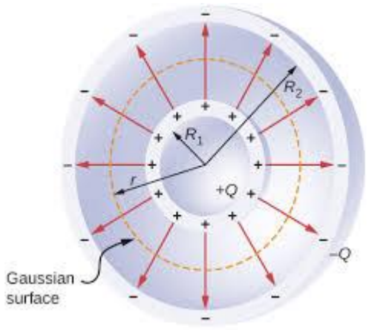
Humboldt Kolleg
Frontiers in Physics: From Electroweak to the Planck Scales
Corfu Summer Institute 18/09/2019

Soft degrees of freedom, Gibbons-Hawking contribution & Casimir effect



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Gravity Euclidean methods

$$\mathrm{Tr}\,\exp(-\beta H) = \int d[\phi]\exp(iI[\phi])\,, \qquad (3.3)$$

where the path integral is now taken over all fields which are periodic with period β in imaginary time. The left-hand side of (3.3) is just the parti-

Gibbons & Hawking 1977, Action integrals and partition functions in QG

What Hilbert space gives this background contribution ?

Can one do the trace on the LHS ?

What is the simplest model ?

NB: Classical black hole thermodynamics comes entirely from background

$$\ln Z = iI[g_0,\phi_0] + \ln\int d[\tilde{g}]\exp(iI_2[\tilde{g}])$$

But the normal thermodynamic argument

$$\ln Z = -WT^{-1}\,,$$

where $W = M - TS - \sum_i \mu_i C_i$ is the "thermodynamic potential" of the system. One can therefore regard $iI[g_0,\phi_0]$ as the contribution of the background to $-WT^{-1}$ and the second as the contributions arising from thermal gravitons

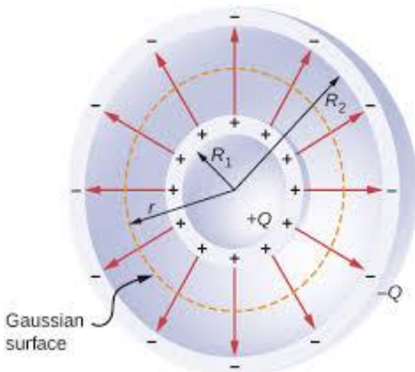
What is the simplest model ?

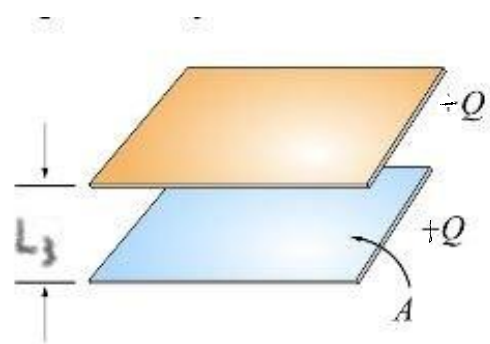
Linear theory

Linearized gravity similar to
electromagnetism

Vacuum capacitor Semi-classical contribution

Charged spherical or planar capacitor





$$R_1 < r < R_2$$

$$+q \qquad -q$$

$$A_0 = -\phi = -\frac{q}{4\pi r}, \quad \pi^i = -\frac{q x^i}{4\pi r^3}$$

$$P_1 : z = 0 \quad P_2 : z = L_3$$

$$+\frac{q}{A} \qquad -\frac{q}{A}$$

$$\phi = -\frac{q}{A} z, \quad \pi^i = -\delta^i_3 \frac{q}{A}$$

$$I = \int d^4x [\dot{A}_i \pi^i - \mathcal{H}_0 + A_0 \partial_i \pi^i]$$

$$\mathcal{H}_0 = \frac{1}{2}(\pi^i \pi_i + B^i B_i)$$

Improved action with boundary term

$$I' = I + \int dt \phi_2 Q - \int dt \phi_1 Q \qquad Q = -\int_S d\sigma_i \, \pi^i_L$$

Hawking & Ross 1995, Duality between electric and magnetic BHs

Vacuum capacitor   Semi-classical contribution

Gibbons & Hawking: contribution to partition function from Euclidean action evaluated at classical saddle point

$$\ln Z(\beta,\mu) = -I'_E + f(\beta) \qquad \mu = \phi_1 - \phi_2$$

$$-I'_E = 2\pi\beta\mu^2 \frac{R_1 R_2}{R_2 - R_1} \qquad -I'_E = \frac{\beta\mu^2 A}{2L_3}$$

