Beyond the Standard Model
(a personal view)

G. Ross, CORFU Summer Institute, Sept 2019

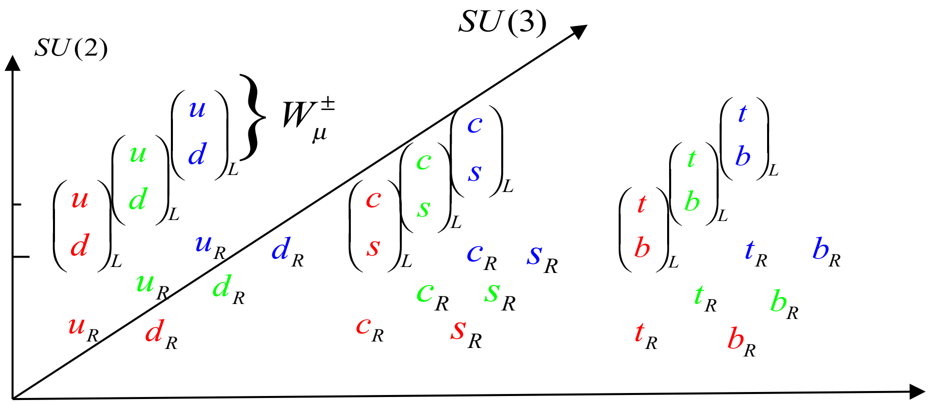
The Standard Model – unanswered questions

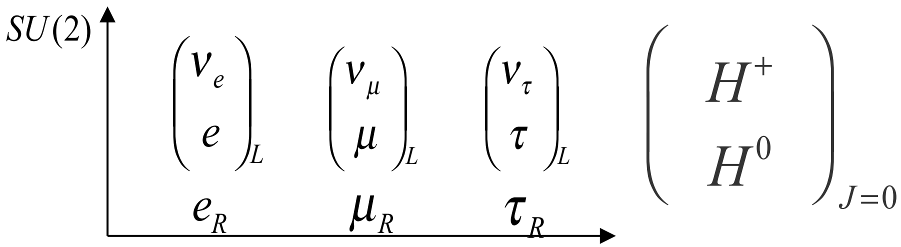
- SM structure
  - Complicated choice of multiplets
  - Fractional and integral charges?
  - Only partial unification
  - The hierarchy problem $\Omega_\Lambda^{1/4} \sim 10^{-3} eV,\ m_H \sim 10^{11} eV \ll M_{\text{Planck}} \sim 10^{27} eV$
- SM unknowns
  - Many parameters 19 (28) $g_i, m_i, \theta_i, \delta_i, M_{W,H}, \theta_{QCD}$
  - Neutrino masses?
  - Strong CP problem
- Dark matter, baryogenesis, inflation.....

- SM structure

Unification incomplete:

$$SU(3)\times SU(2)\times U(1)$$



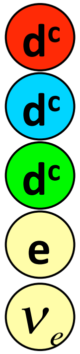


Gravity?

Grand Unification

$SO(10) \supset SU(5) \supset SU(3) \otimes SU(2) \otimes U(1)$

$(\bar{5})_L$:



$SU(3)$

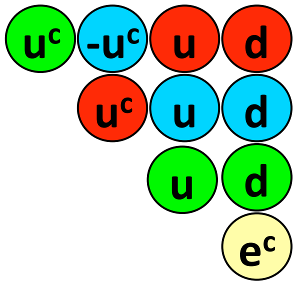
$SU(2)$

$3Q_{d^c} + Q_{e^-} = 0$

$Q_{d^c} = 1/3$ ✔

$(\bar{3},1)+(1,2)_L \times 3$ ✔

$(10)_L$:



$(\bar{3},1)+(3,2)_L+(1,1) \times 3$ ✔

$\nu_{e,L}{}^c \equiv \nu_{e,R}$

$(16)_L = (10)_L + (\bar{5})_L + (1)_L$

Higgs $\bar{5}:(1,2)+(3,1)$ $M_T > 10^{12}\,GeV$ Doublet-triplet splitting problem

Spontaneous symmetry breaking

$$SU(5)\xrightarrow[\Sigma_{24}]{M_X} SU(3)\times SU(2)\times U(1)\xrightarrow[H_{\bar{5}}]{M_W} SU(3)\times U(1)$$

$$\langle\Sigma\rangle = \mathrm{v}_3\mathrm{Diagonal}(2,2,2,-3,-3)$$

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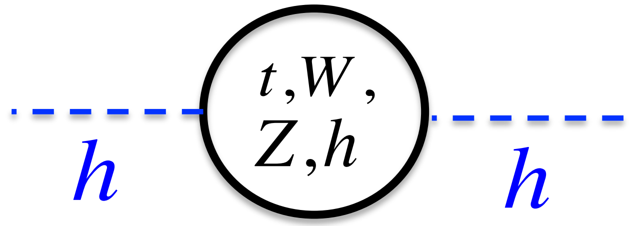
Leptoquark interactions lead to nucleon decay

$$\Rightarrow M_X > 10^{16}\,GeV$$

.... the hierarchy problem

The SM hierarchy problem

$$\delta m_h^2 = \frac{3G_F}{4\sqrt{2}\pi^2}\left(4m_t^2 - 2m_W^2 - m_Z^2 - m_h^2\right)\Lambda^2$$



but….

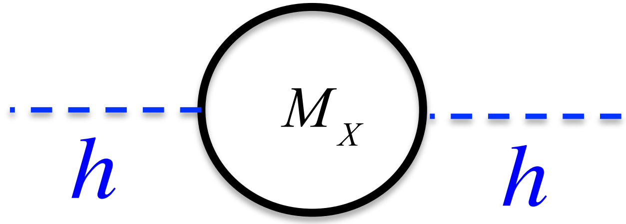
δm^2 not measureable …only $m^2 = m_0^2 + \delta m_h^2$ "physical"

$$\frac{d\, m_h^2}{d \ln \mu} = \frac{3m_h^2}{8\pi^2}\left(2\lambda + y_t^2 - \frac{3g_2^2}{4} - \frac{3g_1^2}{20}\right) + \ldots$$ ✔

"SM doesn't have an hierarchy problem"

The real hierarchy problem

$$\delta m_h^2 \propto M_X^2 \ln\left(\frac{Q^2+M_X^2}{\Lambda^2}\right)$$



$$m^2(Q^2=\Lambda^2)=m_0^2+\delta m^2=0 \quad \not\Rightarrow \quad m^2(0)\approx 0$$

(M_X large)

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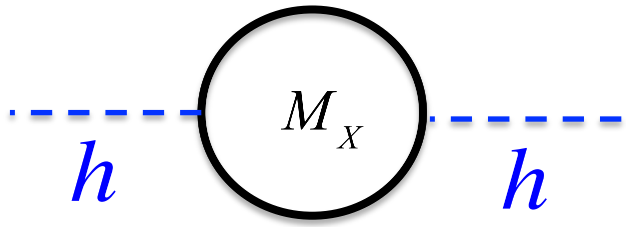


$$m^2(Q^2=\Lambda^2)=m_0^2+\delta m^2=0 \not\Rightarrow m^2(0)\approx 0$$

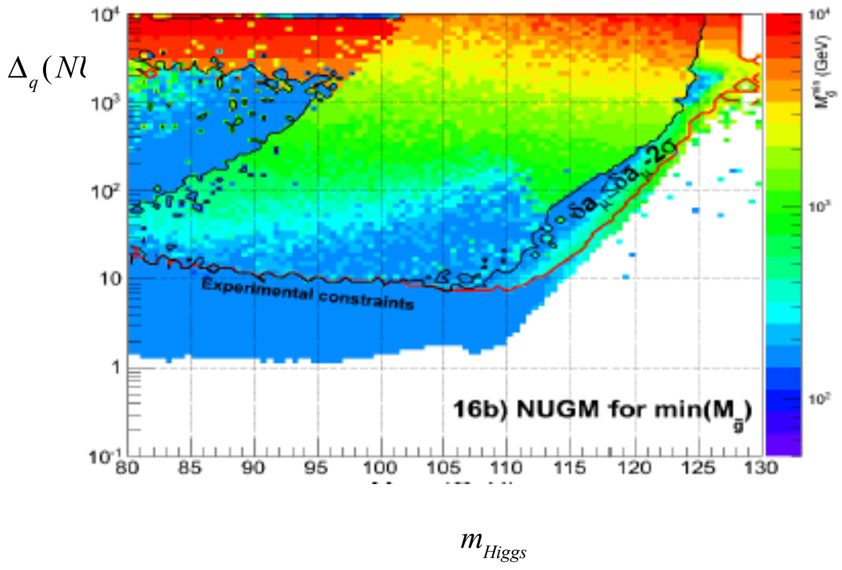
(M_X large)

