Lattice simulations of supersymmetric theories

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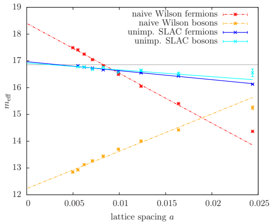
Prospects of supersymmetry on the lattice

Non-perturbative physics from first principles:

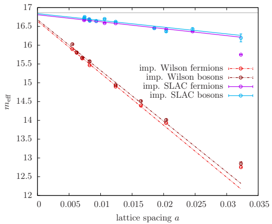
1. SUSY BSM physics: non-perturbative breaking scenarios, metastable vacua
2. SUSY theories for a general understanding of strong interactions. What can we learn from the “exact” analytical approaches? What lessons can supersymmetry teach us about strong interactions?
3. Gauge $\leftrightarrow$ Gravity duality:
    - $\leftarrow$ Predictions for strongly interacting (maximally) supersymmetric gauge theories to be verified and extended with numerical methods.
    - $\rightarrow$ Insights into quantum gravity from SUSY gauge theories.

Progress of supersymmetry on the lattice: Some historical notes

- Easy and simple toy models: 1D Wess-Zumino models
  - failure of naive approach even in 1D, [Catterall, Gregory (2000)],[Giedt, Koniuk, Poppitz, Yavin (2004)], [GB, Kaestner, Uhlmann, Wipf (2008)],. . .
  - principle applicability shown, restoration of SUSY in continuum limit



- Simple, but non-trivial: 2D Wess-Zumino and SUSY gauge theories in 1D and 2D
  - 2D $\mathcal{N} = 1$ Wess-Zumino model: spontaneous SUSY breaking [Golterman, Petcher (1989)],[Beccaria, Campostrini, Feo (2004)], [Steinhauer, Wenger (2014)],. . .
  - 1D SUSY gauge theories: restoration of SUSY ensured



Progress of supersymmetry on the lattice: Current efforts

- 1D and 2D supersymmetric Yang-Mills theory: Gauge/Gravity duality from Matrix Models
  - many talks in this conference (Nishimura, Hanada, Filev ...)
- 4D under control, but (solved) technical difficulties: $\mathcal{N} = 1$ supersymmetric Yang-Mills theory $\rightarrow$ in this talk
- 4D working approaches, techniques still under development: $\mathcal{N} = 4$ supersymmetric Yang-Mills theory [Kaplan, Ünsal], [Catterall, Schaich],. . .

Progress of supersymmetry on the lattice: Future obstacles

- 4D Supersymmetric QCD: several current proposals, seems challenging, but practicable [Giedt (2009)],[Costa, Panagopoulos (2018)],. . .
- general approach for 4D extended SUSY gauge theories: interesting proposals, might be practicable [Giedt (2009)]
- principal unsolved problems: higher dimensional SUSY theories, chiral SUSY gauge theories

Lattice simulations of SUSY theories

Lattice simulations would be the ideal method to investigate non-perturbative sector of SUSY theories . . .

Theory: → next part

- Can we define a lattice SUSY?
- Can we control SUSY breaking?

Practical Simulations: → example SYM

- SUSY theories have nice properties, but require to rework numerical methods

. . . but are challenging from theoretical and practical point of view.

[G.B., S. Catterall, arXiv:1603.04478]

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SUSY breaking and the Leibniz rule on the lattice

Like Nielsen-Ninomiya theorem: locality contradicts with SUSY

On the lattice:

There is no Leibniz rule for a discrete derivative operator. The action can only be invariant with a non-local derivative and non-local product rule. [GB],[Kato,Sakamoto,So],[Nicolai,Dondi]

Further problems:

- fermonic doubling problem, Wilson mass term
- gauge fields represented as link variables

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"The lattice is the only valid non-perturbative definition of a QFT and it can not be combined with SUSY. Therefore SUSY can not exist!" (Lattice theorist)

General solution by generalized Ginsparg-Wilson relation?

