Entanglement and the Infrared

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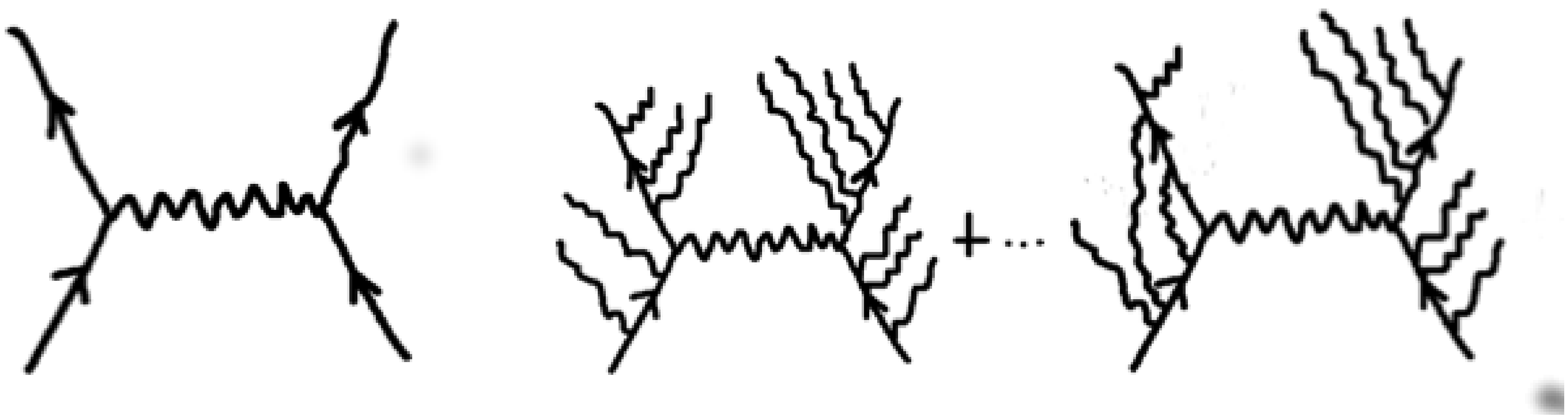
Infrared divergences in quantum electrodynamics

– **Quantum electrodynamics is, for all practical purposes, solvable by perturbation theory.**

– **Asymptotic expansions in fine structure constant** $\alpha \sim 1/137$ **converges rapidly.**

– **However, there is a subtlety due to infrared divergences:**



Two approaches

1. **Inclusive probabilities**

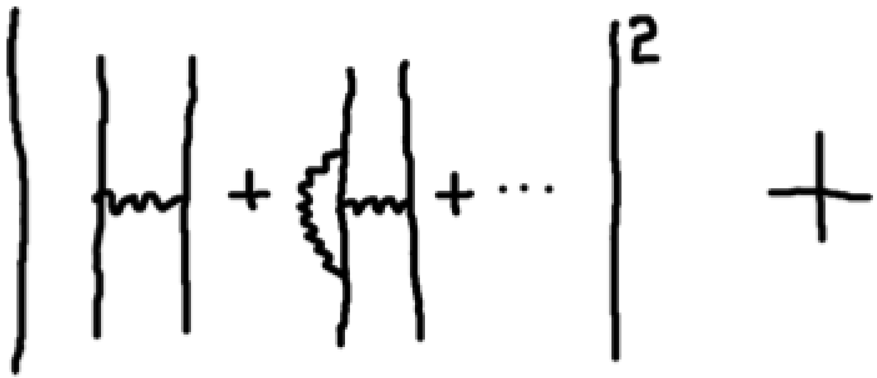
   F.Bloch, A.Nordsieck, Phys.Rev.52, 54 (1937)
   D.R.Yennie, S.C.Frautschi, H.Suura, Ann.Phys.13, 379 (1961)
   S.Weinberg, Phys.Rev.140, B516 (1965)

