Non-linear supersymmetry and TT-bar

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Introduction and plan

What is TT-bar?

→ For a 2D theory we ask that is satisfies

$$\frac{\partial \mathcal{L}^\lambda}{\partial \lambda} = -\det T_{mn}[\mathcal{L}^\lambda]\,,$$

for T_{mn} the energy-momentum tensor. *Zamolodchikov '04*

→ Deformation of free pure boson action gives Nambu–Goto

$$\frac{1}{2\lambda}\left(-1+\sqrt{1+4\lambda\,\partial\phi\partial\phi}\right)\,.$$

Cavaglià, Negro, Szécsényi, Tateo '16, Bonelli, Doroud, Zhu '18

→ Many properties: Controlled spectrum, Relation to ST, Holography, J–T gravity, etc.

Is TT-bar supersymmetric?

→ 2D N=(1,0), N=(2,0) and N=(1,1) already studied.
Baggio, Sfondrini, Tartaglino-Mazzucchelli, Walsh '18, Chang, Ferko, Sethi '18, Jiang, Sfondrini, Tartaglino-Mazzucchelli '19 Coleman, Aguilera-Damia, Freedman, Soni '19

→ 2D N=(2,2) extended recently.
Chang, Ferko, Sethi, Sfondrini, Tartaglino-Mazzucchelli '19

→ Beyond 2D SUSY and partial breaking also under investigation. *Chang, Ferko, Jiang, Sethi, Sfondrini, Tartaglino-Mazzucchelli '19*

Plan:

$\rightarrow$ 2D N=(2,2) supersymmetry and TT-bar

$\rightarrow$ 2D Volkov–Akulov

$\rightarrow$ Volkov–Akulov as TT-bar

2D N=(2,2) supersymmetry

- We will use light-cone space-time coordinates

$$x^{+\!\!+} = (x^0 + x^1)/\sqrt{2}\,, \quad x^{=} = (x^0 - x^1)/\sqrt{2}\,.$$

- For N=(2,2) we have the complex Grassmann variables θ^+ and θ^- on which supersymmetry acts as a shift: $\theta \to \theta + \epsilon$.

- We build the superspace derivatives

$$\mathrm{D}_+ = \frac{\partial}{\partial\theta^+} - \frac{i}{2}\bar{\theta}^+\partial_{+\!\!+}\,, \quad \mathrm{D}_- = \frac{\partial}{\partial\theta^-} - \frac{i}{2}\bar{\theta}^-\partial_{=}\,,$$

which realize the algebra

$$\{\mathrm{D}_+, \overline{\mathrm{D}}_+\} = i\partial_{+\!\!+}\,, \quad \{\mathrm{D}_-, \overline{\mathrm{D}}_-\} = i\partial_{+\!\!+}\,,$$

while the rest (anti) commutators are zero.

- We will work with chiral superfields

$$\overline{\mathrm{D}}_{\pm}\Phi = 0$$

with expansion

$$\Phi = \phi + \theta_+\chi_- + \theta_-\chi_+ + \theta_+\theta_- F\,,$$

from which we also deduce the supersymmetry transformations $\delta\phi = \epsilon\chi$, etc.

- The Lagrangian we will need to consider is

$$\mathcal{L} = \int d^4\theta K(\Phi^i, \overline{\Phi}^j) + \left(\int d^2\theta W(\Phi^i) + c.c.\right)\,,$$

where $K = \delta_{ij}\Phi^i\overline{\Phi}^j$ and W in holomorphic.

- By varying with respect to the Φ^i we can derive the superspace equations of motion

$$\overline{\mathrm{D}}_-\overline{\mathrm{D}}_+\overline{\Phi}_i = -\partial_i W\,.$$

- We can derive the F–Z supercurrent superfields

$$J_{\#} = \frac{1}{2}[\mathrm{D}_+, \overline{\mathrm{D}}_+]K + i\frac{\partial K}{\partial \Phi^i}\partial_{\#}\Phi^i + c.c.\,, \quad J_{=} = \ldots$$

with conservation laws

$$\overline{\mathrm{D}}_+ J_{=} = -\mathrm{D}_- Z\,, \quad \overline{\mathrm{D}}_- J_{\#} = \mathrm{D}_+ Z\,,$$

where Z is a chiral $N = (2,2)$ superfield defined as

$$Z = 2W(\Phi^i)\,.$$

TT-bar in N=(2,2)

Chang, Ferko, Sethi, Sfondrini, Tartaglino-Mazzucchelli '19

- The full expression is

$$\det(T) = T_{++}T_{=} - \Theta^2 ,$$

with

$$T_{++} = \frac{1}{2}[\mathrm{D}_+, \overline{\mathrm{D}}_+]J_{+}\Big|_{\theta=0} , \quad \Theta = -\frac{1}{2}[\mathrm{D}_-, \overline{\mathrm{D}}_-]J_{+}\Big|_{\theta=0} ,$$

and similarly for $J_=$.