"Mrs. RG, the good physics teacher..."
(Peter Hasenfratz)

Symmetry in the continuum ($S[(1+\varepsilon\tilde{M})\varphi] = S[\varphi]$) implies relation for lattice action S_L:

Generalized Ginsparg-Wilson relation

$$M^{ij}_{nm}\phi^j_m \frac{\delta S_L}{\delta \phi^i_n} = (M\alpha^{-1})^{ij}_{nm}\left(\frac{\delta S_L}{\delta \phi^j_m}\frac{\delta S_L}{\delta \phi^i_n} - \frac{\delta^2 S_L}{\delta \phi^j_m \delta \phi^i_n}\right)$$

$$\Phi[\tilde{M}\varphi] = M_{nm}\Phi_m[\varphi]$$

Still open problem how to find solutions. [GB, Bruckmann, Pawlowski]

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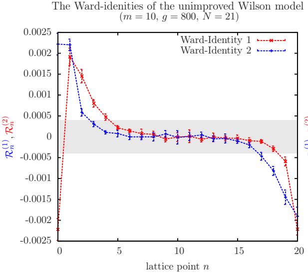
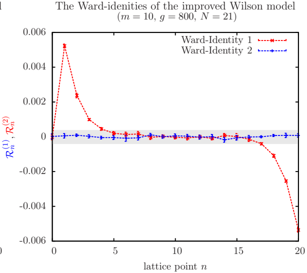
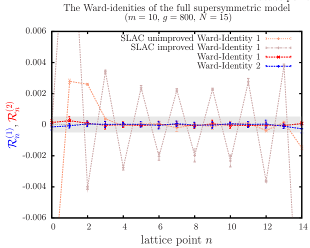
…but we still don't completely understand her lesson.

Sketch of solutions







- only model dependent solutions
- partial realization of extended SUSY
- non-local actions
- otherwise: fine tuning.

Super Yang-Mills theory

Supersymmetric Yang-Mills theory:

$$\mathcal{L} = \mathrm{Tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\bar{\lambda}\not{D}\lambda - \frac{m_g}{2}\bar{\lambda}\lambda\right]$$

- supersymmetric counterpart of Yang-Mills theory; but in several respects similar to QCD
- λ Majorana fermion in the adjoint representation
- SUSY transformations: $\delta A_\mu = -2i\bar{\lambda}\gamma_\mu\varepsilon$, $\delta\lambda = -\sigma_{\mu\nu}F_{\mu\nu}\varepsilon$

Why study supersymmetric Yang-Mills theory on the lattice ?

1. extension of the standard model
   - gauge part of SUSY models
   - understand non-perturbative sector: check effective actions etc.
2. controlled confinement [Ünsal,Yaffe, Poppitz] :
   - compactified SYM: continuity expected
   - small R regime: semiclassical confinement
3. connection to QCD [Armoni,Shifman]:
   - orientifold planar equivalence: SYM $\leftrightarrow$ QCD
   - Remnants of SYM in QCD ?
   - comparison with one flavor QCD

Supersymmetric Yang-Mills theory: Symmetries

SUSY

- gluino mass term m_g $\Rightarrow$ soft SUSY breaking

$U_R(1)$ symmetry, "chiral symmetry": $\lambda \to e^{-i\theta\gamma_5}\lambda$

- $U_R(1)$ anomaly: $\theta = \frac{k\pi}{N_c}$, $U_R(1) \to \mathbb{Z}_{2N_c}$
- $U_R(1)$ spontaneous breaking: $\mathbb{Z}_{2N_c} \overset{\langle\bar{\lambda}\lambda\rangle \neq 0}{\to} \mathbb{Z}_2$

