar{A}_i^\dagger(t))$$



SSB : SO(5) → SO(3) occurs at some point in time.

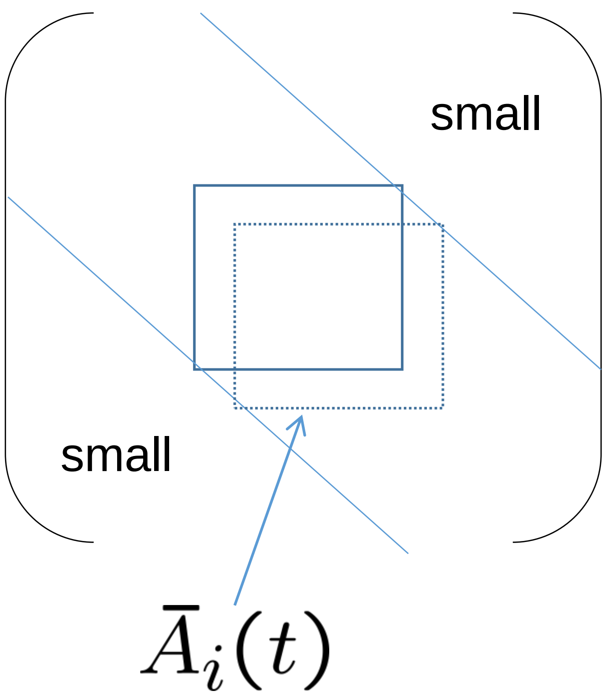
The mechanism of the SSB

$$S = N\beta \left\{ -\frac{1}{2}\mathrm{tr}\,[A_0, A_i]^2 + \frac{1}{4}\mathrm{tr}\,[A_i, A_j]^2 \right\}$$

favors A_j close to diagonal

favors maximal non-commutativity between A_j

$A_i =$



maximize NC $= -\mathrm{tr}\,[\bar{A}_i(t), \bar{A}_j(t)]^2$
for $\mathrm{tr}\,(\bar{A}_i(t))^2 =$ const.

$$\bar{A}_i(t) \propto \sigma_i \qquad \text{for } i = 1, 2, 3$$
$$\bar{A}_i(t) = 0 \qquad \text{for } i \geq 4$$

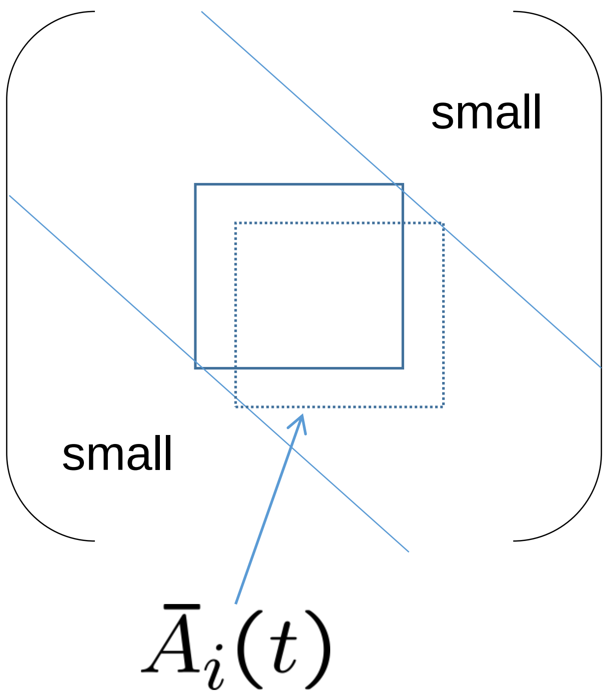
up to SO(5) rotation

Aoki-Hirasawa-J.N.-Ito-Tsuchiya, arXiv:1904.05914 [hep-th]

