Treatment of γ_5 in Dimensionally-Regularized Chiral Yang-Mills Theory with Scalar Fields

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[Work In Progress], with Amon Ilakovac, Marija Mađor-Božinović (PMF Zagreb), and Dominik Stöckinger (IKT, TU Dresden)

Context & Motivation

- Realistic models in 4D contain **chiral** fermions (e.g. Standard Model and extensions; SUSY models, ...).
- Dimensional Regularization (DReg) with MS(bar) subtraction scheme widely used in calculations, in the literature, in automated codes, etc., because it doesn't break (explicit) gauge and Lorentz symmetries.
- ∃ numerous schemes in DReg for treating γ_5 matrix (see e.g. the reviews [Gnendiger...–2017,Bruque...–2018], and [Larin–1993,Trueman–1995,Jegerlehner–2000] – *non-exhaustive*) → perform consistent loop calculations, reproduce known results (ABJ anomaly, etc.).
- However it is not clear what happens at all loop orders. In certain situations, at > 1-loop level some "naive" γ_5 treatment is used, supplemented with manual restoration of symmetries (using e.g. Ward IDs, etc.). Other schemes [Kreimer–1990,'94] even abandon the cyclicity property of the Trace operation...

Our aims

In this talk:

- Propose using ***algebraic renormalization*** techniques in Dimensional Regularization (DReg) with the 't Hooft-Veltman-Breitenlohner-Maison scheme (BMHV), following [Martin,Sanchez-Ruiz–1999];
- Obtain a treatment of γ_5 matrix and other related intrinsically 4-dimensional Lorentz objects that is **consistent by construction** at any loop order.

- **Revisit** the renormalization of a generic Chiral Yang-Mills Theory supplemented with real scalar fields; example on an explicit calculation at 1-loop.
- **Restore** BRST (and possible other) symmetries; **recover** known results concerning gauge anomalies.
- **Work In Progress** – One of our aims is to obtain RGEs and compare them against the usual ones, i.e. method of [Machacek,Vaughn–1983,'84,'85].
- These results are also **needed for 2+ - loop calculations**.

Additionally...

- **Note:** A similar procedure may need to be followed in principle in Dimensional-Reduction scheme for SUSY theories... (open question)

- **Disclaimer:** **We do NOT claim** that the other schemes are inconsistent; however **a more solid proof of these in the context of algebraic renormalization** would be welcome so as to have more confidence in them. Exhibiting relations between these schemes?

Our status wrt. the work [Martin,Sanchez-Ruiz–1999] (and related ones):

	No scalars	With scalars
Abelian	/	[Sanchez-Ruiz–2002]
Non-abelian YM	[Martin,Sanchez-Ruiz–1999]	w/o VEV: Our work w/ VEV: ?
N-ab. YM in Bkgd-field gauge	··· Extension of ···	··· previous works ···

Outline

Outline

Dimensional regularization (DReg) ['t Hooft,Veltman–1972] (1/3)

DReg introduced by 't Hooft and Veltman (1972) as a way to elegantly regularize loop amplitudes in (non-SUSY) Yang-Mills theories.

- Gauge and Lorentz symmetries explicitly preserved ($\neq$ in e.g. Pauli-Villars or other cut-offs regs.).

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Observations:

- Lorentz structures considered as whole objects without any mention of explicit values for their Lorentz indices.
- d-dimensional loop integrals are formal and made to obey the "usual" rules of linearity, translation, scaling, etc.

(See also [Collins–1986].)

Dimensional regularization (DReg) ['t Hooft,Veltman–1972] (2/3)

Formally extend 4D space-time into $d = 4 - 2\epsilon$-"dimensions": $\mathbb{M}_d \supset \mathbb{M}_4$ with $\mathbb{M}_d = \mathbb{M}_4 \oplus \mathbb{M}_{-2\epsilon}$. Small $\epsilon > 0$ regularizes UV divergences ($\epsilon < 0$ for IR divs.).

- **Metrics:** $g_{\mu\nu} = \bar{g}_{\mu\nu} + \hat{g}_{\mu\nu}$. One can construct an "inverse" metric $g^{\mu\nu}$ (mathematically via dual space). Similarly for the other subspaces. The $\hat{\cdot}$ quantities are the so-called ***evanescent*** objects.

- **Properties:**

$$g_{\mu\nu} g^{\nu\rho} = \delta^\rho_\mu \; (d\text{-dimensions}) \,, \; g_{\mu\nu} \bar{g}^{\nu\rho} = \bar{g}_{\mu\nu} \bar{g}^{\nu\rho} = \bar{\delta}^\rho_\mu \; (4\text{-dimensions}) \,,$$
$$g_{\mu\nu} \hat{g}^{\nu\rho} = \hat{g}_{\mu\nu} \hat{g}^{\nu\rho} = \hat{\delta}^\rho_\mu \; (-2\epsilon\text{-dimensions}) \,, \; \bar{g}_{\mu\nu} \hat{g}^{\nu\rho} = 0 \; (\text{"orthogonality"}) \,,$$

  and

$$g_{\mu\nu} g^{\nu\mu} = d \,, \; \bar{g}_{\mu\nu} \bar{g}^{\nu\mu} = 4 \,, \; \hat{g}_{\mu\nu} \hat{g}^{\nu\mu} = -2\epsilon \,.$$

  Similar behaviour with other tensorial quantities, e.g.:

$$\bar{g}_{\mu\nu} k^\nu = \bar{k}_\mu \,, \; \hat{g}_{\mu\nu} k^\nu = \hat{k}_\mu \,, \; \bar{g}_{\mu\nu} \hat{k}^\nu = 0 \,.$$

Dimensional regularization (DReg) ['t Hooft,Veltman–1972] (3/3)

Q1? DimReg OK for bosonic fields and also 4-component fermions, but what about Dirac γ matrices? Usual γ_μ can be constructed (see [Collins–1986]). Convention for $\mathrm{Tr}\,\mathbb{1}$ (spinor space): $\mathrm{Tr}\,\mathbb{1} = 4$ so as to make results simpler.

Q2? What about intrinsically 4-dimensional objects: γ_5 and $\epsilon_{\mu\nu\rho\sigma}$?
(Inconsistency if γ_5 anticommute:
$\mathrm{Tr}(\gamma_5\gamma_{\mu_1}\cdots\gamma_{\mu_4}) \propto (d-4)\,\mathrm{Tr}(\gamma_5\gamma_{\mu_1}\cdots\gamma_{\mu_4}) = 0$ when $d \to 4$
instead of $4i\epsilon_{\mu_1\mu_2\mu_3\mu_4}$.)

Properties for Dirac matrices:
$\gamma_5 = (i/4!)\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$, $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$, $\{\gamma_5, \bar{\gamma}^\mu\} = 0$, but $[\gamma_5, \hat{\gamma}^\mu] = 0$, and: $\{\gamma_5, \gamma^\mu\} = \{\gamma_5, \hat{\gamma}^\mu\}$, $[\gamma_5, \gamma^\mu] = [\gamma_5, \bar{\gamma}^\mu]$.

Consistency: Scheme proved to be axiomatically consistent at all orders by Breitenlohner and Maison [Breitenlohner,Maison–1975, Breitenlohner,Maison–1977].
$\Rightarrow$ **'t Hooft - Veltman - Breitenlohner - Maison scheme** ("BMHV" scheme).
Together with MS(bar) subtraction (subtracting the ϵ poles, possibly with some finite part) $\Rightarrow$ **"Dimensional renormalization"** (DimRen).