Supersymmetric Yang-Mills theory on the lattice

Lattice action:

$$\mathcal{S}_L = \beta \sum_P \left(1 - \frac{1}{N_c} \Re U_P\right) + \frac{1}{2} \sum_{xy} \bar{\lambda}_x \left(\mathrm{D}_w(m_g)\right)_{xy} \lambda_y$$

- Wilson fermions:

$$\mathrm{D}_w = 1 - \kappa \sum_{\mu=1}^{4} \left[(1-\gamma_\mu)_{\alpha,\beta} T_\mu + (1+\gamma_\mu)_{\alpha,\beta} T_\mu^\dagger\right] + \text{clover}$$

gauge invariant transport: $T_\mu \lambda(x) = V_\mu \lambda(x+\hat{\mu})$;

$$\kappa = \frac{1}{2(m_g+4)}$$

- links in adjoint representation: $(V_\mu)_{ab} = 2\mathrm{Tr}[U_\mu^\dagger T^a U_\mu T^b]$ of SU(2), SU(3)

Lattice SYM: Symmetries

Wilson fermions:

- explicit breaking of symmetries: ~~chiral Sym. ($U_R(1)$)~~, ~~SUSY~~

fine tuning:

- add counterterms to action
- tune coefficients to obtain signal of restored symmetry

special case of SYM:

- tuning of m_g enough to recover chiral symmetry [1]
- same tuning enough to recover supersymmetry [2]

[1] [Bochicchio et al., Nucl.Phys.B262 (1985)]

[2] [Veneziano, Curci, Nucl.Phys.B292 (1987)]

Recovering symmetry

Fine-tuning:

chiral limit = SUSY limit $+O(a)$, obtained at critical $\kappa(m_g)$

- no fine tuning with Ginsparg-Wilson fermions (overlap/domainwall) fermions[3]; but too expensive

practical determination of critical κ:

- limit of zero mass of adjoint pion $(a-\pi)$

$\Rightarrow$ definition of gluino mass: $\propto (m_{a-\pi})^2$

- cross checked with SUSY Ward identities

[3] [Fleming, Kogut, Vranas, Phys. Rev. D 64 (2001)], [Endres, Phys. Rev. D 79 (2009)], [JLQCD, PoS Lattice 2011]

Low energy effective theory

	multiplet[1]	multiplet[2]
scalar	meson a $-f_0$	glueball 0^{++}
pseudoscalar	meson a $-\eta'$	glueball 0^{-+}
fermion	gluino-glue	gluino-glue

- confinement: colourless bound states
- symmetries $+$ confinement $\rightarrow$ low energy effective theory
- glueballs, gluino-glueballs, gluinoballs (mesons)
- build from chiral multiplet type

[1] [Veneziano, Yankielowicz, Phys.Lett.B113 (1982)]

[2] [Farrar, Gabadadze, Schwetz, Phys.Rev. D58 (1998)]

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Supersymmetry

Particles must have same mass.

- confinement: colourless bound states
- symmetries + confinement → low energy effective theory
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Bound states on the lattice

- like in YM and QCD: glueball bound states of gluons
- meson states (like flavour singlet mesons in QCD)

$$\text{a–}f_0 \;:\quad \bar{\lambda}\lambda\,; \qquad \text{a–}\eta' \;:\quad \bar{\lambda}\gamma_5\lambda$$

- gluino-glue spin-1/2 state

$$\sum_{\mu,\nu} \sigma_{\mu\nu} \mathrm{tr}\left[F^{\mu\nu}\lambda\right]$$

Quite challenging to get good signal for the correlators of these operators. Mass determined from exponential decay of the correlator.