$\Rightarrow$ GUTs need SUSY

$$\delta m_h^2 \to M_{SUSY}^2 \ln\left(\frac{Q^2+M_{SUSY}^2}{\Lambda^2}\right)$$

(little hierarchy problem)

SUSY fine tuning



$$\Delta_q = \left(\sum \Delta^2_{\gamma_i}\right)^{1/2}, \quad \Delta_{\gamma_i} = \frac{\partial \ln v}{\partial \ln \gamma_i}, \quad \gamma_i = m_0, m_{1/2}, \mu_0, A_0, B_0.$$

Analysis of various models :

CMSSM, NUHM1, NUHM2, NUGM, NUGM2

$$\Delta > 500, m_{Higgs} = 125GeV$$

$$\Delta > 200, m_{Higgs} = 123GeV$$

Ghilencea, Lee, Park

The (SUSY) little hierarchy problem

(talk by Kim)

- Accept some fine tuning $\left(\Delta = 100 \quad \Rightarrow \quad \delta\chi^2 \,|_{d.f.} \sim 1\right)$

  (or new structure e.g.NMSSM...)

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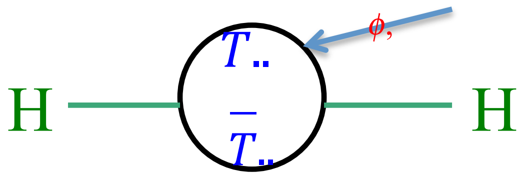
- Further symmetry protection

Light top partners

Nambu - Goldstone



$\rightarrow$ (SUSY) UV completion

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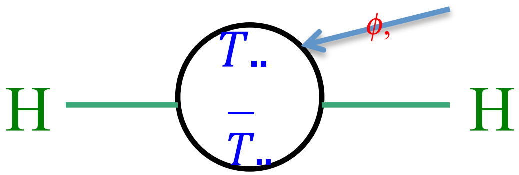
Nambu - Goldstone 

$\longrightarrow$ (SUSY) UV completion

$$-\frac{\phi}{4f}\left(g_2^2 W\widetilde{W} - g_1^2 B\widetilde{B}\right)$$

- Scanning....e.g. relaxion, ϕ, slowed by emission of SM gauge bosons when it is kinematically allowed. Minimal particle extension. sub Plankian vevs, $M_{SUSY} \rightarrow 200\ TeV$

Servant et al

$$V(\phi, h) = \Lambda^4 - g\Lambda^3\phi + \frac{1}{2}\left(-\Lambda^2 + g'\Lambda\phi\right)h^2 + \frac{\lambda}{4}h^4 + \Lambda_b^4\cos\left(\frac{\phi}{f'}\right)$$

The (SUSY) little hierarchy problem

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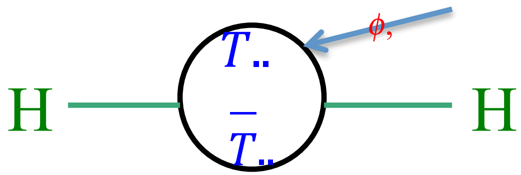
  Nambu - Goldstone 

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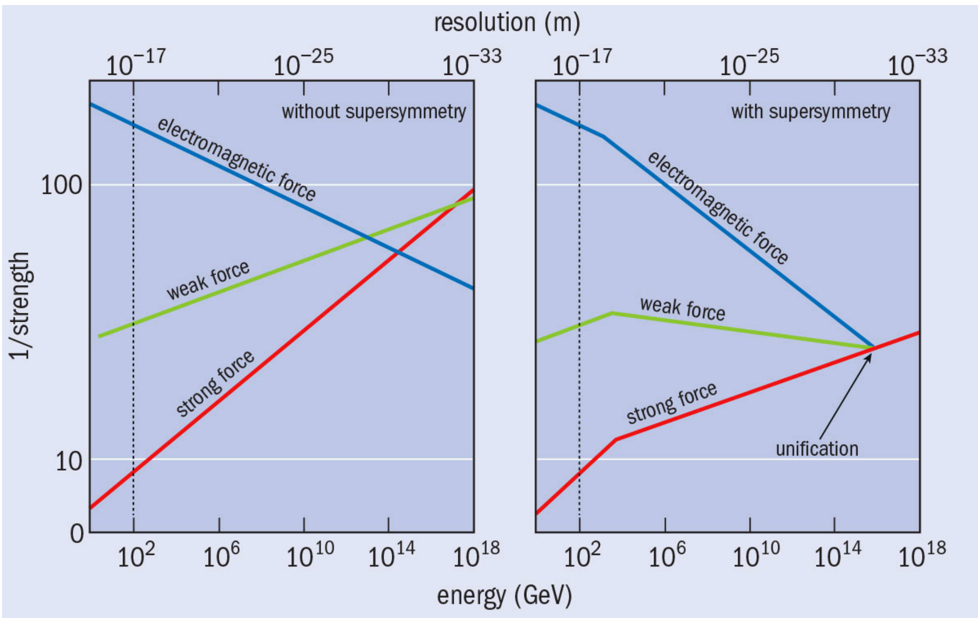
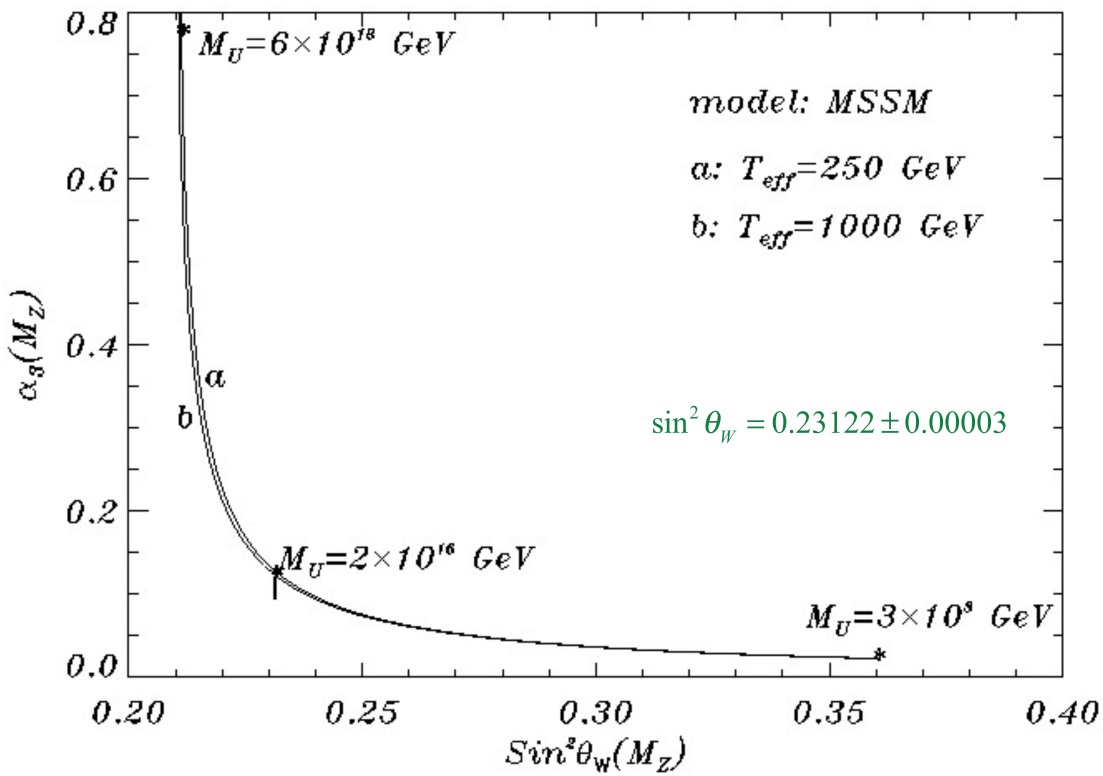
- Unification of gauge couplings :



