Holographic RG flows for 6D SCFTs

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based on 1810.00013
and work in progress with
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Motivations

- Superconformal field theories
  - UV Completion of Quantum Field Theories
  - Obtained through branes engeneering
- Supersymmetric AdS vacua in various dimensions
  - Landscape of vacua from String/M-theory
  - Dual to Superconformal Field Theories
- A string theory construction allows to keep track of the fields dynamics and symmetries, and allows us to perform holographic tests

$\rightarrow$ Deformations of the field theory out of the critical point: the RG flow is triggered by nontrivial scalar vevs, in the gravity picture.
Interpolating between two asymptotic AdS regions

$$AdS_d \quad + \quad \langle \varphi^i \rangle \qquad \leftrightarrow \qquad AdS_d \quad + \quad \langle \varphi'^i \rangle$$

satisfying an extremization condition $\quad \partial_i \mathcal{W}(\Theta, \phi^i) = 0$

Branes setup

[Intriligator, Brunner&Karch, Hanany&Zaffaroni, '97]

6D SCFTs from branes constructions engineered in type IIA theory classified as

$$\mathcal{T}^N_{G,\mu_{\rm L},\mu_{\rm R}}$$

Given N and G, and different μ_L, μ_R they are related by Higgs RG flows as:

$$\mathcal{T}^N_{G,\mu_{\rm L},\mu_{\rm R}} \xrightarrow{\text{RG}} \mathcal{T}^N_{G,\mu'_{\rm L},\mu_{\rm R}} \qquad \Leftrightarrow \qquad \mu_{\rm L} < \mu'_{\rm L}\,,$$

[Gaiotto, Tomasiello, Heckman, Rudelius '14-'16]

Consider $G = SU(k)$. A nilpotent element, up to conjugation, is of the form

$$\mu = \begin{pmatrix} J_{d_1} & & \\ & J_{d_2} & \\ & & \ddots \end{pmatrix}\,, \qquad J_d \equiv \left.\begin{pmatrix} 0 & 1 & & \\ & 0 & 1 & \\ & & \ddots & \ddots \end{pmatrix}\right\} d\,.$$

with $\sum_a d_a = k$. One can associate a Young tableau by assigning each d_a with the number of rows of the diagram.

Branes setup

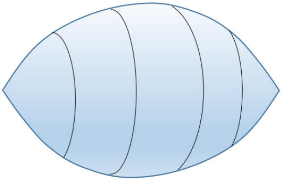
The UV theory at the top of the RG flow corresponds to $\mu_L = \mu_R = 0$

$$\mathcal{T}^{N}_{G,0,0} \qquad d_i = 1 \qquad \mu = \begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline \\ \hline \\ \hline \\ \hline \end{array}$$

This SCFT has $\mathcal{N} = (1,0)$ supersymmetry and flavor symmetry $G_L \times G_R$.
Consider the case $G = SU(k)$.

- Realized in type IIA as $N+1$ NS5-branes on k D6-branes
- Flavor symmetries $SU(k) \times SU(k)$.

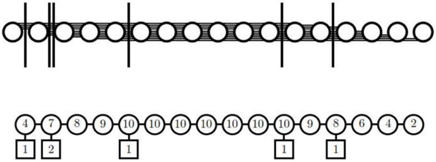
$\rightarrow$ No $D8$ branes in the top UV theory.



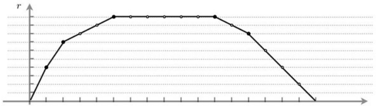
Branes setup

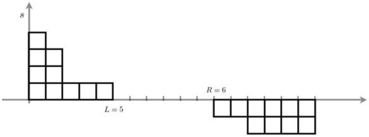
[pics from Cremonesi, Tomasiello, '15]

The branes configurations in the 10 dimensional space gives rise to matter content captured by the *linear* quiver



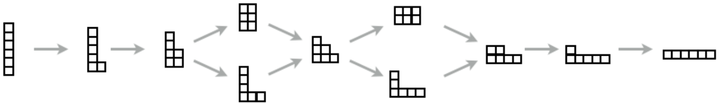
The rank of the flavor symmetry groups give rise to a convex curve to which two Young Tableau are associated. They represent the commutant of the D8 branes on top of the D6