The status of the project

Advanced methods of lattice QCD required:

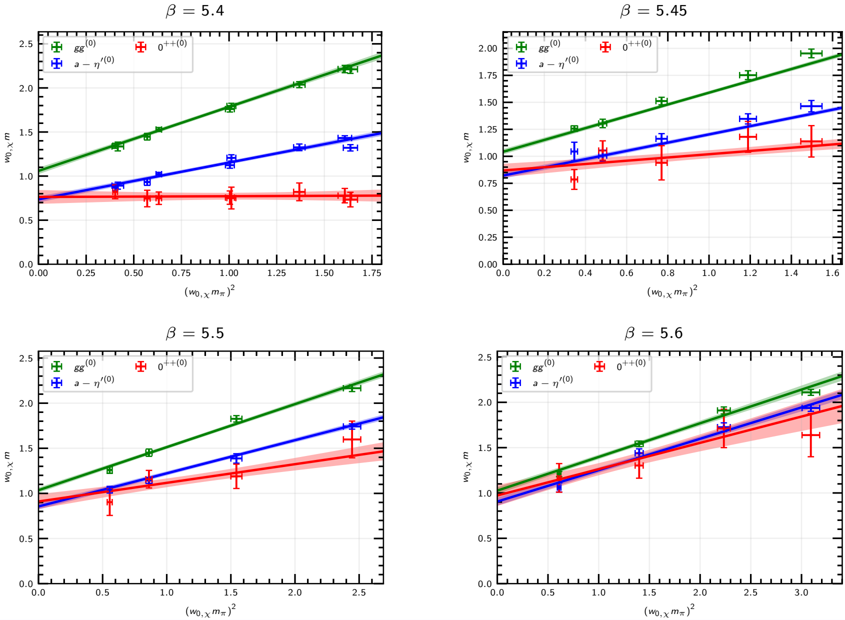
- disconnected contributions [LATTICE2011]
- eigenvalue measurements [GB,Wuilloud]
- variational methods (including mixing of glueball and meson operators) [LATTICE2017]

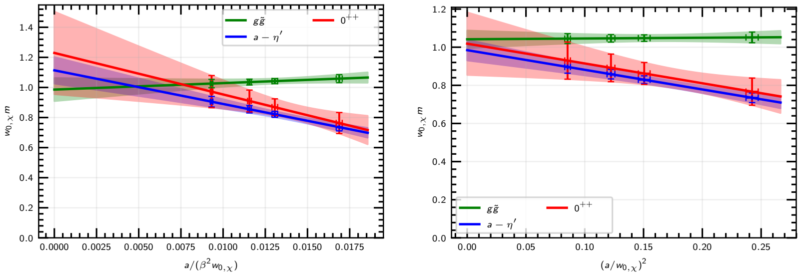
SU(2) SYM:

- multiplet formation found in the continuum limit of SU(2) SYM [JHEP 1603, 080 (2016)]

SU(3) SYM:

- adjoint representation much more demanding than fundamental one (limited to small lattice sizes)
- first SU(3) simulations [LATTICE99,LATTICE2016,LATTICE2017]
- results presented here: [arXiv:1801.08062],[PRL (2019)]


β = 5.4
β = 5.45
β = 5.5
β = 5.6
gg(0)
a − η′(0)
0++(0)
w0,χ m
(w0,χ mπ)2



Fit	$w_0 m_{g\tilde{g}}$	$w_0 m_{0^{++}}$	$w_0 m_{\mathrm{a}-\eta'}$
linear fit	0.917(91)	1.15(30)	1.05(10)
quadratic fit	0.991(55)	0.97(18)	0.950(63)
SU(2) SYM	0.93(6)	1.3(2)	0.98(6)

More details about Ward identities to appear soon.

