# Matter Fields in AdS Model of Noncommutative Gravity

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# Plan of the talk

1. $SO(2,3)_\star$ NC gravity (AdS gauge theory of gravity and its canonical NC deformation)

2. Matter fields (Dirac field, gauge fields)

3. NC $OSp(2|4)$ SUGRA (canonical deformation of $N = 2$ AdS SUGRA in $D = 4$)

# AdS algebra

AdS algebra $\mathfrak{so}(2,3)$ has ten generators $M_{AB} = -M_{BA}$ $(A, B = 0, 1, 2, 3, 5)$

$$[M_{AB}, M_{CD}] = i(\eta_{AD} M_{BC} + \eta_{BC} M_{AD} - \eta_{AC} M_{BD} - \eta_{BD} M_{AC})$$

$\eta_{AB} = (+, -, -, -, +)$. Split the generators into six AdS rotations $M_{ab}$ and four AdS translations $M_{a5}$ $(a, b = 0, 1, 2, 3)$. Introduce $P_a := l^{-1} M_{a5}$, where $l$ is AdS radius parameter.

$$\begin{aligned}
[P_a, P_b] &= -il^{-2} M_{ab} \\
[M_{ab}, P_c] &= i(\eta_{bc} P_a - \eta_{ac} P_b) \ , \\
[M_{ab}, M_{cd}] &= i(\eta_{ad} M_{bc} + \eta_{bc} M_{ad} - \eta_{ac} M_{bd} - \eta_{bd} M_{ac})
\end{aligned}$$

In the $l \to \infty$ limit, AdS algebra $\to$ Poincaré algebra (**WI contraction**).

Representation

$$M_{AB} = \frac{i}{4}[\Gamma_A, \Gamma_B] \ , \qquad \{\Gamma_A, \Gamma_B\} = 2\eta_{AB} \ , \qquad \Gamma_A = (i\gamma_a\gamma_5, \gamma_5)$$

In this particular representation, $M_{ab} = \frac{i}{4}[\gamma_a, \gamma_b] = \frac{1}{2}\sigma_{ab}$ and $M_{a5} = -\frac{1}{2}\gamma_a$.

# AdS gauge theory of gravity

AdS gauge parameter, AdS gauge field and AdS field strength :

$$\epsilon = \frac{1}{2}\epsilon^{AB}M_{AB} = \frac{1}{4}\epsilon^{ab}\sigma_{ab} - \frac{1}{2}\epsilon^{a5}\gamma_a \,, \quad \omega_\mu = \frac{1}{2}\omega_\mu{}^{AB}M_{AB} = \frac{1}{4}\omega_\mu{}^{ab}\sigma_{ab} - \frac{1}{2}\omega_\mu{}^{a5}\gamma_a$$

$$F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu - i[\omega_\mu, \omega_\nu] = \left(R_{\mu\nu}{}^{ab} - (\omega_\mu{}^{a5}\omega_\nu{}^{b5} - \omega_\mu{}^{b5}\omega_\nu{}^{a5})\right)\frac{\sigma_{ab}}{4} - F_{\mu\nu}{}^{a5}\frac{\gamma_a}{2}$$

$$R_{\mu\nu}{}^{ab} = \partial_\mu\omega_\nu{}^{ab} - \partial_\nu\omega_\mu{}^{ab} + \omega_\mu{}^a{}_c\,\omega_\nu{}^{cb} - \omega_\nu{}^a{}_c\,\omega_\mu{}^{cb} \,, \qquad F_{\mu\nu}{}^{a5} = D_\mu^L\omega_\nu{}^{a5} - D_\nu^L\omega_\mu{}^{a5}$$