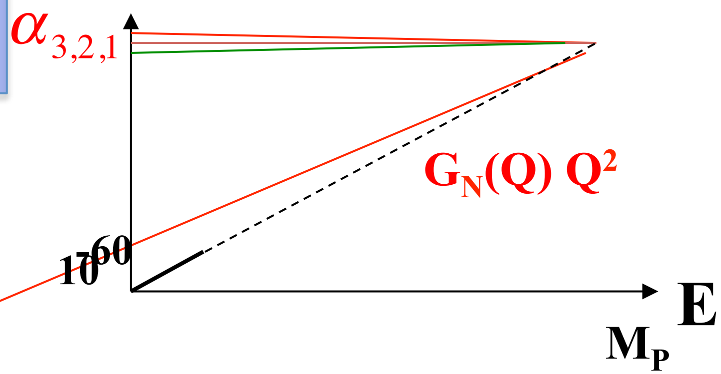
SUSY gauge coupling unification

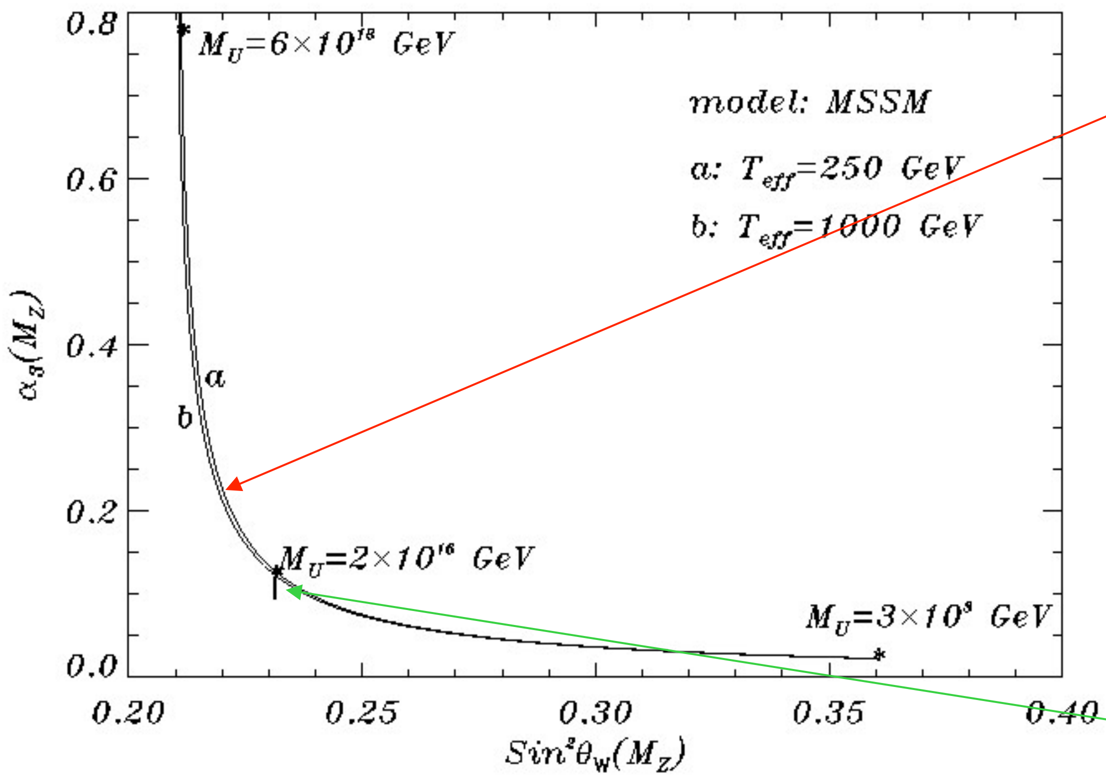


$\alpha_s = 0.126$ c.f. $\alpha_s = 0.118 \pm 0.001 \quad (Expt)$

SUSY gauge coupling unification

Unification with gravity?





$$M_U = (2.6 \pm 2).10^{16}\,GeV$$

$\alpha_s = 0.126$ c.f. $\alpha_s = 0.118 \pm 0.001$ $(Expt)$

Gauge unification - Heterotic String

$$L_{eff}^{HS} = \int d^{10}x\sqrt{g}e^{-\phi}\left(\frac{4}{\alpha'^4}R + \frac{k_i}{\alpha'^3}TrF_i^2 + \ldots\right)$$

$k_i \rightarrow 1$

$\int d^{10}x \rightarrow \int d^4x\, V$ $\qquad e^{-\phi} \rightarrow \alpha_{10}^{-1}$

$\alpha' = 1/M_{string}^2$ only scale

$$G_N = \frac{\alpha_{10}\alpha'^4}{64\pi V}, \quad \alpha_{String} = \frac{\alpha_{10}\alpha'^3}{16\pi V} \quad \Rightarrow \quad G_N = \frac{\alpha_{String}\alpha'}{4}$$

$$\frac{1}{g_i^2(M_Z)} = \frac{k_i}{g_{string}^2} + b_i \ln\left(\frac{M_{string}}{M_Z}\right) + \Delta_i$$

$$M_{string} = g_{string}.M_{Planck} = 3.6\times 10^{17}\,GeV \qquad c.f.\, M_U^{\text{"expt"}} = (2.6\pm 2).10^{16}\,GeV$$

$$\alpha_s = 0.126 \qquad c.f. \quad \alpha_s = 0.118 \pm 0.001 \quad (Expt)$$

$$M_{string} = 3.6 \times 10^{17}\, GeV \quad c.f. \quad M_U^{"expt"} = (2.6 \pm 2).10^{16}\, GeV$$

GUT scale uncertainties?

c.f. talk by Kazuki Sakurai

Doublet -triplet splitting

Higher dimensions (String unification)

