Spontaneous Dark-Matter Mass Generation in Field and String Theory

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September 15, 2019

Based on work done in collaboration with

- Thibaut Coudarchet and Lucien Heurtier [JHEP 1903 (2019) 117]
- Lucien Heurtier and Felix Kling [to appear]

Conference on Recent Developments in Strings and Gravity, Corfu

Introduction

- To explain motion of galaxies
  - Change Gravity at large distances or,
  - Cold Dark Matter : Non-Relativistic *i.e.* Mass $> T$

- In Λ-CDM, the Dark-Matter Mass = cst parameter

- But all Masses in SM arise by Higgs Mechanism

$\Longrightarrow$ More natural to assume an Higgs-like mechanism to generate DM mass

- Consequences :
  - The DM-Mass vary in time
  - Cross-sections with SM can be much larger : May allows detection

- Simplest model in QFT and in String Theory

Standard Constant Mass Scenario

■ At some early epoch, a fermion ψ (Dirac, Majorana,...) is in thermal equilibrium

$\Longrightarrow$ The density of particles n_ψ follows Boltzmann Distribution

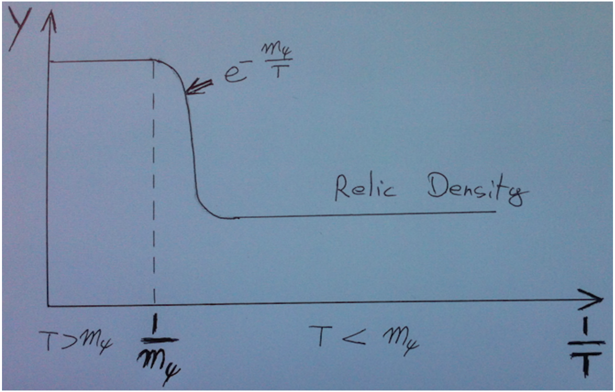
■ This is possible if ψ interacts (weakly) with the SM (lighter)

$$\psi + \bar{\psi} \longrightarrow f + \bar{f}$$

An excess of $\psi \Longrightarrow$ Annihilation into SM

■ The Yield is proportional to the number of particles

$$Y = \frac{n_\psi}{s} = \frac{a^3 n_\psi}{S} \qquad \text{where} \qquad s = \text{Entropy Density}$$

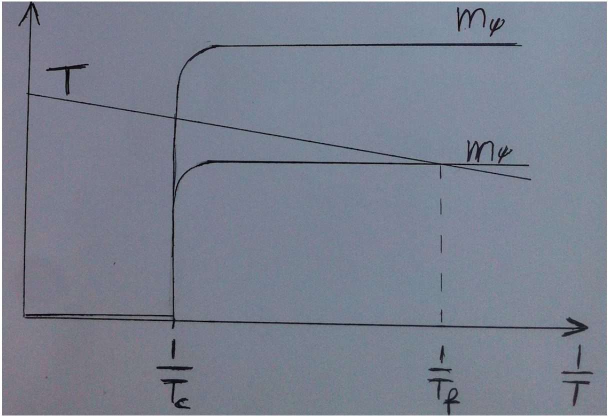


■ The Annihilation stops maintaining Thermal Equilibrium when the Universe expands too fast

$$n_\psi \langle \sigma v \rangle < 3H \qquad \text{where} \quad \sigma = \text{cross section}, \;\; v = \text{relative velocity}$$

■ When $T \lesssim m_\psi$, $n_\psi \simeq n_\psi^{\text{eq}}$ falls exponentially
$\Longrightarrow$ DM deouples from Themal Bath = Freezes Out

Varying DM Mass Scenario



- Either $T_c \gg T_f \simeq m_{\psi,f} \simeq m_{\psi,\text{today}}$: Standard Paradigm
- Or $T_c \gtrsim T_f \simeq m_{\psi,f} < m_{\psi,\text{today}}$: New Paradigm

Model in QFT

■ **At tree level**

$$\mathcal{L}_{\text{tree}} = i\bar{\psi}\partial\!\!\!/\psi + \frac{1}{2}(\partial\phi)^2 - y\phi\bar{\psi}\psi - \mathcal{V}_{\text{tree}}(\phi) + \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{c.t.}}$$

$$\mathcal{V}_{\text{tree}}(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4$$

- The mass of ψ is $m_\psi(\langle\phi\rangle) = y\langle\phi\rangle$
- $\mathcal{L}_{\text{int}}$ couples weakly SM and ψ (not ϕ)

NB: See talk of Susha Parameswaran for dark energy purposes.