Confirmation of the mechanism

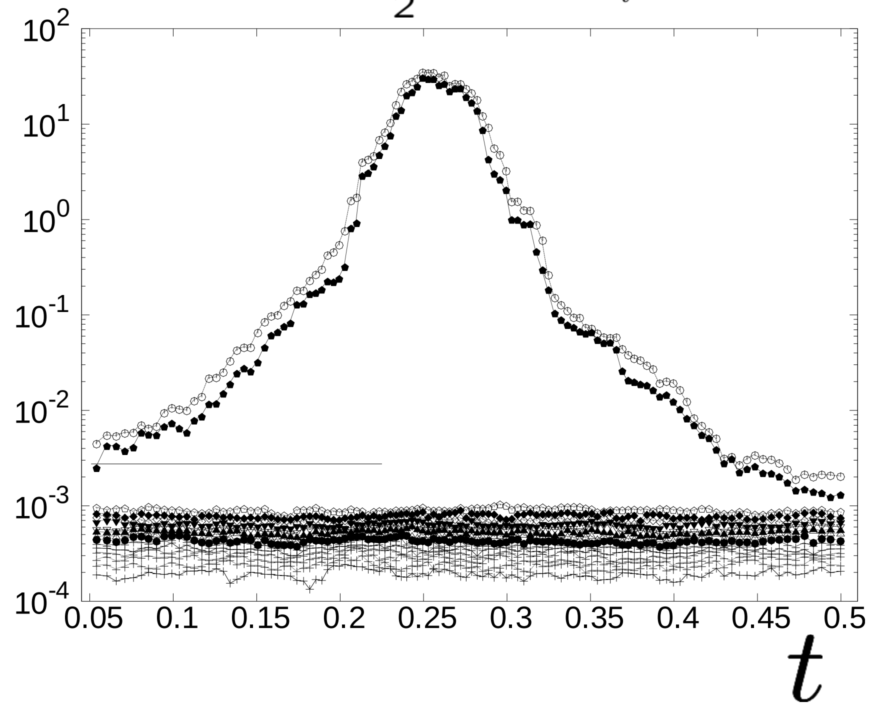
$$N = 128 \ , \quad \kappa = 0.02 \ , \quad \beta = 8 \ , \quad (s, k) = (-1, 0) \ , \quad n = 16$$

$$A_i =$$



eigenvalues of $Q = \sum_{i=1}^{5} \left\{ X_i(t) \right\}^2$

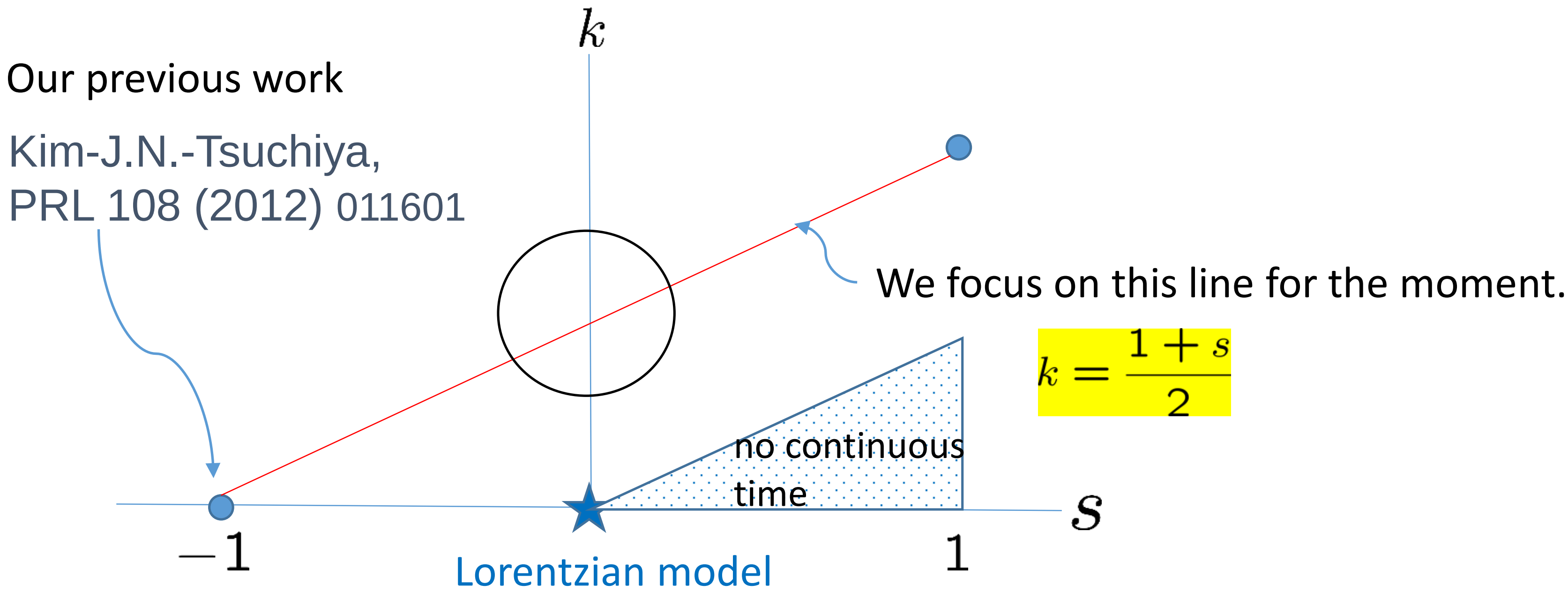
$$X_i(t) = \frac{1}{2}(\bar{A}_i(t) + \bar{A}_i^{\dagger}(t))$$



Only 2 Evs of Q become large suggesting the Pauli-matrix structure.