Outline

The R-Model defining action S_0

Model with generic gauge group $\mathcal{G}$ (usually $SU(N)$; can be something else...) with right-handed (RH) fermions in "right" (R) rep. of $\mathcal{G}$ and scalars in S rep. of $\mathcal{G}$, both coupling to gauge bosons.
Originally defined in 4 dimensions, using either Weyl, or Dirac fermions with projectors $\mathbb{P}_{\mathsf{R/L}} = (1 \pm \gamma_5)/2$.

$$S_0^{(4D)} = \int \mathrm{d}^4 x \left(\mathcal{L}_{\mathsf{YM}}^{(4D)} + \mathcal{L}_{\Psi}^{(4D)} + \mathcal{L}_{\Phi}^{(4D)} + \mathcal{L}_{\mathsf{Yuk}}^{(4D)} + \mathcal{L}_{\mathsf{gh}}^{(4D)} + \mathcal{L}_{\mathsf{g\text{-}fix}}^{(4D)}\right),$$

with:

$$\mathcal{L}_{\mathsf{YM}}^{(4D)} = \frac{-1}{4} F^a_{\mu\nu} F^{a\,\mu\nu}\,,\ \mathcal{L}_{\Phi}^{(4D)} = \frac{1}{2}(D_\mu \Phi^m)^2 - \frac{\lambda_H^{mnop}}{4!}\Phi_m \Phi_n \Phi_o \Phi_p\,,$$

$$\mathcal{L}_{\Psi}^{(4D)} = i\overline{\Psi}_i \partial\!\!\!/ \mathbb{P}_{\mathsf{R}} \Psi_i + g_S T^a_{R\,ij} \overline{\Psi}_i G\!\!\!/^a \mathbb{P}_{\mathsf{R}} \Psi_j \equiv i\overline{\Psi}_i D\!\!\!\!/_R^{ij} \Psi_j\,,$$

$$\mathcal{L}_{\mathsf{Yuk}}^{(4D)} = -(Y_R)^m_{ij} \Phi_m \overline{\Psi}{}_i^C \mathbb{P}_{\mathsf{R}} \Psi_j + \mathsf{h.c.}\,,$$

$$\mathcal{L}_{\mathsf{gh}}^{(4D)} = \partial_\mu \bar{c}_a \cdot D^{ab\,\mu} c_b\,,\ \mathcal{L}_{\mathsf{g\text{-}fix}}^{(4D)} = \frac{\xi}{2} B^a B_a + B^a \partial^\mu G^a_\mu\,.$$

Note the Yukawa interaction with charge-conjugated fermion ($\neq$ Dirac model where left component couples to right component).

The R-Model action S_0 extended to d dimensions

Formally extend S_0 to d dimensions such that **propagators retain their form in d-D**. Trivially done for bosonic fields. RH fermions introduce **two problems**:

1. Kinetic term is **chiral**. In d-D this generates an (inverse) propagator $\propto \overline{\not{p}}$ (see e.g. [Bilal–2008]).
   $\Rightarrow$ We need an actual d-D kinetic term: $i\overline{\Psi}_i \not{\partial} \Psi_i$
   $\Rightarrow$ Equivalent to introducing a "left-handed inert" component to the fermions. (Inert because gets removed in interaction terms due to explicit presence of $\mathbb{P}_{\mathsf{R/L}}$.)
2. How to promote in d-D the $\overline{\Psi}\not{G}\Psi$ interaction term: $g_S T_{R\,ij}^{\,a} \overline{\Psi}_i \not{G}^a \mathbb{P}_{\mathsf{R}} \Psi_j$? While in 4D: $\gamma_\mu \mathbb{P}_{\mathsf{R}} = \mathbb{P}_{\mathsf{L}} \gamma_\mu = \mathbb{P}_{\mathsf{L}} \gamma_\mu \mathbb{P}_{\mathsf{R}}$, it is not so in d-D: they differ by an evanescent term.

$\Rightarrow$ **NO unique way of extending the model to d-dimensions!**
$\Rightarrow$ Use the interaction term that makes calculations **the most simple**:

$$g_S T_{R\,ij}^{\,a} \overline{\Psi}_i \mathbb{P}_{\mathsf{L}} \not{G}^a \mathbb{P}_{\mathsf{R}} \Psi_j \quad (\text{"symmetric"})\,.$$

This choice explicitly conveys the fact that we really started with RH spinors...

Nota about charge conjugation

While it is clear how to define the charge conjugation operation in 4D with e.g. an explicit construction: numerically $C = \begin{pmatrix} \epsilon_{\alpha\beta} & 0 \\ 0 & \epsilon^{\dot\alpha\dot\beta} \end{pmatrix} \sim i\gamma^0\gamma^2$ with the good properties,
In d-D we can define a similar operation only by its action on the fermions – such that it turns fermions to their charge-conjugate and back: $\Psi^C = C\overline{\Psi}^T$ –, and its action on Dirac 4-spinor bilinears:

$$(\Psi^C)^C = \Psi\,,\; C^T = -C\,;$$

$$\overline{\Psi}_i^C \Gamma \Psi_j^C = -\Psi_i^T C^{-1} \Gamma C \overline{\Psi}_j^T = \overline{\Psi}_j C \Gamma^T C^{-1} \Psi_i = \eta_\Gamma \overline{\Psi}_j \Gamma \Psi_i\,,$$

$$\text{with: } \eta_\Gamma = \begin{cases} +1 & \text{for } \Gamma = \mathbb{1}\,,\, \gamma_5\,,\, \gamma^\mu\gamma_5\,, \\ -1 & \text{for } \Gamma = \gamma^\mu\,,\, \sigma^{\mu\nu}\,,\, \sigma^{\mu\nu}\gamma_5\,. \end{cases}$$

(See e.g. Appendix A of [Tsai–2011].)

Outline

The BRST symmetry [Becchi,Rouet,Stora–1975,Tyutin–1975]




There remains a residual symmetry *even after fixing the gauge*: **BRST symmetry** ($\approx$ "weak" or "generalized" version of the gauge symmetry).
From infinitesimal gauge transfo. of fields: $\varphi_i \to \delta_\alpha \varphi_i$ linear in the (small) gauge parameter α, replace $\alpha^a \to \theta c^a$, with θ: Grassmann parameter, and c^a: (anticommuting) ghost.
$\Rightarrow$ BRST transformation of the fields: $\delta_{\text{BRST}} \varphi = \theta s \varphi \equiv \delta_\alpha \varphi|_{\alpha^a \to \theta c^a}$.
Some consequences:

- Gauge-invariant terms in the action S_0 are BRST-invariant, while the allowed gauge-fixing terms are only BRST-invariant and have the form: $s\Psi[\Phi]$.
- Any BRST-invariant term built from the existing fields can be added to S_0 without spoiling physical predictions made out of it.

BRST transformations of fields of R-Model

The d-dimensional BRST transformations on the fields are as follows:

$$s_d G^a_\mu = D^{ab}_\mu c^b = \partial_\mu c^a + g_S f^{abc} G^b_\mu c^c \,,$$
$$s_d \Psi_i = s_d \Psi_{Ri} = i c^a g_S T_{R}{}^a_{ij} \Psi_{Rj} \,,\; s_d \overline{\Psi}_i = s_d \overline{\Psi_{Ri}} = +i \overline{\Psi_{Rj}} c^a g_S T_{R}{}^a_{ji} \,,$$
$$s_d \Phi_m = i c^a g_S \theta^a_{mn} \Phi_n \,,$$
$$s_d c^a = -\frac{1}{2} g_S f^{abc} c^b c^c \equiv i g_S c^2 \,,$$
$$s_d \bar{c}^a = B^a \,,\; s_d B^a = 0 \;\leftrightsquigarrow\; (\bar{c}^a, B^a) \text{ is a BRST doublet} \,,$$

with a similar form (noted s in what follows) in 4D.
The BRST operator s_d is nilpotent: $s_d(s_d\phi) = 0$, similarly to its 4D counterpart.

