Neutrino oscillations in Unruh radiation: the proton's testimony

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Workshop on Connecting Insights in Fundamental Physics: Standard Model and Beyond

Corfù, September 4, 2019

Literature

- M. Blasone, G. Lambiase and **G. L.**, Phys. Rev. D **96**, 025023 (2017)
- M.Blasone, G.Lambiase, **G. L.** and L.Petruzziello, Phys. Rev. D **97**, 105008 (2018)
- M.Blasone, G.Lambiase, **G. L.** and L.Petruzziello, arXiv:1903.03382

Contents

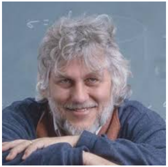
Contents

The Unruh effect

"*...the behavior of particle detectors under acceleration a is investigated where it is shown that an accelerated detector even in flat spacetime will detect particles in the vacuum...*

...This result is exactly what one would expect of a detector immersed in a thermal bath of temperature [Unruh (1976)]

$$T_{\mathrm{U}} = a/2\pi$$ "



The Unruh effect

- Rindler coordinates

$$x^0 = \xi \sinh \eta, \quad x = \xi \cosh \eta$$

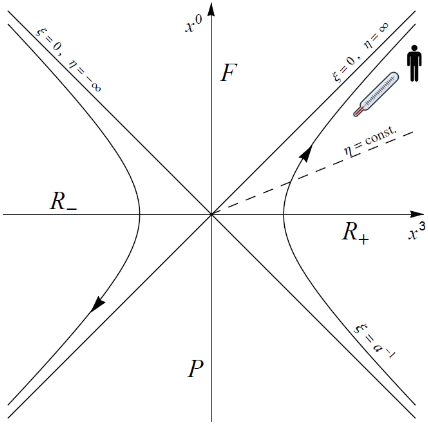
- Rindler vs Minkowski

$$ds^2_{\mathrm{M}} = \left(dx^0\right)^2 - \left(dx\right)^2$$

$$\implies ds^2_{\mathrm{R}} = \xi^2 d\eta^2 - d\xi^2$$

- Rindler worldline

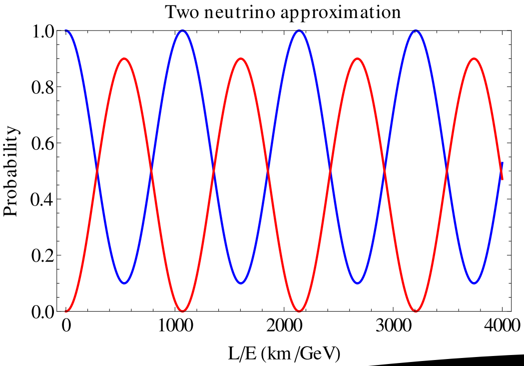
$$\eta = a\tau, \quad \xi = \mathrm{const} \equiv a^{-1}$$



Quantum Mechanics [*Pontecorvo et al. (1978)*]

$$|\nu_e\rangle = |\nu_1\rangle \cos\theta + |\nu_2\rangle \sin\theta$$

$$|\nu_\mu\rangle = -|\nu_1\rangle \sin\theta + |\nu_2\rangle \cos\theta$$



- **Quantum Field Theory** [*Blasone and Vitiello (1995)*]

$$\nu_e(x) \;=\; \nu_1(x)\,\cos\theta \;+\; \nu_2(x)\,\sin\theta$$

$$\nu_\mu(x) \;=\; -\nu_1(x)\,\sin\theta \;+\; \nu_2(x)\,\cos\theta$$

- **Mass** and **flavor** field expansions

$$\nu_i = \sum_{\mathbf{k},\sigma} \left[\alpha^{\sigma}_{\mathbf{k},i}\, u^{\sigma}_{\mathbf{k},i}\, e^{-ik\cdot x} \;+\; \beta^{\sigma\,\dagger}_{\mathbf{k},i}\, v^{\sigma}_{\mathbf{k},i}\, e^{+ik\cdot x}\right], \quad i = 1,2$$

$$\nu_\chi = \sum_{\mathbf{k},\sigma} \left[\alpha^{\sigma}_{\mathbf{k},\chi}(t)\, u^{\sigma}_{\mathbf{k},j}\, e^{-ik\cdot x} \;+\; \beta^{\sigma\,\dagger}_{\mathbf{k},\chi}(t)\, v^{\sigma}_{\mathbf{k},j}\, e^{+ik\cdot x}\right], \quad (\chi, j) = (e,1), (\mu,2)$$

