On Non-Supersymmetric String Model Building

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I. R, I. Florakis and K. Violaris-Gkountonis,
arXiv:1608.04582 [hep-th], Nucl. Phys. B 913 (2016) 495
arXiv:1703.09272 [hep-th], Nucl. Phys. B 921 (2017) 1, work in progress

The Standard Model

The Standard Model of particle interactions has been proved remarkably successful in interpreting the results of recent experiments. However, it is considered as a low energy effective theory as it leaves a number of unanswered questions:
Mass origin, dark matter, charge quantization, hierarchy problem, gravity...)

Supersymmetry is a well studied, compelling Standard Model extension that could help to resolve some of the above SM problems. The introduction of SUSY at a few TeV leads also to coupling unification.

However, to date, experiments have not provided any evidence for supersymmetry.

Non-supersymmetric strings

String theory provides a unified framework of all interactions including the Standard Model.

Space-time supersymmetry is not required for consistency in string theory.

From the early days of the first string revolution it was known that heterotic strings include both the supersymmetric $E_8 \times E_8$ and $SO(32)$ models and the non-supersymmetric tachyon free $SO(16) \times SO(16)$ theory.

However, non-supersymmetric string phenomenology has not received much attention until recently.

see e.g.
S. Abel, K. R. Dienes and E. Mavroudi (2015,2017) , J. R. and I. Florakis (2016,2017) , Y. Sugawara, T. Wada (2016) , A. Lukas, Z. Lalak and E. E. Svanes (2015) , S.G. Nibbelink, O. Loukas, A. Mütter, E. Parr, P. K. S. Vaudrevange (2017)

SUSY breaking in String Theory

Any scenario of supersymmetry breaking in the context of string theory has to address some important issues, as

- Resolve M_W/M_P hierarchy
- Compatibility with gauge coupling evolution (“unification”)
- Account for the smallness of the cosmological constant
- Resolve possible instabilities (tachyons)
- Moduli field stabilisation

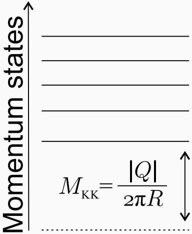
Coordinate dependent compactifications

The Scherk–Schwartz compactification provides an elegant mechanism break SUSY in the context of String Theory. The implementation of a stringy Scherk–Schwartz mechanism requires an extra dimension X^5 and a conserved charge Q. Upon compactification

$$\Phi\left(X^5 + 2\pi R\right) = e^{iQ}\Phi\left(X^5\right)$$

we obtain a shifted tower of Kaluza–Klein states for charged fields, starting at $M_{KK} = \frac{|Q|}{2\pi R}$

$$\Phi(X^5) = e^{\frac{i\,QX^5}{2\pi R}} \sum_{n\in Z} \Phi_n \, e^{i\,nX^5/R}$$



Coordinate dependent compactifications

$Q =$ Fermion number $\Rightarrow$ leads to different masses for fermions-bosons (lying in the same supermultiplet) and thus to spontaneous breaking of supersymmetry.

SUSY breaking related to the compactification radius $M \sim \frac{1}{R}$

We consider the implementation of this mechanism in a class of phenomenologically interesting $SO(10)$ heterotic string models.

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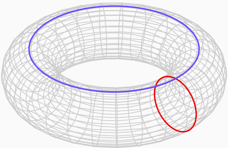
see e.g. J. Scherk and J. H. Schwarz (1978,1979) , R. Rohm (1984) , C. Kounnas and M. Porrati (1988) , S. Ferrara, C. Kounnas, M. Porrati and F. Zwirner (1989) , C. Kounnas and B. Rostand, (1990)

Gravitino mass

We consider compactifications of the six internal dimensions in three separate two-tori parametrised by the $T^{(i)}, U^{(i)}, i = 1, 2, 3$ moduli. For simplicity, we will consider realising the Scherk–Schwartz mechanism utilising the $T^{(1)}, U^{(1)}$ torus.

At tree level the gravitino receives a mass

$$m_{3/2} = \frac{|U^{(1)}|}{\sqrt{T_2^{(1)} U_2^{(1)}}} = \frac{1}{R_1}$$



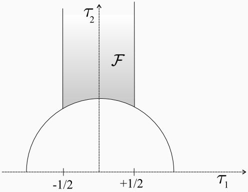
for a square torus: $T = \imath R_1 R_2, U = \imath R_2 / R_1$
All $T^{(i)}, U^{(i)}$ moduli remain massless.

