Flavour and CP Violation

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Organization of the talk

- Brief review of the flavour sector of the SM. Unitarity triangles
- The case of Majorana neutrinos. Majorana-type phases. Dirac and Majorana unitarity triangles.
- Spontaneous CP violation: dangerous FCNC. The NFC principle of Glashow and Weinberg. Natural Suppression of FCNC. BGL models.

- A viable Model of Spontaneous CP violation with 2 Higgs doublets, and a Flavoured Z_2 symmetry

1. Brief review of the Standard Model (SM)

The SM unifies electroweak and strong interactions and is based on the gauge group:

$$G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

It has 12 generators, to each generator corresponds a gauge field.

The introduction of gauge fields is crucial in order to have invariance under local gauge transformations of G_{SM}

$$SU(3)_C \rightarrow G_\mu^k \qquad k = 1 \ldots 8$$

$$SU(2)_L \rightarrow W_\mu^j \qquad j = 1 \ldots 3$$

$$U(1)_Y \rightarrow B_\mu$$

$$W_\mu^j,\ B_\mu \longrightarrow W_\mu^+,\ W_\mu^-,\ Z_\mu,\ A_\mu$$

All the SM fermionic fields carry weak hypercharge Y defined as

$$Y = Q - T_3$$

where Q is the charge operator and T_3 is the diagonal generator of $SU(2)_L$.

Since experiments only provide evidence for left-handed charged currents, the right-handed components of fermion fields are put in $SU(2)_L$ singlets.

Fermion Spectrum

$$q_L = \begin{bmatrix} u_i \\ d_i \end{bmatrix}_L \qquad (3, 2, 1/6)$$

$$u_{iR} \qquad (3, 2, 2/3)$$

$$d_{iR} \qquad (3, 1, -1/3)$$

$$\ell_{iL} \equiv \begin{pmatrix} \nu_i \\ e^-_i \end{pmatrix} \qquad (1, 2, -1/2)$$

$$e^-_{iR} \qquad (1, 1, -1)$$

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Gauge interactions are determined by the covariant derivative which is dictated by the transformation properties of the various fields:

$$D_\mu = \partial_\mu - i g_s L^k G_\mu^k - i g T^j W_\mu^j - i g' Y B_\mu$$

T^j are the 3 $SU(2)$ generators

L^k are the 8 $SU(3)$ generators

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$$T^j = \begin{cases} 0 & \text{singlet} \\ \tau_j/2 & \text{fundamental} \end{cases}$$

$$L^k = \begin{cases} 0 & \text{singlet} \\ \lambda_k/2 & \text{fundamental} \end{cases}$$

τ_j , λ_k are the Pauli and Gell-Mann matrices

Important feature of the SM

no right-handed neutrinos are introduced!

In order to account for the massive gauge bosons $W_\mu^\pm$ and Z_μ, without destroying renormalizability, the gauge symmetry must be spontaneously broken. One introduces a complex doublet scalar

$$\phi = \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix} \sim (1, 2, 1/2)$$

which achieves the breaking:

$$SU(3)_c \times SU(2)_L \times U(1)_Y \longrightarrow SU(3)_c \times U(1)_{em}.$$

The most general gauge invariant renormalisable scalar potential is:

$$\boxed{V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2}$$

$\lambda > 0$ so that the potential is bounded below and for $\mu^2 < 0$ the minimum is at

$$\langle 0 | \Phi | 0 \rangle = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} v \end{bmatrix}$$

$Q = T_3 + Y \Rightarrow$ for the Higgs: $Q = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

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$$Q\begin{bmatrix}0\\ \frac{1}{\sqrt{2}}v\end{bmatrix}=0\,; \quad e^{i\alpha Q}\begin{bmatrix}0\\ \frac{1}{\sqrt{2}}v\end{bmatrix}=\begin{bmatrix}0\\ \frac{1}{\sqrt{2}}v\end{bmatrix}$$

Electric charge is automatically conserved in the SM. This is no longer true in supersymmetric extensions of the SM or in general Two Higgs Doublet Models.

In order to describe spontaneous symmetry breaking in the SM, it is convenient to parametrize the Higgs doublet as:

$$\Phi=\begin{bmatrix}G^+\\ \frac{1}{\sqrt{2}}(v+H+iG_0)\end{bmatrix}$$

$G^\pm$ and G^0 are the Nambu-Goldstone bosons which are aborbed as the longitudinal components of $W_\mu^\pm$ and Z^0 through the Brout-Englert-Higgs mechanism

$W_\mu^\pm$, Z^0 acquire a mass:

$$M_W = \frac{g v}{2} \, ; \quad M_Z = \sqrt{g^2+g'^2}\,\frac{v}{2} = \frac{M_W}{\cos\theta_W}$$

$$Z_\mu = \cos\theta_W W_\mu^3 - \sin\theta B_\mu$$

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W W_\mu^3$$

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Fermion Masses and Mixing

In the SM one cannot write directly a mass term for any of the fundamental fermions because the terms would violate gauge symmetry. Quark and Charged Lepton masses are generated through Yukawa interactions:

$$-\mathcal{L}_Y = (Y_u)_{ij}\, \bar{q}_{iL}\, \tilde{\phi}\, u_{Rj} + (Y_d)_{ij}\, \bar{q}_{iL}\, \phi\, d_{jR} + Y_{\ell_{ij}}\, \bar{l}_{iL}\, \phi\, e_{Rj} + h.c$$

Y_u, Y_d, Y_ℓ arbitrary complex matrices

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The quark mass terms are:

$$-\mathcal{L}_m = (m_u)_{ij}\, \bar{u}^0_{iL}\, u^0_{jR} + (m_d)_{ij}\, \bar{d}^0_{Li}\, d^0_{Rj} + (m_\ell)_{ij}\, \bar{e}^0_{iL}\, e^0_{jR} + h.c.$$

$$m_u = \frac{v}{2} Y_u \; ; \; m_d = \frac{v}{\sqrt{2}} Y_d \; ; \; m_\ell = \frac{v}{\sqrt{2}} Y_\ell$$