- Vacuum annihilator in the flavor basis

$$\alpha^{\sigma}_{\mathbf{k},e}(t) = \underbrace{\cos\theta\ \alpha^{\sigma}_{\mathbf{k},1} \ + \ \sin\theta}_{\textit{Pontecorvo rotation}} \underbrace{\left(\rho^{\mathbf{k}\,*}_{12}(t)\ \alpha^{\sigma}_{\mathbf{k},2} \ + \ \varepsilon^{\sigma}\ \lambda^{\mathbf{k}}_{12}(t)\ \beta^{\sigma\dagger}_{-\mathbf{k},2}\right)}_{\textbf{Bogoliubov transformation}}$$

- Bogoliubov coefficients

$$\rho^{\mathbf{k}}_{12}(t) \ \equiv \ u^{\sigma\dagger}_{\mathbf{k},2}(t)\, u^{\sigma}_{\mathbf{k},1}(t) \ = \ v^{\sigma\dagger}_{-\mathbf{k},1}(t)\, v^{\sigma}_{-\mathbf{k},2}(t)$$

$$\lambda^{\mathbf{k}}_{12}(t) \ \equiv \ \varepsilon^{\sigma}\ u^{\sigma\dagger}_{\mathbf{k},1}(t)\, v^{\sigma}_{-\mathbf{k},2}(t) \ = \ -\varepsilon^{\sigma}\ u^{\sigma\dagger}_{\mathbf{k},2}(t)\, v^{\sigma}_{-\mathbf{k},1}(t)$$

- **Inequivalent** representations $\Longrightarrow$ **Inequivalent** physics

$$\lim_{V\to\infty}\ {}_{1,2}\langle 0|0(\theta,t)\rangle_{e,\mu} = 0$$

Under the magnifying glass

Asymptotic neutrino states: flavor or mass?

$$\mathcal{A} \simeq {}_{\text{out}}\langle \bar{\ell}, \textcircled{\nu}, \ldots \,|\, \hat{S}_I \,|\, \ldots \rangle_{\text{in}}$$

$$\searrow$$

$$|\nu_{e,\mu}\rangle \quad \text{or} \quad |\nu_{1,2}\rangle \quad ?$$

Flavor states vs mass states

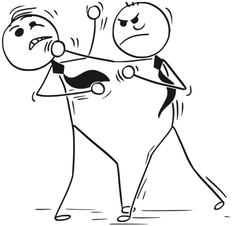
Flavor states

M. Blasone, G. Vitiello (1995)

C. Ji et al. (2002)

C. Lee (2017)

⋮



Mass states

R. E. Shrock (1980)

C. Giunti (2005)

C. Kim et al. (2007)

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Neutrinos as **virtual states**?

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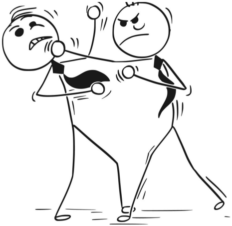

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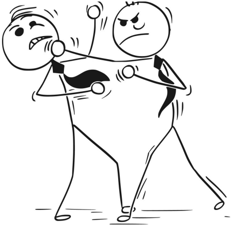

Mass states

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⋮

Neutrinos as **virtual states**? $\Longrightarrow$ Divergent event rates [*Cardall (1999)*]

