

# Janus and J-folds

Friðrik Freyr Gautason

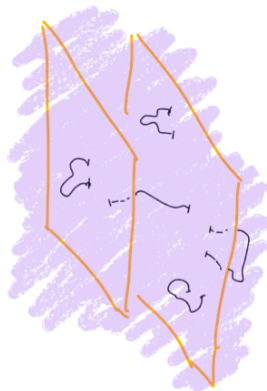
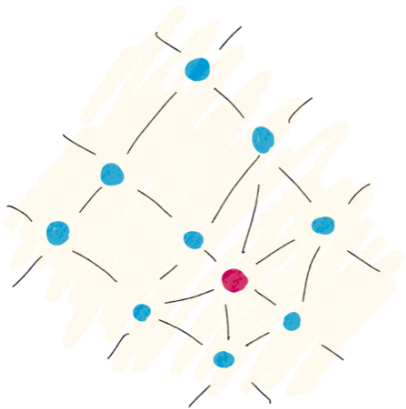
Recent Developments in Strings and Gravity  
EISA, Corfu Summer Institute. September 14, 2019

based on [1907.11132] with Nikolay Bobev, Krzysztof Pilch, Minwoo Suh, and  
Jesse van Muiden



**KU LEUVEN**

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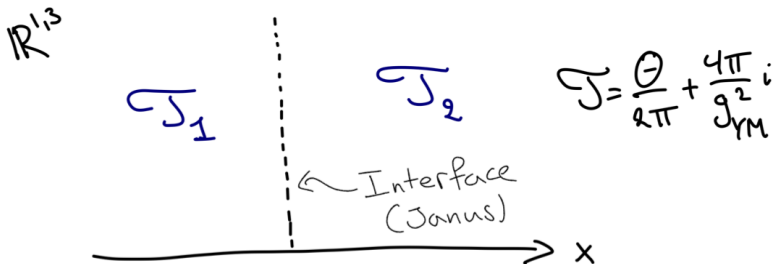
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In  $\mathcal{N} = 4$  SYM they arise from studying the theory with a position dependent coupling.



# JANUS IN $\mathcal{N} = 4$

The Janus configurations can preserve a lot of supersymmetry

D'Hoker, Estes, Gutperle (2006)

$\mathcal{N}$	supergroup	R-symmetry	Commutant
4	$\text{OSp}(4 4, \mathbf{R})$	$\text{SU}(2) \times \text{SU}(2)$	
2	$\text{OSp}(2 4, \mathbf{R})$	$\text{U}(1)$	$\text{SU}(2)$
1	$\text{OSp}(1 4, \mathbf{R})$		$\text{SU}(3)$
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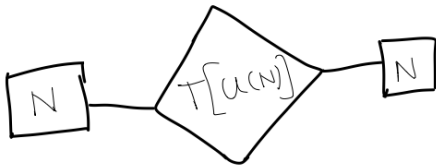
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The  $\text{U}(N)$   $\mathcal{N} = 4$  interface has been studied extensively on the field theory side. The conformally invariant interfaces are closely related to strongly coupled 3D  $\mathcal{N} = 4$  Chern Simons like theories called  $T[\text{U}(N)]$ .

Gaiotto, Witten (2009-10)

# JANUS IN $\mathcal{N} = 4$

The 3D  $\mathcal{N} = 4$  SCFTs of Gaiotto and Witten serve as new strongly coupled building blocks.



The two boxes denote two copies of  $U(N)$  flavor symmetry.



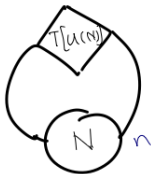
# NEW 3D SCFTs: J-FOLDS

We can build new strongly coupled SCFTs in three dimensions using this building block. For example gauging the diagonal  $U(N)$  flavor symmetries and adding a Chern-Simons level  $n$

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These theories are called J-folds. Sidenote: Naively the CS level breaks supersymmetry from  $\mathcal{N} = 4$  to  $\mathcal{N} = 3$  but there is an IR enhancement to  $\mathcal{N} = 4$ . Recent success by Assel and Tomasiello in matching a supergravity analysis with a localization for the  $\mathcal{N} = 4$  J-fold theories.