■ The simplest implementation of a phase transition is when we have a gas of particles ψ at finite T

$\Longrightarrow$ We add to $\mathcal{V}_{\text{tree}}$ the 1-loop contributions arising from the Matsubara excitations of ψ :

The free energy density + Coleman-Weinberg effective potential

NB : In simplest version, ϕ is treated classically ($\lambda < y^4 < 1$).

$$\mathcal{F}(T,\phi) = -n_F \left[\frac{7\pi^2}{720} T^4 - \frac{T^2}{48} m_\psi^2 - \frac{m_\psi^4}{64\pi^2} \log\left(\frac{m_\psi^2}{\alpha T^2}\right) + \mathcal{O}\left(\frac{m_\psi^6}{T^2}\right) \right]$$

$$\mathcal{V}_{\text{CW}}(\phi) = -n_F \frac{m_\psi^4}{64\pi^2} \left[\log\left(\frac{m_\psi^2}{Q^2}\right) - \frac{3}{2} \right] \qquad \text{where} \quad m_\psi = y\phi$$

$\left(\alpha = \pi^2 \exp(3/2 - 2\gamma_E\right)$

$$\mathcal{V}_{\text{eff}}^{\text{th}}(T,\phi) = -n_F \frac{7\pi^2}{720} T^4 - \frac{\mu_{\text{eff}}(T)^2}{2}\phi^2 + \frac{\lambda_{\text{eff}}(T)}{4!}\phi^4 \,,$$

$$\mu_{\text{eff}}(T)^2 = \mu^2 \left(1 - \left(\frac{T}{T_c}\right)^2\right) \qquad \text{where} \qquad T_c = \frac{2\sqrt{6}}{\sqrt{n_F}}\,\frac{\mu}{y}$$

$$\lambda_{\text{eff}}(T) = \lambda + \frac{3n_F}{4\pi^2}\, y^4 \log \frac{T_c}{T} \qquad \text{where} \qquad Q = \pi e^{-\gamma_E} T_c$$

$$T > T_c: \quad \langle\phi\rangle = 0\,, \qquad m_\phi = |\mu_{\text{eff}}(T)|\,, \qquad m_\psi = 0$$

$$T_c > T: \quad \langle\phi\rangle = \mu_{\text{eff}}(T)\sqrt{\frac{6}{\lambda_{\text{eff}}(T)}}\,, \quad m_\phi = \sqrt{2}\mu_{\text{eff}}(T)\,, \quad m_\psi = y\langle\phi\rangle$$

■ Valid until Freeze-out *i.e.* temperature $T_f \simeq m_\psi(T_f)$

$$T_f = \frac{T_c}{\sqrt{1+u(T_f)}} \qquad \text{where} \qquad u(T) = \frac{4\lambda_{\text{eff}}(T)}{n_F y^4}$$

- $u(T_f) \gg 1$ $\Longrightarrow$ Constant Mass Scenario
- $u(T_f) \simeq 1$ (or lower) $\Longrightarrow$ Varying Mass Scenario

■ At $T < T_f$, we have $N_\psi = \text{cst}$ particles of dust decoupled from the thermal bath.

They have individual trajectories $X^\mu(\tau_i)$, where τ_i is an arbitrary coordinate

$$\begin{aligned} S_{\text{dust}} &= -\text{ Mass} \times \text{Proper Time} \\ &= -\sum_i \int d\tau_i \;\, y\phi(X_i)\, \sqrt{g_{\mu\nu}(X_i)\, \frac{dX_i^\mu}{d\tau_i} \frac{dX_i^\nu}{d\tau_i}} \end{aligned}$$

- $\frac{\delta}{\delta g_{\mu\nu}}$ and $\frac{\delta}{\delta \phi} \quad \Longrightarrow \quad$ Source for gravity and ϕ
- When the particles are Non-Relativistic

$$\rho_{\text{dust}} \simeq n_\psi y \phi\,, \qquad P_{\text{dust}} \simeq 0\,, \qquad \mathcal{V}_{\text{dust}} \simeq n_\psi y \phi$$

■ $\mathcal{V}_{\text{tree}} + \mathcal{V}_{\text{CW}} + \mathcal{V}_{\text{dust}}$ has a minimum at some $\langle\phi\rangle_\infty$ at late times

$$\Longrightarrow \qquad m_\psi \longrightarrow m_\psi^\infty = y\langle\phi\rangle_\infty$$