$$\begin{aligned}
\delta_\epsilon\omega_\mu{}^{ab} &= \partial_\mu\epsilon^{ab} - \epsilon^a{}_c\,\omega_\mu{}^{cb} + \epsilon^b{}_c\,\omega_\mu{}^{ca} - \epsilon^{a5}\,\omega_\mu{}^{5b} + \epsilon^{b5}\,\omega_\mu{}^{5a} \\
\delta_\epsilon\omega_\mu{}^{a5} &= \partial_\mu\epsilon^{a5} - \epsilon^a{}_c\,\omega_\mu{}^{c5} + \epsilon^5{}_c\,\omega_\mu{}^{ca} \\
\delta_\epsilon F_{\mu\nu}{}^{ab} &= -\epsilon^{ac}F_{\mu\nu c}{}^b + \epsilon^{bc}F_{\mu\nu c}{}^a - \epsilon^{a5}F_{\mu\nu 5}{}^b + \epsilon^{b5}F_{\mu\nu 5}{}^a \\
\delta_\epsilon F_{\mu\nu}{}^{a5} &= -\epsilon^{ac}F_{\mu\nu c}{}^5 + \epsilon^{5c}F_{\mu\nu c}{}^a
\end{aligned}$$

$$F_{\mu\nu} = \left(R_{\mu\nu}{}^{ab} - \frac{1}{l^2}(e_\mu{}^a e_\nu{}^b - e_\mu{}^b e_\nu{}^a)\right)\frac{\sigma_{ab}}{4} - T_{\mu\nu}{}^a\frac{\gamma_a}{2l}$$

[MacDowell & Mansouri 1977, Townsend 1977, Stelle & West 1980, Wilczek 1998]

# AdS gauge-invariant action

Introduce an auxiliary field $\phi = \phi^A \Gamma_A$; it is a space-time scalar and internal space 5-vector transforming in the adjoint representation of $SO(2,3)$, that is $\delta_\epsilon \phi = i[\epsilon, \phi]$. Constraint $\phi^2 = \eta_{AB}\phi^A\phi^B = l^2$. Cov. derivative $D_\mu \phi = \partial_\mu \phi - i[\omega_\mu, \phi]$.

$$S_1 = \frac{ilc_1}{64\pi G_N} \mathrm{Tr} \int \mathrm{d}^4 x \; \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \phi$$

$$S_2 = \frac{c_2}{28\pi G_N l} \mathrm{Tr} \int \mathrm{d}^4 x \; \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} D_\rho \phi D_\sigma \phi \phi + c.c.$$

$$S_3 = -\frac{ic_3}{128\pi G_N l} \mathrm{Tr} \int d^4 x \; \varepsilon^{\mu\nu\rho\sigma} D_\mu \phi D_\nu \phi D_\rho \phi D_\sigma \phi \phi$$

Physical gauge : $\phi^a = 0$ $(a = 0, 1, 2, 3)$, $\phi^5 = l$ and $\phi|_{\text{g.f.}} = l\gamma_5$. Covariant derivative becomes $(D_\mu \phi)^a|_{\text{g.f.}} = e^a_\mu$ and $(D_\mu \phi)^5|_{\text{g.f.}} = 0$.

$$\begin{aligned} S_{AdS}|_{\text{g.f.}} &= S_1|_{\text{g.f.}} + S_2|_{\text{g.f.}} + S_3|_{\text{g.f.}} \\ &= -\frac{1}{16\pi G_N} \int \mathrm{d}^4 x \left( e\Big( (c_1 + c_2) R - \frac{6}{l^2}(c_1 + 2c_2 + 2c_3) \Big) + \frac{c_1 l^2}{16} \epsilon^{\mu\nu\rho\sigma} R_{\mu\nu}{}^{mn} R_{\rho\sigma}{}^{rs} \epsilon_{mnrs} \right) \end{aligned}$$

1. Einstein-Hilbert $(c_1 + c_2 = 1)$
2. Topological Gauss-Bonet
3. Cosmological constant $\Lambda = -3(c_1 + 2c_2 + 2c_3) l^{-2}$ (vanishes under WI contraction)

# NC deformation

1. *Deformation quantization* (phase space quantum mechanics). Classical system $(\mathcal{M}, \omega, H)$ is deformed by imposing noncommutative (NC) geometry on its phase space ; $\star$-product deformation of commutative algebra $C^\infty(\mathcal{M})$. [Kontsevich, Gelfand–Naimark]

2. NC Field Theory - field theory on NC-deformed space-time. Introduce an abstract algebra of coordinates
$$[\hat{x}^\mu, \hat{x}^\nu] = iC^{\mu\nu}(\hat{x}) \ .$$
*Canonical (or $\theta$-constant) deformation*,
$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu} \sim l_{NC}^2 \ ,$$
with constant deformation parameters $\theta^{\mu\nu} = -\theta^{\nu\mu}$.