([Eur.Phys.J. C78 (2018) no.5, 404])

SU(2) supersymmetric Yang-Mills theory at finite temperature

Deconfinement:

- above $T_c^{\mathrm{deconf.}}$ plasma of gluons and gluinos
- Order parameter: Polyakov loop

Chiral phase transitions:

- above T_c^{chiral} fermion condensate melts and chiral symmetry gets restored
- order parameter: $\langle\bar{\lambda}\lambda\rangle$

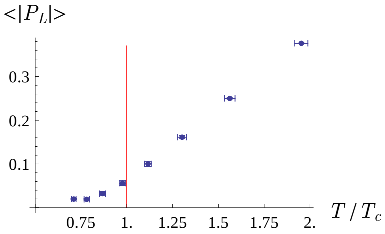
In QCD:

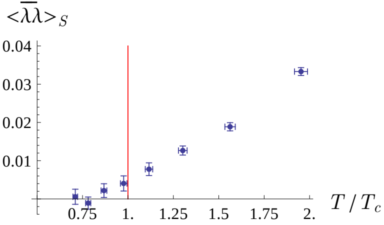
- quarks add screening effects
- explicit chiral symmetry breaking

$\rightarrow$ both transitions become crossover

In SYM: two independent transitions (at $m_g = 0$)

Lattice results SYM at finite T



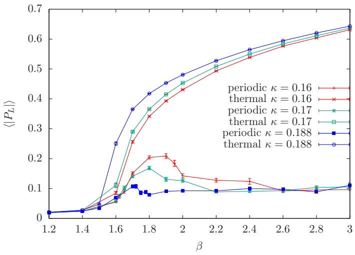


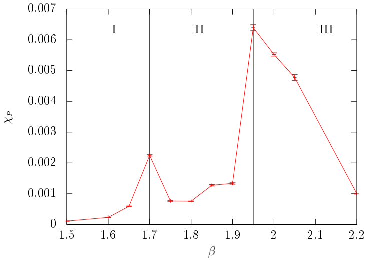
- second order deconfinement transition

$$\frac{T_c(\mathrm{SYM})}{T_c(\text{pure Yang-Mills})} = 0.826(18).$$

- coincidence of deconfinement and chiral transition
  $T_c^{\mathrm{chiral}} = T_c^{\mathrm{deconf.}}$ (within current precision) [JHEP 1411 (2014) 049]

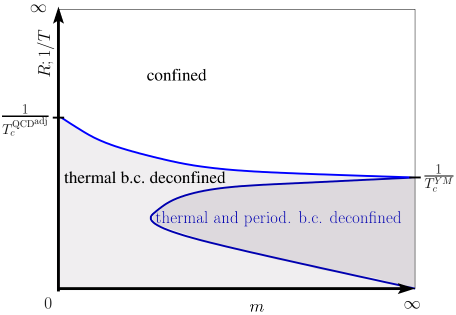
Compactified SYM with periodic boundary conditions





- fermion boundary conditions: thermal $\rightarrow$ periodic
- at small m (large κ) no signal of deconfinement
- intermediate masses: two phase transitions (deconfinement + reconfinement) [GB,Piemonte],[GB, Piemonte, Ünsal]

Phase diagram at finite temperature/compactification



- change of boundary conditions in compact direction $Z(\beta_B) \to \tilde{Z}(\beta_B)$ (Witten index)
- Witten index can not have β_B dependence: states can only be lifted pairwise ⇒ continuity in SYM

From $\mathcal{N}=1$ to $\mathcal{N}=4$ supersymmetric Yang-Mills theory

$\mathcal{N}=4$ supersymmetric Yang-Mills theory is obtained from $\mathcal{N}=1$ supersymmetric Yang-Mills theory in 10 dimensions via dimensional reduction.

- 1 Majorana-Weyl fermion $\rightarrow$ 4 Majorana fermions
- 6 additional gauge fields become scalars X_i
- Yukawa interactions

Additional bosonic term:

$$S_{\mathrm{B}} = \int d^4x \; \left[\frac{1}{2} D_\mu X^i D^\mu X^i + \frac{1}{4}[X^i, X^j]^2\right]$$

$\mathcal{N} = 4$ supersymmetric Yang-Mills theory

Interesting theory:

- gauge-gravity duality, string theory...