The fermion masses can be diagonalised by:

$$u^0_{iL} = U^u_{L_{ij}} u_{jL} \; ; \quad u^0_{iR} \to U^u_{R_{ij}} u_{jR}$$

$$d^0_{iL} = U^d_{ij}\, d_{jL} \; ; \quad d^0_{iR} = U^d_{R_{ij}}\, d_{jR}$$

$$e^0_{iL} = U^e_{L_{ij}}\, e_{jL} \; ; \quad e^0_{iR} = U^e_{R_{ij}}\, e_{j}R$$

The unitary matrices $U_{L,R}^{u,d,e}$ are chosen such that:

$$\begin{cases} U_L^{u\dagger}\, m_u\, U_R^u = \text{diag.}(m_u, m_c, m_t) \\ U_L^{d\dagger}\, m_d\, U_R^d = \text{diag.}(m_d, m_s, m_b) \\ U^{e\dagger}\, m_\ell\, U_R^e = \text{diag.}(m_e, m_\mu, m_\tau) \end{cases}$$

$u^\circ, d^\circ, e^\circ \longrightarrow$ weak eigenstates

$u, d, e \longrightarrow$ mass eigenstates

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In the weak eigenstate basis:

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}}\left[\overline{u}^0_{iL}\gamma^\mu d^0_{iL} + \overline{\nu}^0_{iL}\gamma^\mu e^0_{iL}\right]W^+_\mu + h.c.$$

In the mass-eigenstate basis:

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}}\left[\overline{u}_L\gamma^\mu U^{u\dagger}_L U^d_L d_L + \overline{\nu}^0_L\gamma^\mu U^e_L e_L\right]W^+_\mu + h.c.$$

$$V \equiv U^{u\dagger}_L U^d_L \rightarrow \text{Cabibbo, Kobayashi, Maskawa matrix}$$

In the SM, the matrix U_L^e is physically meaningless. Since neutrinos are massless in the SM, one can always redefine neutrino fields

$$\nu_L^0 \to \nu_L = U_L^e \nu_L$$

So

$$\bar{\nu}_L^0 \gamma^\mu U_L^e e_L \text{ becomes } \bar{\nu}_L \gamma^\mu e_L$$

In the SM there is no leptonic mixing and therefore no neutrino oscillations.

The experimental discovery of neutrino oscillations rules out the SM. Note that in the SM neutrinos are exactly massless

no Dirac mass — ν_R is not introduced

no Majorana mass — due to exact B-L conservation. Note that a Majorana mass

$$\nu_L^T M C \nu_L$$

would violate B-L.

Fortunately (or unfortunately) it is relatively simple to construct extensions of the SM (νSM) where neutrinos acquire a mass. Simplest possibility: Just introduce ν_R!! Then if one writes the most general Lagrangian consistent with gauge invariance the following term should be included:

$$\nu_R^T \, C \, M \, \nu_R$$

M is naturally large, since it is not protected by gauge invariance.

$$M \gg v$$

Then one obtains naturally small masses for the light neutrinos:

$$m_\nu \approx \frac{m_D^2}{M}$$

This is the "seesaw mechanism"

Why did Glashow, Weinberg, Salam put ν_R in their pocket?

Fundamental Properties of V^{CKM}

$$\mathcal{L}_{cc} = (\bar{u}\ \bar{c}\ \bar{t})_L \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

V^{CKM} is complex but some of the phases have no physical meaning. One has the freedom to rephase the mass eigenstates quark fields

$$u_\alpha \Rightarrow e^{i\varphi_\alpha} u'_\alpha \ ; \quad d_k = e^{i\varphi_k} d'_k$$

Under rephasing $\quad V'_{\alpha k} = e^{i(\varphi_k - \varphi_\alpha)} V_{\alpha k}$

Only rephasing invariant quantities have physical meaning. The simplest examples are the moduli $|V_{\alpha k}|$ and rephasing invariant quartets: $Q_{\alpha i \beta j} \equiv V_{\alpha i} V_{\beta j} V^*_{\alpha j} V^*_{\beta i}$ where $\alpha \neq \beta$; $i \neq j$. Invariants of higher order may in general as funtions of quartets and moduli.

$$V = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix};$$

$$\underbrace{V_{us} V_{cb} V^*_{cs} V^*_{ub}}_{Q_{uscb}}$$

For n_g generations one has a $n_g \times n_g$ unitary matrix. One has n_g^2 parameters but through rephasing one can eliminate $(2n_g - 1)$ phases. So one

$$N_{\text{parameters}} = n_g^2 - (2n_g - 1) = (n_g - 1)^2$$

For $n_g = 3$ one has 4 independent parameters. One can parametrize V^{CKM} in various ways: i) 3 moduli and 1 phase (ii) 4 independent moduli; (iii) 4 independent phases

Consider the orthogonality of the first two rows of V_{CKM}:

$$V_{ud}V_{cd}^* + V_{us}V_{cs}^* + V_{ub}V_{cb}^* = 0$$

Multiplying by $V_{us}^* V_{cs}$ and taking imaginary parts:

$$\boxed{\operatorname{Im} Q_{udcs} = -\operatorname{Im} Q_{ubcs}}$$

One can show that for 3 generations, the imaginary part of all quantities has the same value.