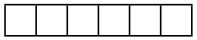


The RG chain

From UV to IR: $\mu > \mu'$ if μ' can be obtained from μ by removing a box from a higher row and adding it to a lower row.
Possible RG flows, from smaller to larger Young diagrams



On the left, the vertical diagram  corresponds to the partition $[1^6]$,

which belongs to the smallest possible orbit.
At the right extremum the horizontal Young diagram ,
corresponds to the largest possible orbit $[1^6]^t = [6]$.

Nilpotent element and $su(2) \subset \mathfrak{g}$ embedding

[Jacobson-Morozov theorem]

Given a nilpotent element $\mu \in G$, find two other elements in G such that they satisfy $sl(2,\mathbb{C})$ comm rel (with μ a creation operator). Up to a change of basis, bring this triple to Hermitian matrices

$$[\sigma^i, \sigma^j] = \epsilon^{ijk}\sigma^k \,.$$

$\rightarrow$ μ defines an embedding $\sigma : \mathrm{su}(2) \subset \mathfrak{g}$, each σ^i is sum of irreducible representations of spins ℓ_1, ℓ_2, ... such that $2\ell_a + 1 = d_a$

$$\sigma^i = \begin{pmatrix} \sigma^i_1 & & \\ & \sigma^i_2 & \\ & & \ddots \end{pmatrix}$$

A normalization useful for the explicit solutions

$$\mathrm{Tr}(\sigma^i_a \sigma^j_a) = -\kappa_a^2 \delta^{ij} \,, \qquad \kappa_a^2 \equiv \frac{\ell_a(\ell_a+1)(2\ell_a+1)}{3} \,.$$

AdS$_7$ dual vacua

10D configuration dual to the branes worldvolume theory [Apruzzi et al., '13-'15], [Cremonesi & Tomasiello, '15]

$$\frac{1}{\pi\sqrt{2}}ds^2 = 8\sqrt{-\frac{\alpha}{\ddot{\alpha}}}ds^2_{\mathrm{AdS}_7} + \sqrt{-\frac{\ddot{\alpha}}{\alpha}}\left(dz^2 + \frac{\alpha^2}{\dot{\alpha}^2 - 2\alpha\ddot{\alpha}}ds^2_{S^2}\right) ;$$

$$B = \pi\left(-z + \frac{\alpha\dot{\alpha}}{\dot{\alpha}^2 - 2\alpha\ddot{\alpha}}\right)\mathrm{vol}_{S^2} , \qquad F_2 = \left(\frac{\ddot{\alpha}}{162\pi^2} + \frac{\pi F_0 \alpha\dot{\alpha}}{\dot{\alpha}^2 - 2\alpha\ddot{\alpha}}\right)\mathrm{vol}_{S^2} ;$$

$$e^{\phi} = 2^{5/4}\pi^{5/2}3^4\frac{(-\alpha/\ddot{\alpha})^{3/4}}{(\dot{\alpha}^2 - 2\alpha\ddot{\alpha})^{1/2}} .$$

- Internal space has S^3 topology.
- $\ddot{\alpha}$ is defined on a closed interval I and D6 branes can sit at its endpoints.
- D8/D6 bound states at z_a loci where the piecewise function $\ddot{\alpha}$ changes slopes.

$\rightarrow$ Can be all described as vacua of 7D minimal gauged SG, as a universal sector.