Aim : microscopic explanation of semi-classical contribution

$$Z(\beta,\mu) = \mathrm{Tr}\, e^{-\beta(H^{\mathrm{phys}} - \mu Q)}$$

Planar vacuum capacitor Boundary conditions

perfectly conducting boundary conditions
as in Casimir effect

$$\vec{n}\cdot\vec{B} = 0 = \vec{n}\times\vec{E}$$

do not use heat kernel techniques but mode expansion
possible because of very simple geometry

$$x^i = (x^a, x^3) \qquad a = 1,2 \qquad k_i = \frac{\pi n_i}{L_{(i)}}$$

Dirichlet, sines only for x,y components

$$A_c(x^i) = \sum_{n_a}\sum_{n_3>0} A^S_{c,k_a,k_3}\sin k_3x^3 e^{ik_ax^a}, \pi^d(x^i) = \sum_{n_a}\sum_{n_3>0}\pi^{Sd}_{k_a,k_3}\sin k_3x^3 e^{ik_ax^a},$$

Neumann, cosines + zero mode for z components

$$A_3(x^i) = \sum_{n_a}[\underline{A^C_{3,k_a,0}} + \sum_{n_3>0} A^C_{3,k_a,k_3}\cos k_3x^3]e^{ik_ax^a},$$

$$\pi^3(x^i) = \sum_{n_a}[\underline{\pi^{C3}_{k_a,0}} + \sum_{n_3>0}\pi^{C3}_{k_a,k_3}\cos k_3x^3]e^{ik_ax^a}.$$

2d FT !

Planar vacuum capacitor Edge modes/Soft degrees of freedom

Detailed Hamiltonian analysis

Put the mode expansion in the canonical Hamiltonian and perform the constraint analysis after having taken the boundary conditions into account

$$H_c = \int d^3x [\frac{1}{2}\pi^i \pi_i + \frac{1}{4}F_{ij}F^{ij} - A_0 \partial_i \pi^i]$$

$k_3 \neq 0$ modes related to standard black body result (corrected because of Casmir effect at non-zero temperature)

constraints

$$H_W = \frac{V}{2} \sum_{n_a, n_3 > 0} \left[A^S_{0,k_a,k_3} (i k_b \pi^{*S}_{b,k_a,k_3} + k_3 \pi^{*3C}_{b,k_a,k_3}) \right]$$

standard discussion applies, only two independent polarizations

Planar vacuum capacitor   Extra partition function

$k_3 = 0$ are physical : not affected by the constraints nor by gauge transformations

$$A_i^{NPG}(x,y,0) = \delta_i^3 \phi = \partial_i [z\phi], \quad \pi^i_{NPG}(x,y,0) = \delta_3^i \pi = \partial_i [z\pi],$$

not longitudinal because $z \in [0, L_3]$

$k_3 = 0$ modes give extra dynamics of massless scalar in 2+1 dimensions

$$H_{NPG} = \frac{V}{2} \sum_{n_a} \left[\pi^{3C}_{k_a,0} \pi^{3C*}_{k_a,0} + \omega^2_{k_a} A^C_{3,k_a,0} A^{C*}_{3,k_a,0} \right], \quad \omega_{k_a} = \sqrt{k_1^2 + k_2^2}.$$

$$S^{NPG} = \frac{L_3}{2} \int dt \int_{-L_1}^{L_1} dx \int_{-L_2}^{L_2} dy \Big[(\dot{\phi})^2 - \partial_a \phi \partial^a \phi \Big].$$

Electric charge observable $Q = -\pi^{3C}_{0,0,0} A = - \int_{-L_1}^{L_1} dx \int_{-L_2}^{L_2} dy \, \pi, \quad A = 4 L_1 L_2.$

Planar vacuum capacitor   Extra partition function

$k_a \neq 0$ modes give $Z'_{NPG}(\beta) = \mathrm{Tr}\, e^{-\beta \hat{H}'_{NPG}},$

$$\ln Z'_{NPG}(\beta) = f_A(\beta) = \frac{1}{2} b_A \beta^{-2}, \quad b_A = \frac{\zeta(3)}{\pi} A.$$