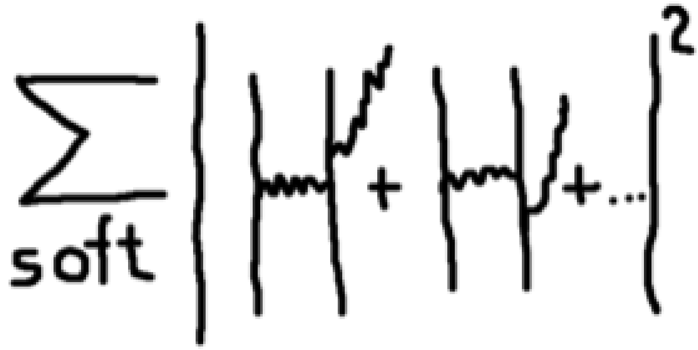
2. **Dressed states**

   V.Chung, Phys.Rev.140, B1110 (1965)
   P.Kulish, L.D.Faddeev, Theor.Math.Phys.4, 745 (1970)

Asymptotic symmetries
S.Hawking, M.Perry, A.Strominger Phys.Rev.Lett.116, 231301 (2016)
R.Bousso, M.Porrati, arXiv:1706.00436 [hep-th]

I. The Inclusive probability is infrared finite





= finite

Cancellation of infrared divergences is guaranteed by unitarity of infrared cutoff S-matrix.

$$|\text{in} >< \text{in}| \implies S^\dagger|\text{in} >< \text{in}|S\ ,\ \ \rho_{\text{out}} = \sum_{\text{soft}\gamma} < \gamma|S^\dagger|\text{in} >< \text{in}|S|\gamma >$$

2. Dressed States

Combine the soft photons which are produced with the charged particles to make "dressed states"

The elements of the usual S-matrix in dressed states are infrared finite

Dressed states

Hard and soft physics are decoupled by a canonical transformation

$$|p_1^{\mu_1}, p_2^{\mu_2}, \ldots > \quad \Longrightarrow \quad |p_1^{\mu_1}, p_2^{\mu_2}, \ldots >_{\mathbf{dressed}}$$

$$\equiv \; \exp\left(-e \int_{\mathbf{m}_{\mathrm{ph}}}^{\Lambda} \frac{d^3k}{\sqrt{2|k|}} \sum_j \frac{p_j^{\mu} \epsilon^s(k)}{p_j^{\nu} k_{\nu} + i\epsilon} a_s(k) - \text{ h.c.} \right) |p_1^{\mu_1}, p_2^{\mu_2}, \ldots >$$

$$a_s(k) \Longrightarrow a_s(k) + \sum_j \frac{p_j^{\mu} \epsilon_s^*(k)}{p_j^{\nu} k_{\nu} - i\epsilon} \; , \quad \mathbf{m}_{\mathrm{ph}} < |k| < \Lambda < p_1^{\mu_1}, p_2^{\mu_2}, \ldots$$

– matrix elements of the S-matrix are independent of $\mathbf{m}_{\mathrm{ph}}$

– Probabilities for processes agree with inclusive approach

– Λ appears as the usual "detector resolution"

– when $\mathbf{m}_{\mathrm{ph}} \to 0$, improper canonical transformation

– not Lorentz invariant

Cutoffs, soft photons and hard particles

regularize infrared divergences: fundamental IR cutoff $\mathbf{m}_{\mathrm{ph}}$

$$|\alpha><\alpha| \implies \left[\sum_{\beta\gamma} |\beta,\gamma> \mathcal{S}^{\mathbf{m}_{\mathrm{ph}}*}_{\alpha,\beta\gamma}\right] \left[\sum_{\tilde\beta\tilde\gamma} \mathcal{S}^{\mathbf{m}_{\mathrm{ph}}}_{\alpha,\tilde\beta\tilde\gamma} < \tilde\beta,\tilde\gamma|\right]$$

$$\left[\mathbf{m}_{\mathrm{ph}}\right] < \left[\textbf{soft} \quad \gamma\right] < \left[\textbf{detector resolution } \Lambda\right] < \left[\textbf{hard } \alpha,\beta\right]$$

$$\rho_{\mathrm{out}} = \sum_{\mathbf{m}_{\mathrm{ph}}<\gamma'<\Lambda} <\gamma'| \left[\sum_{\beta\tilde\beta\gamma\tilde\gamma} |\beta,\gamma> \mathcal{S}^{\mathbf{m}_{\mathrm{ph}}*}_{\alpha,\beta\gamma}\; \mathcal{S}^{\mathbf{m}_{\mathrm{ph}}}_{\alpha,\tilde\beta\tilde\gamma} < \tilde\beta,\tilde\gamma|\right] |\gamma'>$$