7D Supergravity theory

Minimal gauged 7D SG (e_μ^m , ψ_μ^A , A_μ^i , χ^A , $B_{\mu\nu}$, σ) . To describe RG flows we need a reduction that keeps more modes of the internal manifold, and more information about the physics of the SCFTs:

- Each of the D6- and D8-brane stacks should contribute in seven dimensions a non-abelian vector multiplet, coming from the gauge fields living on them in ten dimensions. [Passias, Rota, Tomasiello, '15]
- For a given theory with flavor symmetry G, couple the minimal 7D theory to k vector multiplets, realizing the group G as gauged group

$$(A_{\mu\, R}\, ,\, \lambda_R^A\, ,\, \phi_{iR})\, .$$

Scalars parameterize the moduli space

$$\frac{\mathrm{SO}(3,n)}{\mathrm{SO}(3)\times\mathrm{SO}(n)}\, .$$

[E. Bergshoeff, I. Koh, and E. Sezgin, '85]

$\rightarrow$ uplift?? [E. Malek, H. Samtleben, V. Vall Camell, '19]

7D Supergravity theory

Lagrangian

$$e^{-1}\mathcal{L} = \frac{R}{2} - \frac{5}{8}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}P^{\mu ir}P_{\mu ir} - V(\sigma, L(\phi)) \ ,$$

with scalar potential

$$V \quad = \tfrac{1}{4}e^{-\sigma}\left(C^{iR}C_{iR} - \tfrac{1}{9}C^2\right) + 16h^2e^{4\sigma} - \tfrac{4\sqrt{2}}{3}\,h\,e^{3\sigma/2}C\,,$$

Relevant BPS equations are

$$\begin{aligned}\delta\psi_\mu &= 2D_\mu\epsilon - \frac{\sqrt{2}}{30}e^{-\sigma/2}C\gamma_\mu\epsilon - \frac{4}{5}he^{2\sigma}\gamma_\mu\epsilon\,,\\ \delta\chi &= -\frac{1}{2}\gamma^\mu\partial_\mu\sigma\epsilon + \frac{\sqrt{2}}{30}e^{-\sigma/2}C\epsilon - \frac{16}{5}e^{2\sigma}h\epsilon\,,\\ \delta\lambda^R &= i\gamma^\mu P^{iR}_\mu\sigma^i\epsilon - \frac{i}{\sqrt{2}}e^{-\sigma/2}C^{iR}\sigma^i\epsilon\,.\end{aligned}$$

[Karndumri'14, Louis, Lust '15]

BPS vacua

Choice of gauge group has to contain

$$G = \mathrm{SU}(2)_{\mathcal{R}_0} \times \mathrm{SU}(k) \times \mathrm{SU}(k)\,.$$

Thus the structure constants will split as

$$f_{IJK} = \{g_3 \epsilon_{ijk}\,,\; g_\mathrm{L} f_{rst}\,,\; g_\mathrm{R} f_{\hat{r}\hat{s}\hat{t}}\}$$

AdS vacua will have to:

- be in one-to-one correspondence with a choice of two Young diagrams, which are the main data in the SCFTs
- be those on which the residual gauge symmetries reproduce the flavor symmetries of the dual SCFTs.

In this talk: Restrict to a single Young diagram $\mu_\mathrm{L} \equiv \mu$, so the second copy of $\mathrm{SU}(k)$ gauge group is unbroken.

BPS vacua

The scalar fields at the vacuum satisfy the ansatz

$$\phi^i = \psi \sigma^i \, .$$

- R-symmetry: diagonal $\mathrm{SU}(2)_{\mathcal{R}}$ of the original $\mathrm{SU}(2)_{\mathcal{R}_0}$ and of $\mathrm{SU}(2)$ subgroup of the rest of the gauge group.
- Scalars can be expanded on a basis of generators T_{f}^r of the gauge algebra
$$\phi^i = \phi^i_r T_{\mathrm{f}}^r \, ,$$
- The coset representative are
$$L^I{}_J = (L^i{}_J, L^r{}_J) = \exp \begin{bmatrix} 0 & \phi^i_r \\ \phi^s_j & 0 \end{bmatrix} .$$

BPS vacua

A first check: recover the result of Karndumri '15, where a new vacuum is derived

- Gauge group $SO(3) \times SO(3)$, corresponds to $k = 2$.
- The only nontrivial partition is $\square\!\square$, for which simply

$$\phi_r^i \sim \phi\,\delta_r^i$$

- Nontrivial vacuum at

$$\sigma = -\frac{1}{5}\log\left[\frac{g^2 - 256h^2}{g^2}\right] \,, \qquad V_* = -240e^{4\sigma}h^2$$

$$\tanh(\phi) = \frac{16h}{g}$$

with masses $m_\sigma^2 L^2 = -8$, $m_\phi^2 L^2 = 40$, consistent with BF bound $m^2 L^2 \geq -9$.

