Toward emergent spacetime in quantum gravity: Quantum Black Holes from scratch

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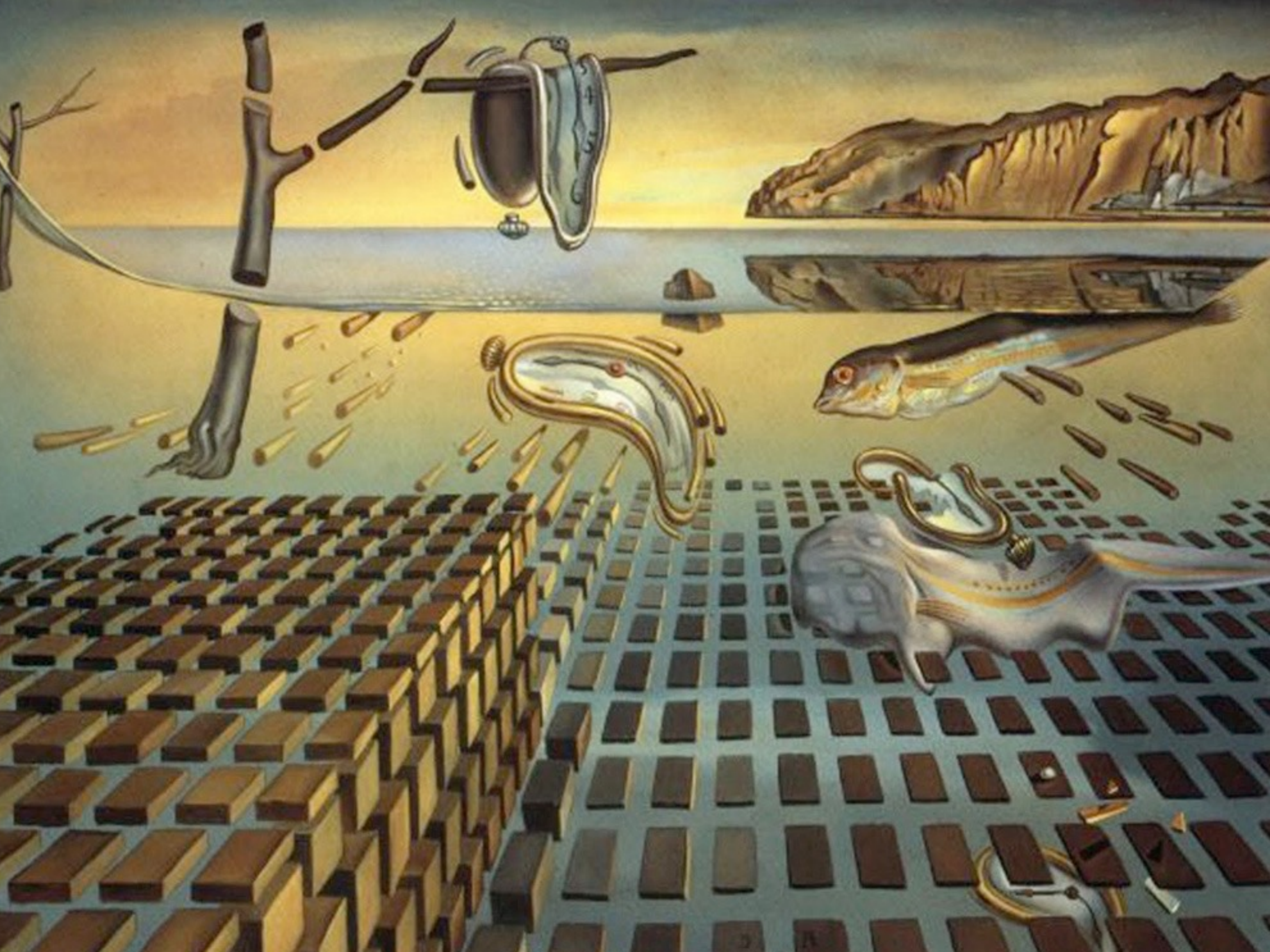
Quantum Gravity
and
the emergence of spacetime