For *canonical noncommutativity*, we use the Moyal $\star$-product,
$$\begin{aligned}(\hat{f} \star \hat{g})(x) &= e^{\frac{i}{2}\,\theta^{\mu\nu}\,\frac{\partial}{\partial x^\mu}\,\frac{\partial}{\partial y^\nu}} f(x)g(y)|_{y\to x} \\ &= f(x)g(y) + \frac{i}{2}\theta^{\mu\nu}\partial_\mu f(x)\partial_\nu g(x) + \mathcal{O}(\theta^2) \ .\end{aligned}$$
The leading term is the commutative point-wise multiplication, and the higher order terms represent (non-classical) NC corrections.

# NC gauge field theory

Let $\{T_A\}$ satisfy some Lie algebra relations $[T_A, T_B] = i f_{AB}{}^C T_C$. Closure rule holds

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] = \delta_{-i[\epsilon_1, \epsilon_2]} \,.$$

If NC gauge parameter $\hat{\Lambda}(x) = \hat{\Lambda}^A(x) T_A$,

$$\begin{aligned}
[\delta_1^\star \overset{\star}{,} \delta_2^\star]\hat{\Psi} &= (\hat{\Lambda}_1 \star \hat{\Lambda}_2 - \hat{\Lambda}_2 \star \hat{\Lambda}_1) \star \hat{\Psi} \\
&= \frac{1}{2}\Big([\hat{\Lambda}_1^A \overset{\star}{,} \hat{\Lambda}_2^B]\{T_A, T_B\} + \{\hat{\Lambda}_1^A \overset{\star}{,} \hat{\Lambda}_2^B\}[T_A, T_B]\Big) \star \hat{\Psi} = i\hat{\Lambda}_3 \star \hat{\Psi} = \delta_3^\star \hat{\Psi} \,.
\end{aligned}$$

1. *Universal enveloping algebra (UEA) approach*; infinite number of dofs.
2. *Seiberg-Witten (SW) map*; $\hat{\Lambda} = \hat{\Lambda}(\epsilon, V_\mu; \partial\epsilon, \partial V_\mu, \ldots)$ and $\hat{V}_\mu = \hat{V}_\mu(V_\mu, \partial V_\mu, \ldots)$; induced NC gauge transformations

$$\delta_\Lambda^\star \hat{V}_\mu = \hat{V}_\mu(V_\mu + \delta_\epsilon V_\mu) - \hat{V}_\mu(V_\mu) \,.$$

SW map between NC and classical fields :

$$\hat{\Lambda}_\epsilon = \epsilon - \frac{1}{4}\theta^{\rho\sigma}\{V_\rho, \partial_\sigma \epsilon\} + \mathcal{O}(\theta^2) \,, \quad \hat{V}_\mu = V_\mu - \frac{1}{4}\theta^{\rho\sigma}\{V_\rho, \partial_\sigma V_\mu + F_{\sigma\mu}\} + \mathcal{O}(\theta^2)$$

$$\hat{F}_{\mu\nu} = F_{\mu\nu} - \frac{1}{4}\theta^{\rho\sigma}\{V_\rho, (\partial_\sigma + D_\sigma)F_{\mu\nu}\} + \frac{1}{2}\theta^{\rho\sigma}\{F_{\rho\mu}, F_{\sigma\nu}\} + \mathcal{O}(\theta^2)$$

$$\hat{\Psi} = \Psi - \frac{1}{4}\theta^{\rho\sigma} V_\rho(\partial_\sigma + D_\sigma)\Psi + \mathcal{O}(\theta^2)$$

