Consistent truncations & Kaluza-Klein spectra from Exceptional Field Theory

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Consistent truncations & AdS

- Product manifold

$$AdS_D \times M_{int}$$

- Domain-wall solutions, AdS-black holes easier in D-dimensional (gauged) SUGRA
- $\Rightarrow$ Use “consistent truncation” around $\text{AdS}_D \times M_{int}$

$$\boxed{\text{10/11-d SUGRA}} \;\underset{\text{Every solution uplifts to solution}}{\overset{\text{Truncation on } \mathrm{M}_{\text{int}}}{\rightleftarrows}}\; \boxed{\text{D-dim gSUGRA}}$$

Consistent truncations are / used to be hard

- Consistent truncations rare & difficult to construct
- Until recently, few systematic constructions
- No scale separation ⇒ No effective action! [Kim, Romans, Nieuwenhuizen '85]

IIB on S^5:

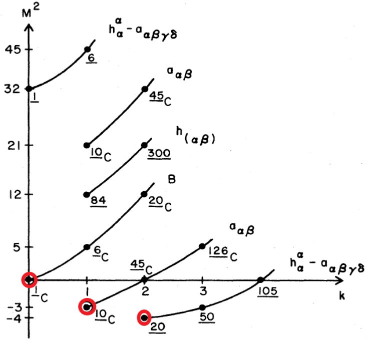

FIG. 2. Mass spectrum of scalars.

- Non-linear mixing of metric & fluxes!

Outline

- Intro to Exceptional Field Theory (ExFT)
- Consistent truncations easy from ExFT
- New method for Kaluza-Klein spectra
- Conclusions

$E_{6(6)}$ Exceptional field theory

[Berman, Perry '10], [Berman, Godazgar^2, Perry '11], [Coimbra, Strickland-Constable, Waldram '11], [Berman, Cederwall, Kleinschmidt, Thompson '12], ...

- Consider KK-split of 11-d SUGRA:

$$M_{11} = M \times M_5 \,.$$

- Unify diffeos + gauge symmetries of 11-d SUGRA on M

$$\delta g = L_v g \,, \quad \delta C_{(3)} = L_v C_{(3)} + d\lambda_{(2)} \,, \quad \delta C_{(6)} = L_v C_{(6)} + d\lambda_{(5)}$$

- Generalised vector field

$$V = v + \lambda_{(2)} + \lambda_{(5)} \in \mathbf{27} \text{ rep of } E_{6(6)}$$

Generalised metric and other fields

- Internal bosonic fields on M:

$$\{g,\ C_{(3)},\ C_{(6)}\} = \mathcal{M}_{MN} \in \frac{E_{6(6)}}{\mathrm{USp}(8)}\,.$$

- Fields with mixed legs:

$$\{g^{ij}g_{\mu\,j},\ C_{\mu\,ij},\ \ldots\} = \mathcal{A}_\mu{}^M \in \mathbf{27} \text{ of } E_{6(6)}\,,$$
$$\{C_{\mu\nu\,i},\ C_{\mu\nu\,ijkl}\ldots\} = \mathcal{B}_{\mu\nu\,M} \in \overline{\mathbf{27}} \text{ of } E_{6(6)}$$

- Spinors form reps of $\mathrm{USp}(8)$ [Coimbra, Strickland-Constable, Waldram '11]

Generalised Lie derivative

- Generalised Lie derivative: local $E_{6(6)}$ action

$$\mathcal{L}_V = V^M \partial_M + (\partial \times_{adj} V) = \text{diffeo} + \text{gauge transf}\,,$$

with $\partial_M = (\partial_i\,,\ \partial^{ij}\,,\ \ldots) = (\partial_i\,,\ 0\,,\ \ldots,\ 0)$.

- E.g. $\mathcal{L}_V \mathcal{M}_{MN} = \left\{ L_v g\,,\ L_v C_{(3)} + d\lambda_{(2)}\,,\ L_v C_{(6)} + d\lambda_{(5)} + \ldots \right\}\,.$
- "Exceptional geometry": generalised tensors, etc.