Beyond spacetime? hints from various corners

- challenges to "localization" in semi-classical GR — minimal length scenarios
- spacetime singularities in GR — breakdown of continuum itself?
- black hole thermodynamics — space itself is a thermodynamic system
- black hole information paradox — some fundamental principle has to go: locality?
- Einstein's equations as equation of state — GR dynamics is effective equation of state for any microscopic dofs collectively described by a spacetime, a metric and some matter fields
- entanglement ~ geometry — geometric quantities defined by quantum (information) notions (examples from AdS/CFT, and various quantum many-body systems)

fundamental discreteness of spacetime? breakdown of locality?
is spacetime itself "emergent" from non-spatiotemporal,
non-geometric, quantum building blocks ("atoms of space")?



Spacetime and its atomic constituents

Spacetime is emergent

and made out of non-spatiotemporal quantum building blocks

("atoms of space")

supporting (indirect) evidence/arguments:

- QG approaches (e.g. LQG spin networks)
- BH entropy (finite) and thermodynamics
- GR singularities (breakdown of continuum?)

quantum space as a (background-independent) quantum many-body system

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→ black holes are crucial testing ground

quantum space as a (background-independent) quantum many-body system

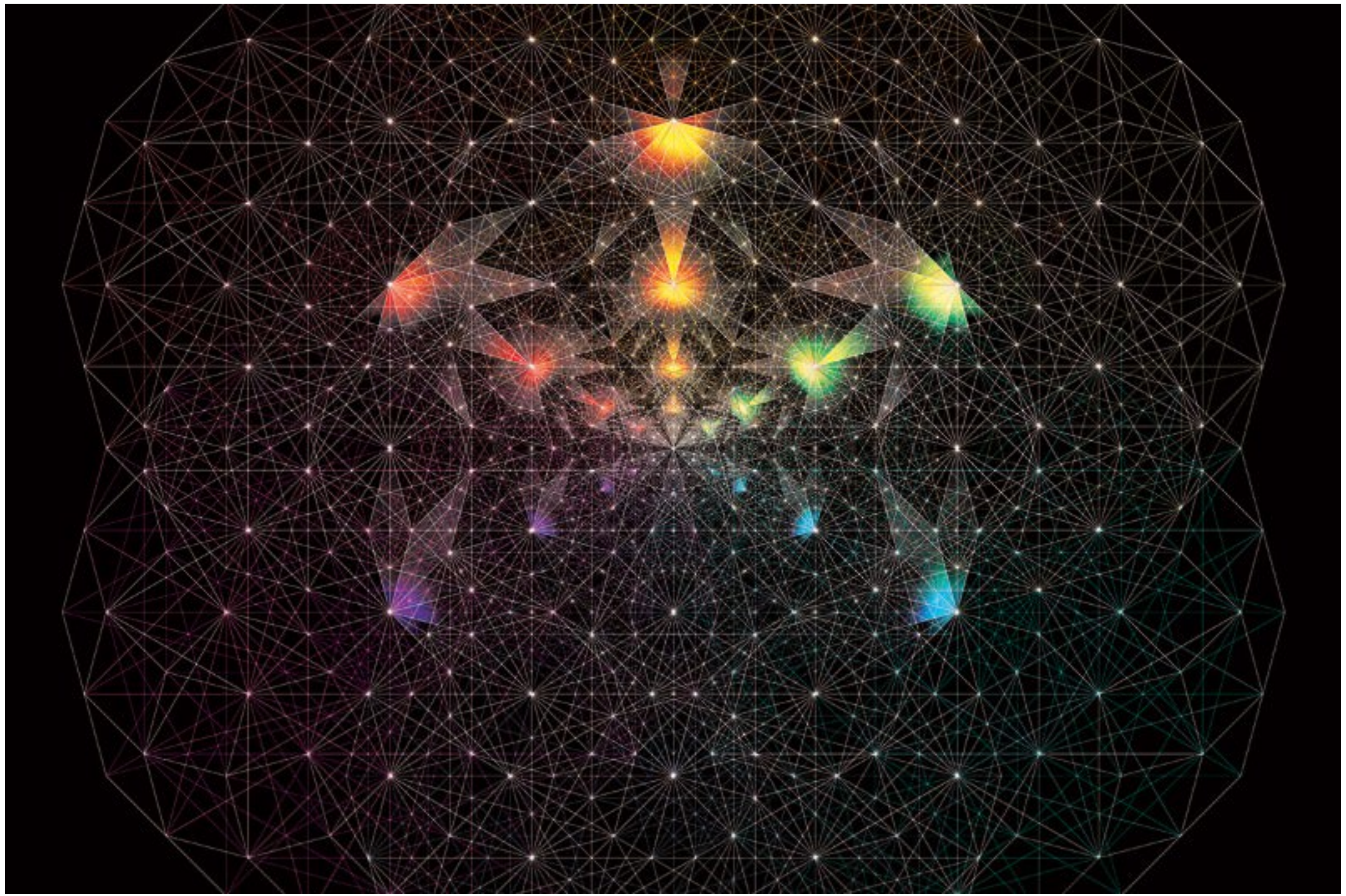
Geometry from Quantum

geometri properties of emergent spacetime to be extracted from quantum properties of building blocks

- fundamental quantum operators directly corresponding to (proto-)geometric quantities
- collective quantum properties corresponding to continuum geometric quantities