# $SO(2,3)_\star$ model of pure NC gravity

NC action invariant under $SO(2,3)_\star$ NC gauge transformations :

$$S_1^\star = \frac{ilc_1}{64\pi G_N}\mathrm{Tr}\int \mathrm{d}^4x\epsilon^{\mu\nu\rho\sigma}\hat{F}_{\mu\nu}\star\hat{F}_{\rho\sigma}\star\hat{\phi}$$

$$S_2^\star = \frac{c_2}{128\pi G_N l}\mathrm{Tr}\int \mathrm{d}^4x\epsilon^{\mu\nu\rho\sigma}\hat{\phi}\star\hat{F}_{\mu\nu}\star D_\rho\hat{\phi}\star D_\sigma\hat{\phi} + c.c.$$

$$S_3^\star = -\frac{ic_3}{128\pi G_N l}\mathrm{Tr}\int \mathrm{d}^4x\,\varepsilon^{\mu\nu\rho\sigma}D_\mu\hat{\phi}\star D_\nu\hat{\phi}\star D_\rho\hat{\phi}\star D_\sigma\hat{\phi}\star\hat{\phi}$$

After SW expansion and SB (in that order !) :

$$S^\star_{AdS}|_{\mathrm{g.f.}} = -\frac{1}{16\pi G_N}\int \mathrm{d}^4x\, e\Big(R(e,\omega) - \frac{6}{l^2}(1+c_2+2c_3) + \theta^{\alpha\beta}\theta^{\gamma\delta}\Big((-2+12c_2+38c_3)R_{\alpha\beta\gamma\delta}$$
$$-(5-\frac{9}{2}c_2-7c_3)T_{\alpha\beta}{}^{\rho}T_{\gamma\delta\rho} + \frac{1}{l^2}(6+28c_2+56c_3)g_{\alpha\gamma}g_{\beta\delta}+\ldots\Big)\Big)$$

M. Dimitrijević Ćirić, B. Nikolić & V. Radovanović, *NC $SO(2,3)_\star$ gravity : noncommutativity as a source of curvature and torsion*, Phys. Rev. D **96**, 064029 (2017).

P. Aschieri, L. Castellani, & M. Dimitrijević, *Noncommutative gravity at second order via Seiberg-Witten map*. Phys. Rev. D **87**, 024017 (2013).

# $SO(2,3)_\star$ model of pure NC gravity - solutions

1. NC equations of motion in the *low energy limit* ($\sim \partial^2$) :

$$\delta e^a_\mu : \; R_{\alpha\beta}{}^{cb} e^\alpha_a e^\beta_b e^\mu_c - \frac{1}{2} e^\mu_a R + \frac{3}{l^2}(1 + c_2 + 2c_3) e^\mu_a = \tau_a{}^\mu = -\frac{8\pi G_N}{e} \delta S^{(2)} / \delta e^a_\mu \sim \theta^2$$

$$\delta \omega^{ab}_\mu : \; T_{ac}{}^c e^\mu_b - T_{bc}{}^c e^\mu_a - T_{ab}{}^c e^\mu_c = S_{ab}{}^\mu = -\frac{16\pi G_N}{e} \delta S^{(2)} / \delta \omega^{ab}_\mu \sim \theta^2$$

**Noncommutativity is a source of curvature and torison**.

2. Canonical deformation of *Minkowski space* ($R \sim \theta^2$) :

$$g_{00} = 1 - R_{0m0n} x^m x^n \; , \quad g_{0i} = -\frac{2}{3} R_{0min} x^m x^n \; , \quad g_{ij} = -\delta_{ij} - \frac{1}{3} R_{imjn} x^m x^n \; .$$

Canonical (or $\theta$-constant) deformation $[x^\mu \stackrel{\star}{,} x^\nu] = i\theta^{\mu\nu} \rightarrow$ Fermi inertial coordinates