At $R_1 \to \infty$ we have $m_{3/2} = 0$ and the supersymmetry is restored.

One loop potential

The effective potential at one loop, as a function moduli $t_I = T^{(i)}, U^{(i)}$, is obtained by integrating the string partition function $Z(\tau_1, \tau_2; t_I)$ over the worldsheet torus Σ_1

$$V_{\mathrm{one-loop}}(t_I) = -\frac{1}{2(2\pi)^4} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^3} Z(\tau, \bar{\tau}; t_I) \,,$$



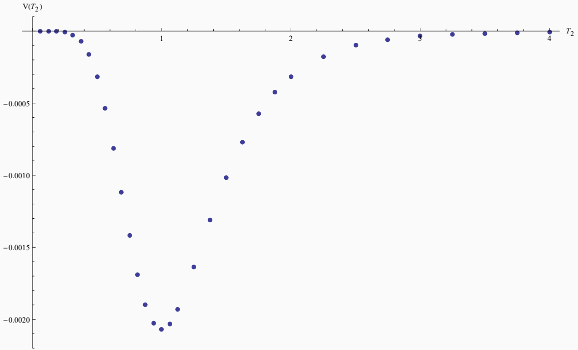
where $\tau = \tau_1 + i\tau_2$ and $\mathcal{F}$ is the fundamental domain .

For given values of the moduli

$$Z = \sum_{\substack{n\in\mathbb{Z}/2 \\ n\geq -1/2}} \sum_{m\in\mathbb{Z}} Z_{n,m}\, q_{\mathrm{r}}^n\, q_{\mathrm{i}}^m = \sum_{\substack{n\in\mathbb{Z}/2 \\ n\geq -1/2}} \left[\sum_{m=-[n]-1}^{[n]+2} Z_{n,m}\, q_{\mathrm{i}}^m \right] q_{\mathrm{r}}^n \,.$$

where $q_{\mathrm{r}} = e^{-2\pi\tau_2}$ and $q_{\mathrm{i}} = e^{2\pi i\tau_1}$

One loop moduli potentials



Typical one-loop potential as a function of the modulus $T_2 = R^2$.

Undesirable features: SUSY breaking at the string scale, huge cosmological constant, ...

One loop potential: Analytic results

$$Z = \frac{1}{2^8}\frac{1}{\eta^{12}\bar\eta^{24}} \sum_{H_1,G_1=0,1} \Gamma_{2,2}^{\text{shift}}\begin{bmatrix} H_1 \\ G_1 \end{bmatrix}\left(T^{(1)}, U^{(1)}\right)$$
$$\times \sum_{\substack{h_2,H=0,1 \\ g_2,G=0,1}} \sum_{\substack{k,\rho,\gamma_2,\gamma_3=0,1 \\ \ell,\sigma,\delta_3,\delta_4=0,1}} (-1)^{\hat\Phi'} \times \vartheta\begin{bmatrix} 1+H_1+h_2 \\ 1+G_1+g_2 \end{bmatrix}^2 \vartheta\begin{bmatrix} 1+H_1 \\ 1+G_1 \end{bmatrix}^2$$
$$\times \bar\vartheta\begin{bmatrix} k \\ \ell \end{bmatrix}^6 \bar\vartheta\begin{bmatrix} k+h_2 \\ \ell+g_2 \end{bmatrix}^2 \bar\vartheta\begin{bmatrix} \rho \\ \sigma \end{bmatrix}^4 \bar\vartheta\begin{bmatrix} \rho+H \\ \sigma+G \end{bmatrix}^4 \vartheta\begin{bmatrix} \gamma_2 \\ \delta_2 \end{bmatrix} \vartheta\begin{bmatrix} \gamma_2+h_2 \\ \delta_2+g_2 \end{bmatrix}$$
$$\times \bar\vartheta\begin{bmatrix} \gamma_2 \\ \delta_2 \end{bmatrix} \bar\vartheta\begin{bmatrix} \gamma_2+h_2 \\ \delta_2+g_2 \end{bmatrix} \vartheta\begin{bmatrix} \gamma_3 \\ \delta_3 \end{bmatrix} \vartheta\begin{bmatrix} \gamma_3-h_3 \\ \delta_3-g_3 \end{bmatrix} \bar\vartheta\begin{bmatrix} \gamma_3 \\ \delta_3 \end{bmatrix} \bar\vartheta\begin{bmatrix} \gamma_3-h_3 \\ \delta_{-}g_3 \end{bmatrix}$$