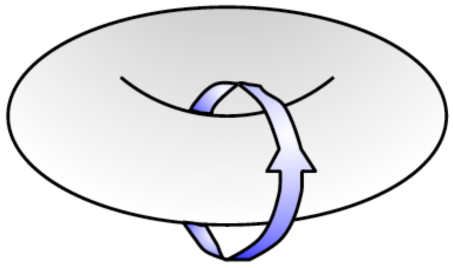
Compactification: $K = K_0 / H$

freely acting discrete group

Wilson line breaking: $W: \quad \overline{H} \subset G$

embedding of H into gauge group G

Massless states: $H \otimes \overline{H}$ singlets



$$W = P\exp\left(-i\int_\gamma T^a A_m^a\, dx^m\right)$$

Breit, Ovrut, Segre

$$e.g. \quad SU(5): \quad H = Z_3, \quad \overline{H} = Diag(\alpha,\alpha,\alpha,1,1), \quad \alpha = e^{2i\pi/3}$$

$$\left(H \otimes \overline{H}\right): \quad (1 \otimes \overline{5}) \to \begin{pmatrix} H^- \\ \overline{H}^0 \end{pmatrix}_1, \quad \left(3,\overline{5}\right) \to \begin{pmatrix} e \\ \nu_e \end{pmatrix}_1 \oplus \begin{pmatrix} d^c \\ d^c \\ d^c \end{pmatrix}_{\alpha^2}, \quad \text{Matter} \quad \to \quad \left(3,\overline{5}+10\right)$$

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	$n=1$ $n_H=0$	$n=1$ $n_H=2$	$n=2$ $n_H=0$	$n=2$ $n_H=2$
$\frac{M_X}{M_X^0}$	5.8	4.4	4.5	3.4
$\Delta\alpha_3^{-1}$	-0.33	0.47	-0.29	0.52
$\frac{M_X}{M_X^0}$	4.7	6.0	3.7	4.8
$\Delta \sin^2\theta$	0.0011	-0.0017	0.001	-0.0018

Table 1: The change in the string predictions for $M_s R = 2$. The calculation refers to the 1D case, eq(5), and is very close to the 2D case with $\rho_1 = \rho$ and $\rho_2 = 0$. The Wilson line group element, eq(3), is specified by $a = e^{i2\pi n/3}$. The columns with $n_H = 0$ have no Higgs KK excitations while the columns with $n_H = 2$ have KK excitations for both Higgs multiplets. Finally the first two rows are obtained using α_{EM} and $sin^2\theta$ as input while the last two rows are obtained using α_{EM} and α_3 as input.

$\alpha_s = 0.126$ c.f. $\alpha_s = 0.118 \pm 0.001 \quad (Expt)$

$$\Delta\alpha_s^{-1} \equiv \alpha_s^{-1} - \alpha_s^{-1} = -0.54$$

- **SM unknowns** - masses and mixing angles :

$$g_i, m_i, \theta_i, \delta_i, M_{W,H}, \theta_{QCD}$$ (+9 in neutrino sector)

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IR/UV stable fixed points

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⇐

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The data for quarks is *consistent* with a very symmetric structure :

$$\frac{M^{d,u}}{m^{b,t}} \simeq \begin{pmatrix} <\varepsilon^4 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^3 & a\varepsilon^2 & a\varepsilon^2 \\ \varepsilon^3 & a\varepsilon^2 & 1 \end{pmatrix}$$

$$\varepsilon^{d} = 0.15, \quad a^{d} = 1$$

$$\varepsilon^{u} = 0.05, \quad a^{u} = 1$$

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$\not{C}P$ SM phase?

$$|V_{us}| = \left| \sqrt{\frac{m_d}{m_s}} - e^{i\delta} \sqrt{\frac{m_u}{m_c}} \right|$$

$$0.217 - 0.222 \quad c.f. \ |(0.216 - 0.214) - (0.07 - 0.076) e^{i\delta}|$$

$$= 0.213 - 0.223, \ \delta = 90^0$$

Gatto, Sartori, Tonin

$q \leftrightarrow l$ symmetry?

Charged leptons are consistent with a similar form

$$\frac{M^{d,l,u}}{m^{b,\tau,t}} \simeq \begin{pmatrix} <\varepsilon^4 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^3 & a\varepsilon^2 & a\varepsilon^2 \\ \varepsilon^3 & a\varepsilon^2 & 1 \end{pmatrix}$$

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GUT relations

Georgi-Jarlskog

e.g. $SU(4)\times SU(2)_L \times SU(2)_R \subset SO(10)$, $\quad (1,2,2), (15,2,2) \subset 120$

$$\psi_\alpha = \begin{pmatrix} d \\ d \\ d \\ l \end{pmatrix}$$

$$\overline{\psi}^\alpha \psi_\alpha$$

$$\frac{m_b}{m_\tau}(M_X) = 1$$

$$\frac{M^{d,l}}{m_3} = \begin{pmatrix} <\varepsilon^4 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^3 & a\varepsilon^2 & a\varepsilon^2 \\ \varepsilon^3 & a\varepsilon^2 & 1 \end{pmatrix}$$

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$$\overline{\psi}^\alpha \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -3 \end{pmatrix} \psi_\alpha$$

$$\frac{m_s}{m_\mu}(M_X) = \frac{1}{3}$$

$$\frac{M^{d,l}}{m_3} = \begin{pmatrix} < \varepsilon^4 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^3 & a\varepsilon^2 & a\varepsilon^2 \\ \varepsilon^3 & a\varepsilon^2 & 1 \end{pmatrix}$$

$\varepsilon^d = 0.15$, $a^d = 1$

$\varepsilon^l = 0.15$, $a^l = -3$

GUT relations

Georgi-Jarlskog $\left(L^5\right)_{33+12+21} + \left(L^{45}\right)_{22}$

$Det(M^l) = Det(M^d)|_{M_X}$ (Texture zero)

$\frac{m_b}{m_\tau}(M_X) = 1$

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$$\frac{M^{d,l}}{m_3} = \begin{pmatrix} \sim 0 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^3 & a\varepsilon^2 & a\varepsilon^2 \\ \varepsilon^3 & a\varepsilon^2 & 1 \end{pmatrix}$$

$\varepsilon^d = 0.15, \ a^d = 1$

$\varepsilon^l = 0.15, \ a^l = -3$

$m_b = 3 m_\tau$ ✔

$m_s = 3 . \frac{1}{3} . m_\mu$ ✔

$m_d = 3 . 3 . m_e$ ✔

Parameters	Input SUSY Parameters					
$\tan\beta$	1.3	10	38	50	38	38
γ_b	0	0	0	0	-0.22	$+0.22$
γ_d	0	0	0	0	-0.21	$+0.21$
γ_t	0	0	0	0	0	-0.44
Parameters	**Corresponding GUT-Scale Parameters with Propagated Uncertainty**					
$y^t(M_X)$	6^{+1}_{-5}	0.48(2)	0.49(2)	0.51(3)	0.51(2)	0.51(2)
$y^b(M_X)$	$0.0113^{+0.0002}_{-0.01}$	0.051(2)	0.23(1)	0.37(2)	0.34(3)	0.34(3)
$y^\tau(M_X)$	0.0114(3)	0.070(3)	0.32(2)	0.51(4)	0.34(2)	0.34(2)
$(m_u/m_c)(M_X)$	0.0027(6)	0.0027(6)	0.0027(6)	0.0027(6)	0.0026(6)	0.0026(6)
$(m_d/m_s)(M_X)$	0.051(7)	0.051(7)	0.051(7)	0.051(7)	0.051(7)	0.051(7)
$(m_e/m_\mu)(M_X)$	0.0048(2)	0.0048(2)	0.0048(2)	0.0048(2)	0.0048(2)	0.0048(2)
$(m_c/m_t)(M_X)$	$0.0009^{+0.001}_{-0.00006}$	0.0025(2)	0.0024(2)	0.0023(2)	0.0023(2)	0.0023(2)
$(m_s/m_b)(M_X)$	0.014(4)	0.019(2)	0.017(2)	0.016(2)	0.018(2)	0.010(2)
$(m_\mu/m_\tau)(M_X)$	0.059(2)	0.059(2)	0.054(2)	0.050(2)	0.054(2)	0.054(2)
$A(M_X)$	$0.56^{+0.34}_{-0.01}$	0.77(2)	0.75(2)	0.72(2)	0.73(3)	0.46(3)
$\lambda(M_X)$	0.227(1)	0.227(1)	0.227(1)	0.227(1)	0.227(1)	0.227(1)
$\bar\rho(M_X)$	0.22(6)	0.22(6)	0.22(6)	0.22(6)	0.22(6)	0.22(6)
$\bar\eta(M_X)$	0.33(4)	0.33(4)	0.33(4)	0.33(4)	0.33(4)	0.33(4)
$J(M_X)\times 10^{-5}$	$1.4^{+2.2}_{-0.2}$	2.6(4)	2.5(4)	2.3(4)	2.3(4)	1.0(2)
Parameters	**Comparison with GUT Mass Ratios**					
$(m_b/m_\tau)(M_X)$	$1.00^{+0.04}_{-0.4}$	0.73(3)	0.73(3)	0.73(4)	1.00(4)	1.00(4)
$(3m_s/m_\mu)(M_X)$	$0.70^{+0.8}_{-0.05}$	0.69(8)	0.69(8)	0.69(8)	0.9(1)	0.6(1)
$(m_d/3\,m_e)(M_X)$	0.82(7)	0.83(7)	0.83(7)	0.83(7)	1.05(8)	0.68(6)
$(\frac{\det Y^d}{\det Y^e})(M_X)$	$0.57^{+0.08}_{-0.26}$	0.42(7)	0.42(7)	0.42(7)	0.92(14)	0.39(7)