- geometry as effective understanding of non-geometric quantum properties

entanglement/geometry correspondence

(entanglement —-> geometry)



supporting evidence:

- many results within AdS/CFT
  - QI aspects of boundary CFT —-> geometric quantities in bulk AdS
- several results in QG
  - entanglement “builds” QG states

Quantum Gravity: new perspective

many current approaches suggest a change of perspective on the quantum gravity problem

traditional perspective:
quantise gravity (i.e. spacetime geometry)

↓

new perspective:
identify quantum structures/building blocks of non-spatiotemporal nature from which spacetime and geometry "emerge" dynamically

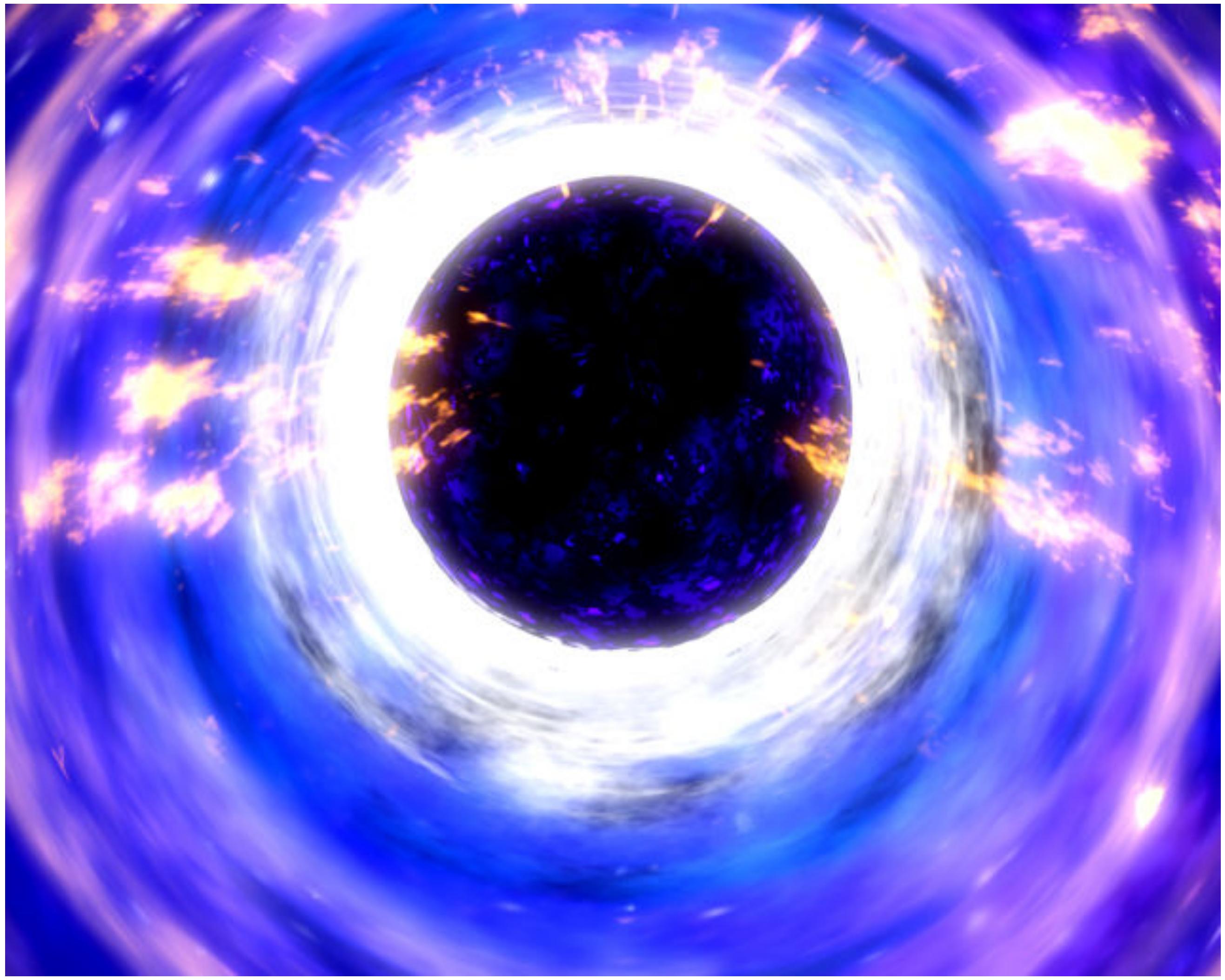
problem becomes similar to the typical one in condensed matter theory (from atoms to macroscopic physics)