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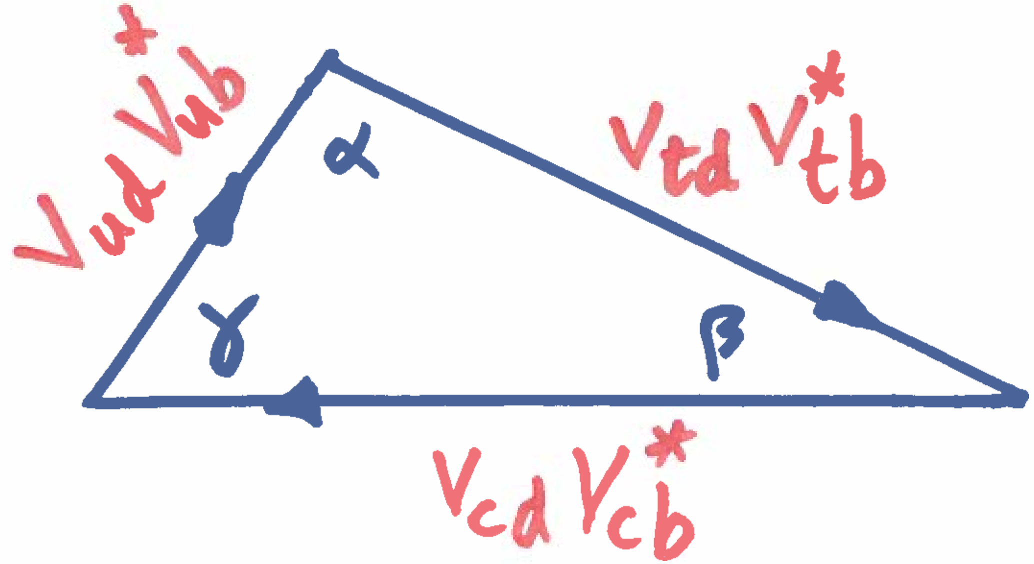
The strength of CP violation in the SM is given by $|\mathrm{Im}\, Q|$ which is small.

Consider orthogonality of 1st and 3rd column:

$$\left(V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*}\right) = 0$$

See Review talks on Unitarity Triangle



$$\alpha \equiv \arg\left[-V_{td} V_{ub} V_{ud}^{*} V_{tb}^{*}\right] = \arg\left[-Q_{ubtd}\right]$$

$$\beta \equiv \arg\left[-V_{cd} V_{tb} V_{cb}^{*} V_{td}^{*}\right] = \arg\left(-Q_{tbcd}\right)$$

$$\gamma \equiv \arg\left[-V_{ud} V_{cb} V_{ub}^{*} V_{cd}^{*}\right] = \arg\left(-Q_{cbud}\right)$$

The following relation is true by definition !!!

$$\boxed{\alpha + \beta + \gamma = \arg(-1) = \pi}$$

No test of unitarity !!

How does one test unitarity of V^{CKM}?

- Without imposing unitarity the V^{CKM} contains 9 moduli and 4 rephasing invariant phases, a total of 13 independent quantities. Within the SM, with V^{CKM} complex but unitary one predicts a series of exact relations among these 13 independent quantities.

(F. Botella, M.N. Rebelo, M. Nebot, GCB)

If one does not impose unitarity, there are 9 phases in the 3×3 V^{CKM}. Five of these phases can be eliminated by rephasing of quark fields. One is left with $9-5=4$ rephasing invariant phases.

A possible choice is: β and γ (defined previously)

$$\chi \equiv \beta_s = \arg\left(-V_{cb} V_{ts} V_{cs}^* V_{tb}^*\right)$$

$$\chi' \equiv \beta_K = \arg\left(-V_{us} V_{cd} V_{ud}^* V_{cs}^*\right)$$

Let us adopt the following phase convention

$$\arg V = \begin{bmatrix} 0 & \chi' & -\gamma \\ \pi & 0 & 0 \\ -\beta & \pi+\chi & 0 \end{bmatrix}$$

Exact relations:

$$\boxed{\sin\chi \equiv \sin\beta_s = \frac{|V_{td}||V_{cd}|}{|V_{ts}||V_{cs}|}\sin\beta.}$$

$$\boxed{\frac{|V_{ub}|}{V_{td}} = \frac{\sin\beta}{\sin\gamma}\frac{|V_{tb}|}{|V_{ud}|}}\ , \text{etc}$$

See F. Botella, M. Nebot, M.N. Rebelo, GCB

- Alternatively, one may look for a region where the location of the vertex of the unitarity triangle where all experimental constraints are satisfied. V^{CKM} is overdetermined!!

$|V_{us}|, |V_{cb}|, |V_{ub}|, \gamma, \beta, \beta_s, \varepsilon_K$

$B_d - \overline{B}_d$ mixing $\rightarrow |V_{td}|$

$B_s - \overline{B}_s$ mixing $\rightarrow |V_{ts}|$

But, hadronic uncertainties!

Leptonic Mixing

In the case of Dirac neutrinos, leptonic mixing is entirely analogous to quark mixing and the V_{PMNS} is analogous to V_{CKM}. Let us consider Majorana neutrinos and work in the physical basis where :

$$\boxed{\begin{aligned} m_\ell &= \mathrm{diag.}(m_e, m_\mu\ m_\tau) \\ m_\nu &= \mathrm{diag}(m_1\ \ m_2\ m_3) \end{aligned}}$$

In this basis, there is still the freedom to rephase the charged lepton fields:

$$\ell_j \longrightarrow \ell'_j = \exp(i\phi_j)\,\ell_j$$

One cannot rephase Majorana fields since the rephasing would not leave invariant the Majorana mass terms:

$$\nu_{Lk}^T\, C\, m_k\, \nu_{Lk}$$

The leptonic weak charged currents can be written :

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} \bar{\ell}_{jL} U^{PMNS} \gamma_\mu \nu_{kL} + h.c.$$

where

$$U^{PMNS} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix}$$

Without introducing the constraints of unitarity U^{PMNS} has 9 moduli and six phases. If one assumes unitarity, one has

$$N_p = (n_g^2 - n_g) \text{ parameters}$$

For $n_g = 3$, $N_p = 6$ parameters

The 6 parameters can be

3 angles + 3 phases.

In the case of quarks or Dirac neutrinos the only meaningful phases are the invariant quartets of V^{CKM} or U^{PMNS}.

Geometrically, these phases correspond to the internal angles of the unitarity triangles. Under rephasing, the unitarity triangles rotate. Their orientation has no meaning.