Comparison with Standard Scenario

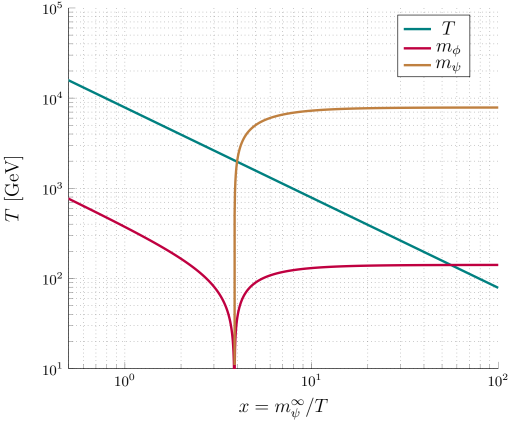
■ Solve Boltzmann equation for ψ

$$\frac{dY_\psi}{d(1/T)} = \langle \sigma v \rangle \, \frac{T s}{H} \left(Y_{\psi \mathrm{eq}}^2 - Y_\psi^2 \right)$$

For interactions with SM fermions

$$G_V \, \bar{\psi} \gamma_\mu \psi \, \bar{f} \gamma^\mu f \qquad \Longrightarrow \qquad \langle \sigma v \rangle \simeq G_V^2 \, \frac{1}{2\pi} \left(1 + \frac{T}{m_\psi(T)} \right) m_\psi^2(T)$$

$$G_S \, \bar{\psi} \psi \bar{f} f \qquad \Longrightarrow \qquad \langle \sigma v \rangle \simeq G_S^2 \, \frac{3}{8\pi} \, T \, m_\psi(T)$$



Before the transition at T_c, ψ is thermalized at temperature $T > m_\phi(T)$

$\Longrightarrow \quad \psi + \psi$ can annihilate into $\phi + \phi$ (t-channel)

$\Longrightarrow$ Compute $\langle \sigma_{\psi\psi\to\phi\phi} v \rangle$ and solve a second Boltzmann equation for Y_ϕ, when ϕ is out of equilibrium = "Freeze-In Mechanism"

■ Choose randomly μ, λ, y, $G_{V,S}$ and run the numerical simulation.

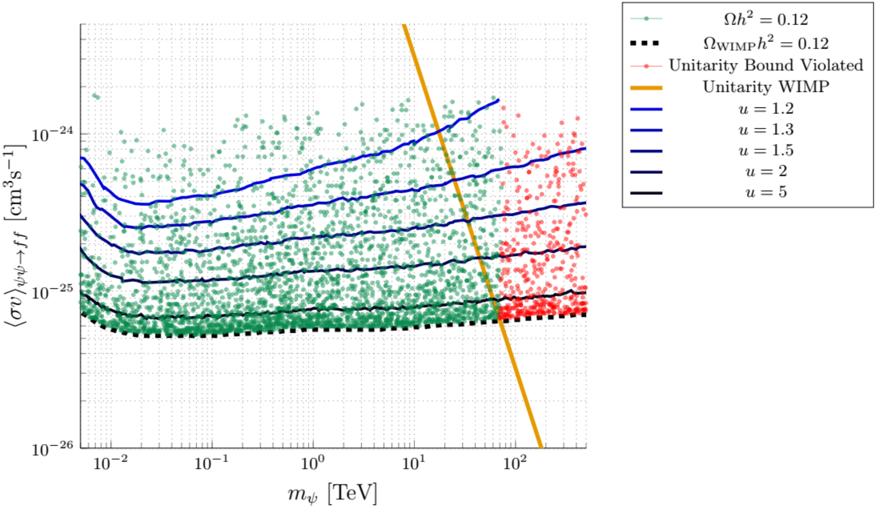
■ For $G_V\,\bar{\psi}\gamma_\mu\psi\,\bar{f}\gamma^\mu f$, select the cases where

- The Relic density is correct : $(\Omega_\psi + \Omega_\phi)h^2 \equiv 0.12$
- ϕ does not thermalize

■ For $G_S\,\bar{\psi}\psi\bar{f}f$: The lifetime of ϕ is short (annihilation into SM at 1-loop)

- The Relic density is correct : $\Omega_\psi h^2 \equiv 0.12$

■ Cross-Section with SM and m_ψ Today :



■ $\simeq$ same bound on m_ψ

■ $\langle \sigma v \rangle$ can be 30 times larger than for the constant mass WIMP