$k_a = 0$ mode corresponds to free particle

$q = A^C_{3,0,0}\sqrt{V}, p = \pi^{3C}_{3,0,0}\sqrt{V}$

$H^0_{NPG} = \frac{1}{2}p^2, \quad Q = -\sqrt{\frac{A}{L_3}}p.$

turn on chemical potential for electric charge $Z^0_{NPG}(\beta, \mu) = \mathrm{Tr} e^{-\beta \hat{H}^0_{NPG} + \beta\mu\hat{Q}},$

$$\ln Z^0_{NPG}(\beta, \mu) = \ln \Delta q - \frac{1}{2} \ln (2\pi\beta) + \beta\mu^2 \frac{A}{2L_3},$$

reproduces the semi-classical result !

Helmholtz free energy

$$\mathcal{F}(\beta) = -\beta^{-1}\ln Z(\beta) = \mathcal{F}(0) + \mathcal{F}_2(\beta)$$

Zero temperature Casimir energy

$$\mathcal{F}(0) = \frac{1}{2}\sum_{\vec{k}}\hbar\omega_{\vec{k}} = -A\frac{\hbar c\pi^2}{720L_3^3}$$

Thermal contribution

$$\mathcal{F}_2(\beta) = \sum_{\vec{k}}\beta^{-1}\ln\left(1 - e^{-\beta\hbar\omega_{\vec{k}}}\right)$$

Subtraction of empty space BB result

$$= \underbrace{\frac{L^2\pi}{d^2}\Bigg[-\frac{1}{2\beta}\left(\frac{d}{\pi\hbar\beta c}\right)^2}_{\boxed{-\beta^{-1}f_A(\beta)}}\zeta(3) + \sum_{n=1}^{\infty} b(d,T,n) + \overbrace{\frac{2}{\beta}\left(\frac{d}{\pi\hbar\beta c}\right)^3\zeta(4)}\Bigg]$$

Sernelius, Surface modes

$$b(d,T,n) = \frac{1}{2}\frac{1}{\beta}\int_{n^2}^{\infty} ds\ln\left[1 - \exp\left(-\pi\hbar\beta c\sqrt{s}/d\right)\right]$$

Summary

Main claim

microstates responsible for BH entropy related
to non-proper gauge (=soft) DoF
rather than physical gravitons

What the physical DoF freedom are can only
be decided after taking the boundary
conditions (or the topology) into account, not
before

Further arguments

- Gauge sector of electromagnetism as topological field theory
- Quantum Coulomb solution as coherent state of unphysical photons
- linearized Schwarzschild solution involves temporal/longitudinal DoF
- no physical gravitons in 3d but BTZ black hole
- observables = ADM surface charges involve unphysical DoF

Electromagnetism — DoF & reduced quantization

Hamiltonian formulation

$$S = \int dt \left[\int d^3x \, (\dot{A}_\mu \pi^\mu - \lambda_1 \pi^0 + A_0 \partial_i \pi^i) - H_0 \right]$$

$$H_0 = \int d^3x \, (\frac{1}{2}\pi^i \pi_i + \frac{1}{4} F^{ij} F_{ij}) \qquad \pi^i = -E^i$$

first class constraints $\pi^0 = 0 = \partial_i \pi^i$

physical DoF (A_i^T, π_T^i)

unphysical DoF $(A_i^L, \pi_L^i), \quad (A_0, \pi^0)$

Quantization

reduced phase space: transverse DoF
in positive definite Hilbert space

with charged sources: quantize transverse
fluctuations around classical charged solution