Majorana-type phases

In the case of Majorana neutrinos the fundamental physical phases are the arguments of rephasing invariant of the type $U_{\ell\alpha} U^*_{\ell\beta}$, with $\alpha \neq \beta$.

$$\arg\left(U_{\ell\alpha} U^*_{\ell\beta}\right) \equiv \text{Majorana-type phases}$$

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There are only six independent Majorana type phases even in the general case where unitarity is not imposed on U.

Let us choose the six independent Majorana-type phases:

$$\beta_1 \equiv \arg(U_{e1} U_{e2}^*) \; ; \; \beta_2 \equiv \arg(U_{\mu 1} U_{\mu 2}^*)$$

$$\beta_3 \equiv \arg(U_{\tau 1} U_{\tau 2}^*) \; ; \; \gamma_1 \equiv \arg(U_{e1} U_{e3}^*)$$

$$\gamma_2 \equiv \arg(U_{\mu 1} U_{\mu 3}^*) \; ; \; \gamma_3 \equiv \arg(U_{\tau 1} U_{\tau 3}^*)$$

M.N. Rebelo, GCB, (2008)

From the 6 independent-type phases one can construct the four independent Dirac-type phases.

Define:

$$\sigma_{e\mu}^{12} \equiv \arg\left(U_{e1} U_{\mu 2} U_{e2}^{*} U_{\mu 1}^{*}\right)$$

$$\sigma_{e\tau}^{12} \equiv \arg\left(U_{e1} U_{\tau 2} U_{e2}^{*} U_{\tau 1}^{*}\right)$$

$$\sigma_{e\mu}^{13} \equiv \arg\left(U_{e1} U_{\mu 3} U_{e3}^{*} U_{\mu 1}^{*}\right)$$

$$\sigma_{e\tau}^{13} \equiv \arg\left(U_{e1} U_{\tau 3} U_{e3}^{*} U_{\tau 1}^{*}\right)$$

$$\sigma^{12}_{e\mu} = \beta_1 - \beta_2 \; ; \; \sigma^{12}_{e\tau} = \beta_1 - \beta_3$$

$$\sigma^{13}_{e\mu} = \gamma_1 - \gamma_2 \; ; \; \sigma^{13}_{e\tau} = \gamma_1 - \gamma_3$$

There are Dirac-type unitarity triangles and Majorana-type unitarity triangles. Examples:

Majorana $U_{e1} U^*_{e2} + U_{\mu 1} U^*_{\mu 2} + U_{e1} U^*_{\tau 2} = 0$

Dirac $U_{e1} U^*_{\mu 1} + U_{e2} U^*_{\mu 2} + U_{e3} U^*_{\mu 3} = 0$

What is the Origin of CP Violation?

Still an open question!

There are two distinctive ways of introducing CP violation:

- At the Lagrangian level

For 3 or more generations one can have CP violation in the SM by introducing complex Yukawa couplings

Kobayashi-Maskawa (1973)

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Spontaneous CP Violation

One can impose CP invariance at the Lagrangian level but have a vacuum which is not CP invariant

T D Lee (1973)

Lee has shown that it is possible to have spontaneous CP violation in the context of a minimal extension of the SM with Two Higgs Doublets (THDM)

CP Violation at the Lagrangian level

What is the simplest (and safest) procedure to check whether a given $\mathcal{L}$ violates CP? Write the Lagrangian as:

1) Write the Lagrangian as:

$$\mathcal{L} = \mathcal{L}^{CP} + \mathcal{L}^{remaining}$$

where $\mathcal{L}^{CP}$ conserves CP (Typically gauge interaction)

2) Write the most general CP transformation which leaves $\mathcal{L}^{CP}$ invariant.

Study whether CP invariance defined in this way imply any restrictions on $\mathcal{L}^{\text{remaining}}$. If such non-trivial CP restrictions exist, one can derive necessary conditions for CP invariance.

Violations of any of these necessary conditions imply CP violation.

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Application to the Standard Model

The most general CP transformation which leaves $\mathcal{L}^{CP}$ invariant is:

$$(CP)\, u_L(t,\vec{r})(CP)^\dagger = U_L \gamma^0 C\, \bar{u}_L^T(t,-\vec{r})$$

$$(CP)\, d_L(t,\vec{r})(CP)^\dagger = U_L \gamma^0 C\, \bar{d}_L^T(t,-\vec{r})$$

$$(CP)\, u_R(t,\vec{r})(CP)^\dagger = U_R^u \gamma^0 C\, \bar{u}_R^T(t,-\vec{r})$$

$$(CP)\, d_R(t,\vec{r})(CP)^\dagger = U_R^d \gamma^0 C\, \bar{d}_R^T(t,-\vec{r})$$

U_L, U_R^u U_R^d unitary matrices acting in flavour space.

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CP invariance of Yukawa couplings imply:

J. Bernabeu
G.C.B, M. Gronau
1986

$$U_L^\dagger Y_u U_R^u = Y_u^*$$
$$U_L^\dagger Y_d U_R^d = Y_d^*$$

$$\Downarrow$$

$$U_L^\dagger (Y_u Y_u^\dagger) U_L = (Y_u Y_u^\dagger)^*$$
$$U_L^\dagger (Y_d Y_d^\dagger) U_L = (Y_d Y_d^\dagger)^*$$

$$\Rightarrow \quad \mathrm{tr}\left[H_{Y_u}, H_{Y_d}\right]^3 = 0$$

for any n° of generations!!

$$H_{Y_{u,d}} \equiv \left(Y_{u,d} Y_{u,d}^\dagger\right)$$

$$I_Y^{CP} \equiv \mathrm{tr}\left[H_{Y_u}, H_{Y_d}\right]^3 = 0$$

Necessary Conditions for CP invariance for any number of fermion generations

For $n=3$ $I_Y^{CP} = 0$ necessary and sufficient conditions for CP invariance at Lagrangian level. After gauge sym. breaking

$$I^{CP} \equiv \Delta_{cu}\Delta_{ct}\Delta_{tu}\Delta_{sd}\Delta_{bs}\Delta_{bd}\,\mathrm{Im}\,Q$$

$$\Delta_{cu} = m_c^2 - m_u^2 \quad ; \quad Q = V_{us}V_{cb}V_{cs}^*V_{ub}^*$$

This result was obtained by us (J. Bernabeu, M. Gronau, GCB) in 1986.