Table 2: The mass parameters continued to the GUT-scale M_X for various values of $\tan\beta$ and threshold corrections $\gamma_{t,b,d}$. These are calculated with the 2-loop gauge coupling and 2-loop Yukawa coupling RG equations assuming an effective SUSY scale $M_S = 500$ GeV.

GGR, M. Serna, 0704.1248

Neutrinos ???

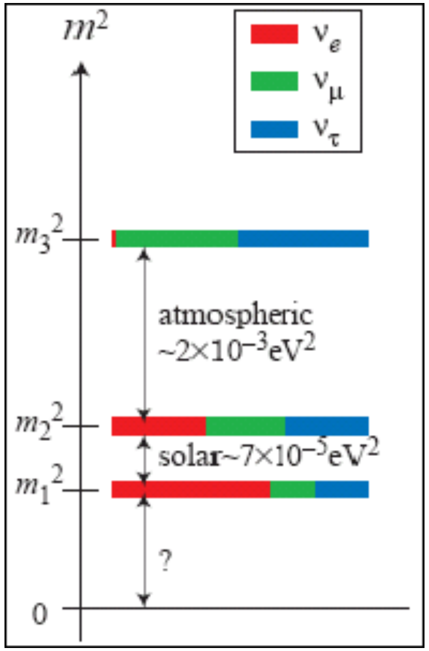
c.f. Steve King's talk

Neutrinos ???

Tri-bi mmaximal mixing

$$L^{\nu}_{eff} \sim m_3\, \bar{\phi}^{i}_{23}\nu_i \bar{\phi}^{j}_{23}\nu_j + m_2\, \bar{\phi}^{i}_{123}\nu_i \bar{\phi}^{j}_{123}\nu_j$$

$$<\bar{\phi}_{23}>^i = (0,1,-1), \quad <\bar{\phi}_{123}>^i = (1,1,1)$$



Can one have a unified description of quark, charged lepton and neutrinos?

$$c.f.\ \ L^{q,l}_{Dirac} = m_{t,b,l}\, \bar{\phi}^{i}_{3}\psi_i \bar{\phi}^{j}_{3}\psi^c_j + \ldots \qquad <\bar{\phi}_3>^i = (0,0,1) \quad ???$$

Neutrinos ???

$$L_{eff}^{\nu} = m_3\, \bar{\phi}_{23}^{i} \nu_i \bar{\phi}_{23}^{j} \nu_j + m_2\, \bar{\phi}_{123}^{i} \nu_i \bar{\phi}_{123}^{j} \nu_j$$

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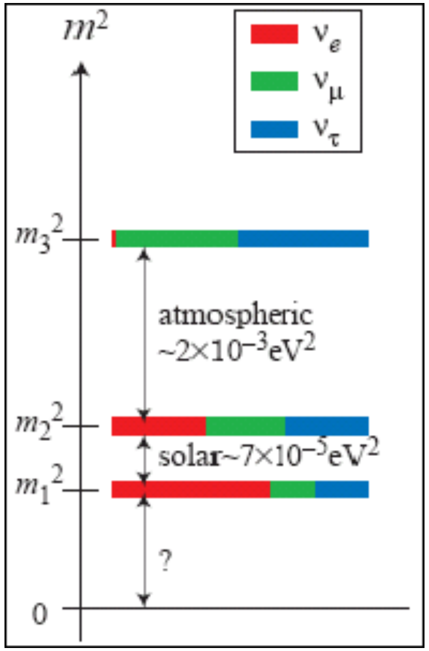


See-Saw

Quarks, charged leptons, neutrinos can have similar Dirac mass :

$$L_{Dirac}^{q,l,\nu} = \alpha^{q,l,\nu}\, \psi_i \bar{\phi}_3^{i} \psi^c_j \bar{\phi}_3^{j} + \beta^{q,l,\nu} \left(\psi_i \bar{\phi}_{123}^{i} \psi^c{}_j \bar{\phi}_{23}^{j} + \psi_i \bar{\phi}_{23}^{i} \psi^c{}_j \bar{\phi}_{123}^{j} \right) + \gamma^{q,l}\, \psi_i \bar{\phi}_{23}^{i} \psi^c{}_j \bar{\phi}_{23}^{j} \qquad \alpha > \beta$$

$$\frac{M^{Dirac}}{m_3} = \begin{pmatrix} 0 & \varepsilon^3 & -\varepsilon^3 \\ \varepsilon^3 & a\varepsilon^2 + \varepsilon^3 & -a\varepsilon^2 + \varepsilon^3 \\ -\varepsilon^3 & -a\varepsilon^2 + \varepsilon^3 & 1 \end{pmatrix}$$

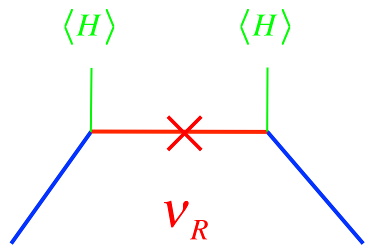
$\varepsilon^{d} = 0.15, \quad a^{d} = 1$

$\varepsilon^{l} = 0.15, \quad a^{e} = -3$

$\varepsilon^{u} = 0.05, \quad a^{u} = 1$

$\varepsilon^{\nu} = 0.05, \quad a^{\nu} = -3$

- "See-saw" with sequential domination



$$M_\nu = M_D^\nu \, M_M^{-1} \, M_D^{\nu T}$$

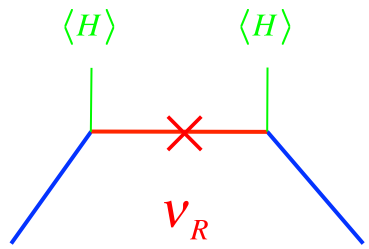
$$L_{Dirac}^{q,l,\nu} = \alpha^{q,l,\nu}\, \psi_i \bar{\phi}_3^i \psi_j^c \bar{\phi}_3^j + \beta^{q,l,\nu} \left(\psi_i \bar{\phi}_{123}^i \psi^c{}_j \bar{\phi}_{23}^j + \psi_i \bar{\phi}_{23}^i \psi^c{}_j \bar{\phi}_{123}^j \right) + \gamma^{q,l}\, \psi_i \bar{\phi}_{23}^i \psi^c{}_j \bar{\phi}_{23}^j$$