Main claim of this talk: Very similar story also holds starting from any  $\mathcal{N} = 1$  SCFT in four dimensions *with* a marginal coupling.

# Holography

## 4D $\mathcal{N} = 1$ SCFTs IN TYPE IIB

Start with type IIB on

$$\mathbf{R}^{1,3} \times \text{CY}_3$$

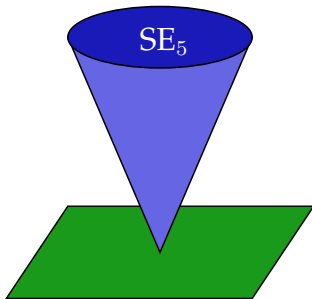
where  $\text{CY}_3 = \mathcal{C}(\text{SE}_5)$ .

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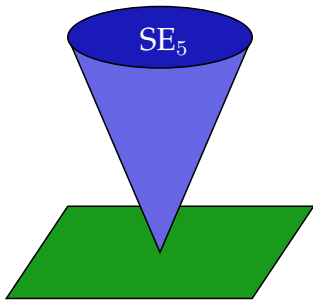


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The holographic dual to D3-branes probing  $\mathcal{C}(\text{SE}_5)$  is

$$\text{AdS}_5 \times \text{SE}_5 .$$



## 4D $\mathcal{N} = 1$ SCFTs IN TYPE IIB

All these SCFTs have a marginal coupling dual to the axion-dilaton in type IIB.

We use a consistent truncation of type IIB on  $SE_5$  to a  $\mathcal{N} = 2$  supergravity in five dimensions coupled to a single hypermultiplet.

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This approach considerably simplifies our search for the dual to a Janus configuration and J-folds starting from the  $\mathcal{N} = 1$  SCFTs.

## 5D SUPERGRAVITY

The Lagrangian is (fermions and gauge fields set to zero)

$$\mathcal{L} = \frac{\sqrt{|g_5|}}{16\pi G_N} \left( R_5 + \frac{1}{4} \text{Tr}[\partial_\mu M \partial^\mu M^{-1}] - \mathcal{P} \right),$$

where  $M$  parametrizes the four scalars taking value in the scalar manifold

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Note that this model was previously used to study  $\mathcal{N} = 1$  preserving Janus solutions in  $\mathcal{N} = 4$  SYM.

Clark, Karch (2005)

With a 10D embedding.

D'Hoker, Estes, Gutperle (2006)

Suh (2011)

## 5D SUPERGRAVITY

We choose a particular representation of the scalar matrix  $M$  such that

$$\begin{aligned}\mathcal{L}_{\text{kin}} &= \frac{1}{4} \text{Tr} [\partial_\mu M \partial^\mu M^{-1}] \\ &= -2(\partial\chi)^2 - \frac{1}{2} \sinh^2 2\chi (\partial\omega - \sinh^2 \varphi \partial c)^2 \\ &\quad - \frac{1}{2} \cosh^2 \chi [4(\partial\varphi)^2 + \sinh^2 2\varphi (\partial c)^2].\end{aligned}$$

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Roughly speaking  $(\varphi, c)$  uplift to the type IIB axion-dilaton, and are thus dual to the marginal couplings of the field theory.  $(\chi, \omega)$  is dual to a complex dimension 3 operator in the QFT. This theory has  $\text{SL}(2, \mathbf{R})_S$  symmetry (descending from the type IIB one) acting on the scalar matrix  $M$ ,  $M \mapsto R^\dagger M R$  where  $R \in \text{SL}(2, \mathbf{R}) \subset \text{SU}(2, 1)$ .