One loop potential: Asymptotic limit $R \to \infty$

The asymptotic behaviour of the one loop potential is

$$\lim_{R\to\infty} V_{\text{one-loop}}(R) = -\frac{(n_B - n_F)}{2^4\pi^7 R^4} \sum_{m_1,m_2\in\mathbb{Z}} \frac{U_2^3}{\left|m_1 + \frac{1}{2} + U\, m_2\right|^6} + \mathcal{O}\left(e^{-\sqrt{2\pi}R}\right)$$

$$\lim_{R\to\infty} V_{\text{one-loop}}(R) = \xi \frac{(n_B - n_F)}{R^4} + \text{exponentially supressed}$$

where ξ is a constant and n_B, n_F stand for the number of bosonic and fermionic degrees of freedom respectively.

Super no scale models $n_B = n_F$. Cosmological constant is exponentially small for large R.

C. Kounnas and H. Partouche (2016), T. Coudarchet and H. Partouche (2018)

A class of models

Consider a big class of semi-realistic $Z_2 \times Z_2$ heterotic string vacua for explicit realisations of the Scherk–Schwarz scenario. Study chirality, moduli potential and thresholds.

To this end we utilise both the free fermionic formulation and orbifold formulation. In the former we have full control of the spectrum in the latter we have explicit moduli dependence.

In the free fermionic formulation we use the model classification techniques developed in

A. Gregori, C. Kounnas and J. R. (1999)
A. E. Faraggi, C. Kounnas, S. E. M. Nooij and J. R. (2004)
A. E. Faraggi, C. Kounnas and J. R. (2007)

The class of $SO(10)$ models

We consider a class of four dimensional $N = 1$ heterotic models spontaneously broken to $N = 0$ via the Scherk–Schwarz mechanism. At technical level, this class is generated by 9 basis vectors in the free fermionic formulation and is parametrised by a set of 36 phases associated with generalised GSO projections. It comprises $2^{9(9-1)/2+1} \sim 10^{11}$ (a priori) distinct models.

The $E_8 \times E_8$ gauge symmetry is reduced to

$$SO(10) \times SO(8)^2 \times U(1)^2$$

We select models using the following criteria

- absence of tachyons
- $SO(10)$ chirality
- compatibility with Scherk–Schwarz breaking of $N = 1$ SUSY

Class of models: Basis vectors

The free fermions in the light-cone gauge are:

left: $\psi^\mu, \chi^{1,\dots,6}, \quad y^{1,\dots,6}, \omega^{1,\dots,6}$

right: $\bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \quad \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \quad \bar{\phi}^{1,\dots,8}$

The class of vacua under consideration is defined by

$$\beta_1 = \mathbf{1} = \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} | \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$\beta_2 = S = \{\psi^\mu, \chi^{1,\dots,6}\}$$

$$\beta_3 = T_1 = \{y^{12}, \omega^{12} | \bar{y}^{12}, \bar{\omega}^{12}\}$$

$$\beta_4 = T_2 = \{y^{34}, \omega^{34} | \bar{y}^{34}, \bar{\omega}^{34}\}$$

$$\beta_5 = T_3 = \{y^{56}, \omega^{56} | \bar{y}^{56}, \bar{\omega}^{56}\}$$

$$\beta_6 = b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\psi}^{1,\dots,5}, \bar{\eta}^{1}\}$$

$$\beta_7 = b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\psi}^{1,\dots,5}, \bar{\eta}^{2}\}$$

$$\beta_8 = z_1 = \{\bar{\phi}^{1,\dots,4}\}$$

$$\beta_9 = z_2 = \{\bar{\phi}^{5,\dots,8}\}$$

and a variable set of $9(9-1)/2+1 = 37$ phases $c\begin{bmatrix}\beta_i \\ \beta_j\end{bmatrix}$.

Chirality

Fermion generations, transforming as SO(10) spinorials, arise from $B^I_{pq} = S + b^I_{pq}$, $I = 1, 2, 3$ where $b^1_{pq} = b^1 + p\,T_2 + q\,T_3$, $b^2_{pq} = b^2 + p\,T_1 + q\,T_2$, $b^3_{pq} = x + b^1 + b^2 + p\,T_1 + q\,T_2$, with $p, q \in \{0, 1\}$, and $x = \mathbf{1} + S + \sum_{i=1}^{3} T_i + \sum_{k=1}^{2} z_k$.