**Diffeomorphism invariance is broken because we implicitly fix the coordinate system**.

# Fermions in $SO(2,3)$ framework

Commutative spinor action :

$$S_{\psi,kin} = \frac{i}{12}\int \mathrm{d}^4x\ \varepsilon^{\mu\nu\rho\sigma}\Big[\bar{\psi}D_\mu\phi D_\nu\phi D_\rho\phi D_\sigma\psi - D_\sigma\bar{\psi}D_\mu\phi D_\nu\phi D_\rho\phi\psi\Big]$$

AdS covariant derivative :

$$D_\mu\psi = \partial_\mu\psi - i\omega_\mu\psi\ , \qquad D_\mu\phi = \partial_\mu\phi - i[\omega_\mu, \phi]$$

For spinors it splits into Lorentz $SO(1,3)$ covariant derivative and AdS extension

$$D_\mu\psi = D^L_\mu\psi + \frac{i}{2l}e^a_\mu\gamma_a\psi$$

Gauge fixing ($\phi^a = 0$ and $\phi^5 = l$) yields

$$S_{\psi,kin}|_{\text{g.f.}} = \frac{i}{2}\int \mathrm{d}^4x\ e\Big[\bar{\psi}\gamma^\mu D^L_\mu\psi - D^L_\mu\bar{\psi}\gamma^\mu\psi\Big] - \frac{2}{l}\int \mathrm{d}^4x\ e\,\bar{\psi}\psi$$

Dirac action in curved space-time for spinors, with universal mass-like term $2/l$ that vanishes under WI contraction ($l \to \infty$).

- for massive fermions we introduce supplementary bilinear $(\bar{\psi}...\psi)$ $SO(2,3)$ gauge-invariant terms.

# NC deformation with fermions

NC-deformed $SO(2,3)_\star$ invariant spinor action before SB :

$$S_\psi^\star = \frac{i}{12}\int \mathrm{d}^4x\, \varepsilon^{\mu\nu\rho\sigma}\Big[\hat{\bar{\psi}}\star(D_\mu\hat\phi)\star(D_\nu\hat\phi)\star(D_\rho\hat\phi)\star(D_\sigma\hat\psi)$$
$$-(D_\sigma\hat{\bar{\psi}})\star(D_\mu\hat\phi)\star(D_\nu\hat\phi)\star(D_\rho\hat\phi)\star\hat\psi\Big] + S^\star_{\psi,m}$$

SW expansions :

$$\hat\psi = \psi - \tfrac{1}{4}\theta^{\alpha\beta}\omega_\alpha(\partial_\beta + D_\beta)\psi + \mathcal{O}(\theta^2)$$
$$\hat\phi = \phi - \tfrac{1}{4}\theta^{\alpha\beta}\{\omega_\alpha,(\partial_\beta + D_\beta)\phi\} + \mathcal{O}(\theta^2)$$
$$D_\mu\hat\psi = D_\mu\psi - \tfrac{1}{4}\theta^{\alpha\beta}\omega_\alpha(\partial_\beta + D_\beta)D_\mu\psi + \frac{1}{2}\theta^{\alpha\beta}F_{\alpha\mu}D_\beta\psi + \mathcal{O}(\theta^2)$$
$$D_\mu\hat\phi = D_\mu\phi - \tfrac{1}{4}\theta^{\alpha\beta}\{\omega_\alpha(\partial_\beta + D_\beta)D_\mu\phi\} + \tfrac{1}{2}\theta^{\alpha\beta}\{F_{\alpha\mu},D_\beta\phi\} + \mathcal{O}(\theta^2)$$