4. Emergence of a smooth space-time

Exploring the phase diagram near $s = 0$



$$S = -N\beta \left\{ \frac{1}{2} \mathrm{tr}\, [A_0, A_i]^2 + \frac{1}{4} e^{-i\frac{\pi}{2}(1-s)} \mathrm{tr}\, [A_i, A_j]^2 \right\}$$

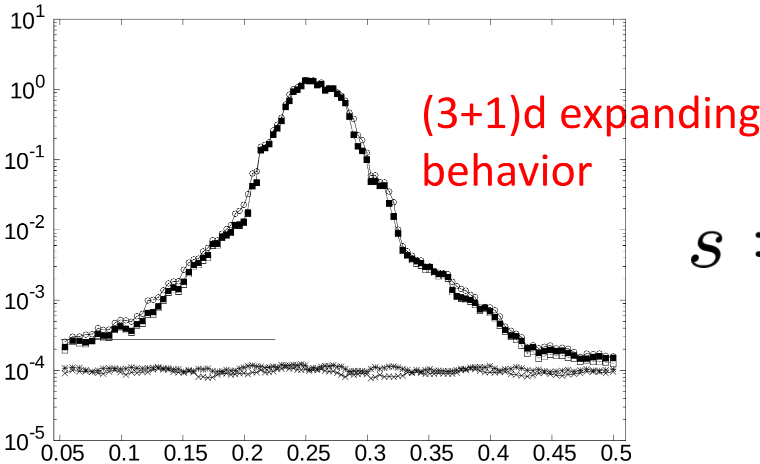
Real part changes sign at $s = 0$.

Can we obtain (3+1)-dim. expanding behavior
with a smooth space-time structure ?

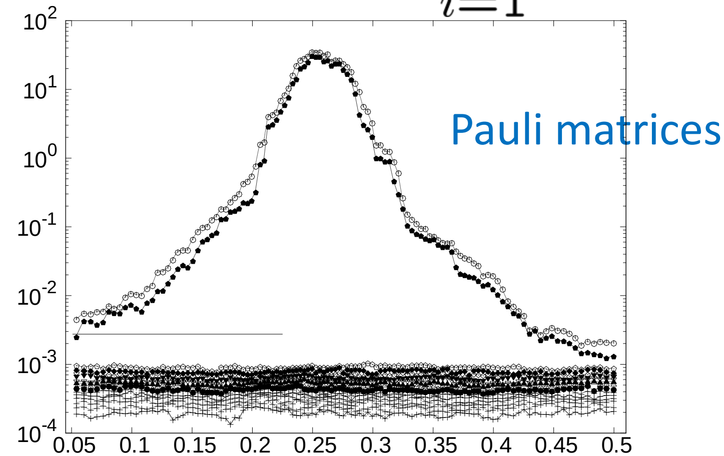
Note: Pauli-matrix structure is obtained by maximizing tr $(F_{ij})^2$!

$s = -1$ v.s. $s \sim 0$

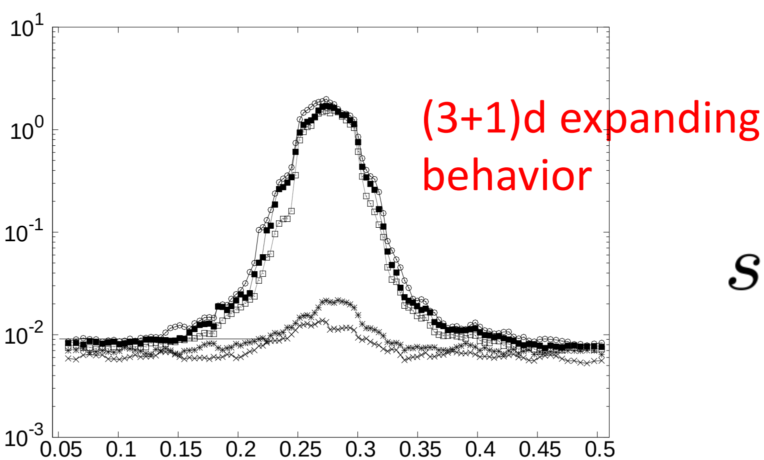
eigenvalues of $T_{ij}(t) = \frac{1}{n}\mathrm{tr}\left\{X_i(t)X_j(t)\right\}$ eigenvalues of $Q = \sum_{i=1}^{5}\left\{X_i(t)\right\}^2$