Effective action Γ: Interpretation & notation (1/2)

Effective action: Generating functional for 1-particle irreducible (1PI) Green's functions [Weinberg–1996]:

$$\Gamma[\Phi] = \sum_{n\geq 2} \frac{1}{|n|!} \int \left(\prod_{i=1}^{n} \mathrm{d}^4 x_i \; \phi_i(x_i) \right) \Gamma_{\phi_n \cdots \phi_1}(x_1, \ldots, x_n)$$

$$\left(\begin{smallmatrix}\text{Fourier}\\ \text{transform}\end{smallmatrix}\right) = \sum_{n\geq 2} \frac{1}{|n|!} \int \left(\prod_{i=1}^{n} \frac{\mathrm{d}^4 p_i}{(2\pi)^4} \widetilde{\phi}_i(p_i) \right) \Gamma_{\phi_n \cdots \phi_1}(p_1, \ldots, p_n) \overbrace{(2\pi)^4 \delta^4\left(\textstyle\sum_{j=1}^{n} p_j\right)}^{\text{Momentum conservation}} ,$$

$\Gamma_{\phi_n \cdots \phi_1}$ are the 1PI Green's functions defined by:

$$i\Gamma_{\phi_n \cdots \phi_1}(x_1, \ldots, x_n) = \left. \frac{i\delta^n \Gamma[\Phi]}{\delta\phi_n(x_n) \cdots \delta\phi_1(x_1)} \right|_{\phi_i = 0} = \langle \Omega | \mathbb{T}[\phi_n(x_n) \cdots \phi_1(x_1)] | \Omega \rangle^{\text{1PI}}$$

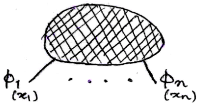
$$\equiv \langle \phi_n(x_n) \cdots \phi_1(x_1) \rangle^{\text{1PI}} ,$$

and $i\Gamma_{\phi_n \cdots \phi_1}(p_1, \ldots, p_n) \equiv \left\langle \widetilde{\phi_n}(p_n) \cdots \widetilde{\phi_1}(p_1) \right\rangle^{\text{1PI}}$ is defined similarly.

Effective action Γ: Interpretation & notation (2/2)

$$\Gamma[\Phi] = \sum_{n\geq 2} \frac{-i}{|n|!} \int \left(\prod_{i=1}^{n} \mathrm{d}^4 x_i \; \phi_i(x_i) \right) \langle \phi_n(x_n) \cdots \phi_1(x_1) \rangle^{\text{1PI}}$$

$$= \sum_{n\geq 2} \frac{-i}{|n|!}$$



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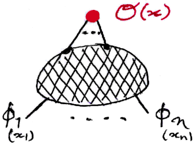
<u>Field-Operator insertion</u> in $\Gamma[\Phi]$ [Piguet,Rouet–1981]:
(e.g. counterterm insertions in loop diagrams...)

$$\mathcal{O}(x) \cdot \Gamma[\Phi] = \sum_{n\geq 2} \frac{-i}{|n|!} \int \left(\prod_{i=1}^{n} \mathrm{d}^4 x_i \; \phi_i(x_i) \right) \langle \mathcal{O}(x) \phi_n(x_n) \cdots \phi_1(x_1) \rangle^{\text{1PI}}$$

$$= \sum_{n\geq 2} \frac{-i}{|n|!}$$



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Notation:
$\mathcal{O} \cdot \Gamma[\Phi] = \int \mathrm{d}\, x \; \mathcal{O}(x) \cdot \Gamma[\Phi]\,.$

BRST invariance at any loop order?

Aim: Formulation for verifying/enforcing BRST invariance $\forall$ orders of perturbation.

- Reformulate BRST sym. invariance using **quantum effective action** Γ (up to $\mathcal{O}(\hbar^n)$) $\to$ **Slavnov-Taylor Identities** (STI). ($\sim$ Ward IDs with gauge transfos.)
- Employ formalism similar to **Batalin-Vilkovisky**: [Batalin,Vilkovisky–1977,'81,'84] $\forall\phi$, introduce in S_0 external sources ("antifields") K_ϕ coupling linearly to $s_d\phi$. (When setting these sources to adequate values, S-matrix remains unchanged.)
- BRST invariance of Γ translates into a condition, the **Zinn-Justin equation**: $(\Gamma, \Gamma) \equiv \mathcal{S}(\Gamma) = 0$, where $\mathcal{S}(\Gamma)$ is the "Slavnov-Taylor" (ST) functional:

$$\mathcal{S}(\Gamma) \equiv \int \mathrm{d}\,x \left(\sum_\Phi \mathrm{Tr}\, \frac{\delta\Gamma}{\delta K_\Phi(x)} \frac{\delta\Gamma}{\delta\Phi(x)} + B^a(x) \frac{\delta\Gamma}{\delta\bar{c}_a(x)} \right) .$$

This functional can be defined in either 4 or d dimensions (noted $\mathcal{S}$ and $\mathcal{S}_d$ resp.), that will either act on the renormalized Γ_{ren} or on the dim-regularized Γ_{DReg} resp.

The completed R-Model defining action S_0 in d-D

Our complete defining action in d dimensions, including the antifields, reads:

$$S_0 = \int \mathrm{d}^d x \left(\mathcal{L}_{\mathsf{YM}} + \mathcal{L}_\Psi + \mathcal{L}_\Phi + \mathcal{L}_{\mathsf{Yuk}} + \mathcal{L}_{\mathsf{gh}} + \mathcal{L}_{\mathsf{g\text{-}fix}} + \mathcal{L}_{\mathsf{ext}}\right),$$

with:
$$\mathcal{L}_{\mathsf{YM}} = \frac{-1}{4} F^a_{\mu\nu} F^{a\,\mu\nu}, \quad \mathcal{L}_\Phi = \frac{1}{2}(D_\mu \Phi^m)^2 - \frac{\lambda_H^{mnop}}{4!} \Phi_m \Phi_n \Phi_o \Phi_p,$$

$$\mathcal{L}_\Psi \Rightarrow i\overline{\Psi}_i \not{D}^{ij}_R \Psi_j = i\overline{\Psi}_i \not{\partial} \Psi_i + g_S T_{R\,ij}^{\,a} \overline{\Psi_R}_i \mathbb{P}_{\mathsf{L}} \not{G}^a \mathbb{P}_{\mathsf{R}} \Psi_{Rj},$$

$$\mathcal{L}_{\mathsf{Yuk}} = -(Y_R)^m_{ij} \Phi_m \overline{\Psi_R}^C_i \mathbb{P}_{\mathsf{R}} \Psi_{Rj} + \text{h.c.},$$

$$\mathcal{L}_{\mathsf{gh}} = \partial_\mu \bar{c}_a \cdot D^{ab\,\mu} c_b, \quad \mathcal{L}_{\mathsf{g\text{-}fix}} = \frac{\xi}{2} B^a B_a + B^a \partial^\mu G^a_\mu,$$

$$\mathcal{L}_{\mathsf{ext}} = \rho^\mu_a s_d G^a_\mu + \zeta_a s_d c^a + \bar{R}^i s_d \Psi_{Ri} + s_d \overline{\Psi_R}_i R^i + \mathcal{Y}^m s_d \Phi_m.$$

Quantum numbers (mass dimension, ghost number and (anti)commutativity):

	G^a_μ	$\overline{\Psi}_i$, Ψ_i	Φ_m	c^a	$\bar{c}^a$	B^a	ρ^μ_a	ζ_a	R^i, $\bar{R}^i$	$\mathcal{Y}^m$	∂_μ	s
mass dim.	1	3/2	1	0	2	2	3	4	5/2	3	1	0
ghost #	0	0	0	1	-1	0	-1	-2	-1	-1	0	1
comm.	+	−	+	−	−	+	−	+	+	−	+	−

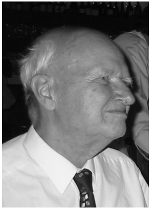
BRST invariance of the R-model @ tree-level?