BPS vacua

Having specified a scalar ansatz, the coset representative can be written explicitly as

$$L^I{}_J = \begin{pmatrix} \cosh\alpha\delta^{ij} & \frac{\sinh\alpha}{\alpha}\phi^i_r \\ \frac{\sinh\alpha}{\alpha}\phi^s_j & \delta^{rs} + \frac{\cosh\alpha - 1}{\alpha^2}P^{rs} \end{pmatrix} ,$$

where the normalization α comes from the choice of partition

$$\phi^i_r\phi^j_r = -\mathrm{Tr}\left(\phi^i\phi^j\right) = \psi^2\sum_a \kappa_a^2\delta^{ij} = \alpha^2\delta^{ij} , \qquad \kappa^2 = \sum_a \kappa_a^2$$

Vacuum conditions can be easily read from the BPS variations and yield

$$\tanh(\psi\kappa) = \frac{\kappa\, g_3}{g_{\mathrm{L}}} , \qquad e^{\frac{5\sigma}{2}} = \frac{g_3\, g_{\mathrm{L}}}{16\, h\sqrt{g_{\mathrm{L}}^2 - g_3^2\,\kappa^2}} .$$

- Each vacua has $V_* = -240e^{4\sigma}h^2$.
- Scalars masses are $m^2_\sigma L^2 = -8$, $m^2_\psi L^2 = 40$.

Cosmological constant

The relation between the cosmological constants $\left(\frac{V_{\mu_{\rm L}}}{V_0}\right)^{5/4} = \frac{1}{1-\kappa^2 \frac{g_3^2}{g_{\rm L}^2}}$

is related to the UV-IR ratio of anomalies $a_{\mu_{\rm L}} = N^3 \frac{k^2}{12} - N \frac{k}{6} \sum_a a^3 f_a + \dots$

[Cremonesi, Tomasiello '15]

- Limit $N \to \infty$ yields a pure D6 theory, dual to $\mathcal{T}^N_{SU(k),0,0}$.
- One has to consider the nontrivial scaling

$$N \to \infty\,, \qquad d_a \to \infty\,, \qquad d_a/N \equiv \delta_a \text{ finite,}$$

this induces

$$\kappa^2 \sim \sum_a \frac{d_a^3}{12} \sim \frac{1}{12} \sum_a a^3 f_a \sim N^3 \delta\,.$$

- The supergravity matches the field theory computation for $\frac{g_3^2}{g_{\rm L}^2} = \frac{1}{Nk^2}$.
- This matching is significant when the ... terms are in fact subleading e.g. when $\delta_a / \langle \phi^i_a \rangle \sim 1$.

Masses, dual operators and Higgs mechanism

Dual operators have dimensions $m^2L = \Delta(\Delta - 6)$, in terms of the $su(2)$ representations σ^i, they can be classified as

Δ	$\mathrm{SU}(2)_{\mathcal{R}}$ rep.
6	**d** *if* $d>1$
4l+6 =2d+4	**d-2** *if* $d>2$
4l+4 =2d+2	**d+2**

$\rightarrow$ For every choice of partition, there will be operators of dimension $\Delta = 6$, which would correspond to marginal operators of the 6D SCFT.

- General arguments in [Louis, Lust '15] and [Cordova, Dumitrescu, Intriligator '17] forbids a continuous class of BPS AdS_7 vacua/CFT$_6$

$\rightarrow$ The scalars corresponding to $\Delta = 6$ operators are eaten by the gauge vectors that become massive at the vacuum. Notice that the trivial vacuum $\mu = [1^6]$ has only $\mathbf{d} = 1$, there is no Higgsing.

- *to* There can be other $\Delta = 6$ from the $d-2$, $d+2$ rows, but they are not BPS operators thus they do not spoil the arguments above.