$$L_{Majorana}^{\nu} = M_3\, \psi_i^c \bar{\phi}_3^i \psi_j^c \bar{\phi}_3^j + M_2 \left(\psi^c{}_i \bar{\phi}_{123}^i \psi^c{}_j \bar{\phi}_{23}^j + \psi^c{}_i \bar{\phi}_{23}^i \psi^c{}_j \bar{\phi}_{123}^j \right) + M_1 \psi^c{}_i \bar{\phi}_{23}^i \psi^c{}_j \bar{\phi}_{23}^j$$

$$\beta, \gamma < \alpha \qquad c.f. \qquad M_1 \sim M_2 << M_3$$

- "See-saw" with sequential domination

$$M_\nu = M_D^\nu \, M_M^{-1} \, M_D^{\nu T}$$



$$L_{Dirac}^{q,l,\nu} = \alpha^{q,l,\nu} \psi_i \bar{\phi}_3^i \psi_j^c \bar{\phi}_3^j + \beta^{q,l,\nu} \left(\psi_i \bar{\phi}_{123}^i \psi^c{}_j \bar{\phi}_{23}^j + \psi_i \bar{\phi}_{23}^i \psi^c{}_j \bar{\phi}_{123}^j \right) + \gamma^{q,l} \psi_i \bar{\phi}_{23}^i \psi^c{}_j \bar{\phi}_{23}^j$$

$$L_{Majorana}^{\nu} = M_3 \psi_i^c \bar{\phi}_3^i \psi_j^c \bar{\phi}_3^j + M_2 \left(\psi^c{}_i \bar{\phi}_{123}^i \psi^c{}_j \bar{\phi}_{23}^j + \psi^c{}_i \bar{\phi}_{23}^i \psi^c{}_j \bar{\phi}_{123}^j \right) + M_1 \psi^c{}_i \bar{\phi}_{23}^i \psi^c{}_j \bar{\phi}_{23}^j$$

$$M_{Dirac} = \begin{pmatrix} 0 & \beta \\ \beta & \beta + \gamma \end{pmatrix}, \quad M_{Majorana} \propto \begin{pmatrix} 0 & M_2 \\ M_2 & M_2 + M_1 \end{pmatrix}$$

$$M_\nu = M_D^\nu \, M_M^{-1} \, M_D^{\nu T} = \begin{pmatrix} 0 & x \\ x & y \end{pmatrix}$$

$\psi_i \theta_{123}^i, \psi_i \theta_{23}^1$ basis

A simple example

$$\Delta(27)\times U(1)$$

	Q
S,θ_{23}	$+1$
Σ,θ_{123}	-2

$$L^{D,eff}_{a,mass}=\psi_i\left(\frac{1}{M^2_{3,a}}\theta^i_3\theta^j_3+\frac{1}{M^3_{23,a}}\theta^i_{23}\theta^j_{23}\Sigma+\frac{1}{M^3_{123,a}}(\theta^i_{123}\theta^j_{23}+\theta^i_{23}\theta^j_{123})S\right)\psi^c_j H_5,\ a=u,d,l,\nu$$

$$\langle\Sigma\rangle\propto B-L$$

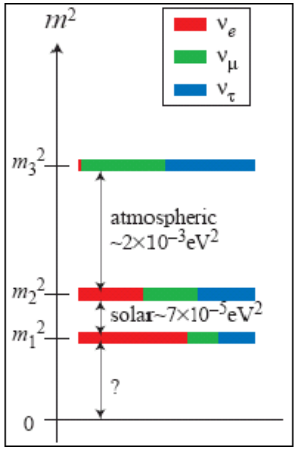
$$L^{D,eff}_{\nu\ mass}=\psi_i\left(\frac{1}{M^2_{3,a}}\theta^i_3\theta^j_3+\frac{1}{M^3_{123,a}}(\theta^i_{123}\theta^j_{23}+\theta^i_{23}\theta^j_{123})S\right)\psi^c_j H_5$$

$$L^{\nu}_{Majorana\ mass}=\psi^c_i\left(\frac{1}{M}\theta^i\theta^j+\frac{1}{M^4}[\alpha\theta^i_{23}\theta^j_{23}(\theta^a\theta^a\theta^a_{123})+\beta(\theta^i_{23}\theta^j_{123}+\theta^i_{123}\theta^j_{23})(\theta^a\theta^a\theta^a_{23})]\right)\psi^c_j$$

Medeiros Varzielas, GGR, Talbert

Texture zero

$$M_\nu = \begin{pmatrix} 0 & x \\ x & y \end{pmatrix}$$



$$\frac{m_2}{m_1} \approx \frac{x^2}{y}, \qquad \nu_1 \propto \nu_{23} - e^{i\eta}\sqrt{\frac{m_2}{m_1}}\nu_{123}$$

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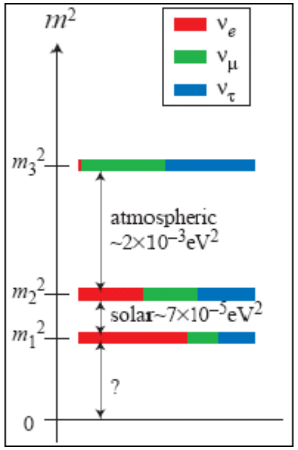

$$\sin\theta_{13}|_\nu = \sqrt{\frac{m_2}{3m_1}} = 0.24$$

$$\delta \sin\theta_{13}|_{lepton} = \frac{\sin\theta_c}{3} = 0.075$$

$$\Bigg\} \quad c.f.\ 0.15|_{exp}$$

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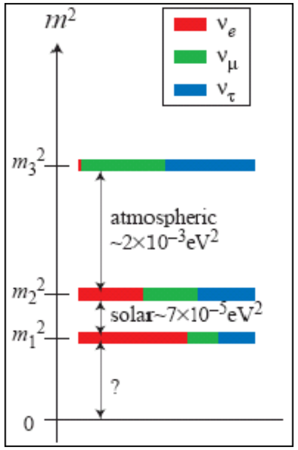

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$$\sin^2\theta_{12} = \tfrac{1}{3}, \quad c.f. \quad \sin^2\theta_{12}|_{exp} = 0.308^{+0.013}_{-0.012}$$

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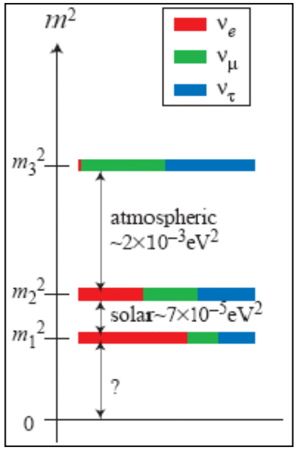

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$\sin^2\theta_{12} = \frac{1}{3}$, $\quad c.f. \quad \sin^2\theta_{12}|_{\exp} = 0.308^{+0.013}_{-0.012}$

$$\sin\theta_{23} = \frac{1}{\sqrt{2}} - e^{i\delta}\sin\theta_{13}|_\nu \qquad c.f. \quad \sin^2\theta_{23} = 0.574^{+0.026}_{-0.144}$$

$$75^0 < \delta < 138 \ or\ 222^0 < \delta < 285^0.$$

A simple example

$$\Delta(27)\times U(1)$$

	Q
S,θ_{23}	$+1$
Σ,θ_{123}	-2

$$L^{D,eff}_{a,mass}=\psi_i\left(\frac{1}{M^2_{3,a}}\theta^i_3\theta^j_3+\frac{1}{M^3_{23,a}}\theta^i_{23}\theta^j_{23}\Sigma+\frac{1}{M^3_{123,a}}(\theta^i_{123}\theta^j_{23}+\theta^i_{23}\theta^j_{123})S\right)\psi^c_j H_5,\ a=u,d,l,\nu$$

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Summary:

Numerical fit: 18 observables well fitted by 9 parameters

- SM unknowns - masses and mixing angles :

$$g_i, m_i, \theta_i, \delta_i, M_{W,H}, \theta_{QCD}$$ (+9 in neutrino sector)

The strong CP problem $\theta_{QCD} G\tilde{G}, \quad \theta_{QCD} < 10^{-10}$

CP violation

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- Axion solution — Make θ_{QCD} a dynamical variable with small vev at potential minimum

CP violation

The strong CP problem $\theta_{QCD} G\tilde{G}, \quad \theta_{QCD} < 10^{-10}$

- Axion solution

  Make θ_{QCD} a dynamical variable with small vev at potential minimum

- Symmetry solution

  … if CP exact at high scale, $\theta_{QCD}(M_X) = 0$

  (e.g. string symmetry-relic of higher dimension Lorentz symmetry)

  Spontaneous symmetry breaking in the flavour changing sector

  $$\theta_{QCD}(M_Z) = O(10^{-16})$$

  Ellis, Gaillard

  (also suppresses dangerous CP violating gaugino, higgsno and trilinear terms in SUSY)

- **SM unknowns** - masses and mixing angles :

$$g_i, m_i, \theta_i, \delta_i, M_{W,H}, \theta_{QCD}$$ (+9 in neutrino sector)

Anarchy or Order?

Symmetries : GUT, family

IR/UV stable fixed points ⬅

UV/IR fixed points - Reduction of couplings

IR fixed points in the SM :

$$\mathcal{D} \ln g_{QCD} = -7 g_{QCD}^2$$

$$\mathcal{D} \ln g_t = \frac{9}{2} g_t^2 - 8 g_{QCD}^2$$

$$\mathcal{D} \ln \left(\frac{g_t}{g_{QCD}} \right) = \frac{9}{2} g_t^2 - g_{QCD}^2 = 0 \,|_{IRFP}$$

$$\mathcal{D} = 16\pi^2 \frac{\partial}{\partial t}$$

$$t = \frac{1}{2} \ln \frac{\mu^2}{\mu_0^2}$$

$$g_{t,IRFP}^2 = \frac{2}{9} g_{QCD}^2$$

$$\left(\Rightarrow m_t = 110 GeV \right)$$

UV/IR fixed points – Reduction of couplings

Generalisation to multiple couplings:

$$\Phi(g_1, \cdots, g_A) = \text{const.}$$

$$\mu \frac{d\Phi}{d\mu} = \vec{\nabla}\Phi \cdot \vec{\beta} = \sum_{a=1}^{A} \beta_a \frac{\partial \Phi}{\partial g_a} = 0$$

$$\Rightarrow \quad \beta_g \frac{dg_a}{dg} = \beta_a \ , \ a = 1, \cdots, A$$

Zimmerman

Review:
Heinemeyer, Mondragon, Tracas, Zoupanos

UV/IR fixed points – Reduction of couplings

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$$\Rightarrow \quad \beta_g \frac{dg_a}{dg} = \beta_a \ , \ a = 1, \cdots, A$$

Finite theories: $\beta_a = 0, a = 1..A$

Can solve these equations perturbatively

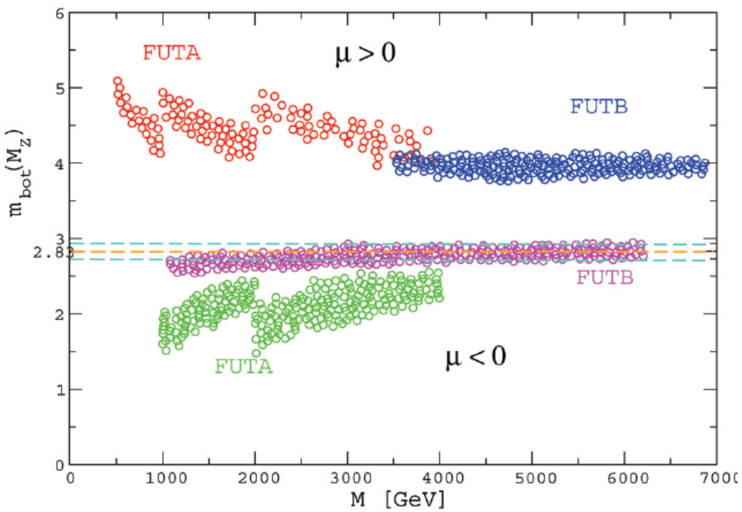
c.f. talk by Patellis

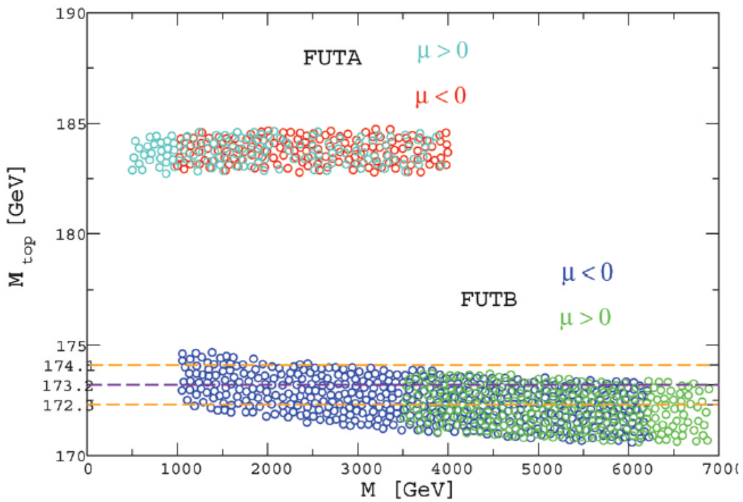
e.g. Finite Unified theory:

SUSY SU(5) : $3\times\left[\bar{5}+10\right]_{matter}+\left[4\times(5+\bar{5})+24\right]_{Higgs}$

Reduction of couplings – enhanced symmetry: $Z_4\times Z_4\times Z_4$

Top and bottom quark masses:





Inclusion of gravity?

Asymptotic safety:

c.f. talk by Held

Nonperturbative UV fixed point:

$$\frac{M^2_{Planck}}{k^2} \to \text{constant}$$

Weinberg, Reuter

Inclusion of gravity?

Asymptotic safety:

Nonperturbative UV fixed point: $\frac{M^2_{Planck}}{k^2} \to$ constant

$$M_P^2\left(k\right) = M_P^2 + 2\xi_0 k^2 \quad \Rightarrow \quad G_N\left(k^2\right)|_{k^2 \gg M_P^2} \sim \frac{1}{16\pi\xi_0 k^2}$$

Regular behaviour of high energy amplitudes

i.e. Gravitational radiative corrections contribute to RG equations

$$\beta_{x_j}^{grav} = \frac{a_j}{8\pi}\frac{k^2}{M_P^2(k)}x_j, \quad x_j = g_1, g_2, g_3, h_t, \ldots$$

Robinson, Wilczek +…

Ellis, Mavromatis+… ?