Electromagnetism　　　　Ghosts

quantize all polarizations in indefinite metric Hilbert space

a) Gupta-Bleuler $$[a_\mu(\vec{k}), a_\nu^\dagger(\vec{k}')] = \eta_{\mu\nu}\delta^{(3)}(\vec{k}-\vec{k}')$$

physical state condition $$(\partial_\mu A^\mu)^+|\psi\rangle^{\rm phys} = 0$$ additional (null) states decouple

b) BRST quantization $$(\mathcal{P}, C), \quad (\bar{C}, \rho)$$

spurious fermionic DoF

cancel contributions from longitudinal and temporal photons

BRST charge $$\Omega = \int d^3x \, (\pi^0\rho - \partial_i\pi^i C)$$

gauge fixation $$K = \int d^3x(\bar{C}\partial_k A^k + \mathcal{P}A_0 - \frac{1}{2}\bar{C}\pi^0),$$

path integral $$\int \mathcal{D}(A_\mu\pi^\mu C\mathcal{P}\rho\bar{C})e^{iS^H_{\rm BRST}}$$

$$S^H_{\rm BRST} = \int dt[\int d^3x \, (\dot{A}_\mu\pi^\mu + \dot{C}\mathcal{P} + \dot{\bar{C}}\rho) - H_0 - \{\Omega, K\}]$$

Electromagnetism Topological sector

"physical" sector (A_i^T, π_T^i) $$H^{\rm ph} = \int d^3x \, \frac{1}{2}(\pi_T^i \pi_i^T - A_i^T \Delta A_T^i),$$

topological sector $$(A_i^L = \partial_i A, \pi_i^L = \frac{1}{\Delta}\partial^i \pi), (A_0 \pi^0)$$

$$H^{\rm gs} = \int d^3x \, \mathcal{H}^{\rm gs} = -\frac{1}{2} i \{\Omega, \bar{\Omega}\}, \quad \bar{\Omega} = 2iK - i\int d^3x \, \mathcal{P}\frac{1}{\Delta}\pi$$

contains electric charge observable

Electromagnetism BRST quantization

expand all fields in terms of oscillators

physical transverse $[a_a(\vec{k}), a_b^\dagger(\vec{k}')] = \delta_{ab}\delta^{(3)}(\vec{k}-\vec{k}'), \quad a,b=1,2$

unphysical bosonic (null) $[a(\vec{k}), b^\dagger(\vec{k}')] = \delta^{(3)}(\vec{k}-\vec{k}')$

unphysical ghost $[c(\vec{k}), \bar{c}^\dagger(\vec{k}')] = \delta^{(3)}(\vec{k}-\vec{k}')$

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \longrightarrow \eta_{\mu\nu} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

BRST charge $\Omega = \int d^3k \, (c^\dagger a + a^\dagger c)$

physical states $\Omega|\psi\rangle^{\text{phys}} = 0$ BRST exact states decouple ${}^{\text{phys}}\langle\psi|\Omega|\chi\rangle = 0$

in particular $\Omega|0\rangle = 0$ vacuum state is physical

Hamiltonian $H_0 + [\Omega, K] = \int d^3k \, \omega_{\vec{k}} a_a^\dagger a^a + [\Omega, K']$

responsible for black body entropy

ghosts and unphysical bosonic DoF drop out

$(\mathrm{s})\mathrm{Tr}\, e^{-\beta[\Omega, K']} = 0$

Gauge sector always trivial ? No: cf. topological field theories

Electromagnetism

Coupling to a source

static charge at the origin $$S_T = S^{EM} - \int d^4x \, j^\mu A_\mu, \quad j^\mu = \delta^\mu_0 Q \delta^{(3)}(x)$$

only Gauss law modified $$\partial_i \pi^i = j^0$$

modified BRST charge $$\Omega^Q = \int d^3k \, [c^\dagger (a - q_{\vec{k}}) + (a^\dagger - q_{\vec{k}})c]$$

c-number $$q_{\vec{k}} = \frac{Q}{\sqrt{2}(2\pi)^{3/2}\omega_{\vec{k}}^{3/2}}$$ Fourier transform of j^0

old vacuum no longer physical $$\Omega^Q |0\rangle \neq 0$$

new vacuum $$(a - q_{\vec{k}})|0\rangle^Q = 0 \qquad (b, c, \bar{c}, a_a)|0\rangle^Q = 0 \quad \longrightarrow \quad \Omega^Q |0\rangle^Q = 0$$

in terms of old vacuum $$|0\rangle^Q = e^{\int d^3k \, q_{\vec{k}} b^\dagger(\vec{k})} |0\rangle$$

coherent state of null photons

Electromagnetism   Quantum Coulomb solution

$$[b(\vec{k}), b^\dagger(\vec{k}')] = 0 \longrightarrow$$

unusual ‘classical’ properties $\; {}^Q\langle 0|0\rangle^Q = 1$

instead of $\; \langle\alpha|\beta\rangle = e^{\alpha^*\beta}$

Ehrenfest theorem

$${}^Q\langle 0|\vec{E}(x)|0\rangle^Q = -\frac{Q\vec{x}}{4\pi r^3} \qquad {}^Q\langle 0|\vec{\nabla}\times\vec{A}(x)|0\rangle^Q = 0$$