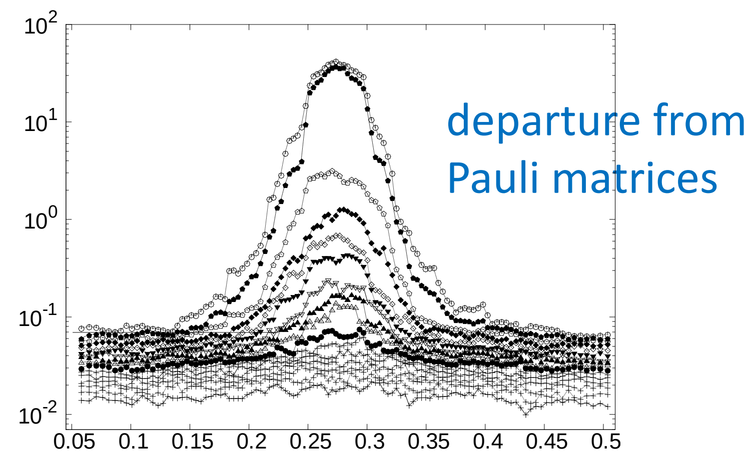
$s = -1$



$N = 128\ ,\quad \kappa = 0.02\ ,\quad \beta = 8\ ,\quad (s,k) = (-1, 0)\ ,\quad n = 16$



$s \sim 0$

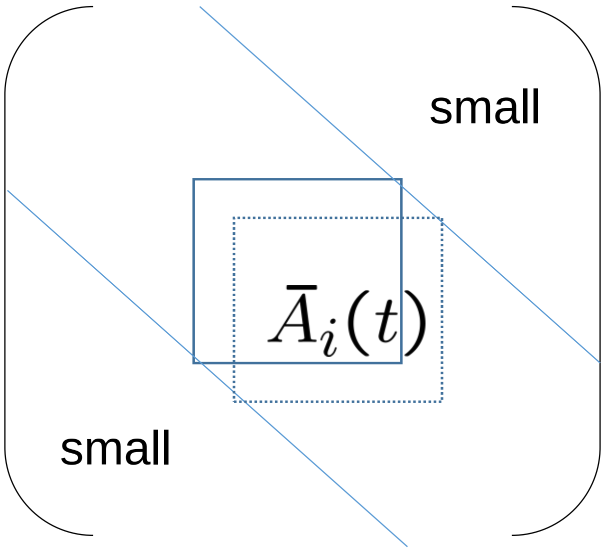


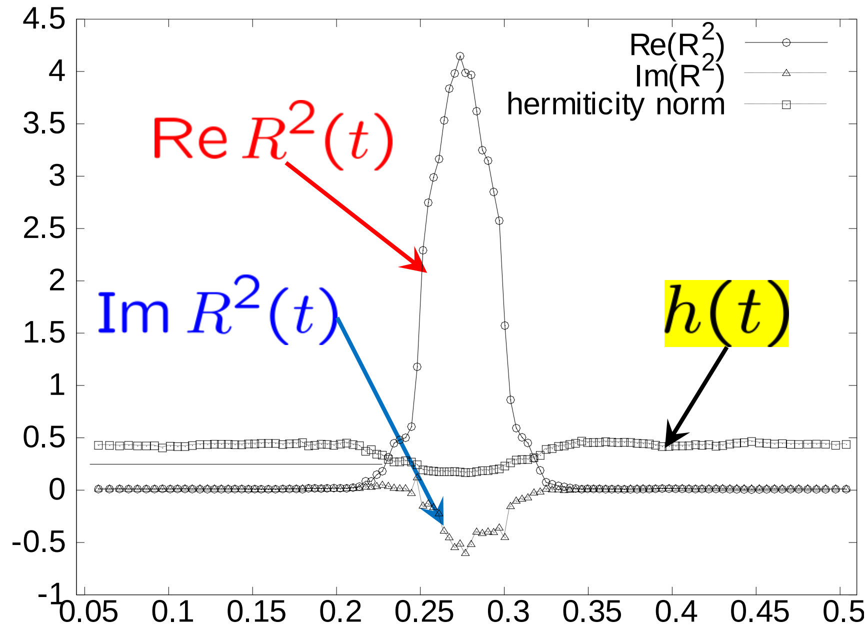
$N = 128\ ,\quad \kappa = 0.02\ ,\quad \beta = 8\ ,\quad (s,k) = (-0.004, 0.498)\ ,\quad n = 16$