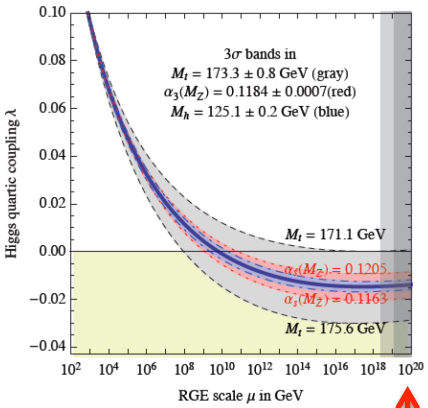
$$\sim \begin{cases} f_j\, x_j, & k^2 > k_{tr}^2 \\ 0, & k^2 < k_{tr}^2 \end{cases}$$

$$f_j = \frac{a_j}{16\pi\xi_0}$$

Asymptotic safety - prediction

- Higgs mass $$V(H) = -m^2 |\phi|^2 + \lambda |\phi|^4$$

λ



$$a_\lambda > 0, \lambda(k_{tr}) \to 0$$

"Infra red" fixed point

Wetterich, Shaposhnikov

$$k_{tr} \sim 10^{19}\, GeV$$

Buttazzo et al

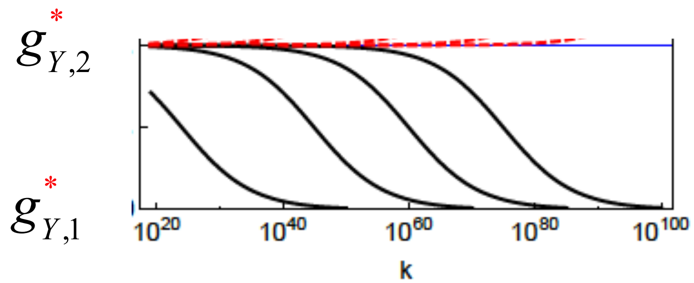
Asymptotic safety - postdictions

- $U(1)_Y$: taming the Landau pole

$$\beta_{g_y}^{k^2>k_{tr}^2} = -f_g g_Y + \frac{41}{6}\frac{g_Y^2}{16\pi^2} + \dots$$

$$g_{Y,1}^* |_{UVSFP} = 0, \quad g_{Y,2}^* |_{IRSFP} = \sqrt{\frac{6.16\pi^2 f_g}{41}}$$

$$g_{Y,1}^* \leq g_Y(k_{tr}) \leq g_{Y,2}^*$$



$f_g > 0$

Test of asymptotic safety?

Asymptotic safety - postdictions

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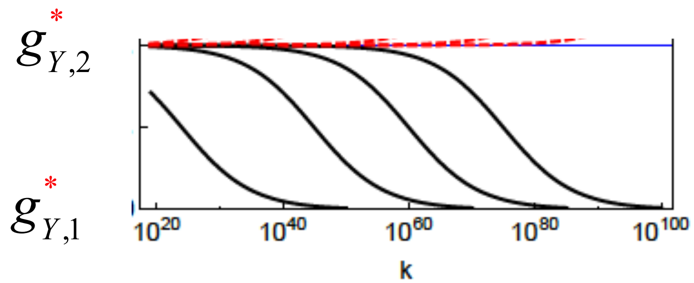


$f_g > 0$

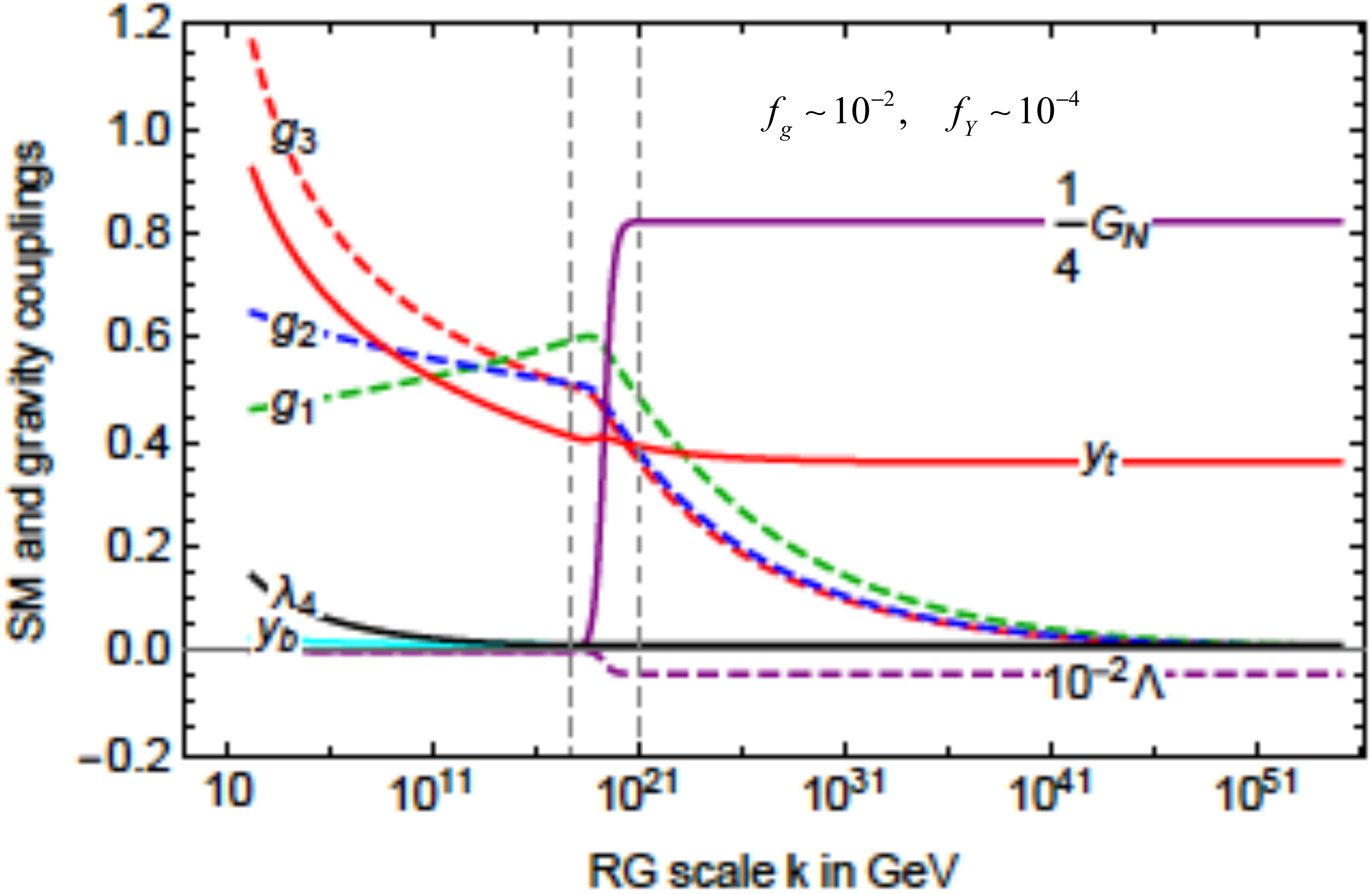
$$g_{Y,1}^* \leq g_Y(k_{tr}) \leq g_{Y,2}^*$$

Test of asymptotic safety?

- $$\beta_{y_{t\,(b)}} = \frac{y_{t\,(b)}}{16\,\pi^2}\left(\frac{3y_{b\,(t)}^2}{2} + \frac{9y_{t\,(b)}^2}{2} - \frac{9}{4}g_2^2 - 8g_3^2\right) - f_y\, y_{t\,(b)} - \frac{3y_{t\,(b)}}{16\,\pi^2}\left(\frac{1}{36} + Y_{t\,(b)}^2\right) g_Y^2$$

$+\ldots$

Asymptotic safety - postdictions



Eichorn, Held

See Eichhorn review 1810.07615 and references therein

Summary

- Complicated multiplet structure of SM elegantly explained by GUT

- The "real" hierarchy problem requires SUSY GUT leaving a little hierarchy problem…suggesting further structure needed

- Support for SUSY GUT comes from gauge coupling unification $\left(\alpha_s, M_X\right)$

- Anarchy/Order in fermion masses and mixing…it is possible to describe all masses and imixings by a very symmetric structure

- Inclusion of gravity - could it explain outstanding puzzles?

$\Omega_\Lambda, m_{Higgs}, g_i, h_i$

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DOMINUS ILLUMINATIO MEA