Case ϕ and ψ thermalized

■ We add to $\mathcal{V}_{\text{tree}}$ the 1-loop free energy density + Coleman-Weinberg effective potential of ϕ and ψ. For ϕ they are :

- Taking 2 derivatives of $\mathcal{V}_{\text{tree}}$ $\implies$ $m_0(\phi)^2 = -\mu^2 + \frac{\lambda}{2}\phi^2$

$$\mathcal{F}(T,\phi) = -\frac{\pi^2}{45}T^4 + \frac{T^2}{24}m_0^2 - \frac{T}{12\pi}\left(m_0^2\right)^{\frac{3}{2}} - \frac{m_0^4}{64\pi^2}\log\left(\frac{m_0^2}{16\alpha T^2}\right) + \mathcal{O}\left(\frac{m_0^6}{T^2}\right)$$
$$\mathcal{V}_{\text{CW}}(\phi) = \frac{m_0^4}{64\pi^2}\left[\log\left(\frac{m_0^2}{Q^2}\right) - \frac{3}{2}\right]$$

- High temperature restores the $\mathbb{Z}_2$ symmetry of the potential $\implies$ thermal loops dominate over the zero-temperature tree level contribution : Perturbation theory breaks down.

■ The high T quantum corrections are dominated by the "ring diagrams", with arbitrary number of loops [Dolan, Jackiw, '74] [Carrington, '92][Delaunay, Grojean, Wells, '07][Martin, '14][Elias-Miró, Espinosa, Konstandin, '14]...

$$\Longrightarrow \quad V^{\rm th}_{\rm ring}(T,\phi) = \frac{T}{12\pi}\Big[\big(m_0(\phi)^2\big)^{\frac{3}{2}} - \big(m_0(\phi)^2 + \Pi_\phi(T)\big)^{\frac{3}{2}}\Big]$$

The Mass Shift Π_ϕ is the dominant thermal correction to m_0^2 arising from $\mathcal{F}_\phi + \mathcal{F}_\psi$

$$\Pi_\phi(T) = \frac{T^2}{24}(\lambda + n_F y^2)$$

■ Derive effective $\mu_{\rm eff}(T)$ and $\lambda_{\rm eff}(T)$ $\quad\Longrightarrow\quad$ Phase transition at T_c as before...

Model in String Theory

■ We have to care about $\simeq$ flatness of the Universe.
(In QFT, we did not solve the cosmological equations of motion. We solved Boltzmann equation for the yield.)

■ At tree level : Choose background in d dimensions such that :

- $\langle \mathcal{V}_{\text{tree}} \rangle = 0$
- Supersymmetry is broken

■ At 1-loop : Thermal effective potential

$$\mathcal{V}_{\text{th}} = \mathcal{V}_{\text{CW}} + \text{Free Energy}$$

$\Longrightarrow$ induce sources ρ, P for gravity

$\Longrightarrow$ The static background is no more solution = Cosmology

■ At 1-loop, $\mathcal{V}_{\text{th}}$ is expressed in terms of tree level masses.

■ These masses depend on moduli fields (flat directions of $\mathcal{V}_{\text{tree}} = 0$).

E.g. : In Heterotic string, states generically massive can become massless when $R \equiv e^{\phi} = 1$

$$\left| R - \frac{1}{R} \right| \simeq 2\phi$$

■ We want the 1-loop thermal effective potential $\mathcal{V}_{\text{th}}$ to imply :

- High Temperature $\implies$ $\langle \phi \rangle = 0$
- Low Temperature $\implies$ $\langle \phi \rangle \neq 0$

$$S^1_{\mathrm{E}}(R_0) \times \mathbb{R}^{d-1} \times S^1(R_d) \times T^{8-d} \times S^1(R_9)\,, \qquad R_d \equiv e^{\phi}$$

■ Finite temperature : Matsubara (=Kaluza-Klein) states with momentum

$$\frac{m_0 + \frac{F}{2}}{R_0}\,, \qquad T = \frac{1}{2\pi R_0}$$

■ Supersymmetry breaking via Sherk-Schwarz mechanism along X^9 :
The states that can become massless at $R_d = 1$ have masses

$$\frac{\left| m_9 + \frac{F + B_{9d}\, n_d}{2} \right|}{R_9}\,, \qquad M = \frac{1}{2R_9}$$

where B_{9d} is Antisymmetric tensor, $\quad n_d = \pm 1$

- $B_{9d} = 0 \implies$ 2 bosons are massless
- $B_{9d} = 1 \implies$ 2 fermions are massless