Black Holes
and
Quantum Gravity

Black holes call Quantum Gravity

Black holes theoretical challenges: a testbed for quantum gravity

- black hole entropy and thermodynamics
  Jacobson, '99; Sorkin, '05
- black hole microstructure and statistical treatment
  Wallace, '17
- black hole singularity
- black holes and information — Marolf, '17



need Quantum Gravity description

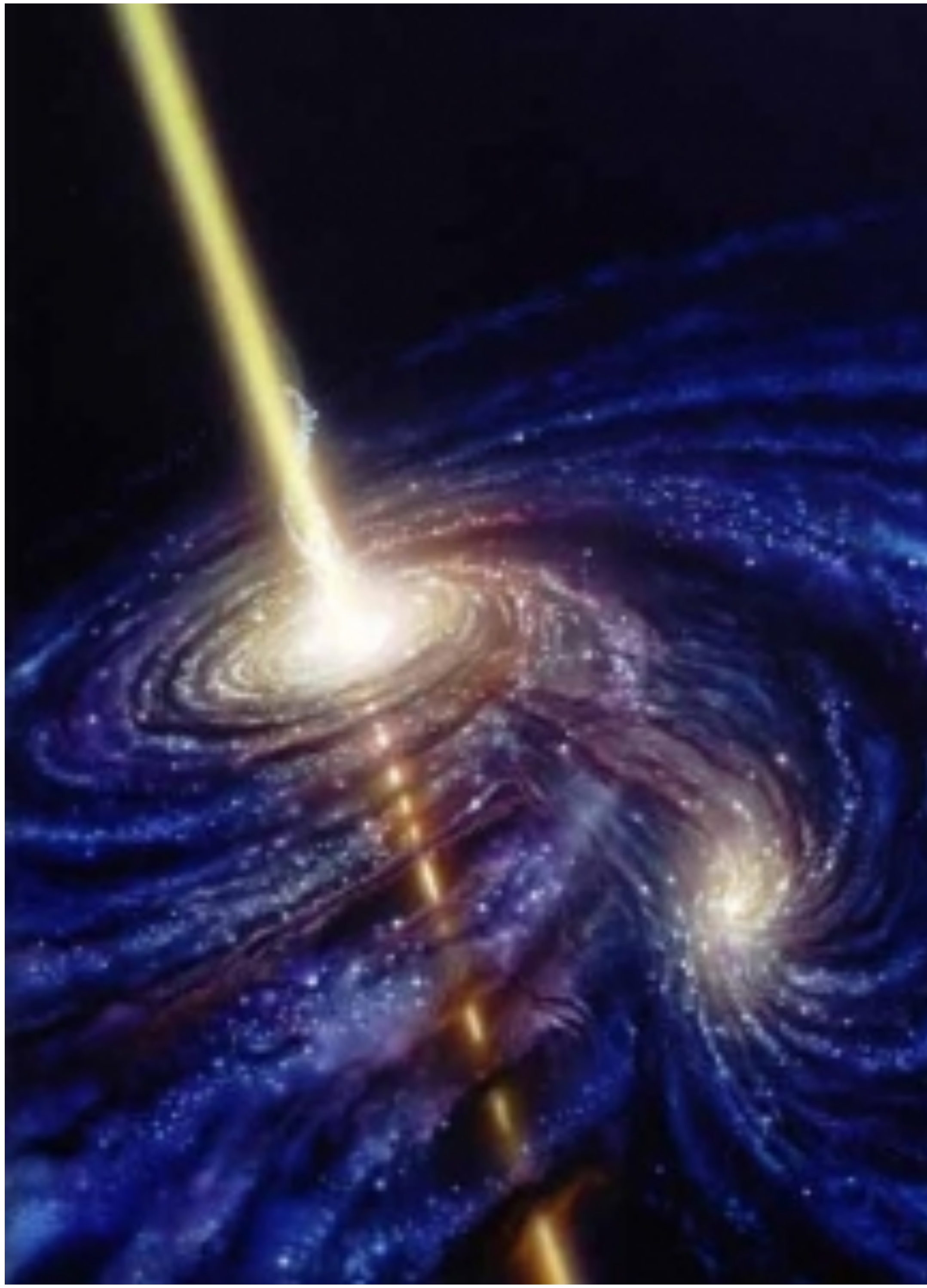
QG modifications to standard spacetime, gravity and QFT → challenges to very definition of black holes

no global causal structure, approx. locality, approx. decomposition int/ext, modifications at horizon scales (consistent with "emergent spacetime" idea)

Quantum Gravity calls Black Holes

Black holes: door to quantum gravity phenomenology

- modified Hawking radiation
- modified horizon structure (eg quantum atmosphere) and gravitational waves
  Giddings, '16; Dey, Liberati, Pranzetti, '17; Cardoso, Pani, '17
- relics of (primordial) BH evaporation as dark matter candidates
  Carr, '16
- modified QFT (eg dispersion relations)
  Liberati, '11
- BH-WH transition. - possibly detectable in (fast) radio bursts
- consequences for gravitational waves - modified quasi-normal modes spectrum
  Barrau, Martineau, Moulin, '18
- ....



Quantum Gravity modelling of Black Holes

intense modelling activity in various quantum gravity formalisms

- in string theory via D-branes (entropy counting, extremal case) Strominger, Vafa, ’96; ….
- BH as quantum condensates at critical point Dvali, Gomez, ‘12
- BHs in causal sets (global causal structure, entanglement entropy) He, Rideout, ‘08
- semi-classical black holes from dual CFT in AdS/CFT correspondence (…….)
- other semi-classical guess-work of BH interior dofs Nomura, Sanchez, Weinberg, ‘15
- Giddings’ statistical approach (challenges to standard entropy picture) Giddings, ‘12
- ……….