Linear NC correction to Dirac action in curved spacetime after g.f. :

$$S_\psi^{(1)} = \theta^{\alpha\beta}\int \mathrm{d}^4x\, e\,\Big(-\frac{1}{8}R_{\alpha\mu}{}^{ab}e^\mu_a(\bar\psi\gamma_b D^L_\beta\psi) + \frac{i}{8l}T_{\alpha\mu}{}^a e^\mu_a(\bar\psi D^L_\beta\psi)$$
$$-\frac{1}{4l}(\bar\psi\sigma_\alpha{}^\sigma D^L_\beta D^L_\sigma\psi) + \frac{1}{96l}R_{\alpha\beta}{}^{ab}(\bar\psi\sigma_{ab}\psi) - \frac{1}{16l^2}T_{\alpha\mu}{}^a e^\mu_a(\bar\psi\gamma_\beta\psi) - \frac{1}{3l^3}(\bar\psi\sigma_{\alpha\beta}\psi) + \ldots\Big)$$

**[P. Aschieri, L. Castellani (2012), (2014) ; Majorana $\sim \theta^2$ and Dirac $\sim \theta$]**

# Minkowski space

Residual NC effects in flat spacetime (we set $\Lambda = 0$)

$$S^{(1)}_{\psi,\mathrm{flat}} = \theta^{\alpha\beta} \int \mathrm{d}^4 x \left[ -\frac{1}{2l}(\bar{\psi}\sigma_\alpha{}^\sigma \partial_\beta \partial_\sigma \psi) + \frac{7i}{24 l^2} \varepsilon_{\alpha\beta}{}^{\rho\sigma} (\bar{\psi}\gamma_\rho \gamma_5 \partial_\sigma \psi) - \left( \frac{m}{4l^2} + \frac{1}{6l^3} \right) (\bar{\psi}\sigma_{\alpha\beta}\psi) \right]$$

**NC Dirac equation** : $\left(i\gamma^\mu \partial_\mu - m + \theta^{\alpha\beta} \mathcal{M}_{\alpha\beta}\right)\psi = 0$

For "free" electron, $[H, \mathbf{p}] = 0 \Rightarrow$ plane wave ansatz $\psi(x) = u(\mathbf{p}) e^{-ip\cdot x}$

For $\mathbf{p} = (0, 0, p_z)$ and $[\hat{x}^1, \hat{x}^2] = i\theta^{12}$, $\theta^{12} \equiv \theta \neq 0$, we have four independent energy functions :

Electron's energy depends on its *helicity* → NC **"Zeeman effect"**

$$E_{1,2} = E_\mathbf{p} \mp \left[ \frac{m^2}{12 l^2} - \frac{m}{3 l^3} \right] \frac{\theta}{E_\mathbf{p}} + \mathcal{O}(\theta^2)$$

$$E_{3,4} = -E_\mathbf{p} \pm \left[ \frac{m^2}{12 l^2} - \frac{m}{3 l^3} \right] \frac{\theta}{E_\mathbf{p}} + \mathcal{O}(\theta^2)$$

with classical energy $E_\mathbf{p} = \sqrt{m^2 + p_z^2}$.

Background NC space behaves as a *birefringent* medium for electrons propagating in it.

# Gauge fields in $SO(2,3)$ framework

For gauge group $SO(2,3) \times U(1)$, gauge field and field strength :

$$\Omega_\mu = \frac{1}{2}\omega_\mu{}^{AB} M_{AB} \otimes \mathbb{I} + \mathbb{I} \otimes A_\mu \ , \quad \mathbb{F}_{\mu\nu} = \frac{1}{2} F_{\mu\nu}{}^{AB} M_{AB} \otimes \mathbb{I} + \mathbb{I} \otimes \mathcal{F}_{\mu\nu}$$

Commutative model (**no Hodge dual**)

$$S_A = -\frac{1}{16l} \mathrm{Tr} \int \mathrm{d}^4 x \ \varepsilon^{\mu\nu\rho\sigma} \left( f \mathbb{F}_{\mu\nu} D_\rho \phi D_\sigma \phi \phi + \frac{i}{3!} f^2 D_\mu \phi D_\nu \phi D_\rho \phi D_\sigma \phi \phi \right) + c.c.$$

Auxiliary field $f = \frac{1}{2} f^{AB} M_{AB}$ ; EoMs $f_{a5} = 0$ and $f_{ab} = -e_a^\mu e_b^\nu \mathcal{F}_{\mu\nu}$.