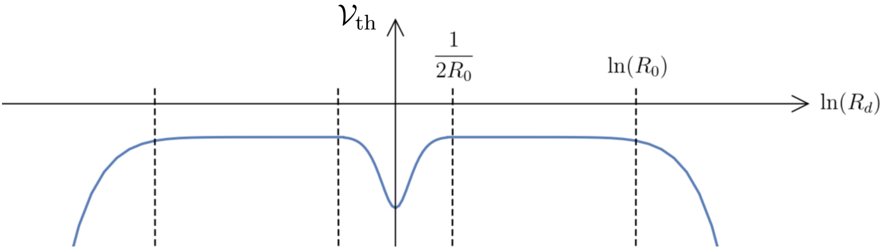
■ When T, $M \gg M_{\text{string}}$, the thermal potential is dominated by the KK modes

$$\mathcal{V}_{\text{th}}(T, M, \phi) = -\,(N_F + N_B)\, T^d\, f_T(M/T) + (N_F - N_B)\, M^d\, f_V(M/T)$$
$$+\,\phi^2 N_{\text{extra}}\Big[T^{d-2}\, \tilde{f}_T(M/T) + (-1)^{B_{9d}}\, M^{d-2}\, \tilde{f}_V(M/T)\Big] + \cdots$$

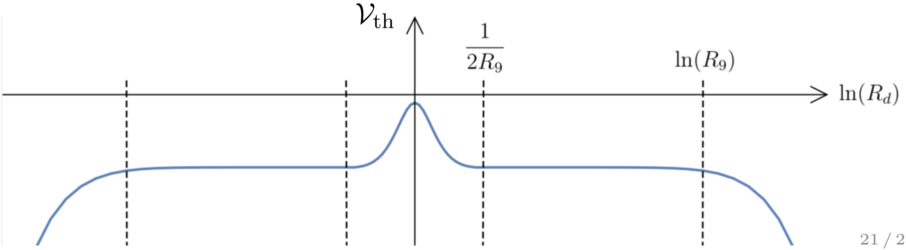
- N_F, N_B are the numbers of massless Fermions and Bosons
- $N_{\text{extra}} = 2$ is the number of extra massless states

- Extra Bosons

or Extra Fermions when $T > M$



- Extra Fermions when $T < M$



■ Solve equations of motions for $a(t)$, $\Phi_{\rm dil}(t)$, $M(t)$, $T(t)$ and $\phi(t)$

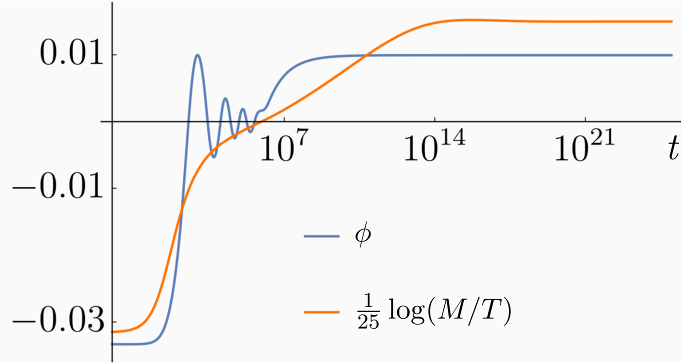
• If $\quad 0 < \dfrac{N_F - N_B}{N_F + N_B} < \dfrac{1}{2^d - 1}$, then $\mathcal{V}_{\rm th}$ as a function of the M/T admits a minimum

$$\Longrightarrow \qquad \frac{M(t)}{T(t)} \longrightarrow r \;\; \text{(depends on } N_F, N_B\text{)}$$

• If $T(t_i) > M(t_i)$ at initial time

• If $r > 1$, the attractor forces $T(t) < M(t)$ at later times

• If Extra Fermions



- ϕ oscillates in the well
- It is destabilized when $M > T$
- $\phi \to \langle\phi\rangle$ along flat direction (friction due to Universe Expansion)
- The 2 fermions Freeze-Out

Conclusion

■ DM with Mass acquired by Higgs Mechanism generalizes the paradigm of the WIMP

■ For a given DM-mass today, the Cross-Section DM+DM$\rightarrow$ SM+SM can be 1 order of magnitude larger

■ Such large cross-sections may allow DM detection in the near future