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Loop Quantum Gravity modelling of Black Holes

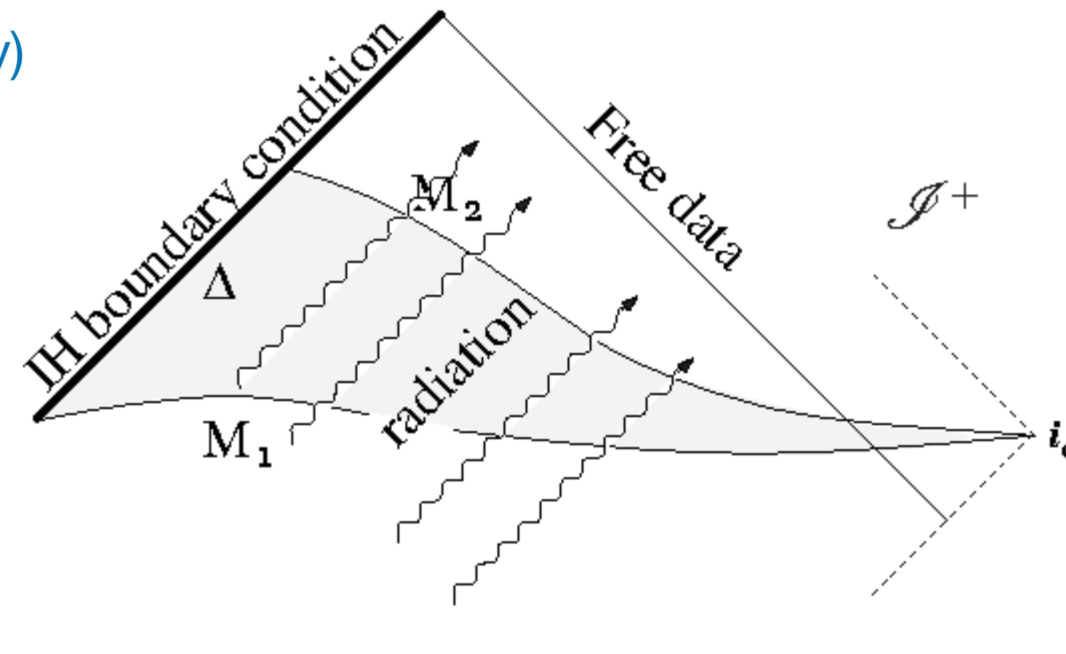
Barbero, Perez, ’15; Diaz-Polo, Pranzetti, ’12; Perez, ‘17

- quantization of isolated horizons (sector of GR, induced CS theory)

Ashtekar, Beetle, Fairhurst, ‘99

+ : quasi-local, microstates counting, entropy calculations (area law, correct coefficient, log corrections)

- : quasi-local, semi-classical modelling, no quantum control over bulk dofs, no control over quantum dynamics