We just followed the idea of Lee and Yang.

∴ CP is defined by the part of the Lagrangian which respects CP.

Lee could have derived the above result in 1973 and would have won a second Nobel prize, with Kobayashi and Maskawa ...

T. D. Lee has shown that if one introduces 2 Higgs doublets and imposes CP invariance of the Lagrangian then there is a region of parameters of the Higgs potential where the minimum is at

$$\langle \phi_1 \rangle = \begin{bmatrix} 0 \\ v_1 e^{i\theta_1} \end{bmatrix} ; \quad \langle \phi_2 \rangle = \begin{bmatrix} 0 \\ v_2 e^{i\theta_2} \end{bmatrix}$$

For generic $\theta \equiv \theta_2 - \theta_1$ the vacuum is not CP invariant $\Rightarrow$ spontaneous CP violation

T.D.Lee (1973)

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In general two Higgs doublet models have FCNC

Neutral currents have played an important rôle in the construction and experimental tests of unified gauge theories

EPS Prize in 2009 to Gargamelle, CERN

In the Standard Model Flavour changing neutral currents (FCNC) are forbidden at tree level

- in the gauge sector, no ZFCNC

- in the Higgs sector, no HFCNC

Two Dogmas likely to be violated

Models with two or more Higgs doublets have potentially large HFCNC

Strict limits on FCNC processes!

In two Higgs Doublet Models (2HDM) Flavour Changing Neutral Currents (FCNC) have to be eliminated at tree level or naturally suppressed, in order to conform to experiment.

- Z_2 symmetry leading to Natural Flavour Conservation (NFC)

  Glashow and Weinberg (1977)

- Attempt at generalising NFC : R. Gatto

Can one have a framework where there are FCNC at tree level, but naturally suppressed?

Is it possible to have a framework where the FCNC exist, but are only functions of V^{CKM} and the ratio v_2/v_1?

The suppression of FCNC could be related to the smallness of some of the V^{CKM} elements.

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- Naturally suppressed FCNC as a result of a symmetry of the Lagrangian. The suppression is due to small V^{CKM} elements

G.C.B, Grimus, Lavoura (1996) (BGL)

- Extension to the leptonic sector

F. Botella, GCB, MNRebelo
F. Botella, GCB., MNRebelo, M.Nebot

- Thorough Phenomenological analysis

F. Botella, GCB, A. Carmona, M Nebot, L. Pedro, M.N. Rebelo

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Notation

Yukawa Interactions

$$\mathcal{L}_Y = -\overline{Q_L^0}\,\Gamma_1\Phi_1 d_R^0 - \overline{Q_L^0}\,\Gamma_2\Phi_2 d_R^0 - \overline{Q_L^0}\,\Delta_1\tilde{\Phi}_1 u_R^0 - \overline{Q_L^0}\,\Delta_2\tilde{\Phi}_2 u_R^0 + \text{h.c.}$$

$$\tilde{\Phi}_i = -i\tau_2\Phi_i^*$$

Quark mass matrices

$$M_d = \frac{1}{\sqrt{2}}(v_1\Gamma_1 + v_2 e^{i\theta}\Gamma_2), \quad M_u = \frac{1}{\sqrt{2}}(v_1\Delta_1 + v_2 e^{-i\theta}\Delta_2),$$

Diagonalised by:

$$U_{dL}^\dagger M_d U_{dR} = D_d \equiv \text{diag}\,(m_d, m_s, m_b)\,,$$
$$U_{uL}^\dagger M_u U_{uR} = D_u \equiv \text{diag}\,(m_u, m_c, m_t)\,.$$

Leptonic Sector

$$-\overline{L_L^0}\,\Pi_1\Phi_1\ell_R^0 - \overline{L_L^0}\,\Pi_2\Phi_2\ell_R^0 + \text{h.c.}$$

Charged leptons

$$\left(-\overline{L_L^0}\,\Sigma_1\tilde{\Phi}_1\nu_R^0 - \overline{L_L^0}\,\Sigma_2\tilde{\Phi}_2\nu_R^0 + \text{h.c.} \right)$$

Neutrino Dirac

$$\left(\frac{1}{2}{\nu_R^0}^T C^{-1} M_R \nu_R^0 + \text{h.c.} \right)$$

Neutrino Majorana

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Expansion around the vev's

$$\Phi_j = \begin{pmatrix} \phi_i^+ \\ \frac{e^{i\alpha_j}}{\sqrt{2}}(v_j + \rho_j + i\eta_j) \end{pmatrix}, \qquad j = 1, 2$$

We perform the following transformation:

$$\begin{pmatrix} H^0 \\ R \end{pmatrix} = U \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}; \quad \begin{pmatrix} G^0 \\ I \end{pmatrix} = U \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}; \quad \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = U \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix},$$

$$U = \frac{1}{v} \begin{pmatrix} v_1 e^{-i\alpha_1} & v_2 e^{-i\alpha_2} \\ v_2 e^{-i\alpha_1} & -v_1 e^{-i\alpha_2} \end{pmatrix}; \qquad v = \sqrt{v_1^2 + v_2^2} = (\sqrt{2} G_F)^{-\frac{1}{2}} \simeq 246\mathrm{GeV}$$