- The model is BRST-invariant at tree-level in 4D due to gauge symmetry: $s_4 S_0^{(4\text{D})} = 0$.
- Is it still so in d-dimensions? $\Rightarrow$ **No!** $\exists$ BRST breaking $\widehat{\Delta}$ at tree-level:

$$s_d S_0 = \int \mathrm{d}^d x \, \left(\frac{g_S}{2} T_{R/L}{}^a_{ij}\right) c^a \left\{ \partial_\mu (\overline{\Psi}_i \widehat{\gamma}^\mu \Psi_j) \mp \overline{\Psi}_i \overset{\leftrightarrow}{\widehat{\not{\partial}}} \gamma_5 \Psi_j \right\} \equiv \widehat{\Delta} \, .$$

(R/L: results for right-handed / left-handed fermions resp.)

Notation: "Normal Products" $N[\mathcal{O}(x)]$ [Zimmermann–1973]



Introduced by Zimmermann. (See also [Lowenstein–1971].)
For a field-product operator $\mathcal{O}(x)$, a normal product $N[\mathcal{O}(x)]$ is defined as the "finite part" of $\mathcal{O}(x)$, i.e. via the finite part of the time-ordered Green's functions of $\mathcal{O}(x)$:
$\langle N[\mathcal{O}] \prod_i \phi_i(x_i) \rangle^{\text{1PI}} = \text{Fin.} \left(\langle \mathcal{O} \prod_i \phi_i(x_i) \rangle^{\text{1PI}} \right)$.
[Piguet,Rouet–1981]

They depend on the chosen renormalization scheme:

- In BPHZ renormalization (original): done by subtracting the first terms of a Taylor expansion of loop integrands up to a given order ("degree" of subtraction). $\rightarrow \exists$ different normal products associated to the choice of the "degree" of subtraction. [Piguet,Rouet–1981]
- In dimensional renormalization (DimRen): the normal products are defined with respect to the ϵ-pole subtraction. [Collins–1974]

BRST restoration, Renormalized action (1/2)

Restore BRST symmetry, i.e. **remove the irrelevant anomalies**, if possible.
The equation for renormalized 4D action, $\mathcal{S}\Gamma_{\text{ren}} = \Delta_{\text{breaking}}$, is generalizable via the *Regularized Action Principle* [Breitenlohner,Maison–1977] for Γ_{DReg}:

$$\mathcal{S}_d \Gamma_{\text{DReg}} = \widehat{\Delta} \cdot \Gamma_{\text{DReg}} + \widehat{\Delta}_{\text{ct}} \cdot \Gamma_{\text{DReg}} + \int \mathrm{d}^d x \sum_{\Phi} \mathrm{Tr}\,[\frac{\delta S^{(n)}_{\text{ct}}}{\delta K_\Phi(x)} \cdot \Gamma_{\text{DReg}}] \frac{\delta \Gamma_{\text{DReg}}}{\delta \Phi(x)} \,,$$

with the relation: $\Delta_{\text{breaking}} = \underset{d\to 4}{\text{LIM}}(\mathcal{S}_d \Gamma_{\text{DReg}}) = \mathcal{S}\Gamma_{\text{ren}}$.
(LIM is by taking $d \to 4$ and cancelling the remaining evanescent parts.)
At one-loop $\mathcal{O}(\hbar)$, we have, using $\Gamma_{\text{ren}} = \underset{d\to 4}{\text{LIM}}(\Gamma_{\text{DReg}})$:

$$\Delta^{(1)}_{\text{breaking}} = \underset{d\to 4}{\text{LIM}}\{\widehat{\Delta} \cdot \Gamma_{\text{DReg}}\big|^{(1)}_{\text{sing.}} + b_d S^{(1)}_{\text{sct}}\} + N[\widehat{\Delta}] \cdot \Gamma_{\text{ren}}\big|^{(1)} + b_4 S^{(1)}_{\text{fct}} \,.$$

The finite counterterm action $S^{(1)}_{\text{fct}}$ is computed so that $b_4 S^{(1)}_{\text{fct}}$ cancels the irrelevant anomalies from $N[\widehat{\Delta}] \cdot \Gamma_{\text{ren}}\big|^{(1)}$. $S^{(1)}_{\text{fct}}$ is contained in Γ_{ren}.

BRST restoration, Renormalized action (2/2)

$$\mathcal{S}\Gamma_{\text{ren}}{}^{(1)} = \Delta^{(1)}_{\text{breaking}} = \underset{d\to 4}{\text{LIM}}\{\widehat{\Delta}\cdot\Gamma_{\text{DReg}}|^{(1)}_{\text{sing.}} + b_d S^{(1)}_{\text{sct}}\} + N[\widehat{\Delta}]\cdot\Gamma_{\text{ren}}|^{(1)} + b_4 S^{(1)}_{\text{fct}}\,.$$

Procedure:

1. Evaluate $b_d S^{(1)}_{\text{sct}}$.
2. Evaluate $\widehat{\Delta}\cdot\Gamma_{\text{DReg}}|^{(1)}_{\text{sing.}}$ by computing 1-loop diagrams with insertion of $\widehat{\Delta}$. Check whether it cancels with $b_d S^{(1)}_{\text{sct}}$.
3. Evaluate $N[\widehat{\Delta}]\cdot\Gamma_{\text{ren}}|^{(1)}$ using Bonneau Identities; symbolically:

$$N[\widehat{\mathcal{O}}]\cdot\Gamma_{\text{ren}} = \sum_{\{\overline{\mathcal{O}}\}_i} c_{\overline{\mathcal{O}}_i} N[\overline{\mathcal{O}}_i]\cdot\Gamma_{\text{ren}}\,.$$

4. Keep aside / cancel the relevant anomalies from $N[\widehat{\Delta}]\cdot\Gamma_{\text{ren}}|^{(1)}$, and define $S^{(1)}_{\text{fct}}$ such that $b_4 S^{(1)}_{\text{fct}}$ absorbs the non-zero terms (irrelevant anomalies) from $N[\widehat{\Delta}]\cdot\Gamma_{\text{ren}}|^{(1)}$.

Outline

Calculation machinery

- Calculations performed with `Mathematica`.
- Model programmed using `FeynRules` [Christensen...–2009,Alloul...–2014] **(except with no BRST sources since unsupported)**. Manually patched for supporting arbitrary $SU(N)$ (N not limited to numerical values).
- Loop diagrams **(w/o BRST sources)** generated using `FeynArts` [Hahn–2000]. Amplitudes evaluated using `FeynCalc` [Mertig...–1990,Shtabovenko...–2016]; ϵ-"expansion" obtained using the `FeynCalc`'s interface `FeynHelpers` [Shtabovenko–2016] to `Package-X` [Patel–2017] **(for 1-loop only)**. WARNING! Using development version of `FeynCalc` that includes needed fixes (versions up to 17th June 2019 are OK).
- Diagrams with sources manually generated, then evaluated using `FeynCalc` as described above.
- Semi-automated (manually and computer) evaluation of group-structure invariants, using notations similarly defined as those in Machacek & Vaughn [Machacek,Vaughn–1983,'84,'85].