Diagonal components of ρ_{out} are independent of $\mathbf{m}_{\mathrm{ph}}$

$$<\beta|\rho_{\mathrm{out}}|\beta> = \sum_{\mathbf{m}_{\mathrm{ph}}<\gamma<\Lambda} \mathcal{S}^{\mathbf{m}_{\mathrm{ph}}*}_{\alpha,\beta\gamma}\; \mathcal{S}^{\mathbf{m}_{\mathrm{ph}}}_{\alpha,\beta\gamma} = \mathcal{S}^{\Lambda*}_{\alpha,\beta\gamma}\; \mathcal{S}^{\Lambda}_{\alpha,\beta\gamma}$$

$$= \left|{}_{\mathrm{dressed}} <\alpha|S|\beta>_{\mathrm{dressed}}\right|^2$$

What about off-diagonal elements of $<\beta|\rho_{\rm out}|\tilde\beta>$?

$$<\beta|\rho_{\rm out}|\tilde\beta> \;\; = \sum_{\mathbf{m}_{\rm ph}<\gamma<\Lambda} \mathcal{S}^{\mathbf{m}_{\rm ph}\,*}_{\alpha,\beta\gamma}\;\mathcal{S}^{\mathbf{m}_{\rm ph}}_{\alpha,\tilde\beta\gamma}$$

versus the dressed out-state (which is a pure state)

$$<\beta|\rho^{\rm dressed}_{\rm out}|\tilde\beta> \;\; = \;_{\rm dressed}<\beta|S^\dagger|\alpha>_{\rm dressed}<\alpha|S|\tilde\beta>_{\rm dressed}$$

What about off-diagonal elements of $<\beta|\rho_{\rm out}|\tilde{\beta}>$?

$$< \beta|\rho_{\rm out}|\tilde{\beta} > \;=\; \sum_{\mathbf{m}_{\rm ph}<\gamma<\Lambda} \mathcal{S}^{\mathbf{m}_{\rm ph}\,*}_{\alpha,\beta\gamma}\; \mathcal{S}^{\mathbf{m}_{\rm ph}}_{\alpha,\tilde{\beta}\gamma}$$

versus the dressed out-state (which is a pure state)

$$< \beta|\rho^{\rm dressed}_{\rm out}|\tilde{\beta} > \;=\; {}_{\rm dressed}< \beta|S^{\dagger}|\alpha >_{\rm dressed}< \alpha|S|\tilde{\beta} >_{\rm dressed}$$

Compute von Neumann entropy of $\rho_{\rm out}$, $S = -{\rm Tr}\rho \ln \rho$.

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Compute von Neumann entropy of ρ_{out}, $S = -\mathrm{Tr}\rho \ln \rho$.

Entanglement of soft and hard products of scattering?

What about off-diagonal elements of $<\beta|\rho_{\rm out}|\tilde\beta>$?

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versus the dressed out-state (which is a pure state)

$$<\beta|\rho_{\rm out}^{\rm dressed}|\tilde\beta> \;=\; {}_{\rm dressed}<\beta|S^\dagger|\alpha>_{\rm dressed}<\alpha|S|\tilde\beta>_{\rm dressed}$$

Compute von Neumann entropy of $\rho_{\rm out}$, $S=-{\rm Tr}\rho\ln\rho$.

Entanglement of soft and hard products of scattering?

Computed in perturbation theory, the entanglement entropy is logarithmically infrared divergent.

D.Carney, L.Chaurette, D.Neuenfeld, G.Semenoff, Phys. Rev. Lett. 119, 180502 (2017); Phys. Rev. D 97, 025007 (2018); JHEP 1809, 121 (2018); GWS 2019??