Number of generations $\quad N = \sum_{I=1,2,3} \chi^I$ where

$$\chi^1_{pq} = -4\,c\begin{bmatrix} B^1_{pq} \\ S + b_2 + (1-q)T_3 \end{bmatrix} P^1_{pq}\,,$$

$$\chi^2_{pq} = -4\,c\begin{bmatrix} B^2_{pq} \\ S + b_1 + (1-q)T_3 \end{bmatrix} P^2_{pq}\,,$$

$$\chi^3_{pq} = -4\,c\begin{bmatrix} B^3_{pq} \\ S + b_1 + (1-q)T_1 \end{bmatrix} P^3_{pq}\,,$$

and

$$P^I_{pq} = \frac{1}{2^3}\left(1 - c\begin{bmatrix} B^I_{pq} \\ T_I \end{bmatrix}\right)\left(1 - c\begin{bmatrix} B^I_{pq} \\ z_1 \end{bmatrix}\right)\left(1 - c\begin{bmatrix} B^I_{pq} \\ z_2 \end{bmatrix}\right)$$

Orbifold Partition function

The one-loop partition function at the generic point reads

$$Z = \frac{1}{\eta^{12}\bar{\eta}^{24}} \frac{1}{2^3} \sum_{\substack{h_1,h_2,H \\ g_1,g_2,G}} \frac{1}{2^3} \sum_{\substack{a,k,\rho \\ b,\ell,\sigma}} \frac{1}{2^3} \sum_{\substack{H_1,H_2,H_3 \\ G_1,G_2,G_3}} (-1)^{a+b+HG+\Phi}$$

$$\times\, \vartheta[^a_b]\, \vartheta[^{a+h_1}_{b+g_1}]\, \vartheta[^{a+h_2}_{b+g_2}]\, \vartheta[^{a-h_1-h_2}_{b-g_1-g_2}]$$

$$\times\, \Gamma^{(1)}_{2,2}[^{H_1}_{G_1}|^{h_1}_{g_1}](T^{(1)}, U^{(1)})\, \Gamma^{(2)}_{2,2}[^{H_2}_{G_2}|^{h_2}_{g_2}](T^{(2)}, U^{(2)})\, \Gamma^{(3)}_{2,2}[^{H_3}_{G_3}|^{h_1+h_2}_{g_1+g_2}](T^{(3)}, U^{(3)})$$

$$\times\, \bar{\vartheta}[^k_\ell]^5\, \bar{\vartheta}[^{k+h_1}_{\ell+g_1}]\, \bar{\vartheta}[^{k+h_2}_{\ell+g_2}]\, \bar{\vartheta}[^{k-h_1-h_2}_{\ell-g_1-g_2}]\, \bar{\vartheta}[^\rho_\sigma]^4\, \bar{\vartheta}[^{\rho+H}_{\sigma+G}]^4$$

Where $T^{(i)} = T^{(i)}_1 + iT^{(i)}_2$, $U^{(i)} = U^{(i)}_1 + iU^{(i)}_2$ are the moduli of the three two tori, $\eta(\tau)$ is the Dedekind eta function and $\vartheta[^\alpha_\beta](\tau)$ stand for the Jacobi theta functions.

Connection with fermionic formulation

Fermionic point $T = \imath$ and $U = (1+\imath)/2$

Phase $\Phi\left(c[^{\beta_i}_{\beta_j}]\right)$

Twisted/shifted lattices

$$\Gamma_{2,2}[^{H_i}_{G_i}|^h_g](T,U) = \begin{cases} \left|\frac{2\eta^3}{\vartheta[^{1-h}_{1-g}]}\right|^2 & , \ (H_i,G_i)=(0,0) \text{ or } (H_i,G_i)=(h,g) \\ \Gamma^{\text{shift}}_{2,2}[^{H_i}_{G_i}](T,U) & , \ h=g=0 \\ 0 & , \ \text{otherwise} \end{cases}$$

$$\Gamma^{\text{shift}}_{2,2}[^{H_i}_{G_i}](T,U) = \sum_{\substack{m_1,m_2\\ n_1,n_2}} (-1)^{G(m_1+n_2)}\, q^{\frac{1}{4}|P_L|^2}\, \bar{q}^{\frac{1}{4}|P_R|^2}\,,$$

with

$$P_L = \frac{m_2 + \frac{H_i}{2} - Um_1 + T(n_1 + \frac{H_i}{2} + Un_2)}{\sqrt{T_2U_2}}\,,$$

$$P_R = \frac{m_2 + \frac{H_i}{2} - Um_1 + \bar{T}(n_1 + \frac{H_i}{2} + Un_2)}{\sqrt{T_2U_2}}\,.$$