Flavor states vs mass states

Flavor states

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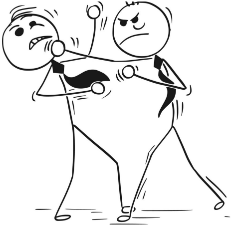
⋮



Mass states

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~~Neutrinos as **virtual states**? $\Longrightarrow$ Divergent event rates [*Cardall (1999)*]~~

A mathematical curiosity? NO

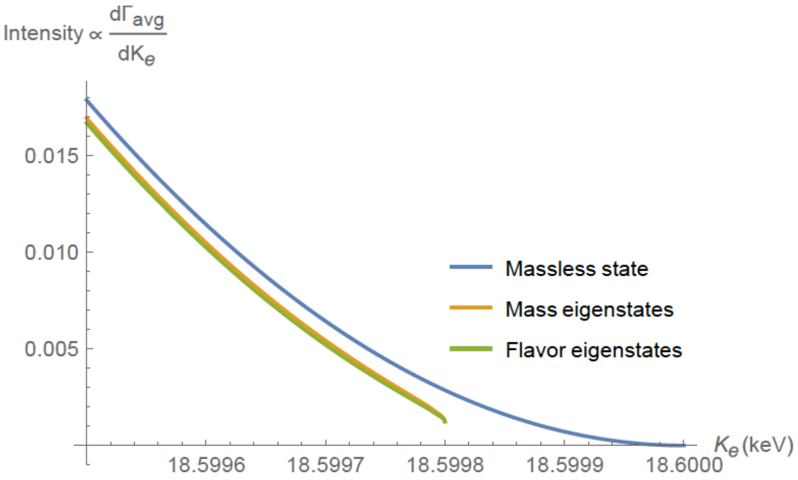


Fig.: Intensity of Tritium β-decay spectrum versus the electron kinetic energy near the end point energy (from arXiv:1709.06306 [hep-ph])

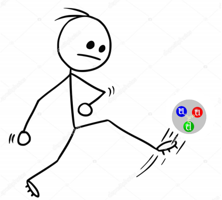
Contents

Decay of accelerated particles

Decay properties are not universal [*Ginzburg (1965)*, *Muller (1997)*]

$$\tau_{proton} \gg \tau_{universe} \sim 10^{10}\,\mathrm{yr}$$

However, if we “kick” the proton…



$$\mathbf{p} \rightarrow \mathbf{n} + \mathbf{e}^{+} + \nu_{\mathbf{e}}$$

acceleration	lifetime
a_{LHC}	$\tau_p \sim 10^{3\times10^{8}}\,\mathrm{yr}$
a_{pulsar}	$\tau_p \sim 10^{-1}\,\mathrm{s}$

Laboratory frame

$$p \to n + e^+ + \nu_e$$

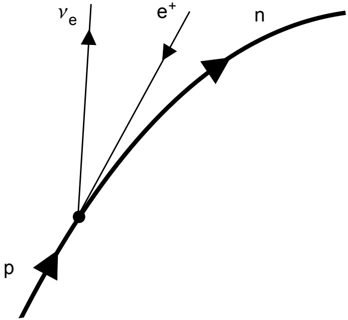


Fig.: The decay occurs since the acceleration supplies the *p*-*n* rest mass difference

Comoving frame

$$p + e \to n + \nu_e \qquad p + \bar{\nu}_e \to n + e^+ \qquad p + e + \bar{\nu}_e \to n$$

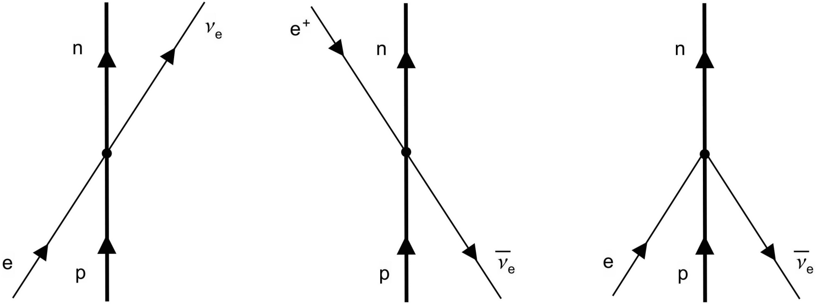


Fig.: The decay occurs since p interacts with the Unruh thermal bath of e^- and ν_e