Hermiticity of the spatial matrices

$$N = 128\ , \quad \kappa = 0.02\ , \quad \beta = 8\ , \quad (s,k) = (-0.004, 0.498)\ , \quad n = 16$$

$A_i =$





$$R^2(t) = \frac{1}{n}\mathrm{tr}\left(\bar{A}(t)^2\right)$$

$$h(t) = \frac{-\mathrm{tr}\left(\bar{A}_i(t) - \bar{A}_i(t)^\dagger\right)^2}{4\mathrm{tr}\left(\bar{A}_i(t)^\dagger \bar{A}_i(t)\right)}$$

$$0 \leq h(t) \leq 1$$

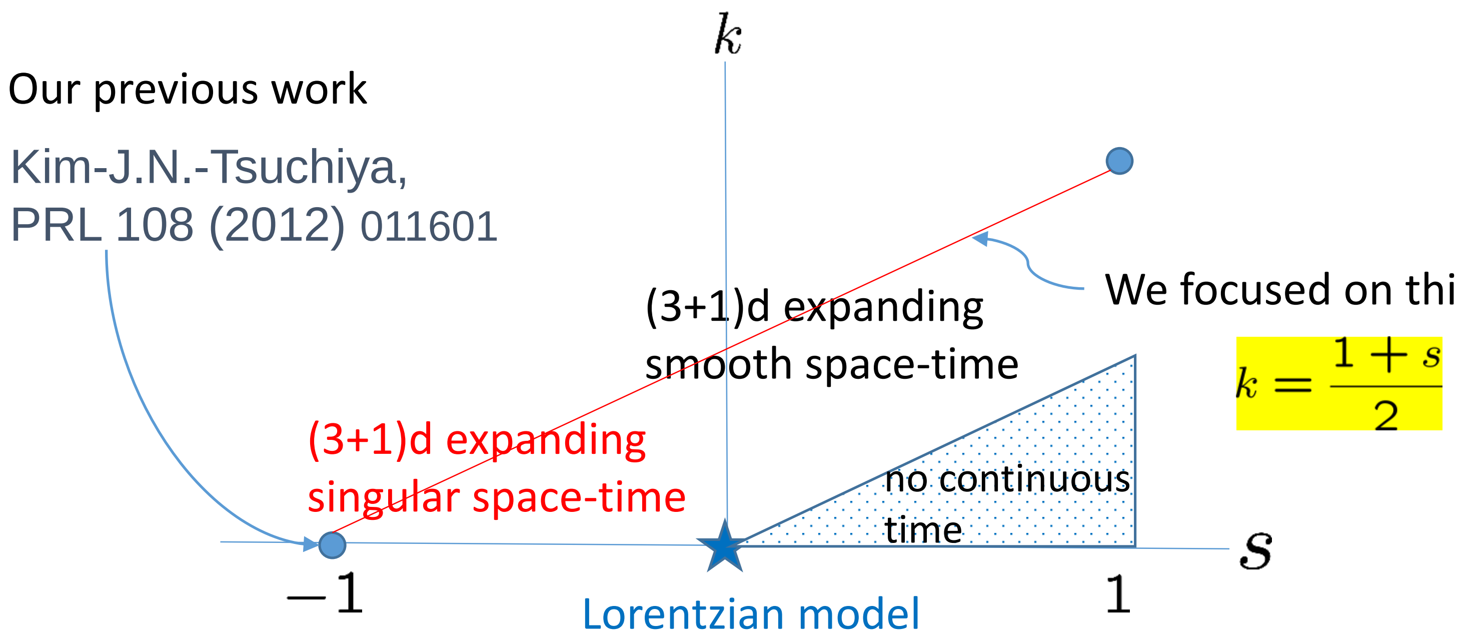
Hermitian anti-Hermitian

Spatial matrices become close to Hermitian near the peak of $\mathrm{Re}\, R^2(t)$.

Classical solution seems to be dominating in this region.

5. Summary and Discussions

Summary



- Transition from the Pauli matrices to a smooth space-time seems to occur as we approach *s=0*.

- Complex Langevin simulation becomes unreliable due to growing non-hermiticity when we decrease *k* from *k*=(1+*s*)/2 too much.

Can we approach the target (s,*k*)=(0,0) at larger N ?
Does the (3+1)d expanding smooth space-time survive there ?

Discussions

- Hermiticity of spatial matrices emerges as the space expands.

  This suggests that a classical solution is dominating there.
  If so, solving the classical eq. of motion is a sensible way
  to explore the late time behavior of this model.

  Possible emergence of the Standard Model from the intersecting branes in the extra dimensions.