- LQG-inspired symmetry reduced models

Ashtekar, Bojowald, ‘05

quantum singularity resolution

- BH modelling (incl. interior) with simple spin network states

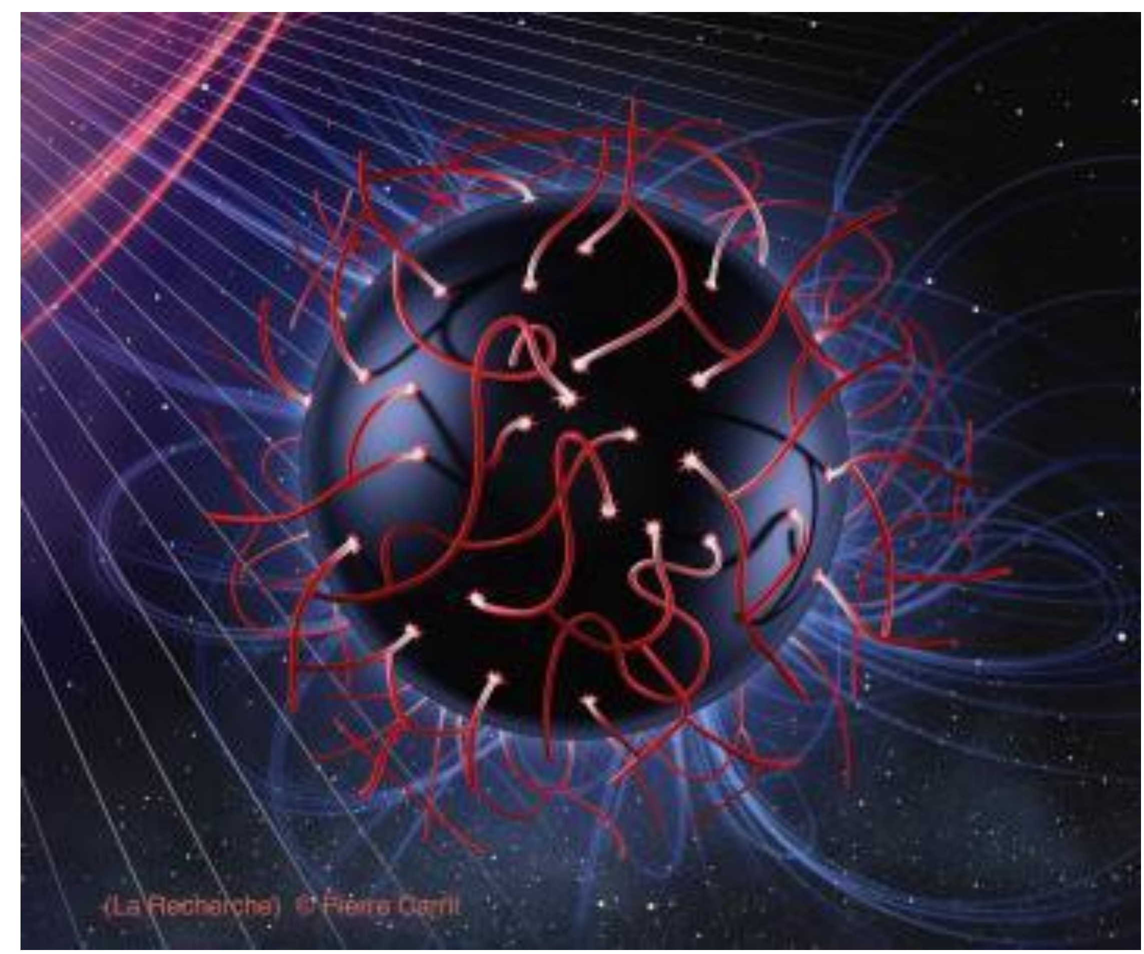
counting of (intertwiner) bulk states, entropy law, entanglement entropy

Livine, Terno, ‘08

nice, compelling picture of BH microstates (spin networks puncturing horizon)

main open issues:

modelling within full theory; superpositions of graphs, continuum limit; quantum dynamics



issue of quantum black holes
from new QG perspective:

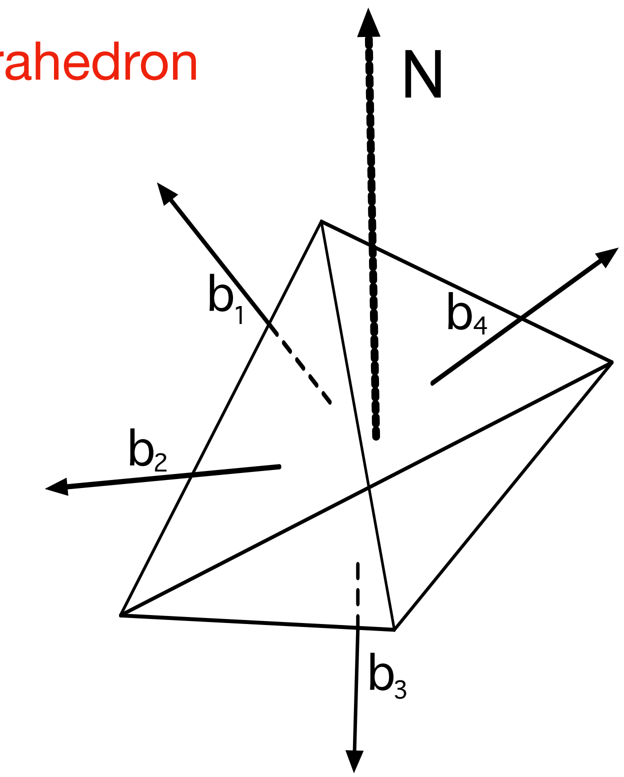
special (and especially challenging) case of
emergent spacetime and geometry