P. Aschieri & L. Castellani, General Relativity and Gravitation, 45(3), 581-598 (2013).

$$S_{A,EoM}|_{\text{g.f.}} = -\frac{1}{4} \int \mathrm{d}^4 x \ \sqrt{-g} \ g^{\mu\rho} g^{\nu\sigma} \mathcal{F}_{\mu\nu} \mathcal{F}_{\rho\sigma}$$

M. Dimitrijević Ćirić, D. G., N. Konjik & V. Radovanović, Eur. Phys. J. C, **78** 548 (2018).

1. $S_A^{(1)} \neq 0$
2. NC-deformed relativistic Landau levels

# NC OSp(1|4) SUGRA

L. Castellani, Physical Review D, 88(2), 025022 (2013).

1. Orthosymplectic supergroup $OSp(1|4)$ has 14 generators - 10 AdS generators $\hat{M}_{AB}$ and 4 fermionic generators $\hat{Q}_\alpha$ comprising a single Majorana spinor.
2. Supermatrix for the $OSp(1|4)$ gauge field $\Omega_\mu$ is given by

$$\Omega_\mu = \frac{1}{2}\omega_\mu^{\ AB}\hat{M}_{AB} + \frac{1}{\sqrt{l}}\bar{\psi}_\mu^\alpha \hat{Q}_\alpha = \left(\begin{array}{c|c} \omega_\mu & \frac{1}{\sqrt{l}}\psi_\mu \\ \hline \frac{1}{\sqrt{l}}\bar{\psi}_\mu & 0 \end{array}\right) .$$

3. Action with $OSp(1|4)$ gauge symmetry :

$$S_{14} = \frac{il}{32\pi G_N}\,\mathrm{STr}\int \mathrm{d}^4x\; \varepsilon^{\mu\nu\rho\sigma}\mathbb{F}_{\mu\nu}\Big(\mathbb{I}_{5\times 5} - \tfrac{1}{2l^2}\Phi^2\Big)\mathbb{F}_{\rho\sigma}\Phi$$

Auxiliary field

$$\Phi|_{\mathrm{g.f.}} \left(\begin{array}{c|c} l\gamma_5 & 0 \\ \hline 0 & 0 \end{array}\right)$$

In the physical gauge, the action **exactly** reduces to $N = 1$ $AdS_4$ SUGRA action

$$S_{14}|_{\mathrm{g.f.}} = -\frac{1}{2\kappa^2}\int \mathrm{d}^4x \left(e\big(R(e,\omega) - 6/l^2\big) + 2\varepsilon^{\mu\nu\rho\sigma}\big(\bar{\psi}_\mu\gamma_5\gamma_\nu\Big(D^L_\rho + \frac{i}{2l}\gamma_\rho\Big)\psi_\sigma\big)\right)$$

4. **The leading non-vanishing NC correction is quadratic in $\theta$.**

# OSp(2|4) SUGRA

Orthosymplectic group $OSp(2|4)$ has 19 generators ($\hat{M}_{AB}$, $\hat{Q}_\alpha^{I=1,2}$, $\hat{T}$).

$$\Omega_\mu = \hat{\omega}_\mu + \frac{1}{\sqrt{l}}\bar{\psi}^I_\mu \hat{Q}^I + \frac{1}{l}\mathcal{A}_\mu \hat{T} = \left(\begin{array}{c|cc} \omega_\mu & \frac{1}{\sqrt{l}}\psi^1_\mu & \frac{1}{\sqrt{l}}\psi^2_\mu \\ \hline \frac{1}{\sqrt{l}}\bar{\psi}^1_\mu & 0 & \frac{i}{l}\mathcal{A}_\mu \\ \frac{1}{\sqrt{l}}\bar{\psi}^2_\mu & -\frac{i}{l}\mathcal{A}_\mu & 0 \end{array}\right).$$