- The proposal for the superspace TT-bar generalization is a "supercurrent squared" expression

$$\mathcal{T}\overline{\mathcal{T}} = \frac{1}{8}\int d^4\theta \left(J_{+}J_{=} - 2Z\overline{Z}\right) ,$$

which matches to TT-bar up to EOM and boundary terms.

2D Volkov–Akulov

- We can break SUSY with a chiral superfield

$$X = A + \theta_+ G_- + \theta_- G_+ + \theta_+\theta_- F\,.$$

- SUSY broken: $\delta G_\pm = -f\,\epsilon_\pm + \cdots$

- In the low energy the scalar generically decouples by imposing the superspace constraint

$$X^2 = 0 \;\rightarrow\; X = \frac{G_- G_+}{F} + \theta_+ G_- + \theta_- G_+ + \theta_+\theta_- F\,.$$

Rocek '78, Casalbuoni, De Curtis, Dominici, Feruglio, Gatto '89, FF, Koci, von Unge '16

- We can then simply study

$$\mathcal{L} = \int d^4\theta X\overline{X} + \left(f \int d^2\theta X + c.c. \right) ,$$

and the component form (after eliminating F) expression is

$$\mathcal{L}_{VA} = -f^2 + iG_-\partial_{\#}\overline{G}_- + iG_+\partial_{=}\overline{G}_+ - \frac{1}{f^2}G_+G_-\Box(\overline{G}_-\overline{G}_+) - \frac{1}{f^6}G_+G_-\overline{G}_-\overline{G}_+\Box(G_+G_-)\Box(\overline{G}_-\overline{G}_+) .$$

- Non-linear V–A SUSY
  - Fermion D.O.F. $\neq$ Boson D.O.F.
  - $\delta G_+ = -f\epsilon_+ - \frac{i}{f}\overline{\epsilon}^+\partial_{\#}G_+G_- + \cdots$

How do we find the supercurrent superfields and super-EOM?

Linearizing the V–A

- We will consider the Lagrangian

$$\mathcal{L} = \int d^4\theta X\overline{X} + \left(\int d^2\theta (fX + MX^2) + c.c. \right) ,$$

where superfields X and M are chiral but otherwise unconstrained.

- This Lagrangian is on-shell equivalent to the V–A

$$\delta\mathcal{L}/\delta M \to X^2 = 0 .$$

- This allows us to use the standard rules for deriving super-EOM and supercurrents for

$$K = X\overline{X} , \quad W = fX + MX^2 .$$

The supercurrents of V–A

- For the supercurrents we find

$$J_{\#} = 2\mathrm{D}_+ X \overline{\mathrm{D}}_+ \overline{X}\,, \quad J_{=} = 2\mathrm{D}_- X \overline{\mathrm{D}}_- \overline{X}\,,$$

  and

$$Z = 2fX\,.$$

- One can check the conservation laws independently by using derivatives of EOM
  - $X^2 = 0$
  - $X\overline{\mathrm{D}}_-\overline{\mathrm{D}}_+\overline{X} = -fX$
  - $\mathrm{D}_\pm X\overline{\mathrm{D}}_-\overline{\mathrm{D}}_+\overline{X} = -f\mathrm{D}_\pm X$
- Notice that M has dropped out.

Volkov–Akulov as TT-bar

Cribiori, FF, von Unge '19

The TT-bar for V–A

- Now we can finally calculate (using EOM)

$$J_{++}J_{=} - 2Z\overline{Z} = -4f^2 X\overline{X}\,.$$

- Then we have (using EOM and B.T.)

$$\mathcal{T}\overline{\mathcal{T}} = -\frac{f^2}{2}\int d^4\theta X\overline{X} = \frac{f^3}{4}\int d^2\theta X + c.c.$$

- On the other hand from the superspace V–A Lagrangian

$$\frac{\partial \mathcal{L}}{\partial f} = \int d^2\theta X + c.c.$$

The flow equation for V–A

- We then set

$$f^2 = 2\lambda^{-1}\,, \quad \lambda > 0\,.$$

- As a result we verify

$$\frac{\partial \mathcal{L}}{\partial \lambda} = -\frac{f^3}{2}\int d^2\theta X = -\det(T[\mathcal{L}])\,,$$

  which means that the 2D Volkov–Akulov model is a TT-bar deformation.

- To see V–A is TT-bar deformation of free fermion we take the limit

$$\mathcal{L}_{VA}\Big|_{f\to\infty/\lambda\to 0} \to iG_-\partial_{++}\overline{G}_- + iG_+\partial_{=}\overline{G}_+\,.$$

Outlook

Future research interests:

$\rightarrow$ Deeper relation between non-linear SUSY and TT-bar?

$\rightarrow$ What is the impact on holography?

$\rightarrow$ What happens for different dimensions/supersymmetry?

Thank you