1-loop singular counterterm (SCT) action

Compute 1-loop diagrams (self-energies, vertices, & sources insertions) from S_0.
From them, define the 1-loop SCT action $S_{\text{sct}}^{(1)} = -\Gamma^{(1)}|_{\text{div}}^{\text{BMHV}}$.

$$
\begin{aligned}
S_{\text{sct}}^{(1)} = \frac{\hbar}{16\pi^2\epsilon} \Bigg\{ & g_S^2 \frac{22C_2(G) - S_2(S)}{6} L_{gS} - g_S^2 \frac{2S_2(F)}{3} \overline{L_{gS}} - g_S^2 \frac{3+\xi}{4} C_2(G) L_G \\
& - \left(g_S^2 \xi C_2(F) + \frac{Y_2(F)}{2} \right) \frac{\overline{L_{\Psi_R}}}{2} - g_S^2 \frac{\xi C_2(G)}{2} L_c \Bigg\} \\
& - \frac{\hbar}{16\pi^2\epsilon} \left\{ g_S^2 \frac{S_2(F)}{3} \int \mathrm{d}^d x \, \frac{1}{2} \bar{G}_\mu^a \widehat{\partial}^2 \bar{G}_\mu^a + \frac{2Y_2(S)}{3} \widehat{S_{\Phi\Phi}^0} \right\} \Leftarrow \text{Evanescent!} \\
& + \\
& \frac{\hbar}{16\pi^2\epsilon} \frac{1}{2} \Big\{ g_S^2 (3-\xi) C_2(S) L_{D\Phi} - Y_2(S) \overline{L_{D\Phi}} + \left(Y_2(F) - 3g_S^2 C_2(F) \right) L_{\bar{\psi}\phi\psi} \\
& - g_S^2 \xi \Lambda_{mnop}^S S_{\Phi^4}^0 + Y_{ik}^n (Y_{kl}^m)^* Y_{lj}^n S^0_{\overline{\Psi_{R_i}^C} \Phi^m \Psi_{R_j}} + \text{h.c.} \\
& + (3g_S^4 A_{mnop} - 4H_{mnop} + \Lambda_{mnop}^2) S_{\Phi^4}^0 \Big\} .
\end{aligned}
$$

L_φ: invariants under the linear BRST transformation b_d, i.e. $b_d L_\phi = 0$ (except for $b_d L_c = \widehat{\Delta}$). Defined as sums/differences of field-counting operators applied on S_0.
(see Backups for details.)

Evaluation of $b_d S_{\text{sct}}^{(1)}$ – Cancellation with $\widehat{\Delta} \cdot \Gamma_{\text{DReg}}|_{\text{sing.}}^{(1)}$ (1/2)

$$\mathcal{S}\Gamma_{\text{ren}}{}^{(1)} = \Delta^{(1)}_{\text{breaking}} = \underset{d \to 4}{\text{LIM}}\{\widehat{\Delta} \cdot \Gamma_{\text{DReg}}|_{\text{sing.}}^{(1)} + b_d S_{\text{sct}}^{(1)}\} + N[\widehat{\Delta}] \cdot \Gamma_{\text{ren}}|^{(1)} + b_4 S_{\text{fct}}^{(1)} \,.$$

Since all the $b_d L_\phi = 0$ except for $b_d L_c = \widehat{\Delta}$ the tree-level breaking, and in addition: $b_d \left\{ Y^n_{ik} (Y^m_{kl})^* Y^n_{lj} S^0_{\overline{\Psi_{R_i}}^C \Phi^m \Psi_{Rj}} + \text{h.c.} \right\} = 0$, we obtain:

$$b_d S_{\text{sct}}^{(1)} = \frac{\hbar}{16\pi^2 \epsilon} \left\{ -g_S^2 \frac{\xi C_2(G)}{2} \widehat{\Delta} - g_S^2 \frac{S_2(F)}{3} b_d \int \mathrm{d}^d x \, \frac{1}{2} \bar{G}^a_\mu \widehat{\partial}^2 \bar{G}^a_\mu - \frac{2 Y_2(S)}{3} b_d \widehat{S^0_{\Phi\Phi}} \right\}$$
$$+ \frac{\hbar}{16\pi^2 \epsilon} \frac{1}{2} b_d \left\{ (3 g_S^4 A_{mnop} - 4 H_{mnop} + \Lambda^2_{mnop}) S^0_{\Phi^4} \right\},$$

with:

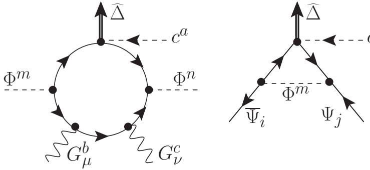
$$b_d \int \mathrm{d}^d x \, \frac{1}{2} \bar{G}^a_\mu \widehat{\partial}^2 \bar{G}^a_\mu = \int \mathrm{d}^d x \, (\overline{\partial}_\mu c_a + g_S f^{abc} \bar{G}^b_\mu c_c) \widehat{\partial}^2 \bar{G}^a_\mu \,,$$
$$b_d \widehat{S^0_{\Phi\Phi}} = b_d \int \mathrm{d}^d x \, \frac{-1}{2} \Phi_m \widehat{\partial}^2 \Phi_m = \int \mathrm{d}^d x \, i g_S \theta^a_{mn} c^a \Phi_m \widehat{\partial}^2 \Phi_n \,.$$

The term in blue should cancel too **(WIP!)**.

Evaluation of $b_d S_{\text{sct}}^{(1)}$ – Cancellation with $\widehat{\Delta} \cdot \Gamma_{\text{DReg}}|_{\text{sing.}}^{(1)}$ (2/2)

On the other hand we calculate the non-zero terms contributing to $\widehat{\Delta} \cdot \Gamma_{\text{DReg}}|_{\text{sing.}}^{(1)}$

Examples of $\widehat{\Delta}$ insertion (left: does not contribute; right: contributes for $i[\widehat{\Delta} \cdot \Gamma_{\text{DReg}}{}^{ji,a}_{\psi\bar{\psi}c}]^{(1)}_{\text{div}}$.