Basic assumptions [*Matsas et al. (1999)*]:

- Massless neutrino
- $|\mathbf{k}_e| \sim |\mathbf{k}_{\nu_e}| \ll M_{p,n}$
- Current-current Fermi theory

$$\widehat{S}_I = \int d^4x \sqrt{-g}\, \widehat{j}_\mu \left(\widehat{\overline{\Psi}}_\nu \gamma^\mu \widehat{\Psi}_e + \widehat{\overline{\Psi}}_e \gamma^\mu \widehat{\Psi}_\nu \right)$$

$$\widehat{j}^\mu = \widehat{q}(\tau)\, u^\mu\, \delta\left(u - a^{-1}\right), \quad \widehat{q}(\tau) = e^{i\widehat{H}\tau}\, \widehat{q}_0\, e^{-i\widehat{H}\tau}$$

$$\widehat{H}\,|n\rangle = m_n\,|n\rangle\,, \quad \widehat{H}\,|p\rangle = m_p\,|p\rangle\,, \quad G_F = |\langle p|\, \hat{q}_0\, |n\rangle|$$

Laboratory frame

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- Tree-level transition amplitude

$$\mathcal{A}^{p\to n} = \langle n| \otimes \langle e^{+}_{k_e\,\sigma_e}, \nu_{k_\nu\,\sigma_\nu}| \widehat{S}_I |0\rangle \otimes |p\rangle$$

- Differential transition rate

$$\frac{d^2\mathcal{P}^{p\to n}_{in}}{dk_e dk_\nu} = \frac{1}{2}\sum_{\sigma_e=\pm}\sum_{\sigma_\nu=\pm} |\mathcal{A}^{p\to n}|^2$$

- Scalar decay rate

$$\Gamma^{p\to n}_{in} \equiv \frac{\mathcal{P}^{p\to n}_{in}}{T} = \frac{4G_F^2 a}{\pi^2 e^{\pi\Delta m/a}}\int_0^\infty d\tilde{k}_e \int_0^\infty d\tilde{k}_\nu K_{2i\Delta m/a}\left[2\left(\tilde{\omega}_e + \tilde{\omega}_\nu\right)\right]$$

Comoving frame

$$\Gamma_{com}^{p\to n} = \frac{G_F^2\, m_e}{a\,\pi^2\, e^{\pi\Delta m/a}} \int_{-\infty}^{+\infty} d\omega\, \frac{K_{i\omega/a+1/2}(m_e/a) K_{i\omega/a-1/2}(m_e/a)}{\cosh\left[\pi\left(\omega-\Delta m\right)/a\right]}$$

Result

$$\Gamma_{in}^{p\to n} = \Gamma_{com}^{p\to n}$$

Remark

The Unruh effect is **mandatory** for the General Covariance of QFT

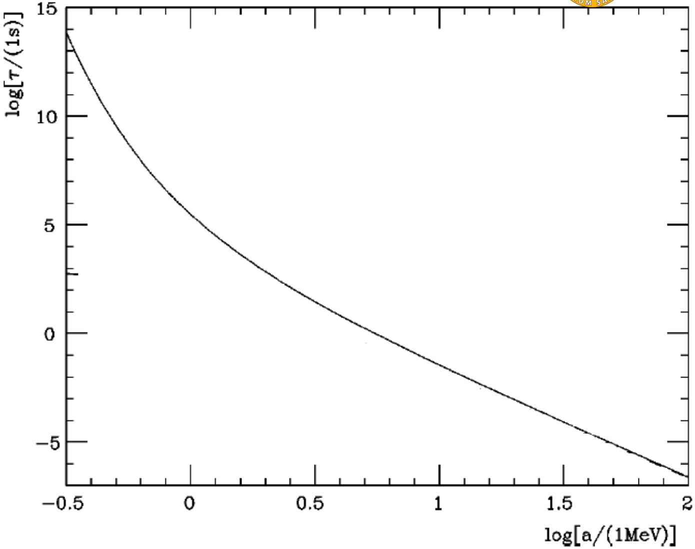


Fig.: The mean proper lifetime τ of proton versus its proper acceleration a.