Typical partition functions

Some typical expansions of partition functions (fermionic point)

$$
\begin{aligned}
Z_{(A)} = {} & \frac{2q_\mathrm{i}}{q_\mathrm{r}} - \frac{16q_\mathrm{i}}{\sqrt{q_\mathrm{r}}} + (-312 + 32q_\mathrm{i} + 56q_\mathrm{i}^2) \\
& + \left(4064 + \frac{6144}{q_\mathrm{i}} + 512q_\mathrm{i} - 416q_\mathrm{i}^2\right)\sqrt{q_\mathrm{r}} \\
& + \left(12288 + \frac{16384}{q_\mathrm{i}^2} + \frac{103680}{q_\mathrm{i}} - 12320q_\mathrm{i} - 256q_\mathrm{i}^2 + 792q_\mathrm{i}^3\right) q_\mathrm{r} + \dots
\end{aligned}
$$

$$
\begin{aligned}
Z_{(B)} = {} & \frac{2q_\mathrm{i}}{q_\mathrm{r}} - \frac{32q_\mathrm{i}}{\sqrt{q_\mathrm{r}}} + \left(8 + 224q_\mathrm{i} + 56q_\mathrm{i}^2\right) + \left(1984 + \frac{2048}{q_\mathrm{i}} - 1024q_\mathrm{i} - 832q_\mathrm{i}^2\right)\sqrt{q_\mathrm{r}} \\
& + \left(30720 + \frac{10240}{q_\mathrm{i}^2} + \frac{92160}{q_\mathrm{i}} + 1760q_\mathrm{i} + 5376q_\mathrm{i}^2 + 792q_\mathrm{i}^3\right) q_\mathrm{r} + \dots
\end{aligned}
$$

$$
\begin{aligned}
Z_{(C)} = {} & \frac{2q_\mathrm{i}}{q_\mathrm{r}} - \frac{16q_\mathrm{i}}{\sqrt{q_\mathrm{r}}} + \left(40 + 64q_\mathrm{i} + 56q_\mathrm{i}^2\right) + \left(224 + \frac{6912}{q_\mathrm{i}} + 768q_\mathrm{i} - 672q_\mathrm{i}^2\right)\sqrt{q_\mathrm{r}} \\
& + \left(14336 + \frac{9216}{q_\mathrm{i}^2} + \frac{118656}{q_\mathrm{i}} - 10144q_\mathrm{i} + 3072q_\mathrm{i}^2 + 792q_\mathrm{i}^3\right) q_\mathrm{r} + \dots
\end{aligned}
$$

Classification

A comprehensive computer scan results in 7×10^4 models that satisfy all criteria.

We expand the partition function in powers of $q_{\mathrm{r}} = e^{-2\pi\tau_2}$

$$Z = \sum_{\substack{n \in \mathbb{Z}/2 \\ n \geq -1/2}} W_n \, q_{\mathrm{r}}^n$$

The constant term at the fermionic point W_0 or the generic point W_0^G is proportional to $n_B - n_F$.

	$W_0 < 0$	$W_0 = 0$	$W_0 > 0$
$W_0^G < 0$	3560	0	1856
$W_0^G = 0$	96	0	8848
$W_0^G > 0$	0	0	62192
Total	3656	0	72896

Table 1: Number of chiral models for the subclasses of models with W_0^G positive/negative/zero and W_0 positive/negative.

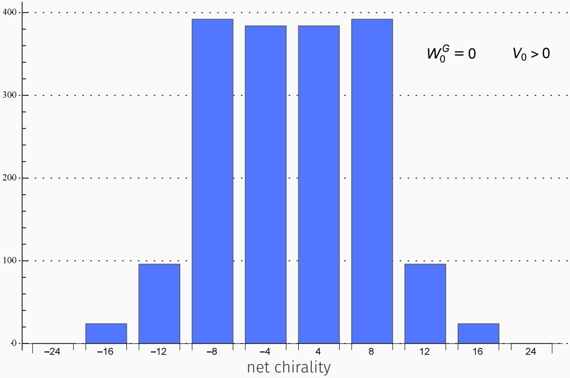
Numerical calculation

n	Model A	Model B
-1	24.4	24.4
$-\frac{1}{2}$	−9.87	−19.7
0	172.	2.11
$\frac{1}{2}$	−29.6	−17.7
1	3.13	−2.73
$\frac{3}{2}$	9.71	8.18
Total	+170.	−5.47