## BPS EQUATIONS

We look for a holographic dual of the conformal Janus. This dictates the following metric ansatz

$$ds_5^2 = dr^2 + e^{2A(r)} ds_{\text{AdS}_4}^2,$$

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and all scalars only depend on  $r$ . I.e. the EOMs we have to solve reduce to ODEs. Supersymmetry reduces second order ODEs to first order BPS equations

$$(\chi')^2 = \frac{1}{4}(\partial_\chi W)^2 - \cosh^2 \chi \sec^2(c + 2\omega)(\varphi')^2,$$

$$\omega' = \sinh^2 \varphi(c'),$$

$$\sinh 2\varphi(c') = -2 \tan(c + 2\omega)(\varphi'),$$

$$A' = -\frac{1}{3} \coth \chi (\chi'),$$

$$\varphi' = 3e^{-A} \cos(c + 2\omega) \operatorname{sech} \chi \tanh \chi.$$

# SOLVING THE BPS EQUATIONS

Three equations are trivial to solve

$$\sin(c + 2\omega) = \frac{\mathcal{J}}{\sinh 2\varphi},$$

$$\cos^2(c - c_0) = \frac{\sinh^2 2\varphi - \mathcal{J}^2}{\sinh^2 2\varphi(1 + \mathcal{J}^2)},$$

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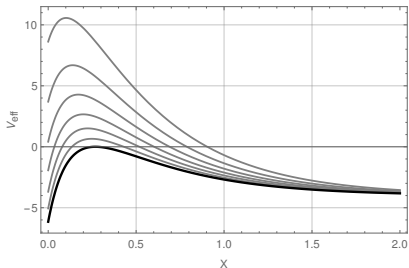
The remaining two equations can be rewritten as a simple classical mechanics problem ( $e^{-3X} = \sinh \chi$ )

$$(X')^2 + V_{\text{eff}} = 0,$$

$$V_{\text{eff}} = 16e^{-2X} \left( \frac{9}{5^{5/3}\mathcal{I}} - e^{-4X} \cosh^2(3X) \right).$$

# SOLVING THE BPS EQUATIONS

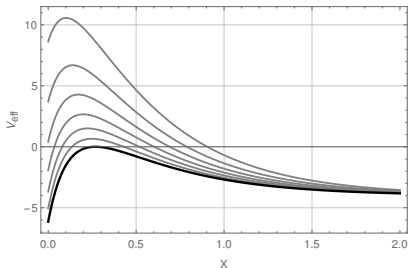
Scattering of a classical particle with zero energy of a potential wall (depending on the integration constant  $0 \leq \mathcal{I} \leq 1$ )



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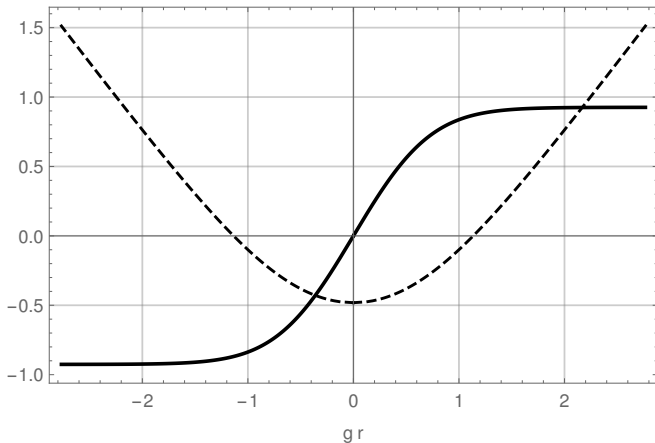


$\text{AdS}_5$  emerges asymptotically as  $X \rightarrow \infty$ . Finally the dilaton is given by a simple integral

$$\cosh 2\varphi = \cosh 2F + \frac{1}{2}e^{-2F} \mathcal{J}^2, \quad F = F_0 \pm \int \frac{9e^{-X} dX}{\cosh(3X) \sqrt{-5^{5/3} \mathcal{I} V_{\text{eff}}}}.$$



## TYPICAL SOLUTION



The function  $4(F(r) - F_0)$  (solid curve) for  $\mathcal{I} = 4/5$  which determines the dilaton, and  $X - 1$  (dashed curve) which determines the metric function  $A$ .