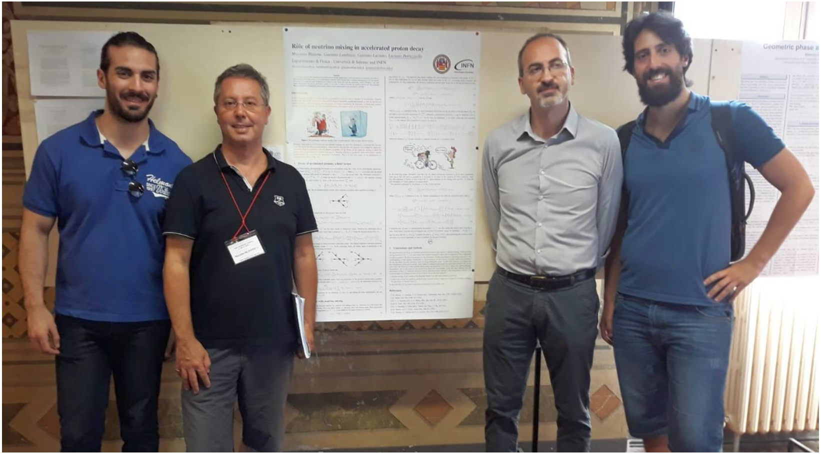
Contents

Neutrino mixing in the inverse β-decay [*Ahluwalia et al. (2016)*]

"*In the laboratory frame, the interaction is the electroweak vertex, hence neutrinos are in* ***flavor eigenstates****. In the comoving frame, the proton interacts with neutrinos in Rindler states, which display an effective thermal weight and are* ***mass eigenstates***".

"*...if charge eigenstates were the asymptotic states also in the accelerating frame, the* ***thermality*** *of the Unruh effect would be* ***violated****"*.

"*...we conclude that the rates in the two frames* ***disagree*** *when taking into account neutrino mixings*".



The **paladins** of General Covariance of QFT

Non-thermal Unruh effect

Unruh spectrum for **mixed neutrinos**

$$ {}_{\mathrm{M}}\langle 0|\mathcal{N}(\theta,\omega)|0\rangle_{\mathrm{M}} = \underbrace{\frac{1}{e^{a\,\omega/T_{\mathrm{U}}}+1}}_{Thermal\ spectrum} + \underbrace{\sin^2\theta\left\{\mathcal{O}\left(\frac{\delta m}{m}\right)^2\right\}}_{Non-thermal\ corrections} $$

Remark

The Unruh spectrum for mixed fields acquires **non-thermal corrections**

Neutrino mixing in the inverse β-decay [*Ahluwalia et al. (2016)*]

"*In the laboratory frame, the interaction is the electroweak vertex, hence neutrinos are in* ***flavor eigenstates****. ~~In the comoving frame, the proton interacts with neutrinos in Rindler states, which display an effective thermal weight and are~~* ~~***mass eigenstates***~~".

$$\mathcal{A}^{p\to n} = \langle n| \otimes \langle e^{+}_{k_e\,\sigma_e}, \nu_{k_\nu\,\sigma_\nu}| \widehat{S}_I \,|0\rangle \otimes |p\rangle$$

$$|\nu_e\rangle = |\nu_1\rangle \cos\theta \,+\, |\nu_2\rangle \sin\theta$$

$$|\nu_\mu\rangle = -|\nu_1\rangle \sin\theta \,+\, |\nu_2\rangle \cos\theta$$

Working with **flavor neutrinos**, in the *laboratory frame*...