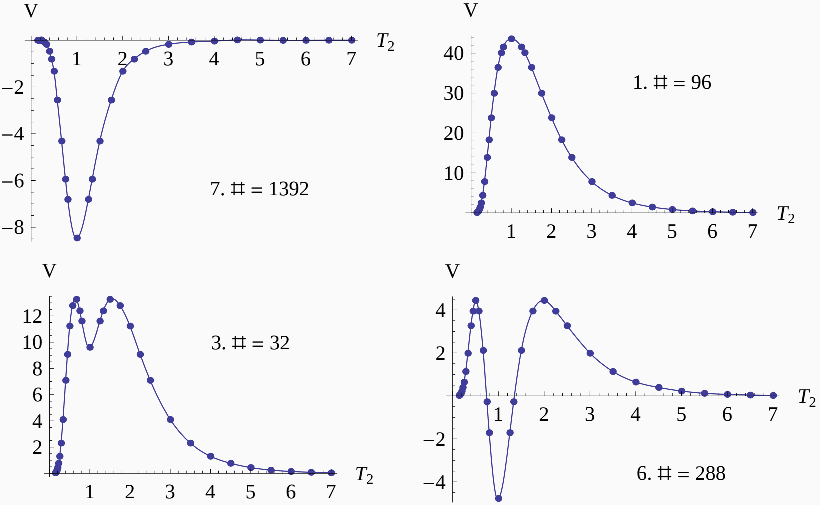
Contributions to the one-loop potential $2(2\pi)^4 V_{1-\mathrm{loop}}$ *arranged according to energy level for two models (A and B) at the fermionic point.*

Chiral super no-scale models

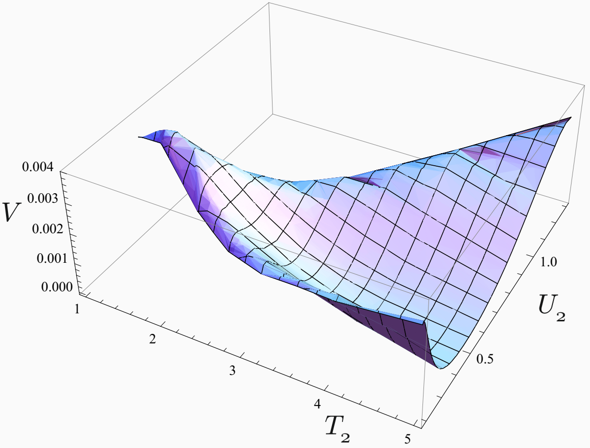
A comprehensive scan shows that a number of approximately 7×10^4 models in the class under consideration satisfy all criteria. Among them we have 9×10^3 super no-scale models. A tedious numerical calculation leads to 1792 models with $V_0 > 0$.



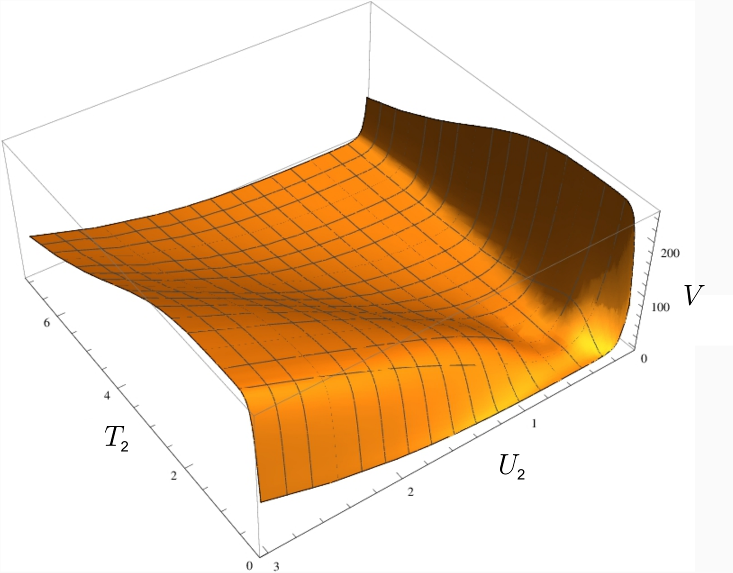
One loop potentials: Super no scale model potentials



One loop potentials: Super no scale model potentials



One loop potential as a function of the T_2, U_2 moduli.