Terms from $b_d S_{\text{sct}}^{(1)} \times \left(\frac{\hbar}{16\pi^2\epsilon}\right)^{-1}$	Terms from $\widehat{\Delta} \cdot \Gamma_{\text{DReg}}|_{\text{sing.}}^{(1)} \times \left(\frac{\hbar}{16\pi^2\epsilon}\right)^{-1}$
$-g_S^2 \frac{S_2(F)}{3} b_d \int d^d x \, \frac{1}{2} \bar{G}^a_\mu \widehat{\partial}^2 \bar{G}^a_\mu =$ $-g_S^2 \frac{S_2(F)}{3} \int d^d x \, (\bar{\partial}_\mu c_a)(\widehat{\partial}^2 \bar{G}^a_\mu)$ $-g_S^3 \frac{S_2(F)}{3} \int d^d x \, f^{abc} c_a (\widehat{\partial}^2 \bar{G}^b_\mu) \bar{G}^c_\mu$	$i[\widehat{\Delta} \cdot \Gamma_{\text{DReg}}{}^{ba,\mu}_{Gc}]^{(1)}_{\text{div}} + i[\widehat{\Delta} \cdot \Gamma_{\text{DReg}}{}^{cba,\nu\mu}_{GGc}]^{(1)}_{\text{div}} \Rightarrow$ $S_{cG} = g_S^2 \frac{S_2(F)}{3} \int d^d x \, (\bar{\partial}_\mu c_a)(\widehat{\partial}^2 \bar{G}^a_\mu)$ $+S_{cGG} = g_S^3 \frac{S_2(F)}{3} \int d^d x \, f^{abc} c_a (\widehat{\partial}^2 \bar{G}^b_\mu) \bar{G}^c_\mu$
$-\frac{2Y_2(S)}{3} b_d \widehat{S^0_{\Phi\Phi}}$	$i[\widehat{\Delta} \cdot \Gamma_{\text{DReg}}{}^{nm,a}_{\Phi\Phi c}]^{(1)}_{\text{div}} \Rightarrow S_{c\Phi\Phi} = \frac{2Y_2(S)}{3} b_d \widehat{S^0_{\Phi\Phi}}$
$-g_S^2 \frac{\xi C_2(G)}{2} \widehat{\Delta}$	$i[\widehat{\Delta} \cdot \Gamma_{\text{DReg}}{}^{ji,a}_{\psi\bar{\psi}c}]^{(1)}_{\text{div}} \Rightarrow S_{c\bar{\psi}\psi} = g_S^2 \frac{\xi_G C_2(G)}{2} \widehat{\Delta}$

Bonneau Identities, graphical interpretation (1/2)

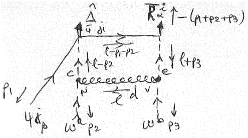
In DimRen, normal products $N[\widehat{\mathcal{O}}]$ of evanescent operators $\widehat{\mathcal{O}}$ of the form $\widehat{\mathcal{O}} \equiv (\hat{g}_{\mu\nu} = g_{\mu\nu} - \bar{g}_{\mu\nu})\mathcal{O}_{\mu\nu\rho\dots}$ are interpreted [Bonneau–1980] as the difference between two ways of performing a "subtraction" in this renormalization scheme.
$\Rightarrow$ "Zimmermann-like" identities: **Bonneau Identities**. (see Amon's talk)

$$N[\widehat{\mathcal{O}}]\cdot\Gamma_{\text{ren}} = -\sum_{n=2}^{n_{\max}=4}\ \sum_{\substack{J=\{j_1,\cdots,j_n\},\\ 0\le r\le\delta(J)}}\ \sum_{\substack{\{i_1,\cdots,i_r\}/\\ 1\le i_j\le n}} \frac{(-i)^r}{r!}\frac{\partial^r}{\partial p_{i_1}^{\mu_1}\cdots\partial p_{i_r}^{\mu_r}}\cdot(-i\hbar)\text{r.s.p.}\left.\overline{\left\langle\prod_{i=1}^{n}\widetilde{\phi_{j_i}}(p_i)N[\check{\mathcal{O}}](q=-\textstyle\sum p_i)\right\rangle}^{\text{1PI}}\right|_{\substack{p_i=0\\ \check{g}=0}}$$
$$\times N\left[\frac{1}{n!}\prod_{k=n}^{1}\left(\prod_{\{\alpha/i_\alpha=k\}}\partial_{\mu_\alpha}\right)\phi_{j_k}\right]\cdot\Gamma_{\text{ren}} + \text{similar with additional BV sources insertions}\,.$$

r.s.p.: residue of simple pole in $\nu = 2\epsilon = 4-d$. Overline: 1PI minimally subtracted.
$\check{g} \sim \hat{g}/\nu$, where this ν is not submitted to Laurent ν-expansion for the r.s.p..

$$N[\widehat{\mathcal{O}}]\cdot\Gamma_{\text{ren}} = \sum_{\{\overline{\mathcal{O}}\}_i} c_{\overline{\mathcal{O}}_i} N[\overline{\mathcal{O}}_i]\cdot\Gamma_{\text{ren}}\,.$$

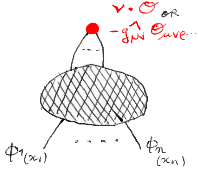
Expands evanescent operators $\widehat{\mathcal{O}_d}$ on a basis of quantum 4D operators of the renormalized 4D theory.

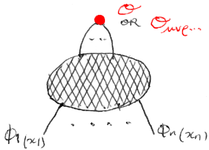


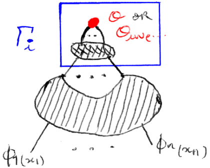
$$-\begin{pmatrix} a \\ p_2 \end{pmatrix} \leftrightarrow \begin{pmatrix} b \\ p_3 \end{pmatrix}\,.$$

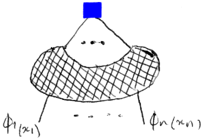
Bonneau Identities, graphical interpretation (2/2)

$$N[\widehat{\mathcal{O}}] \cdot \Gamma_{\text{ren}} = \sum_{\{\overline{\mathcal{O}}\}_i} c_{\overline{\mathcal{O}}_i} N[\overline{\mathcal{O}}_i] \cdot \Gamma_{\text{ren}}$$

$\Bigg($ 

$- \left(\begin{smallmatrix} \nu \text{ or} \\ (-\hat{g}_{\mu\nu}) \end{smallmatrix} \right) \cdot$ 

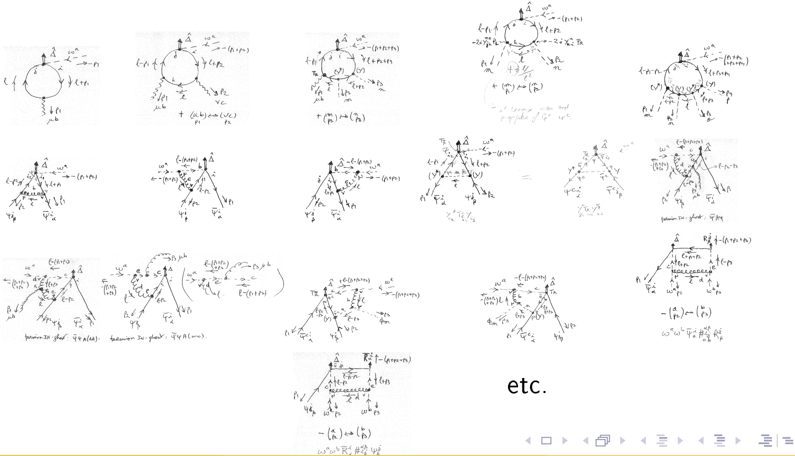
$\Bigg) = \sum_{\Gamma_i}$ 

$= \sum_{\{\overline{\mathcal{O}}\}_i} c_{\overline{\mathcal{O}}_i}$ 

Evaluation of $N[\widehat{\Delta}] \cdot \Gamma_{\text{ren}}|^{(1)}$

$$\mathcal{S}\Gamma_{\text{ren}}{}^{(1)} = \Delta^{(1)}_{\text{breaking}} = \underset{d\to 4}{\text{LIM}}\{\widehat{\Delta} \cdot \Gamma_{\text{DReg}}|^{(1)}_{\text{sing.}} + b_d S^{(1)}_{\text{sct}}\} + N[\widehat{\Delta}] \cdot \Gamma_{\text{ren}}|^{(1)} + b_4 S^{(1)}_{\text{fct}} \,.$$

Using Bonneau Identities. Some considered diagrams *(non-exhaustive...)*:



etc.