  Chatzistavrakidis-Steinacker-Zoupanos (2011)
  Aoki-J.N.-Tsuchiya (2014),
  Hatakeyama-Matsumoto-J.N.-Tsuchiya-Yosprakob, in prep.

- Effects of the fermionic matrices ?

  Not straightforward due to the "singular-drift problem" in the CLM
  caused by the near-zero eigenvalues the Dirac operator.

  Deformation of the Dirac operator (and extrapolations) may be needed.

  Successful in Euclidean type IIB matrix model
  Anagnostopoulos-Azuma-Ito-J.N.-Papadoudis JHEP 1802 (2018) 151

6. Backup slides

Partition function of the Lorentzian type IIB matrix model

Kim-J.N.-Tsuchiya PRL 108 (2012) 011601 [arXiv:1108.1540]

partition function

$$Z = \int dA\, d\Psi\, e^{i(S_{\rm b}+S_{\rm f})} = \int dA\, e^{iS_{\rm b}} \mathrm{Pf}\mathcal{M}(A)$$

This seems to be natural from the connection to the worldsheet theory.

$$S = \int d^2\xi \sqrt{g} \left(\frac{1}{4}\{X^\mu, X^\nu\}^2 + \frac{1}{2}\bar{\Psi}\gamma^\mu \{X^\mu, \Psi\} \right)$$

$$\xi_0 \equiv -i\xi_2$$

The worldsheet coordinates should also be Wick-rotated.

Lorenzian v.s. Euclidean

The reason why no one dared to study
the Lorentzian model for many years:

$$S_{\mathrm{b}} \propto \mathrm{tr}\,(F_{\mu\nu}F^{\mu\nu}) = -2\,\underbrace{\mathrm{tr}\,(F_{0i})^2 + \mathrm{tr}\,(F_{ij})^2}_{\text{opposite sign}}$$

$$F_{\mu\nu} = -i[A_\mu, A_\nu]$$

Once one Euclideanizes it by $A_0 = -iA_{10}$,

$$S_{\mathrm{b}} \propto \mathrm{tr}\,(F_{\mu\nu})^2$$

positive definite!

The flat direction ($[A_\mu, A_\nu] \sim 0$) is lifted
due to quantum effects. Aoki-Iso-Kawai-Kitazawa-Tada '99

Euclidean model is well defined without any need for cutoffs.
Krauth-Nicolai-Staudacher ('98),
Austing-Wheater ('01)

Monte Carlo studies :
e.g., Ambjorn-Anagnostopoulos-Bietenholz-Hotta-J.N., JHEP 0007 (2000) 013

How to treat the IR cutoffs

IR cutoffs
$$\begin{cases} \frac{1}{N}\mathrm{tr}\,(A_0)^2 &= \kappa \\ \frac{1}{N}\mathrm{tr}\,(A_i)^2 &= 1 \end{cases}$$

Use the unconstrained matrices A_μ as fundamental variables and substitute

$$X_0 = \frac{\sqrt{\kappa}A_0}{\sqrt{\frac{1}{N}\mathrm{tr}\,(A_0)^2}}\,, \qquad X_i = \frac{A_i}{\sqrt{\frac{1}{N}\mathrm{tr}\,(A_i)^2}}$$

in the action and observables

$$S = N\beta\, e^{-i\frac{\pi}{2}(1-s)}\left\{\frac{1}{2}e^{-ik\pi}\mathrm{tr}\,[X_0, X_i]^2 - \frac{1}{4}\mathrm{tr}\,[X_i, X_j]^2\right\}$$
$$\boxed{+\frac{1}{2}N\mathrm{tr}\,(A_0)^2 + \frac{1}{2}N\mathrm{tr}\,(A_i)^2}$$

Add some functions of $\frac{1}{N}\mathrm{tr}\,(A_0)^2, \frac{1}{N}\mathrm{tr}\,(A_i)^2$ so that the integral of A_μ converges.

How to introduce the "time ordering"

$$Z = \int dA_0 \, dA_i \, e^{-S} = \int d\alpha \, dA_i \, \Delta(\alpha) e^{-S}$$

$$A_0 = \mathrm{diag}(\alpha_1, \cdots, \alpha_N)$$

$$\alpha_1 < \alpha_2 < \cdots < \alpha_N$$

$$\Delta(\alpha) = \prod_{a>b} (\alpha_a - \alpha_b)^2 \quad : \quad \text{van der Monde determinant}$$

Before complexification, we make the change of variables

$$\alpha_1 = 0 \ , \quad \alpha_2 = e^{\tau_1} \ , \quad \alpha_3 = e^{\tau_1} + e^{\tau_2} \ , \quad \cdots \ , \quad \alpha_N = \sum_{a=1}^{N-1} e^{\tau_a} \ ,$$

in order to introduce the "time ordering".