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- "Exceptional geometry": generalised tensors, etc.
- Higher-dimensional origin? **No?!** Closure of algebra requires "section condition"

$$d^{MNP} \partial_N \otimes \partial_P = 0\,.$$

Covariant restriction to 6 (11-d SUGRA) or 5 (IIB SUGRA) coordinates.

Rewriting the action

Full 10-d/11-d action can be rewritten: [Hohm, Samtleben '13]

$$L = R_g + \frac{1}{24} g^{\mu\nu} \mathfrak{D}_\mu \mathcal{M}^{MN} \mathfrak{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{F}_{\mu\nu}{}^M \mathcal{F}^{\mu\nu\, N} \mathcal{M}_{MN} + L_{top} - \mathcal{R} \,,$$

- "Looks like 5-d gSUGRA"
- $\mathfrak{D}_\mu = \partial_\mu - \mathcal{L}_{\mathcal{A}_\mu}$ "external covariant derivatives"
- $\mathcal{F}_{\mu\nu}{}^M = 2\partial_{[\mu} \mathcal{A}_{\nu]}{}^M - 2 \left[\mathcal{A}_\mu , \mathcal{A}_\nu\right]_E^M + 10 d^{MNP} \partial_N B_{\mu\nu\, P}$
- $dL_{top} \sim F \wedge F \wedge F + H \wedge dH$ "topological term"
- "Scalar potential":

$$\mathcal{R} = \mathcal{M}^{MN} \mathcal{R}_{MN} = \mathcal{M}^{MN} \partial_M \mathcal{M}^{PQ} \partial_N \mathcal{M}_{PQ} + \ldots$$

Consistent truncations are easy in ExFT

[Aldazabal, Baguet, Baron, Berman, Blair, Dibitetto, Fernandéz-Melgarejo, Geissbühler, Graña, Hassler, Hohm, Inverso, Jeon, Lee, Lüst, EM, Marqués, Musaev, Nunez, Park, Perry, Petrini, Pope, Roest, Samtleben, Strickland-Constable, Thompson, Trigiante, Waldram, ...]

- Generalised Scherk-Schwarz Ansatz: $U_M{}^{\bar{A}}(Y) \in E_{6(6)}$

$$
\begin{aligned}
\mathcal{M}_{MN}(x, Y) &= \mathcal{M}_{\bar{A}\bar{B}}(x)\, U_M{}^{\bar{A}}(Y)\, U_N{}^{\bar{B}}(Y)\,, \\
A_\mu{}^M(x, Y) &= A_\mu{}^{\bar{A}}(x)\, U_{\bar{A}}{}^M(Y)\,, \\
B_{\mu\nu\, M}(x, Y) &= B_{\mu\nu,\bar{A}}(x)\, U_M{}^{\bar{A}}(Y)
\end{aligned}
$$

- Consistency condition:

$$
\mathcal{L}_{U_{\bar{A}}} U_{\bar{B}} = X_{\bar{A}\bar{B}}{}^{\bar{C}}\, U_{\bar{C}}\,,
$$

- Consistent truncation to maximal 6-D gSUGRA

New consistent truncations

- Consistent truncation on S^5, use sphere harmonics $\mathcal{Y}_a$

$$\mathcal{Y}^a \mathcal{Y}_a = 1\,.$$

[Lee, Strickland-Constable, Waldram '14], [Hohm, Samtleben '14]

- IIB $\text{AdS}_5 \times S^5$: $U = \begin{pmatrix} \partial_i \mathcal{Y}^a \\ \mathcal{Y}^a - 2\xi^i \partial_i \mathcal{Y}^a \end{pmatrix} \in \mathrm{SL}(6) \subset E_{6(6)}$

  Proof of consistent truncation
- Reproduce S^7, S^4 truncations [deWit, Nicolai '87], [Nastase, van Nieuwenhuizen, Vaman '99]
- Hyperboloids, 4-D dyonic gaugings, ... [EM, Samtleben '15], [Inverso, Samtleben, Trigiante '16], [EM, Samtleben '17]