Putting results together – Finite counter-terms (FCT) (1/3)

$$\mathcal{S}\Gamma_{\text{ren}}{}^{(1)} = \Delta^{(1)}_{\text{breaking}} = \underset{d\to 4}{\text{LIM}}\{\widehat{\Delta}\cdot\Gamma_{\text{DReg}}|^{(1)}_{\text{sing.}} + b_d S^{(1)}_{\text{sct}}\} + N[\widehat{\Delta}]\cdot\Gamma_{\text{ren}}|^{(1)} + b_4 S^{(1)}_{\text{fct}} \,.$$

Find $S^{(1)}_{\text{fct}}$ such that $b_4 S^{(1)}_{\text{fct}} = -N[\widehat{\Delta}]\cdot\Gamma_{\text{ren}}|^{(1)}$ **UP TO relevant anomalies** that cannot be absorbed (if so, BRST symmetry broken and model not renormalizable).
First, the **anomalous terms that cannot be transformed out**:

$$\hbar\frac{g_S^2}{16\pi^2}\frac{S_2(F)}{3}\int \mathrm{d}^4 x\; g_S d_R^{abc}\epsilon^{\mu\nu\rho\sigma} c_a(\partial_\rho G^b_\mu)(\partial_\sigma G^c_\nu)$$
$$+\,\hbar\frac{g_S^2}{16\pi^2}\frac{1}{18}\int \mathrm{d}^4 x\; g_S^2 \mathcal{D}_R^{abcd}\epsilon^{\mu\nu\rho\sigma} c_a\partial_\sigma(G^b_\mu G^c_\nu G^d_\rho)\,,$$

with:

$$d_R^{abc} = \mathrm{Tr}[{T_R}^a\{{T_R}^b,{T_R}^c\}]\,,$$
$$\mathcal{D}_R^{abcd} = (-i)3!\,\mathrm{Tr}[{T_R}^a{T_R}^{[b}{T_R}^c{T_R}^{d]}] \;{\scriptstyle =\frac{1}{2}(d_R^{abe}f^{ecd}+d_R^{ace}f^{edb}+d_R^{ade}f^{ebc})}\,.$$

Cancel the unwanted gauge anomalies due to the RH fermions by e.g. introducing left-handed fermions under suitable group representations; see e.g. the SM.

Putting results together – Finite counter-terms (FCT) (2/3)

$$\mathcal{S}\Gamma_{\text{ren}}{}^{(1)} = \Delta^{(1)}_{\text{breaking}} = \underset{d\to4}{\text{LIM}}\{\widehat{\Delta}\cdot\Gamma_{\text{DReg}}|^{(1)}_{\text{sing.}} + b_d S^{(1)}_{\text{sct}}\} + N[\widehat{\Delta}]\cdot\Gamma_{\text{ren}}|^{(1)} + b_4 S^{(1)}_{\text{fct}}\,.$$

Now the **finite counterterms** $S^{(1)}_{\text{fct}}$ such that $b_4 S^{(1)}_{\text{fct}} = -N[\widehat{\Delta}]\cdot\Gamma_{\text{ren}}|^{(1)}$:

$$S^{(1)}_{\text{fct}} = \hbar\frac{g_S^2}{16\pi^2}\int \mathrm{d}^4x\,\Big(-\frac{S_2(F)}{3}\frac{1}{2}G^a_\mu\partial^2G^a_\mu + S_{GG} + C_2(F)(1+\frac{\xi-1}{6})S_{\overline{\Psi}\Psi}$$
$$-\frac{\xi C_2(G)}{4}(S_{\bar{R}s\Psi} + S_{s\overline{\Psi}R})\Big) + \frac{\hbar}{16\pi^2}\frac{Y_2(S)}{3}S_{\Phi\Phi}$$
$$+\cdots\ \textbf{(Work In Progress!)} + \hbar\left\{\sum_\varphi C^\xi_\varphi L_\varphi + C^\xi_{\Psi_R}\overline{L_{\Psi_R}} + C^\xi_c L_c\right\}.$$

- They are of order $\hbar^1$.
- Terms in red have arbitrary coeffs C_φ because $b_4 L_\varphi = 0$.

Putting results together – Finite counter-terms (FCT) (3/3)

- They won't of course effect in any case the RGEs we can calculate at 1-loop, but they are still there and don't appear to cancel even in the case we adapt manually our model so as to cancel the unwanted gauge anomalies.
- Therefore we expect that their presence will matter for higher-order calculations, e.g. for RGEs, due to 1+ - loop diagrams with insertion of finite counterterms. **Unless**, they precisely cancel extra contributions that would appear at these higher-orders...

What about a L-Model?

How do the results modify for left-handed (LH) fermions? Two approaches:

1. Either note that $\mathbb{P}_\mathsf{R} \leftrightarrow \mathbb{P}_\mathsf{L}$, corresponding to the change $\gamma_5 \leftrightarrow -\gamma_5$, and related change $\epsilon^{\mu\nu\rho\sigma} \leftrightarrow -\epsilon^{\mu\nu\rho\sigma}$.

2. Or, view LH fermions in a "left" (L) representation of $\mathcal{G}$, as being the charge-conjugate of corresponding RH fermions that would belong to the conjugate representation of the "left" ones: $\mathbb{P}_\mathsf{L}\Psi_L \equiv (\mathbb{P}_\mathsf{R}\Psi_R)^C$, and $T_L \leftrightarrow T_R \equiv T_{\overline{L}}$.

WARNING! We have **not yet** taken into account possible mixings between these right-handed and left-handed fermions (in the Yukawa sector...)!

Outline

Towards an RGE – Work In Progress! (1/2)

Suppose gauge anomalies are cancelled (e.g. through suitable field contents).

- **Problem:** Cannot use straightforwardly the technique with bare φ's & g's, and the Z renormalization factors in order to define the β_g and γ_φ functions, because we have (non-physical) evanescent operators for which $\beta_{\widehat{\mathcal{O}}}$ would have to be defined...
  The Renormalization-Group Equations (RGEs) give the expansion of $\mu\partial_\mu\Gamma_{\text{ren}}$.
- **Requirement:** RGEs must be BRST-compatible, i.e.:
  $\mu\partial_\mu(\mathcal{S}\Gamma_{\text{ren}}) = \mathcal{S}_{\Gamma_{\text{ren}}}(\mu\partial_\mu\Gamma_{\text{ren}}) = 0$ up to $\mathcal{O}(\hbar^n)$. It can be shown that a solution is given by a linear combination of functionals $\partial_g\Gamma_{\text{ren}} \equiv \partial\Gamma_{\text{ren}}/\partial g$ and $\mathcal{N}_\varphi\Gamma_{\text{ren}}$ where $\mathcal{N}_\varphi$ are linear combinations of field-counting operators:

$$\mu\partial_\mu\Gamma_{\text{ren}} = \left[-\sum_{g=g_S,Y,\lambda_H}\beta_g\, g\partial_g + \sum_{\varphi=G,\Phi,\Psi_R,c}\gamma_\varphi\mathcal{N}_\varphi\right]\cdot\Gamma_{\text{ren}}\,.$$

  The β_g's and γ_φ's coefficients have the standard interpretation.