U singles out

H^0 with couplings to quarks proportional to mass matrices

G^0 neutral pseudo-Goldstone boson

G^+ charged pseudo-Goldstone boson

Physical neutral fields are combinations of $H^0 \quad R \quad I$

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Neutral and charged Higgs Interactions for the quark sector

$$\begin{aligned}\mathcal{L}_Y(\text{quark, Higgs}) = & -\overline{d_L^0}\frac{1}{v}\left[M_d H^0 + N_d^0 R + i N_d^0 I\right] d_R^0 \\ & -\overline{u_L^0}\frac{1}{v}\left[M_u H^0 + N_u^0 R + i N_u^0 I\right] u_R^0 \\ & -\frac{\sqrt{2}H^+}{v}\left(\overline{u_L^0} N_d^0\, d_R^0 - \overline{u_R^0} N_u^{0\dagger}\, d_L^0\right) + \text{h.c.}\end{aligned}$$

$$N_d^0 = \frac{1}{\sqrt{2}}(v_2\Gamma_1 - v_1 e^{i\theta}\Gamma_2), \quad N_u^0 = \frac{1}{\sqrt{2}}(v_2\Delta_1 - v_1 e^{-i\theta}\Delta_2).$$

Flavour structure of quark sector of 2HDM characterised by:

four matrices $M_d,\ M_u,\ N_d^0,\ N_u^0$.

Likewise for Leptonic sector, Dirac neutrinos:

$$M_\ell,\ M_\nu,\ N_\ell^0,\ N_\nu^0.$$

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Yukawa Couplings in terms of quark mass eigenstates

for H^+, H^0, R, I

$$\mathcal{L}_Y(\text{quark, Higgs}) =$$

$$- \frac{\sqrt{2}H^+}{v}\bar{u}\left(V N_d \gamma_R - N_u^\dagger \, V \gamma_L\right) d + \text{h.c.} - \frac{H^0}{v}\left(\bar{u} D_u u + \bar{d} D_d \, d\right) -$$

$$- \frac{R}{v}\left[\bar{u}(N_u \gamma_R + N_u^\dagger \gamma_L) u + \bar{d}(N_d \gamma_R + N_d^\dagger \gamma_L) \, d\right] +$$

$$+ i\frac{I}{v}\left[\bar{u}(N_u \gamma_R - N_u^\dagger \gamma_L) u - \bar{d}(N_d \gamma_R - N_d^\dagger \gamma_L) \, d\right]$$

$$\gamma_L = (1 - \gamma_5)/2 \qquad \gamma_R = (1 + \gamma_5)/2$$

$$V = V_{CKM}$$

FCNC controlled by:

$$N_d = \frac{1}{\sqrt{2}} U_{dL}^\dagger \left(v_2 \Gamma_1 - v_1 e^{i\alpha} \Gamma_2 \right) U_{dR}$$

$$N_u = \frac{1}{\sqrt{2}} U_{uL}^\dagger \left(v_2 \Delta_1 - v_1 e^{-i\alpha} \Delta_2 \right) U_{uR}$$

For general 2HDM, N_d, N_u are are arbitrary complex 3×3 matrices.

One can rewrite N_d as:

$$N_d = \frac{v_2}{v_1} D_d - \frac{v_2}{\sqrt{2}} \left(t + \frac{1}{t} \right) \underbrace{U_{dL}^\dagger e^{i\alpha} \Gamma_2 U_{dR}}_{\text{leads to FCNC}}$$

$$t \equiv \tan\beta = \frac{v_2}{v_1}$$

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In general N_d, N_u depend on (U_{dL}, U_{dR}), (U_{uL}, U_{uR}) respectively.

Our initial aim was to prove that it is "impossible" to have N_d, N_u to depend only on V_{CKM}.

While looking for the "proof" we (Gérard, Grimus and I) discovered the so called BGL models

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Example of a BGL-type model : Impose the following discrete symmetry :

$$Q^0_{Lj} \to \exp(i\tau) Q^0_{Lj} \; ; \quad u^0_{Rj} \to \exp(2i\tau) u^0_{Rj} \; ;$$

$$\phi_2 \to \exp(i\tau) \phi_2$$

$\tau \neq 0, \pi$ $\left\{ \begin{array}{l} Z_2 \\ \text{excluded} \end{array} \right.$

Γ_j, Δ_j have the form:

$$\Gamma_1 = \begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix} ; \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix} ; \quad \Delta_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 0 \end{bmatrix} ; \quad \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x \end{bmatrix}$$

FCNC only in the down sector

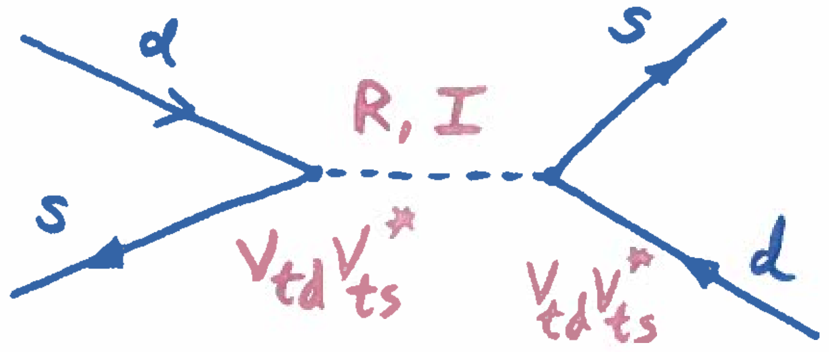
If one imposes $d^0_{Rj} \to \exp(2i\tau) d^0_{Rj}$ instead of $u^0_{Rj} \to \exp(2i\tau) u^0_{Rj}$ only FCNC in up sector

Considering only the Quark sector there are 6 different BGL type models. In the example considered, one has:

$$\left(N_d\right)_{rs} = \frac{v_2}{v_1}\left(D_d\right)_{rs} - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right)\left(V_{CKM}^{\dagger}\right)_{r3}\left(V_{CKM}\right)_{3s}\left(D_d\right)_{ss}$$

$$N_u = -\frac{v_1}{v_2}\,\mathrm{diag}\left(0, 0, m_t\right) + \frac{v_2}{v_1}\,\mathrm{diag.}\left(m_u, m_c, 0\right)$$