Majorana spinors, $\psi^1_\mu$ and $\psi^2_\mu$, can be combined into an $SO(2)$ doublet,

$$\Psi_\mu = \begin{pmatrix} \psi^1_\mu \\ \psi^2_\mu \end{pmatrix}$$

Charged Dirac vector-spinors $\psi^\pm_\mu = \psi^1_\mu \pm i\psi^2_\mu$, related by $C$-conjugation, $\psi^-_\mu = C\bar{\psi}^{+T}_\mu$.

$$S_{24} = \frac{il}{32\pi G_N}\mathrm{STr}\int \mathrm{d}^4x\, \varepsilon^{\mu\nu\rho\sigma}\mathbb{F}_{\mu\nu}\left(\mathbb{I}_{6\times 6} - \frac{1}{2l^2}\Phi^2\right)\mathbb{F}_{\rho\sigma}\Phi$$

**Gauge-fixed action is not complete!**

$$S_{24}|_{\text{g.f.}} = -\frac{1}{2\kappa^2}\int \mathrm{d}^4x \left(e\Big(R(e,\omega) - 6/l^2\Big) + \frac{l^2}{16}R_{\mu\nu}{}^{mn}R_{\rho\sigma}{}^{rs}\varepsilon^{\mu\nu\rho\sigma}\varepsilon_{mnrs}\right)$$

$$+\,\varepsilon^{\mu\nu\rho\sigma}\Big(2\bar{\Psi}_\mu\gamma_5\gamma_\nu(D_\rho + l^{-1}\mathcal{A}_\rho i\sigma^2)\Psi_\sigma + i\mathcal{F}_{\mu\nu}(\bar{\Psi}_\rho\gamma_5 i\sigma^2\Psi_\sigma) - \frac{i}{2}(\bar{\Psi}_\mu i\sigma^2\Psi_\nu)(\bar{\Psi}_\rho\gamma_5 i\sigma^2\Psi_\sigma)\Big)$$

# $OSp(2|4)$ SUGRA

$$
\begin{array}{ccc}
(OSp(2|4) \text{ invariant action}) & + & (SO(2,3)\times U(1) \text{ invariant action}) \\
\text{g.f.}\downarrow & & \downarrow\text{g.f.} \\
(SO(1,3)\times U(1) \text{ invariant action}) & + & (SO(1,3)\times U(1) \text{ invariant action})
\end{array}
$$

$$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}_{\text{N=2 AdS SUGRA in D=4}}$$

$OSp(2|4)$ field strength (bosonic blocks)

$$\mathbb{F}_{\mu\nu} = \left(\begin{array}{c|cc} \widetilde{F}_{\mu\nu} & * & * \\ \hline * & 0 & \frac{i}{l}\widetilde{\mathcal{F}}_{\mu\nu} \\ * & -\frac{i}{l}\widetilde{\mathcal{F}}_{\mu\nu} & 0 \end{array}\right)$$

Bosonic field strength : $\widetilde{f}_{\mu\nu} := \widetilde{F}_{\mu\nu} + \kappa^{-1}\widetilde{\mathcal{F}}_{\mu\nu} = \widetilde{F}_{\mu\nu} + \kappa^{-1}(\mathcal{F}_{\mu\nu} - \bar{\Psi}_\mu i\sigma^2\Psi_\nu)$

$$S_A \sim Tr\int \mathrm{d}^4x\, \varepsilon^{\mu\nu\rho\sigma}\left(f\,\widetilde{f}_{\mu\nu}D_\rho\phi D_\sigma\phi\phi + \frac{i}{6}\, f^2 D_\mu\phi D_\nu\phi D_\rho\phi D_\sigma\phi\phi\right) + c.c.$$

$$S^{(1)}_{\text{low-energy}} = -\frac{9i\kappa\theta^{\mu\nu}}{8l^4}\int \mathrm{d}^4x\; e\,(\bar{\psi}^+_\mu\psi^+_\nu) + \text{surface term}$$

# The last frame

Thank you !