But:

- large supersymmetry group, scalar fields
- naive expectation: large fine tuning

Idea: turn "bug" into a "feature"

- large number of super-charges allows to construct a sub-group that is preserved on the lattice
- subgroup is (nearly) enough to recover the complete symmetry in the continuum limit

Twisted formulation: example $\mathcal{N} = (2,2)$ SYM in two dimensions

Field content:

- 2 Majorana fermions λ^I
- two scalar fields B^I, and two gauge fields A_i

Twisted symmetry group:

- $SO(2)_E$ Lorentz group, $SO(2)_I$ flavour symmetry
- decompose fields according to $SO(2)'$=diag($SO(2)_E \times SO(2)_I$)

Q becomes a matrix:

$$Q = qI + q_\mu \gamma_\mu + q_{12}\gamma_1\gamma_2$$

Scalar supercharge $\{q, q\} = 0$: q can be preserved on the lattice

Example $\mathcal{N} = (2,2)$ SYM in two dimensions

- action is a q-exact form
- scalar fields transform as vectors and are combined with A into complexified gauge field
- Dirac-Kähler fermions $(\eta, \psi_\mu, \chi_{12})$

Lattice structure:

- ψ_μ, A_μ on links, χ_{12} on (backward) diagonal

$\mathcal{N} = 4$ supersymmetric Yang-Mills theory on the lattice

Similar construction:

- $\mathrm{SO}(4)_E$ Lorentz group, SU(4) R-symmetry contains $\mathrm{SO}(4)_R \times \mathrm{U}(1)$ part
- choose diagonal $\mathrm{SO}(4)'$ part
- 5 complex "gauge" fields
- 16 fermionic degrees of freedom $(\eta, \psi_a, \chi_{ab})$
- lattice structure with 5 basis vectors

$\Rightarrow$ interesting approach, needs further work concerning stabilizing the simulations [Catterall, Giedt, Jha (2018)]

Gerneralizing the tuning approach

General tuning approach:

- $O(a)$ SUSY breaking on the lattice
- radiative corrections lead to relevant breaking, compensated by counterterms $\rightarrow$ tuning
- SQCD: depending on the formulation $O(10)$ tuning coefficients [J. Giedt,Int.J.Mod.Phys. A24 (2009)]
- estimate tuning in perturbation theory

$\Rightarrow$ provide a more general approach for 4D SUSY gauge theories

Conclusions

- simulation of supersymmetric theories on the lattice is still in some aspects an open theoretical problem

Simple solutions, non-trivial applications

- matrix models, supersymmetric Yang-Mills in 1D and 2D

4D supersymmetric Yang-Mills theory:

- theoretical problem is solvable, practical challenges
- interesting non-perturbative physics like the phase diagram can be investigated on the lattice

Open challenges and ongoing efforts:

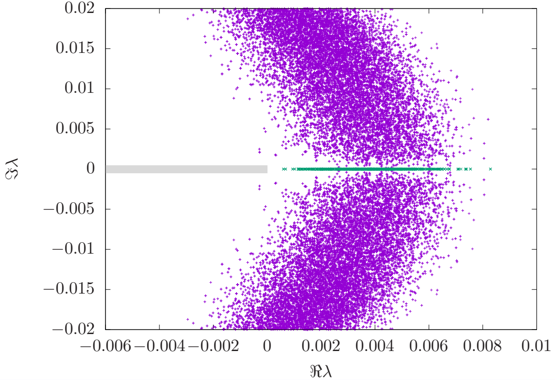
- generalizing the tuning approach: Can we simulate SQCD and $\mathcal{N} = 2$ supersymmetric Yang-Mills?

The sign problem in supersymmetric Yang-Mills

Majorana fermions:

$$\int \mathcal{D}\lambda e^{-\frac{1}{2}\int \bar{\lambda} D \lambda} = \mathrm{Pf}(CD) = (-1)^n \sqrt{\det D}$$

n = number of degenerate real negative eigenvalue pairs



no sign problem
@ current
parameters