Strong and natural suppression of $K^0-\overline{K^0}$ transitions



$\longrightarrow \lambda^{10}$ suppression

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Neutral couplings in BGL models

$$N_u = -\frac{v_1}{v_2}\mathrm{diag}(0,0,m_t) + \frac{v_2}{v_1}\mathrm{diag}(m_u,m_c,0)$$

Explicitely

$$N_d = \frac{v_2}{v_1}\begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} - \left(\frac{v_1}{v_2}+\frac{v_2}{v_1}\right)\begin{pmatrix} m_d|V_{31}|^2 & m_s V_{31}^* V_{32} & m_b V_{31}^* V_{33} \\ m_d V_{32}^* V_{31} & m_s|V_{32}|^2 & m_b V_{32}^* V_{33} \\ m_d V_{33}^* V_{31} & m_s V_{33}^* V_{32} & m_b|V_{33}|^2 \end{pmatrix}$$

<u>It all comes from the symmetry</u>

suggested in 1996!

BGL models have some features in Common with the Minimal Flavour Violation Framework

Buras, Gambino, Gorbahn, Jager, Silvestrini (2001)
D'Ambrosio, Giudice, Isidori, Strumia (2002)

Namely, Flavour dependence of New Physics is completely controlled by V^{CKM}, with no other flavour parameters.

Note: MFV is an "Hypothesis" not a model!!!

An important question :

Can one introduce other discrete symmetries leading to other models with FCNC, completely controlled by V_{CKM}?

Answer : In the framework of 2HDM with Abelian symmetries and the constraint that FCNC only depend on V_{CKM}, BGL models are unique!

Ferreira and Silva 2010.

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The explicit example of a BGL model was written in a weak-basis chosen by the symmetry. How to recognize a BGL model when written in a different WB?

The following relations

$$\Delta_1^\dagger \Delta_2 = 0 \; ; \quad \Delta_1 \Delta_2^\dagger = 0 \; ; \quad \Gamma_1^\dagger \Delta_2 = 0 \; ; \quad \Gamma_2^\dagger \Delta_1 = 0$$

are necessary and sufficient conditions for a set of Yukawa matrices Γ_i, Δ_i to be of the BGL type, with FCNC in the down sector.

- In a certain sense, BGL models are rather unique. They have FCNC either in the up or the down sectors but not in both.
- If one restricts oneself to abelian symmetries, BGL models are the only 2HDM with FCNC at tree level, but no new flavour parameters, apart from V_{CKM}.
- Question – Can one generalize BGL models and construct a 2HDM with non-vanishing but controlled FCNC in both the up and down sectors? These gBGL models would contain BGL models as special cases, corresponding to specific values of the parameters of gBGL

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Answer: Yes!!

gBGL allowing for HFCNC both in up and down sectors

Symmetry: (not flavour blind!!)

$$Q_{L_3} \mapsto -Q_{L_3},$$
$$d_R \mapsto d_R, \qquad \Phi_1 \mapsto \Phi_1,$$
$$u_R \mapsto u_R, \qquad \Phi_2 \mapsto -\Phi_2.$$

- drastic reduction in number of free parameters

-no NFC

one may say that *the principle leading to gBGL constrains the Yukawa couplings so that each line of* Γ_j, Δ_j *couples only to one Higgs doublet.*

$$\Gamma_1 = \begin{pmatrix} \times & \times & \gamma_{13} \\ \times & \times & \gamma_{23} \\ 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \gamma_{31} & \gamma_{32} & \times \end{pmatrix},$$

$$\Delta_1 = \begin{pmatrix} \times & \times & \delta_{13} \\ \times & \times & \delta_{23} \\ 0 & 0 & 0 \end{pmatrix}, \quad \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \delta_{31} & \delta_{32} & \times \end{pmatrix},$$

- renormalisable;

- FCNC both in up and down sectors;

- no longer of MFV type, four additional flavour parameters;

- both up and down type BGL appear as special limits;

gBGL verify:

$$\Gamma_2^\dagger \Gamma_1 = 0, \quad \Gamma_2^\dagger \Delta_1 = 0,$$
$$\Delta_2^\dagger \Delta_1 = 0, \quad \Delta_2^\dagger \Gamma_1 = 0.$$

Structure of Yukawa Couplings

$$\Gamma_1 = \begin{bmatrix} \times & \times & \gamma_{13} \\ \times & \times & \gamma_{23} \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \gamma_{31} & \gamma_{32} & \times \end{bmatrix}$$

$$\Delta_1 = \begin{bmatrix} \times & \times & \delta_{13} \\ \times & \times & \delta_{23} \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \delta_{31} & \delta_{32} & \times \end{bmatrix}$$

For $\delta_{ij} = 0$ one obtains uBGL models, with

$$\Gamma_1 = \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{bmatrix}$$

$$\Delta_1 = \begin{bmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{bmatrix}$$

Similarly for $\gamma_{ij} = 0$, one obtains dBGL

It can be shown that N_d, N_u can be parametrized as:

$$N_d = \left[t_\beta \mathbb{1} - (t_\beta + t_\beta^{-1})\, V^\dagger U P_3 U^\dagger V \right] M_d$$

$$N_u = \left[t_\beta \mathbb{1} - (t_\beta + t_\beta^{-1})\, \mathcal{U} P_3 U^\dagger \right] M_u$$

$V \equiv V^{CKM}$. It is clear that in gBGL one has more freedom, due to the presence of the arbitrary matrix $\mathcal{U}$. Nevertheless, there is much less freedom than one might expect, since the only quantities involving $\mathcal{U}$ are:

$$\left[U P_3 U^\dagger \right]_{ij} = \mathcal{U}_{i3}\, \mathcal{U}_{j3}^*$$

Mass matrices

$$\Gamma_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}, \Delta_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}$$

- Mass matrices

$$M_d^0 = \frac{ve^{i\theta_1}}{\sqrt{2}}(c_\beta \Gamma_1 + e^{i\theta} s_\beta \Gamma_2)\,, \quad M_u^0 = \frac{ve^{-i\theta_1}}{\sqrt{2}}(c_\beta \Delta_1 + e^{-i\theta} s_\beta \Delta_2)$$