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We have found a full class of solutions (upto two numerical integrals) dual to a Janus configuration of any holographic  $\mathcal{N} = 1$  SCFT with a marginal coupling.

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Now let us focus on the special case  $\mathcal{I} = 1$ . The effective potential has a critical point at zero energy. This implies there is a solution for which the metric function  $A$  is constant, and the dilaton  $\varphi$  is linear.

## J-FOLD

The solution is

$$e^A = \frac{5}{6} = \text{constant}, \quad \varphi = \varphi_0 + \rho, \quad \rho = \frac{3r}{\sqrt{5}}.$$

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This solution looks singular, as the dilaton blows up when  $\rho \rightarrow \pm\infty$ . However, we can *compactify* the  $\rho$ -direction à la Scherk-Schwarz using the  $\text{SL}(2, \mathbf{R})$  symmetry of the theory.

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Inverso, Samtleben, Trigiante (2017)  
Assel, Tomasiello (2018)

$$\rho \sim \rho + \rho_0, \quad \mathfrak{J}_n = \begin{bmatrix} 2 \cosh \rho_0 & 1 \\ -1 & 0 \end{bmatrix}.$$



# 10D UPLIFT

The ten-dimensional type IIB solution is completely explicit. For example the Janus metric is

$$ds_{10}^2 = \cosh \chi ds_5^2 + \frac{ds_4^2}{\cosh \chi} + \cosh \chi \zeta^2,$$

where

$$ds_{SE_5}^2 = ds_4^2 + \zeta^2,$$

$ds_4^2$  is a metric on a Kähler-Einstein base, with  $U(1)$  fiber  $\zeta = d\phi + \sigma$ ,  $d\sigma = 2J$ , etc. All other fields are completely specified in terms of  $KE_4$  data  $(J, \Omega)$  and five-dimensional supergravity data.

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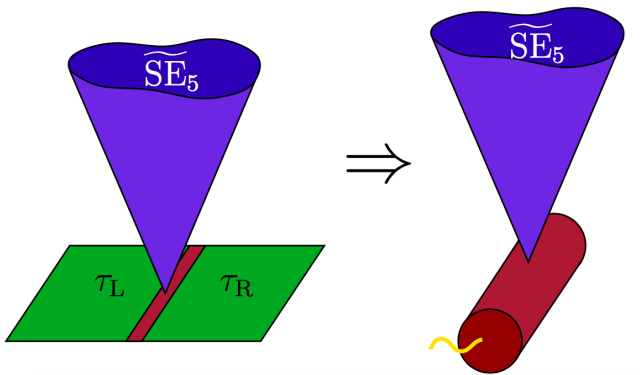
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The local form of the Janus metric is

Lüst, Tsimpis (2009)

$$ds_{10}^2 = \sqrt{\frac{5}{6}} \frac{1}{6} \left( 4d\rho^2 + 5ds_{AdS_4}^2 + 6ds_4^2 + \frac{36}{5}\zeta^2 \right).$$

# CARTOONS



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$$2 \cosh \rho_0 = n \in \mathbf{Z}, \quad n > 2 .$$

This leads to an infinite class of 3D  $\mathcal{N} = 1$  SCFTs with a close connection to the interfaces on  $\mathcal{N} = 1$  SCFTs in four dimensions. In fact the free energy on  $S^3$  is given in terms of the 4D central charge

$$\mathcal{F}_{S^3} = \sqrt{\frac{5^5}{36}} \operatorname{arccosh}(n/2) a_{4d} .$$

## FUTURE DIRECTIONS

- ❖ Understand this universal relation directly in QFT.
- ❖ Similar story for  $\mathcal{N} = 2$  Janus starting from  $\mathcal{N} = 2$  SCFTs in 4D.
- ❖ Dual to  $\mathcal{N} = 2$  Janus and J-folds starting from  $\mathcal{N} = 4$  SYM (work in progress).
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