Soft photon theorem applied to the density matrix

Soft photon theorems imply:

$$\rho_{\beta\tilde{\beta}} = \mathcal{S}^{\lambda *}_{\alpha\beta}\mathcal{S}^{\lambda}_{\alpha\tilde{\beta}} \left(\frac{\mathbf{m}_{\rm ph}}{\lambda}\right)^{\Delta A} \left(\frac{\Lambda}{\lambda}\right)^{\tilde{A}} \quad \Longrightarrow \quad \mathbf{m}_{\rm ph.} \to 0??$$

$$\Delta A = \frac{1}{2} A_{\alpha\beta,\alpha\beta} + \frac{1}{2} A_{\alpha\tilde{\beta},\alpha\tilde{\beta}} - A_{\alpha\beta,\alpha\tilde{\beta}} \geq 0$$

$$A_{X,Y} = -\sum_{n\in X, m'\in Y} \frac{e_n e_{n'} \eta_n \eta'_n}{8\pi\beta_{nn'}} \ln\left[\frac{1+\beta_{nn'}}{1-\beta_{nn'}}\right]$$

$\beta_{nn'}$ =relative relativistic velocity

$$\left\{\frac{e_1 p_1^\mu}{2\omega(p_1)}, \dots, \frac{e_n p_n^\mu}{2\omega(p_n)}\right\} = \left\{\frac{\tilde{e}_1 \tilde{p}_1^\mu}{2\omega(\tilde{p}_1)}, \dots, \frac{\tilde{e}_{\tilde{n}} \tilde{p}_{\tilde{n}}^\mu}{2\omega(\tilde{p}_{\tilde{n}})}\right\}$$

decoherence

momentum eigenstates are "pointer basis"

Example: Compton scattering

$$\rho_{k'q';\tilde{k}'\tilde{q}'} = [\mathbf{m}_{\mathrm{ph}}]^{\frac{e^2}{4\pi^2}\left[\frac{1}{2\beta}\ln\frac{1+\beta}{1-\beta}-1\right]}$$

β =**relative velocity of outgoing electrons** $k', \tilde{k}'$

Exponent ≥ 0**. Exponent** $= 0$ **only when** $\beta = 0$**,** $k' = \tilde{k}'$**.**

$\boxed{\textbf{As } \mathbf{m}_{\mathrm{ph}} \to 0,\ \rho_{k',q';\tilde{k}',\tilde{q}'} = 0 \textbf{ unless } k'_\mu = \tilde{k}'_\mu.}$

Implication for Compton scattering

Diagonal elements of the density matrix are the transition probabilities for QED processes.

$\rho_{k',q';k',q'}$ **= Probability of** $|k,q> \rightarrow |k'q'>$

Off-diagonal elements vanish $\rho_{k',q';\tilde{k}',\tilde{q}'} = 0, \;\; k \neq \tilde{k}'$

Probability $|k,q> \quad \Longrightarrow \quad \frac{1}{\sqrt{2}}|k_1',q_1'> + \frac{1}{\sqrt{2}}|k_2',q_2'>$

equals

$\frac{1}{2}\cdot$**Prob.** $|k,q> \rightarrow |k_1',q_1'> + \frac{1}{2}\cdot$**Prob.** $|k,q> \rightarrow |k_2',q_2'>$

What if the photon has a mass?

$$\rho_{k',q';\tilde{k}',\tilde{q}'} = (\mathbf{m}_{\rm ph})^{\frac{e^2}{4\pi^2}\left[\frac{1}{2\beta}\ln\frac{1+\beta}{1-\beta}-1\right]}$$

$\mathbf{m}_{\rm ph} \sim 10^{-32} m_{\rm el}$

$$\sim e^{-0.1\beta^2} \ \ \beta << 1 \ , \ \ \sim \left(\frac{1-\beta}{2}\right)^{0.1} \ \ \beta \sim 1 \ \ \boxed{(.0001)^{0.1} \sim 0.4}$$

Gravity is even more weakly coupled.

Conclusions

- **Solutions of the infrared problem in quantum electrodynamics lead to either fundamental decoherence of final states or the necessity of using dressed states.**
- **Two theories of QED with different physical predictions, which one describes nature?**
- **All that has been said here for quantum electrodynamics can also be said for perturbative quantum gravity – two theories of perturbative quantum gravity. Which describes nature?**