- **Important:** with $\hat{M}_d^0$ and $\hat{M}_u^0$ *real*

$$M_d^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\theta} \end{pmatrix} \hat{M}_d^0, \qquad M_u^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\theta} \end{pmatrix} \hat{M}_u^0$$

$$M_d^0 = \left[(\mathbf{1} - \mathrm{P}_3) + e^{i\theta}\mathrm{P}_3\right] \hat{M}_d^0, \quad M_u^0 = \left[(\mathbf{1} - \mathrm{P}_3) + e^{-i\theta}\mathrm{P}_3\right] \hat{M}_u^0$$

$$\mathrm{P}_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Bidiagonalisation of M_d^0, M_u^0

$$\mathcal{U}_{d_L}^\dagger M_d^0 \mathcal{U}_{d_R} = \mathrm{diag}(m_{d_i}), \quad \mathcal{U}_{u_L}^\dagger M_u^0 \mathcal{U}_{u_R} = \mathrm{diag}(m_{u_i})$$

- $M_d^0 M_d^{0\dagger}$

$$M_d^0 M_d^{0\dagger} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\theta} \end{pmatrix} \hat{M}_d^0 \hat{M}_d^{0\,T} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\theta} \end{pmatrix}$$

$\hat{M}_d^0 \hat{M}_d^{0\,T}$ *real* and *symmetric*

$$\mathcal{O}_L^{d\,T} \hat{M}_d^0 \hat{M}_d^{0\,T} \mathcal{O}_L^d = \mathrm{diag}(m_{d_i}^2) \qquad \text{with real orthogonal } \mathcal{O}_L^d$$

$$\mathcal{U}_{d_L}^\dagger M_d^0 M_d^{0\dagger} \mathcal{U}_{d_L} = \mathrm{diag}(m_{d_i}^2)\,, \text{ with } \mathcal{U}_{d_L} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\theta} \end{pmatrix} \mathcal{O}_L^d$$

- Similarly

$$\mathcal{U}_{u_L}^\dagger M_u^0 M_u^{0\dagger} \mathcal{U}_{u_L} = \mathrm{diag}(m_{u_i}^2)\,, \text{ with } \mathcal{U}_{u_L} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\theta} \end{pmatrix} \mathcal{O}_L^u$$

- Right-handed transformations

$$M_d^{0\dagger} M_d^0 = \hat{M}_d^{0\,T} \hat{M}_d^0, \qquad M_u^{0\dagger} M_u^0 = \hat{M}_u^{0\,T} \hat{M}_u^0$$

$$\mathcal{O}_R^{d\,T} M_d^{0\dagger} M_d^0 \mathcal{O}_R^d = \mathrm{diag}(m_{d_i}^2), \quad \mathcal{O}_R^{u\,T} M_u^{0\dagger} M_u^0 \mathcal{O}_R^u = \mathrm{diag}(m_{u_i}^2)$$

with real orthogonal $\mathcal{O}_R^d$ and $\mathcal{O}_R^u$

- Finally

$$M_d = \mathrm{diag}(m_{d_i}) = \mathcal{U}_{d_L}^\dagger M_d^0 \mathcal{O}_R^d, \quad M_u = \mathrm{diag}(m_{u_i}) = \mathcal{U}_{u_L}^\dagger M_u^0 \mathcal{O}_R^u$$

- The CKM matrix $V \equiv \mathcal{U}_{u_L}^\dagger \mathcal{U}_{d_L}$ is

$$V = \mathcal{O}_L^{u\,T} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i2\theta} \end{pmatrix} \mathcal{O}_L^d$$

requires $e^{i2\theta} \neq \pm 1$ for CP violation!

The fact that it is necessary to have $\boxed{e^{2i\theta} \neq \pm 1}$ in order to have CP violation, was to be expected, as it can be seen from a close analysis of the scalar potential.

One can show that for $\theta = \pm \pi/2$ the vacuum is CP conserving!!

Scalar sector

- 2HDM potential
  - CP invariant (all couplings are real)
  - $\mathbb{Z}_2$ symmetry, softly broken by $\mu_{12}^2 \neq 0$

$$\mathcal{V}(\Phi_1,\Phi_2) = \mu_{11}^2 \Phi_1^\dagger\Phi_1 + \mu_{22}^2 \Phi_2^\dagger\Phi_2 + \mu_{12}^2(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1)$$
$$+\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + 2\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + 2\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$
$$+\lambda_5[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2]$$

$\cos\theta = \dfrac{-\mu_{12}^2}{2\lambda_5 v_1 v_2}$; For $\theta = \pm\pi/2$ the vacuum is CP invariant!!

Phenomenology implications

We have shown that there a region of the parameters of the model where:

- One can obtain a correct CKM matrix: reproduce the moduli of the first and second ~~[illegible]~~ lines of V_{CKM}; obtain correct γ
- Satisfy the stringent constraints from experiment K^0-$\bar{K}^0$, B_d-$\bar{B}_d$, B_s-$\bar{B}_s$, rare top decays etc

- We point out that there is a deep connection between the generation of a complex CKM matrix from a vacuum phase and the appearance of SFCNC.
- The new scalars are necessarily lighter than 1 TeV
- Possibility of observing New Physics, such as:

$t \to hc, hu \to$ relevant for LHC

$h \to bs, bd \to$ relevant for ILC

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Conclusions

- It is possible to have a realistic 2HDM with spontaneous CP violation and controlled SFCNC. The crucial point is the use of a flavoured Z_2 symmetry where the 3rd family is odd and the first two families are even.
- The model predicts the existence of new scalars lighter than 1 TeV

32/(40)

There is no scientific reason to believe in the $\left\{\begin{array}{l}\text{dogma}\\ \text{myth}\end{array}\right.$ that flavour can only be understood at the Planck scale

- FCC, ILC, etc should be constructed