$$\Gamma^{p\to n}_{in} \;=\; \cos^4\theta\,\Gamma^{p\to n}_{1} \,+\, \sin^4\theta\,\Gamma^{p\to n}_{2} \,+\, \cos^2\theta \sin^2\theta\,\Gamma^{p\to n}_{12}$$

$$\Gamma^{p\to n}_{i} \;\equiv\; \frac{1}{T}\sum_{\sigma_\nu,\sigma_e} {G_F}^2 \int d^3k_\nu \int d^3k_e \left|\mathcal{I}_{\sigma_\nu\sigma_e}(\omega_{\nu_i},\omega_e)\right|^2, \quad i=1,2,$$

$$\Gamma^{p\to n}_{12} \;\equiv\; \frac{1}{T}\sum_{\sigma_\nu,\sigma_e} {G_F}^2 \int d^3k_\nu \int d^3k_e \left[\mathcal{I}_{\sigma_\nu\sigma_e}(\omega_{\nu_1},\omega_e)\,\mathcal{I}^*_{\sigma_\nu\sigma_e}(\omega_{\nu_2},\omega_e) + \mathrm{c.c.}\right]$$

. . . and in the *comoving frame*

$$\Gamma^{p\to n}_{com} = \cos^4\theta\,\widetilde{\Gamma}^{p\to n}_{1} + \sin^4\theta\,\widetilde{\Gamma}^{p\to n}_{2} + \cos^2\theta\sin^2\theta\,\widetilde{\Gamma}^{p\to n}_{12}$$

$$\begin{aligned}
\widetilde{\Gamma}^{p\to n}_{12} \;=\; & \frac{2\,G_F^2}{a^2\,\pi^7\,\sqrt{l_{\nu_1} l_{\nu_2}}\,e^{\pi\Delta m/a}}\int_{-\infty}^{+\infty} d\omega \left\{\int d^2k_e\, l_e \left|K_{i\omega/a+1/2}\left(\frac{l_e}{a}\right)\right|^2\right.\\
&\times \int d^2k_\nu\,\left(\kappa_\nu^2 + m_{\nu_1} m_{\nu_2} + l_{\nu_1} l_{\nu_2}\right)\\
&\times \mathrm{Re}\left\{K_{i(\omega-\Delta m)/a+1/2}\left(\frac{l_{\nu_1}}{a}\right) K_{i(\omega-\Delta m)/a-1/2}\left(\frac{l_{\nu_2}}{a}\right)\right\}\\
&+ m_e \int d^2k_e \int d^2k_\nu\left(l_{\nu_1} m_{\nu_2} + l_{\nu_2} m_{\nu_1}\right)\\
&\times \mathrm{Re}\left\{K^2_{i\omega/a+1/2}\left(\frac{l_e}{a}\right) K_{i(\omega-\Delta m)/a-1/2}\left(\frac{l_{\nu_1}}{a}\right)\right.\\
&\left.\left.\times\, K_{i(\omega-\Delta m)/a-1/2}\left(\frac{l_{\nu_2}}{a}\right)\right\}\right\}, \qquad \kappa_\nu \equiv (k_\nu^x, k_\nu^y)
\end{aligned}$$

Comparing the rates

Laboratory vs comoving decay rates

$$\Gamma_{in}^{p\to n} = \cos^4\theta\,\Gamma_1^{p\to n} + \sin^4\theta\,\Gamma_2^{p\to n} + \cos^2\theta\sin^2\theta\,\Gamma_{12}^{p\to n},$$

$$\Gamma_{com}^{p\to n} = \cos^4\theta\,\widetilde{\Gamma}_1^{p\to n} + \sin^4\theta\,\widetilde{\Gamma}_2^{p\to n} + \cos^2\theta\sin^2\theta\,\widetilde{\Gamma}_{12}^{p\to n}$$

$$\Gamma_i^{p\to n} = \widetilde{\Gamma}_i^{p\to n}, \qquad i = 1, 2$$

What about the "off-diagonal" terms?