NB: requires infrared regularisation $\; FT(\frac{1}{k^2+\nu^2}) = \frac{e^{-\nu r}}{4\pi r}, \quad \nu \to 0$

interpretation : extrapolation of Aharonov-Bohm
effect to quantized electromagnetic field

Dirac 1932
Fock & Podolski 1932
Bronstein 1936
GB 2010

32-3*

On Dirac's Quantum Electrodynamics

V. Fock and B. Podolsky

Phys. Zs. Sowjetunion **1**, 798, 1932 (in English)
Fock57, pp. 52–54

In his new paper,[1] Dirac suggested an original combination of the quantum electrodynamics of vacuum with the wave equation for matter. For a one-dimensional example, he demonstrated how the Coulomb interaction can appear in some approximation.

$$W\psi_2 - i\hbar\frac{\partial\psi_2}{\partial t} = \left(K - \frac{\varepsilon_1\varepsilon_2}{4\pi|\mathbf{r}_1 - \mathbf{r}_2|}\right)\psi_0 = -U\psi_0.$$

[2]We are looking for that part of the wave functional φ (or ψ) that corresponds to the zero-quantum state. (This is denoted symbolically by the factor δ_{j0}, where j is the number of light quanta.) Only the retained operator $a_0^\dagger a_0$ sends this zero-quantum part again to a zero-quantum one, whereas the removed operators $a_0 a_0^\dagger$, $a_0^\dagger a_0^\dagger$ and $a_0 a_0$ either cancel it or transform it into a two-quantum state. (*V. Fock*)

Electromagnetism Quantum Coulomb solution

QUANTUM THEORY OF WEAK GRAVITATIONAL FIELDS[1]

By M. Bronstein.

(Received on 2. January 1936)

§1. General remarks. §2. Hamiltonian form and plane waves. §3. Commutation relations and eigenvalues of the energy. §4. Let us undertake a little gedanken experiment! §5. Interaction with matter. §6. Energy transfer by gravitational waves. §7. Deduction of Newton's law of gravitation.

while in our case another commutation relation applies, namely (cf. (8))

$$[h^{+}_{00,\mathfrak{k}}, h_{00,\mathfrak{k}'}] = -\frac{h}{2\omega}\delta(\mathfrak{k}-\mathfrak{k}').$$

Neither commutation relations are introduced ad hoc, but originated quite naturally from the general quantum-mechanical formalism. As we shall see this suffices to obtain the correct sign of the gravitational interactions. Thus, the fundamental difference between Coulomb and Newtonian forces is explained from quantum mechanics.

Following the idea of Dirac, Fock and Podolsky[7] derived Coulomb's law. Our calculation proceeds exactly parallel to theirs. We start from the equations

$$\left(\frac{1}{2m_1}\mathfrak{p}_1{}^2 + \frac{m_1}{2}h_{00}(\mathfrak{r}_1)\right)\psi + \frac{h}{i}\frac{\partial\psi}{\partial t_1} = 0,$$
$$\left(\frac{1}{2m_2}\mathfrak{p}_2{}^2 + \frac{m_2}{2}h_{00}(\mathfrak{r}_2)\right)\psi + \frac{h}{i}\frac{\partial\psi}{\partial t_2} = 0.$$

The sign of the right-hand side is different than in the Fock - Podolsky formula (42). When we go back to the configuration space we accordingly obtain the Schrödinger equation with the potential energy

$$-\frac{m_1 m_2}{16\pi|\mathfrak{r}_1-\mathfrak{r}_2|},$$

and thus we have recovered Newtonian gravitation as a necessary consequence of the quantum theory of gravity.