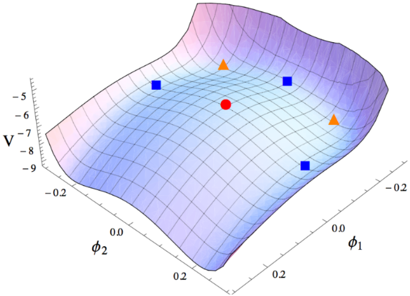
Consistent truncations with less SUSY

- Less SUSY: Twist matrix $U_{\bar{A}}{}^{M}(Y) \in E_{6(6)}$ replaced by other "nice objects" [EM '16, '17], [Cassani, Josse, Petrini, Waldram '19]
- Include matter multiplets & no-go theorems [EM '17]
- Consistent truncations around AdS_6 & AdS_7 vacua of type II SUGRA [EM, Samtleben, Vall Camell '18, '19]
- Proof of [Gauntlett, Varela '07]:
  For any (warped) SUSY AdS vacua / Mink vacua of 10-d/11-d SUGRA $\exists$ *consistent truncation*

Kaluza-Klein spectrum

[EM, Samtleben '19?]

- Often many vacua within maximal gSUGRA, e.g.
  - $\mathcal{N} = 2$ AdS vacua of 5-D $\mathcal{N} = 8$ gSUGRA [Khavaev, Pilch, Warner '98]
  - $\mathcal{N} = 0$ AdS vacua of 4-D $\mathcal{N} = 8$ gSUGRA [Warner '84]



- Can we compute the KK spectrum of these vacua?

Kaluza-Klein spectrum

- Harmonics $\mathcal{Y}_\Sigma$, e.g. on S^d: $\mathcal{Y}_\Sigma = (\mathcal{Y}_a, \mathcal{Y}_a\mathcal{Y}_b, \mathcal{Y}_a\mathcal{Y}_b\mathcal{Y}_c, \ldots)$
- Linear action on harmonics

$$U_{\bar{A}}{}^M \partial_M \mathcal{Y}_\Sigma = T_{\bar{A}\Sigma}{}^\Omega \mathcal{Y}_\Omega \,.$$

- K-K Ansatz: $U_{\bar{A}}{}^M \longrightarrow U_{\bar{A}\,\Sigma}{}^M \equiv U_{\bar{A}}{}^M \mathcal{Y}_\Sigma$

$$\begin{aligned}
\mathcal{M}_{MN}(x, Y) &= \mathcal{M}_{\bar{A}\bar{B}\,\Sigma\Omega}(x)\, U^{\bar{A}\,\Sigma}{}_M(Y)\, U^{\bar{B}\,\Omega}{}_N(Y)\,, \\
A_\mu{}^M(x, Y) &= A_\mu{}^{\bar{A}\,\Sigma}(x)\, U_{\bar{A}\,\Sigma}{}^M(Y)\,, \\
B_{\mu\nu\,M}(x, Y) &= B_{\mu\nu\,\bar{A}\,\Sigma}(x)\, U_M{}^{\bar{A}\,\Sigma}(Y)\,.
\end{aligned}$$

- Mass matrix: e.g. for 2-forms $\quad (\mathcal{L}_{U_{\bar{A}}} U_{\bar{B}} = X_{\bar{A}\bar{B}}{}^{\bar{C}} U_{\bar{C}})$

$$m^{\bar{A}}{}_\Sigma{}^{\bar{B}}{}_\Omega = 2d^{\bar{A}\bar{C}\bar{D}} X_{\bar{C}\bar{D}}{}^{\bar{B}} \delta_{\Sigma\Omega} + 10 d^{\bar{A}\bar{B}\bar{C}} T_{\bar{C},\Omega\Sigma}\,,$$

Conclusions

- ExFT unifies geometry and fluxes
- Useful tool for constructing consistent truncations, e.g.
  - $\mathrm{AdS}_7 \times S^4$, $\mathrm{AdS}_5 \times S^5$, $\mathrm{AdS}_4 \times S^7$
  - SUSY AdS_7, AdS_6 vacua of type II
- Compute Kaluza-Klein spectrum for any vacuum in consistent truncation