$$\Gamma_{12}^{p\to n} \stackrel{?}{=} \widetilde{\Gamma}_{12}^{p\to n}$$

Comparing the rates

Laboratory vs comoving decay rates

$$\Gamma_{in}^{p\to n} = \cos^4\theta\,\Gamma_1^{p\to n} + \sin^4\theta\,\Gamma_2^{p\to n} + \cos^2\theta\sin^2\theta\,\Gamma_{12}^{p\to n},$$

$$\Gamma_{com}^{p\to n} = \cos^4\theta\,\widetilde{\Gamma}_1^{p\to n} + \sin^4\theta\,\widetilde{\Gamma}_2^{p\to n} + \cos^2\theta\sin^2\theta\,\widetilde{\Gamma}_{12}^{p\to n}$$

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What about the "off-diagonal" terms?

$$\Gamma_{12}^{p\to n} \overset{?}{=} \widetilde{\Gamma}_{12}^{p\to n}$$

Non-trivial calculations...

...for $\frac{\delta m}{m} \ll 1$

$$\Gamma_{12}^{p\to n} = \widetilde{\Gamma}_{12}^{p\to n} \quad \text{up to } \mathcal{O}\left(\frac{\delta m}{m}\right)$$

Result

$$\Gamma_{in}^{p\to n} = \Gamma_{com}^{p\to n} \quad \text{up to } \mathcal{O}\left(\frac{\delta m}{m}\right)$$

Non-trivial calculations...

...for $\frac{\delta m}{m} \ll 1$

$$\Gamma_{12}^{p\to n} = \widetilde{\Gamma}_{12}^{p\to n} \quad \text{up to } \mathcal{O}\left(\frac{\delta m}{m}\right)$$

Result

$$\Gamma_{in}^{p\to n} = \Gamma_{com}^{p\to n} \quad \text{up to } \mathcal{O}\left(\frac{\delta m}{m}\right)$$

Solving the controversy with **mass eigenstates** [*Matsas et al. (2018)*] ?

" [. . .] *a physical Fock space for flavor neutrinos cannot be constructed. Flavor states are only phenomenological since their definition depends on the specific considered process.*"

" *We should view the neutrino states with well defined mass as the fundamental ones.* [. . .] *The decay rates calculated in this way are perfectly in agreement".*

Why not mass states?

- A physical Fock space for flavor neutrinos can be rigorously defined [*Blasone and Vitiello (1995)*]

- The use of mass eigenstates *wipes mixing out* of calculations

$$\Gamma^{p\to n+\bar{\ell}_\alpha+\nu_i} = |U_{\alpha,i}|^2\Gamma_i, \qquad i=1,2$$

- Inconsistency with the asymptotic occurrence of *flavor oscillations*

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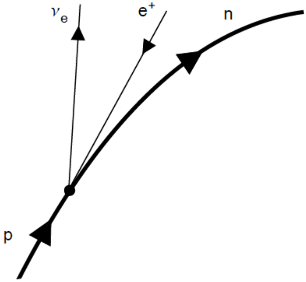
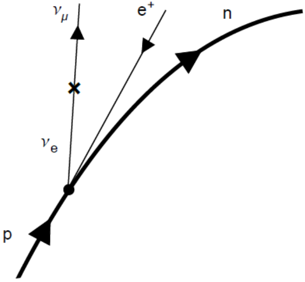
Neutrino oscillations (inertial frame)



a) Without oscillations

$$\Gamma_{in}^{(\nu_e)} = c_\theta^4\,\Gamma_1^{p\to n} + s_\theta^4\,\Gamma_2^{p\to n} + c_\theta^2\,s_\theta^2\,\Gamma_{12}^{p\to n}$$