Towards an RGE – Work In Progress! (2/2)

Sketch: From the **Quantum Action Principle** [Lowenstein–1971,Piguet,Sorella–1995], [Piguet,Rouet–1981], express the differential operators applied on Γ_{ren}:

$$g\partial_g\Gamma_{\text{ren}} = N\left[g\partial_g(S_0+S_{\text{ct}})\right]\cdot\Gamma_{\text{ren}} \quad (\text{for } g = g_S, Y, \lambda_H)\,,$$
$$\mathcal{N}_\varphi\Gamma_{\text{ren}} = N\left[\mathcal{N}_\varphi(S_0+S_{\text{ct}})\right]\cdot\Gamma_{\text{ren}}\,.$$

Cannot do the same for $\mu\partial_\mu\Gamma_{\text{ren}}$ because neither S_0 nor S_{ct} depend on the renormalization scale μ, since the latter is not introduced at the Lagrangian / action level, but as a modification of the loop integration measure: $\mu^\epsilon\int \mathrm{d}^d x$.
$\Rightarrow$ Trick: use modified Bonneau-like IDs for $\mu\partial_\mu\cdot\Gamma_{\text{ren}}$ *(not shown here)*.
Next step: re-express all the operators insertions into a basis of (independent) operator insertions, and do the same for $g\partial_g\Gamma_{\text{ren}}$ and $\mathcal{N}_\varphi\Gamma_{\text{ren}}$:

$$\mu\partial_\mu\Gamma_{\text{ren}} = \sum_{\{\mathcal{O}\}_i} c_i\mathcal{O}_i = \sum_{\{\mathcal{O}\}_i}\left(-\sum_g \beta_g\, g\times d_{g,i} + \sum_\varphi \gamma_\varphi\times e_{\varphi,i}\right)\mathcal{O}_i\,,$$

thus obtaining a system of equations for the β_g's and the γ_φ's to be solved.

Outline

Summary

- Using the algorithm from [Martin,Sanchez-Ruiz–1999], based on algebraic renormalization techniques in DimReg with BMHV scheme and MS(bar) subtraction, we have revisited the renormalization of a Chiral Yang-Mills Theory **with real scalar fields**, doing an explicit calculation at 1-loop. BRST symm. is restored when possible, and gauge anomaly results are recovered as expected.

- Once the d-D extension of the model is chosen, the γ_5 matrix (and other intrinsically 4D Lorentz objects) is treated unambiguously, thus making the method clear of usage and explicitly consistent for any loop order.

- Our current **work in progress** is to obtain RGEs using algebraic techniques and compare the obtained results against the usual ones.

- The scalars appear to just resize contributions of the gauge sector, or supplement them with separate (possibly evanescent) terms, that do not actually break BRST since they can be reabsorbed into finite counterterms.

- These results constitute first steps for next-loop (2+ - loop) calculations.


Thank you!


Backups

The BRST b_d invariants L used in $S_{\mathsf{sct}}^{(1)}$ (1/2)

L quantities, invariant under the linear BRST transformation b_d, are defined:

$$\begin{aligned}
L_G &= b_d \int \mathrm{d}^d x\, \widetilde{\rho}^\mu_a G^a_\mu = \left(N_G - N_{\bar{c}} - N_B - N_\rho + 2\xi \frac{\partial}{\partial \xi} \right) S_0 \\
&= \int \mathrm{d}^d x\, \left(2 S_{GG} + 3 S_{G^3} + 4 S_{G^4} + \overline{S_{\overline{\Psi} G \Psi}} + S_{\Phi G \Phi} + 2 S_{\Phi G G \Phi} - \widetilde{\rho}^\mu_a (\partial_\mu c_a) \right) , \\
L_c &= -\, b_d \int \mathrm{d}^d x\, \zeta_a c^a = (N_c - N_\zeta)\, S_0 \\
&= \int \mathrm{d}^d x\, \widetilde{\rho}^\mu_a s_d G^a_\mu + \zeta_a s_d c^a + \bar{R}^i s_d \Psi_i + s_d \overline{\Psi}_i R^i + \mathcal{Y}^m s_d \Phi_m \,, \\
L_\Phi &= b_d \int \mathrm{d}^d x\, \mathcal{Y}^m \Phi_m = (N_\Phi - N_{\mathcal{Y}})\, S_0 \\
&= \int \mathrm{d}^d x\, \left((D_\mu \Phi^m)^2 \equiv 2 S_{\Phi\Phi} + 2 S_{\Phi G \Phi} + 2 S_{\Phi G G \Phi} \right) + 4 \lambda_{mnop} S_{\Phi^4_{mnop}} \\
&\quad + \left((Y_R)^m_{ij} S_{\overline{\Psi_R}_i^C \Phi^m \Psi_{R j}} + \text{h.c.} \right) , \\
\overline{L_{\Psi_R}} &= -\, b_d \int \mathrm{d}^d x\, \left(\bar{R}^i \mathbb{P}_\mathsf{R} \Psi_i + \overline{\Psi}_i \mathbb{P}_\mathsf{L} R^i \right) - \int \mathrm{d}^d x\, i \overline{\Psi}_i \widehat{\partial\!\!\!/} \Psi_i = -(N^R_\Psi + N^L_{\overline{\Psi}} - N_{\bar{R}} - N_R) S_0 \\
&\quad - \int \mathrm{d}^d x\, i \overline{\Psi}_i \widehat{\partial\!\!\!/} \Psi_i = 2 \int \mathrm{d}^d x\, i \overline{\Psi}_i \overline{\partial\!\!\!/} \mathbb{P}_\mathsf{R} \Psi_i + \overline{S_{\overline{\Psi} G \Psi}} + \left((Y_R)^m_{ij} S_{\overline{\Psi_R}_i^C \Phi^m \Psi_{R j}} + \text{h.c.} \right) ,
\end{aligned}$$

The BRST b_d invariants L used in $S_{\text{sct}}^{(1)}$ (2/2)

In the previous calculations the field-counting operators have been used; they are defined as:

$$N_\varphi = \int \mathrm{d}^d x\, \varphi(x)_i \frac{\delta}{\delta\varphi(x)_i}\,, \text{ for } \varphi_i = G^a_\mu, \Phi^m, c_a, \bar{c}_a, B^a, \rho^\mu_a, \zeta_a, R^i, \bar{R}^i, \mathcal{Y}^m\,,$$

$$N_\Psi^{R/L} = \int \mathrm{d}^d x\, (\mathbb{P}_{\mathsf{R/L}}\Psi_i(x))_s \frac{\delta}{\delta\Psi_i(x)_s}\,,\ N_{\overline{\Psi}}^{L/R} = \int \mathrm{d}^d x\, (\overline{\Psi}_i(x)\mathbb{P}_{\mathsf{L/R}})^s \frac{\delta}{\delta\overline{\Psi}_i(x)^s}\,.$$

Other b_d invariants are:

- the pure Yang-Mills term L_{gS} and its equivalent 4-dimensional version $\overline{L_{gS}}$:

$$L_{gS} = \frac{-1}{4}\int \mathrm{d}^d x\, F^a_{\mu\nu}F^{a\,\mu\nu} = S_{GG} + S_{G^3} + S_{G^4}\,;$$

- the Yukawa interaction: $L_{\bar{\psi}\phi\psi} = (Y_R)^m_{ij} S_{\overline{\Psi_{R_i}}^C \Phi^m \Psi_{R_j}} + \text{h.c.}\,,$
- the four-scalar interaction: $L_{\Phi^4} = \lambda_{mnop} S_{\Phi^4_{mnop}}\,.$

Since $L_\Phi = \int \mathrm{d}^d x\, 2S_{\Phi\Phi} + 2S_{\Phi G\Phi} + 2S_{\Phi GG\Phi} + 4L_{\Phi^4} + L_{\bar{\psi}\phi\psi}$ is a b_d invariant, it follows that the combination $L_{D\Phi} = \int \mathrm{d}^d x\, (D_\mu\Phi^m)^2 = L_\Phi - L_{\bar{\psi}\phi\psi} - 4L_{\Phi^4}$ is also a b_d invariant by itself.

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