The Physical-Technical Institute
and The Physical Institute of the University.
Leningrad, August 1935.

Gravity DoF

linearized GR = massless spin 2 gauge field on Minkowski background

Hamiltonian formulation $S_{PF}[h_{mn},\pi^{mn},n_m,n]=\int dt\Big[\int d^3x\,(\pi^{mn}\dot h_{mn}-n^m\mathcal{H}_m-n\mathcal{H})-H_{PF}\Big],$

$$H_{PF}[h_{mn},\pi^{mn}]=\int d^3x\big(\pi^{mn}\pi_{mn}-\frac{1}{2}\pi^2+\frac{1}{4}\partial^r h^{mn}\partial_r h_{mn}-\\ -\frac{1}{2}\partial_m h^{mn}\partial^r h_{rn}+\frac{1}{2}\partial^m h\partial^n h_{mn}-\frac{1}{4}\partial^m h\partial_m h\big)$$

$$h_{00}=-2n \quad h_{0i}=n_i$$

$$\mathcal{H}_m=-2\partial^n\pi_{mn},\quad \mathcal{H}_\perp=\Delta h-\partial^m\partial^n h_{mn}$$

orthogonal decomposition of symmetric rank 2 tensor

	# of comp D=4	D=3
$\phi_{mn}=\phi^{TT}_{mn}+\phi^{T}_{mn}+\phi^{L}_{mn},$	6	3
$\phi^{L}_{mn}=\partial_m\psi_n+\partial_n\psi_m,$	3	2
$\phi^{T}_{mn}=\frac{1}{2}\left(\delta_{mn}\Delta-\partial_m\partial_n\right)\psi^T$	1	1
$\phi^{TT}_{mn}=\phi_{mn}-\phi^{L}_{mn}-\phi^{T}_{mn}$	2	0

Physical DoF: gravitational waves

ADM 1962, Dynamics of GR, gr-qc/0405109

Gravity   Linearized Schwarzschild solution

canonical pairs $\quad \big(h^{TT}_{mn}(x),\, \pi^{kl}_{TT}(\vec{y})\big), \quad \big(h^{L}_{mn}(\vec{x}),\, \pi^{kl}_{L}(y)\big), \quad \big(h^{T}_{mn}(x),\, \pi^{kl}_{T}(y)\big).$

$$\mathcal{H}_m = 0 = \mathcal{H} \iff \pi^{kl}_L = 0 = h^T_{mn}$$

$$H^R = 0$$ D=3

$$H^R = \int d^3x \left(\pi^{mn}_{TT}\pi^{TT}_{mn} + \frac{1}{4}\partial_r h^{TT}_{mn}\partial^r h^{mn}_{TT}\right).$$ D=4

coupling to a massive particle at rest

$$S_T = \frac{1}{16\pi} S_{PF} + \int d^4x\, h_{\mu\nu} T^{\mu\nu} \qquad T^{\mu\nu} = \delta^\mu_0 \delta^\nu_0 M \delta^{(3)}(x)$$

only Hamiltonian constraint is affected

$$\mathcal{H}_\perp = -16\pi M \delta^{(3)}(x) \iff \begin{cases} h^T_{mn} = M\frac{x_m x_n}{r^3} \\ n = -\frac{M}{r} \end{cases}$$

all other variables 0

after spatial diffeo $\quad h_{rr} = \dfrac{2M}{r} = h_{00}$

linearized Schwarzschild solution, no TT variables involved

Quantum version in linearized gravity:
M. Bronstein (1936)

Gravity	Observable

observable ADM mass only sees h^T_{mn}

surface charge

$$16\pi P^{\perp} = \oint\limits_{S^\infty} d\sigma_m \left(\partial_n h_T^{mn} - \partial^m h_T\right) \approx M$$

related to G&H boundary term that gives non trivial value for Euclidean action

exactly like for electric charge

$$Q = -\oint\limits_{S} d\sigma_m \pi_L^m$$

Summary

Main claims

(i) Interesting edge dynamics in electromagnetism
(ii) Related to non-proper gauge degrees of freedom

(Justified ?) hope: extendable to gravity to understand microstates for black hole entropy