Then we complexify $\tau_a \ (a = 1, \cdots, N-1)$.

Can the Lorentzian type IIB matrix model generate a smooth (3+1)D expanding space-time ?

- It is nice that the generalized model at $(s,k)=(-1,0)$ has good properties such as

  - band-diagonal structure ➡ enables us to extract the time-evolution
  - 3 extended spatial directions which expands with time ➡ can be regarded as a "seed" of a smooth (3+1)D expanding space-time

(These properties can be understood just from the action.)

- Does a smooth (3+1)D expanding space-time appears if one approaches $s=0$ (the target value) with fixed $k=(1+s)/2$?

(Eventually, we have to approach $(s,k)=(0,0)$, the Lorentizan model.)

Summary

The (generalized) Lorentzian type IIB matrix model

$$S = N\beta\, e^{-i\frac{\pi}{2}(1-s)} \left\{ \frac{1}{2}\mathrm{tr}\,[A_0, A_i]^2 - \frac{1}{4}\mathrm{tr}\,[A_i, A_j]^2 \right\}$$

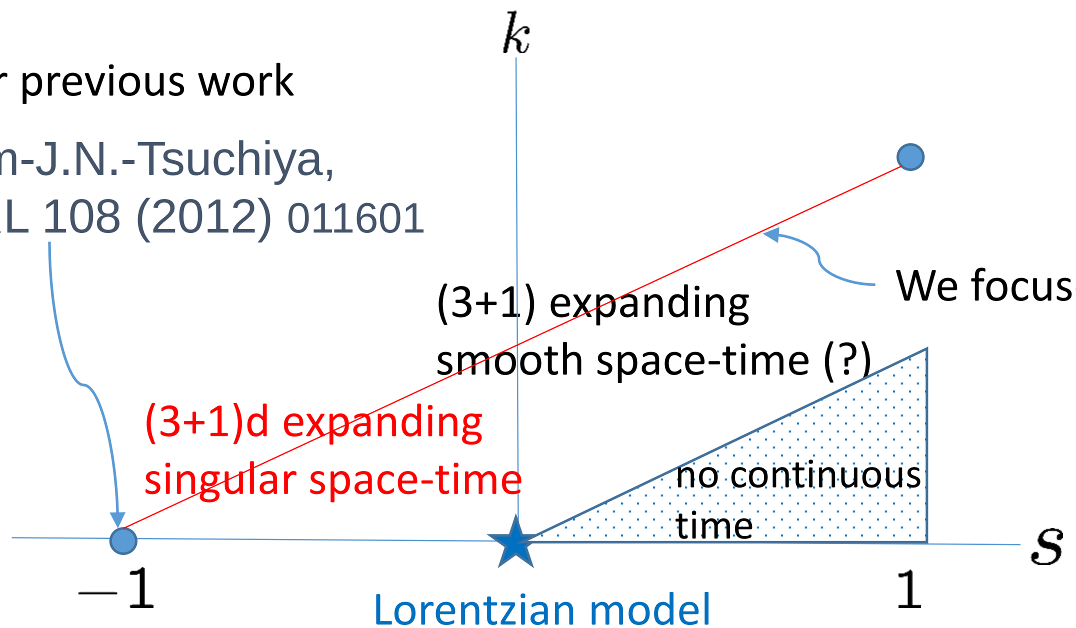
$$A_0 \mapsto e^{-ik\pi/2} A_0$$

$$S = N\beta\, e^{-i\frac{\pi}{2}(1-s)} \left\{ \frac{1}{2} e^{-ik\pi}\mathrm{tr}\,[A_0, A_i]^2 - \frac{1}{4}\mathrm{tr}\,[A_i, A_j]^2 \right\}$$

IR cutoffs : $\frac{1}{N}\mathrm{tr}\,(A_0)^2 = \kappa\ , \quad \frac{1}{N}\mathrm{tr}\,(A_i)^2 = 1$