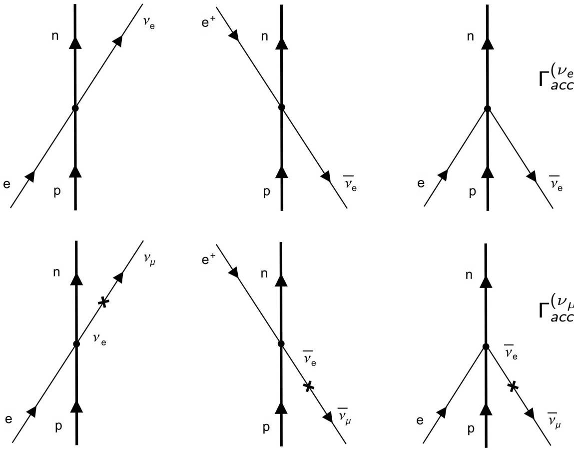
b) With oscillations

$$\Gamma_{in}^{(\nu_\mu)} = c_\theta^2\,s_\theta^2\left(\Gamma_1^{p\to n} + \Gamma_2^{p\to n} - \Gamma_{12}^{p\to n}\right)$$

Total decay rate

$$\Gamma_{in}^{tot} \equiv \Gamma_{in}^{(\nu_e)} + \Gamma_{in}^{(\nu_\mu)} = \cos^2\theta\,\Gamma_1^{p\to n} + \sin^2\theta\,\Gamma_2^{p\to n}$$

Neutrino oscillations (comoving frame)



a) Without oscillations

$$\Gamma^{(\nu_e)}_{acc} = c^4_\theta \, \widetilde{\Gamma}^{p\to n}_1 + s^4_\theta \, \widetilde{\Gamma}^{p\to n}_2 + c^2_\theta \, s^2_\theta \, \widetilde{\Gamma}^{p\to n}_{12}$$

b) With oscillations

$$\Gamma^{(\nu_\mu)}_{acc} = c^2_\theta \, s^2_\theta \left(\widetilde{\Gamma}^{p\to n}_1 + \widetilde{\Gamma}^{p\to n}_2 - \widetilde{\Gamma}^{p\to n}_{12} \right)$$

Total decay rate

$$\Gamma^{tot}_{acc} = \cos^2\theta \, \widetilde{\Gamma}^{p\to n}_1 + \sin^2\theta \, \widetilde{\Gamma}^{p\to n}_2 = \Gamma^{tot}_{in}$$

Contents

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Take-home message

	Ahluwalia's approach	Matsas's approach	Our approach
Asympt. neutrinos in the laboratory frame	Flavor	Mass	Flavor
Asympt. neutrinos in the comoving frame	Mass	Mass	Flavor
Agreement between the decay rates	✗	✓	✓
Consistency with neutrino oscillations	✗	✗	✓

Outlook

- **Beyond** the l.o. approximation

$$\Gamma^{p\to n}_{in} = \Gamma^{p\to n}_{com} \qquad\qquad \Gamma^{p\to n}_{in} \neq \Gamma^{p\to n}_{com}$$

$$\downarrow \qquad\qquad\qquad\qquad \downarrow$$

The paradox would be definitively solved

- Unruh effect for mixed fields is to be revised
- Pontecorvo treatment of mixing is not consistent with QFT

- Generalization to **three flavors** with *CP*-violation
- Extension to **curved background** (gravity effects)
- Application to condensed matter systems


THANK
YOU
GRACIAS
ARIGATO
SHUKURIA
DANKSCHEEN
BIYAN
SHUKRIA
TINGKI
TASHAKKUR
ATU
YAQHANYELAY
SUKSAMA
EKHMET
MEHRBANI
GRAZIE
KOMAPSUMNIDA
MAAKE
PALDIES
GOZAIMASHITA
EFCHARISTO
JUSPAXAR
BOLZÏN
MERCI
SPASSIBO
SNACHALHUYA
NUHUN
CHALTU
WABEEJA
MAITEKA
YUSPAGARATAM
HUI
DENKAUJA
NENACHALHYA
UNALCHEESH
SIKOMO
EKOJU
MERASTAWHY
GAEJTHO
TAVTAPUCH
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