One loop potential as a function of the T_2, U_2 moduli.

Gauge coupling Running - Thresholds

The gauge coupling running is calculable in the context of string theory. It turns out that they depend on the compactification moduli. At one loop level

$$\frac{16\pi^2}{g_i^2(\mu)} = k_a \frac{16\pi^2}{g_s^2} + b_a \log \frac{M_s^2}{\mu^2} + \Delta_a$$

where $M_s = g_s M_P$, $M_P = 1/\sqrt{32 G_N}$.

$b_a \leftrightarrow$ Massless modes $\quad \Delta_a \leftrightarrow$ Massive modes

$\Delta_a = \Delta'_a(t_i) + \hat{\Delta}_a$

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see e.g. L. J. Dixon, V. Kaplunovsky and J. Louis (1991) , I. Antoniadis, E. Gava , K.S. Narain (1992) , C. Angelantonj, I. Florakis and M. Tsulaia (2014) , Florakis (2015)

Decompactification problem

$$\Delta'_a - \Delta'_b = \sum_i \Big\{ -\alpha^i_{ab} \log \left[T^i_2 U^i_2 \, |\eta(T^i)\, \eta(U^i)|^4 \right] \\ -\beta^i_{ab} \log \left[T^i_2 U^i_2 \, |\vartheta_4(T^i)\, \vartheta_2(U^i)|^4 \right] \\ -\gamma^i_{ab} \log \left[|\hat{j}_2(T^i/2) - \hat{j}_2(U^i)|^4 |j_2(U^i) - 24|^4 \right] \Big\} ,$$

$\alpha^i_{ab}, \beta^i_{ab}, \gamma^i_{ab}$ model dependent coefficients. The dominant growth at $T^i_2 \gg 1$

$$\Delta'_a = \alpha^i_a \left(\frac{\pi}{3} T^i_2 - \log T^i_2 \right) + \ldots ,$$

Solutions ? : $a^i_a = 0, \ldots$

Antoniadis (1990)

E. Kiritsis , C. Kounnas, P.M. Petropoulos, J. R. (1996)

C. Angelantonj and I. Florakis (2019)

The full moduli dependent threshold part takes the form

$$\Delta_a = -\frac{k_a}{48}\, Y + \hat{\beta}_a\, \Delta\,, \text{ where}$$

$$\Delta = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2}\, \Gamma_{2,2}(T,U) = -\log\left[T_2 U_2\, \left|\eta(T)\,\eta(U)\right|^4\right]\,.$$

and the universal part Y is defined as

$$Y = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2}\, \Gamma_{2,2}(T,U)\left(\frac{\hat{\bar{E}}_2 \bar{E}_4 \bar{E}_6 - \bar{E}_4^3}{\bar{\Delta}} + 1008\right)\,,$$

At the limit $T_2 \gg 1$

$$Y = 48\pi T_2 + \mathcal{O}(T_2^{-1}) \;\;,\;\; \Delta = \frac{\pi}{3}\, T_2 - \log T_2 + \mathcal{O}(e^{-2\pi T_2})$$

and finally

$$\Delta_a = \left(\frac{\hat{\beta}_a}{3} - k_a\right)\pi T_2 + \mathcal{O}(\log T_2)\,.$$

Computation of the thresholds

A comprehensive scan over a class of 7×10^4 models with $SO(10) \times SO(8)^2 \times U(1)^2$ gauge symmetry yields for the non-abelian gauge couplings

$\hat{b}_{10}$	$\hat{b}_8$	$\hat{b}_{8'}$	# of models	%
3	3	3	29456	38.5
9	-3	-3	15840	20.7
-3	9	9	14000	18.3
.	.	.	...	22.5

In a big class of vacua we have $\hat{\beta}_a = 3k_a$ (decompactification condition) , hence there is no decompactification problem for the gauge couplings g_{10}, g_8, g_8'

$$\Delta_a = \cancel{\left(\frac{\hat{\beta}_a}{3} - k_a\right)} \pi T_2 + \mathcal{O}(\log T_2)$$

Gauge coupling running

For models satisfying the decompactification condition $\hat{\beta}_a = 3k_a$ the coupling running is

$$\frac{16\pi^2}{g_a^2(\mu)} = k_a \frac{16\pi^2}{g_s^2} + \beta_a \log \frac{M_s^2}{\mu^2} + \beta_a' \log \left(\frac{2e^{1-\gamma}}{3\pi\sqrt{3}} \frac{M_{\mathrm{KK}}^2}{M_s^2} \right) + \dots$$