Our previous work

Kim-J.N.-Tsuchiya,
PRL 108 (2012) 011601



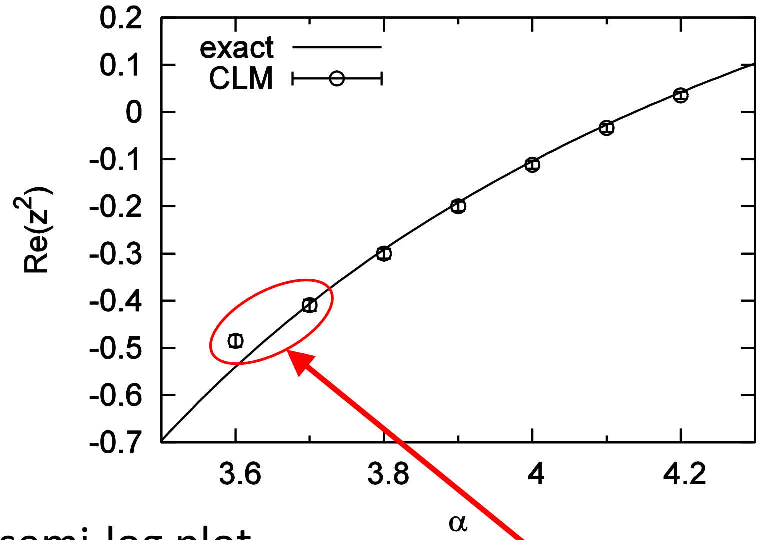
We focused on this line. $k = \frac{1+s}{2}$

The limits

$$\begin{cases} 1) & N \to \infty \\ 2) & (s,k) \to (0,0) \end{cases}$$

should be taken eventually.

Recent development : the condition for correct convergence



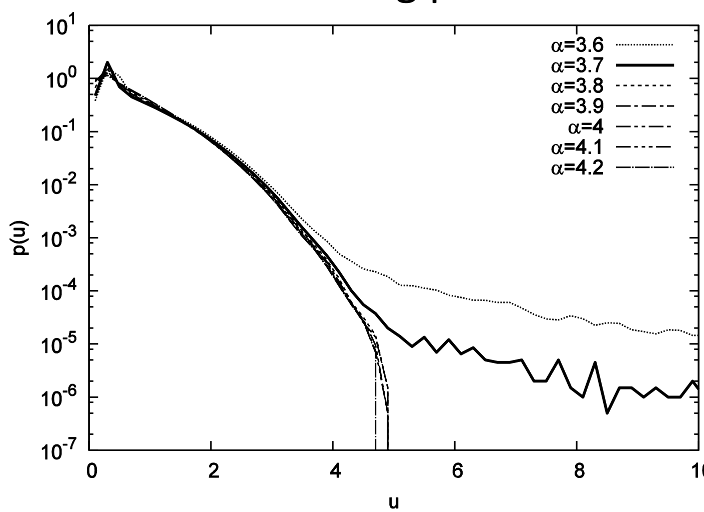
$$Z = \int dx\, w(x)$$

$$w(x) = (x+i\alpha)^p \, \mathrm{e}^{-x^2/2}$$

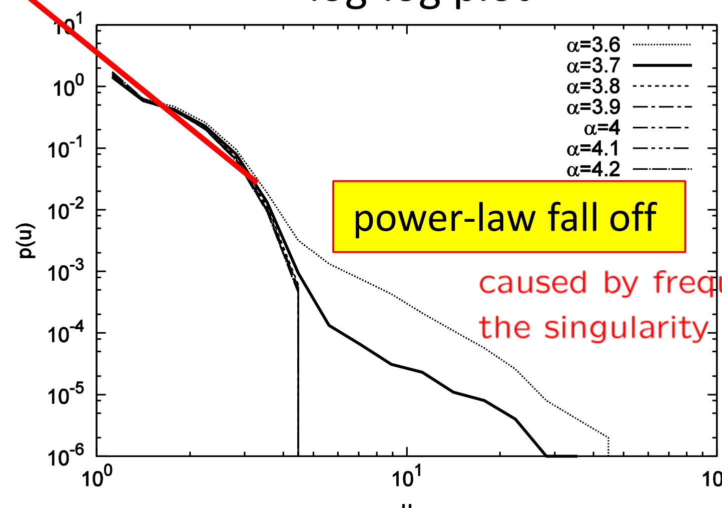
$$p = 4$$

In this model, CLM fails at $\alpha \lesssim 3.7$.

semi-log plot



log-log plot



caused by frequent visits to the singularity at $z = -i\alpha$

The probability distribution of the magnitude of the drift term $u \equiv |v(z)| = \left| \dfrac{p}{z+i\alpha} - z \right|$ should be suppressed exponentially in order for the method to be justified.

Nagata-J.N.-Shimasaki, Phys.Rev. D94 (2016) no.11, 114515.

Side remark: application to finite density QCD

Nagata-J.N.-Shimasaki : arXiv:1805.03964 [hep-lat]
Ito-Matsufuru-J.N.-Shimasaki-Tsuchiya-Tsutsui, work in progress