BPS flow and domain wall solution

The domain wall metric ansatz is

$$ds_7^2 = e^{2A(\rho)} ds^2_{\mathrm{Mink}_6} + e^{2B(\rho)} d\rho^2 \,.$$

We express the fermionic BPS variation in terms of the flow superpotential

$$W(\sigma, \phi^{ir}) = \frac{\sqrt{2}}{30} e^{-\sigma/2} C + \frac{4}{5} h e^{2\sigma} \,.$$

so the BPS flow is the solution to

$$A' = e^B W \,, \qquad \sigma' = -4 e^B \partial_\sigma W \,, \qquad \phi^{ir} t' = -5 e^B \partial_{\phi^i_r} W \,.$$

Based on the superpotential relation

$$V = 5 \left(-3W^2 + 2\partial_\sigma W^2 + \frac{5}{2} \partial_{\phi^i_r} W \partial_{\phi^r_i} W \right) \,.$$

BPS flow and domain wall solution

The flow between fixed points at UV $\rho \to +\infty$ and IR $\rho \to -\infty$ requires

$$\phi^i(-\infty) = \phi^i_{\mu_{\mathrm{L}-}}\,, \qquad \phi^i(+\infty) = \phi^i_{\mu_{\mathrm{L}+}}\,,$$

where $\phi^i_{\mu_{\mathrm{L}\pm}}$ will be proportional to the σ^i representations associated to two partitions $\mu_{\mathrm{L}\pm}$. The scalar ansatz yields the BPS equations

$$\partial_\rho \Phi^i = -g_3 \Phi^i + \frac{1}{2}\left[\Phi^j\,,\,\Phi^k\right]\epsilon^{ijk}\,,$$

→ *Massive* Nahm equations. [Bachas, Koppe, Pioline '00]

- The scalars determine $\alpha(\rho)$, which allows to solve for the remaining fields, σ and A.
- The cosmological constants are

$$\left(\frac{V_+}{V_-}\right)^{5/4} = \frac{g_{\mathrm{L}}^2 - \kappa_-^2 g_3^2}{g_{\mathrm{L}}^2 - \kappa_+^2 g_3^2}\,,$$

  we expect $\kappa_+ < \kappa_-$ from which it follows that $V_+ < V_-$.

→ The scalar field equation should contain information on the hierarchy of κ's

BPS flow and domain wall solution

[Kronheimer et al., 1990]

Redefine $\Phi_i = \frac{1}{g_3 s} T_i$, $s = -\frac{1}{2} e^{-g_3 \rho}$ so the BPS equations is

$$\partial_s T_i = -\frac{1}{2} \epsilon_{ijk} [T_j, T_k] \,.$$

The solution of the field equations with boundary conditions corresponding to the $\phi^i_\pm = \phi_{\mu_{\mathrm{L}\pm}}$ have moduli space given by the intersection:

$$\mathcal{O}(\mu_{\mathrm{L}-}) \cap \mathcal{S}(\mu_{\mathrm{L}+}) \,.$$

- The Slodowy slice is the space

$$\mathcal{S}(\mu_{\mathrm{L}+}) \equiv \{\phi^-_{\mathrm{L}+} + X \mid [X, \phi^+_{\mathrm{L}+}] = 0\} \,,$$

  with $\phi^\pm \equiv \frac{1}{2}(\phi_1 \pm i\phi_2)$, since ϕ^i_μ give the embedding $\mathrm{su}(2) \to \mathrm{su}(k)$ associated to μ.

- The slice $\mathcal{S}(\mu_{\mathrm{L}+})$ intersects the orbit of every other nilpotent element $\mu_{\mathrm{L}-}$ in one point such that

$$\mu_{\mathrm{L}+} < \mu_{\mathrm{L}-} \,,$$

  reproducing the hierarchy of RG flows from the UV ($\mu_{\mathrm{L}+}$) to the IR ($\mu_{\mathrm{L}-}$).

Summary and Outlook

- Infinite many new AdS_7 vacua in one to one correspondence with the 6D theory
- Test of the conjectured hierarchy of RG flows of 6D SCFT
- Identified extremization conditions, solved BPS equations as Nahm equations

- Does a consistent truncation from 10D type IIA exist?
- DBI-like derivation of higher order terms to capture subleading anomaly coefficients
- non-BPS AdS_7 vacua: instability branes origin?