Here, γ is the Euler-Mascheroni constant, $M_{KK} = 1/\sqrt{T_2}$ is the Kaluza-Klein scale. $\beta_a = b_a^{(1)} + b_a^{(2)} + b_a^{(3)}$ and $\beta_a' = b_a^{(1)} + b_a^{(2)}$ with $b_a^{(1)} = \hat{\beta}_a$

A Standard Model scenario

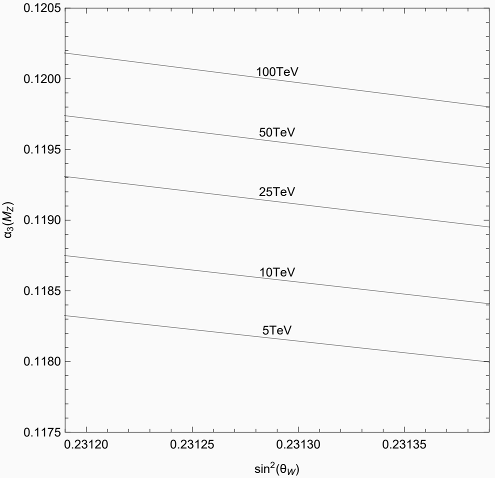
$$\frac{k_2 + k_Y}{\alpha_S} = \frac{1}{\alpha_{\rm em}} - \frac{\beta_2 + \beta_Y}{4\pi} \log \frac{M_S^2}{M_Z^2} - \frac{\beta_2' + \beta_Y'}{4\pi} \log \left(\frac{2e^{1-\gamma}}{3\pi\sqrt{3}} \frac{M_{\rm KK}^2}{M_S^2} \right)$$

$$\sin^2 \theta_W = \frac{k_2}{k_2 + k_Y} + \frac{\alpha_{\rm em}}{4\pi} \left[\frac{k_Y \beta_2 - k_2 \beta_Y}{k_2 + kY} \log \frac{M_S^2}{M_Z^2} + \right.$$
$$\left. \frac{k_Y \beta_2' - k_2 \beta_Y'}{k_2 + kY} \log \left(\frac{2e^{1-\gamma}}{3\pi\sqrt{3}} \frac{M_{\rm KK}^2}{M_S^2} \right) \right]$$

$$\frac{1}{\alpha_3(M_Z)} = \frac{k_3}{\alpha_{\rm em}(k_2 + k_Y)} + \frac{1}{4\pi} \left[\left(\beta_3 - \frac{k_3(\beta_2 + \beta_Y)}{k_2 + k_Y} \right) \log \frac{M_S^2}{M_Z^2} \right.$$
$$\left. + \left(\beta_3' - \frac{k_3(\beta_2' + \beta_Y')}{k_2 + k_Y} \right) \log \left(\frac{2e^{1-\gamma}}{3\pi\sqrt{3}} \frac{M_{\rm KK}^2}{M_S^2} \right) \right]$$

A Standard Model scenario

For $(\beta_Y, \beta_2, \beta_3) = (-7, -\frac{19}{6}, \frac{41}{6})$, $(k_Y, k_2, k_3) = (\frac{5}{3}, 1, 1)$ and $(\beta'_Y, \beta'_2, \beta'_3) = (-\frac{15}{2}, -\frac{43}{6}, -\frac{23}{3})$.



Towards realistic model building

The Pati–Salam model $SU(4) \times SU(2)_L \times SU(2)_R$ can be easily implemented in this framework.

In the free fermionic language it requires the introduction of one additional vector

$$\beta_{10} = \alpha = \{\bar{\psi}^{45}\}$$

This would allow for the study of more realistic scenarios both from the point of view of the spectrum and gauge coupling thresholds.

We have constructed a few consistent models. The full classification is under investigation.

Conclusions

We have analysed a class of non supersymmetric heterotic vacua where SUSY is spontaneously broken via the Scherk–Schwartz mechanism. In this context we have constructed semi-realistic models with the following interesting characteristics

- Fermion chirality
- Exponentially small cosmological constant
- Models where the SUSY breaking scale $M_{susy} \sim \frac{1}{R} \ll M_{Planck}$
- Finite gauge coupling running (no decompactification problem)
- These developments pave the way for non-supersymmetric string phenomenology (consider more realistic models e.g. Pati–Salam)