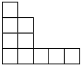
Thank You!

More...

Flavor symmetries of $\mathcal{T}_{G,\mu_L,\mu_R}$

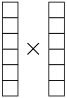
Let's consider $\mu_L, \mu_R \neq 0$. Each theory results from a partially Higgsing of the flavor symmetry $G_{\mathrm{L}} \times G_{\mathrm{R}}$ of $\mathcal{T}^N_{G,0,0}$.

$$f_a^{\mathrm{L,R}} = \#\{\text{blocks } J_a \text{ with dimension } a\}$$



The flavor symmetries are $S(\Pi_a U(f_a^L)) \times S(\Pi_a U(f_a^R))$. For example

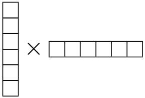
$\mathcal{T}^N_{SU(6),0,0}$



$\rightarrow f_1 = 6$

has flavor symmetry $SU(6) \times SU(6)$ while if we completely Higgs one of the groups

$\mathcal{T}^N_{SU(6),[6],0}$



$\rightarrow \quad f_1^L = 6, f_6^R = 1$

the flavor group si simply $SU(6)$.

Example of one tableau flow

[Bachas, Koppe, Pioline '00]

Consider the case when the UV is the trivial vacuum, $\mu = 0$ (or $[1^k]$). The solution for the scalars is

$$\Phi^i = \frac{g_3}{1 + e^{g_3 \rho}} \phi^i_{\mu_{\mathrm{L}-}}$$

in terms of the normalized fields

$$\alpha = \operatorname{arctanh}\left[\frac{g_3\, \kappa_{\mathrm{L}-}}{g_L} \frac{1}{1 + e^{g_3 \rho}}\right] \quad \Rightarrow \quad \phi^i = \frac{1}{\kappa_{\mathrm{L}-}} \operatorname{arctanh}\left[\frac{g_3\, \kappa_{\mathrm{L}-}}{g_{\mathrm{L}}} \frac{1}{1 + e^{g_3 \rho}}\right] \phi^i_{\mu_{\mathrm{L}-}} .$$

Relevant deformations

- Metric asymptotics

$$ds_7^2 \sim e^{2 \frac{g_3\, \rho}{4}} ds^2_{\mathrm{Mink}_6} + d\rho^2 ,$$

  so an AdS_7 metric of radius $L_+ = g_3/4$.

- Dilaton and scalar fields behave as

$$\phi^i \sim e^{-\frac{4\rho}{L_+}} \phi^i_{\mu_{\mathrm{L}-}} , \quad \sigma \sim e^{-\frac{4\rho}{L_+}} .$$

Example of one tableau flow

To understand the deformation of our solution due to

$$\phi^i \sim e^{-\frac{4\rho}{L_+}} \phi^i_{\mu_{\mathrm{L}-}} ,$$

consider the general AdS_d expansion

$$\delta\varphi \approx \varphi_{\text{non-norm}} e^{-(6-\Delta)\rho} + \varphi_{\text{norm}} e^{-\Delta\rho}$$

with respect to the usual dictionary

$$\varphi_{\text{non-norm}} \to \text{operator } \mathcal{O} \qquad \varphi_{\text{norm}} \to \langle \mathcal{O} \rangle$$

so the theory in the UV is deformed by the vev of an operator of dim. $\Delta = 4$.

RG flows

When a background gauge field is turned on they

$$AdS_d \qquad \leftrightarrow \qquad AdS_p \times M_{d-p}$$

- BPS black holes in 4D and 5D are RG flows across dimensions. States dual to the topologically twisted sector of the 3D ABJM original theory [Benini, Hristov, Zaffaroni, '14], or 4D SYM [Benini, Milan '18] - See C. Toldo's talk.
- Extremization mechanisms in the dual theory
  - 2d c, central charge extremization [Benini, Bobev '13]
  - 3d F, free energy [Jafferis, '10]
  - 6d a anomaly for (1,0) SCFTs [Cordova, Dumitrescu, Intriligator '15]