The Group Field Theory formalism

An “atom of space”

Barbieri ’97; Baez, Barrett, ’99; Rovelli, Speziale, ’06; Bianchi, Dona, Speziale, ’10; ……

Elementary building block of 3d space: single polyhedron - simplest example: a tetrahedron

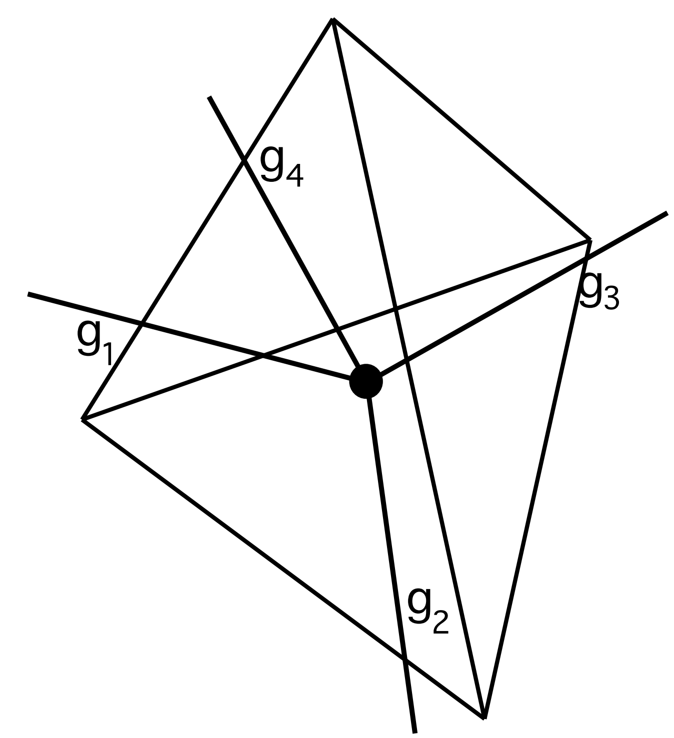


Classical geometry in group-theoretic variables

4 vectors normal to triangles that close (lying in hypersurface with normal N)

$$A_i\, n_i^I = b_i^I \in \mathbb{R}^{3,1} \qquad b_i \cdot N = 0 \qquad \sum_i b_i = 0$$

equivalent formulation in terms of irreps of G



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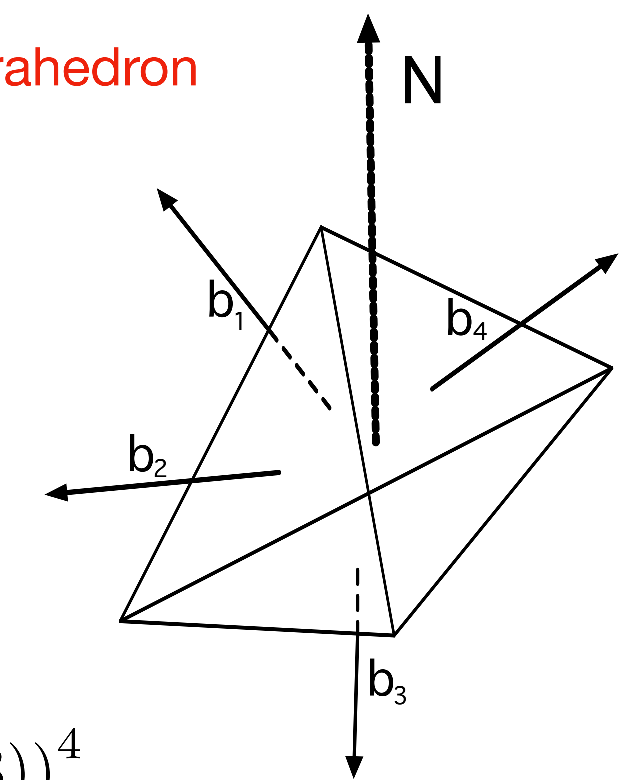

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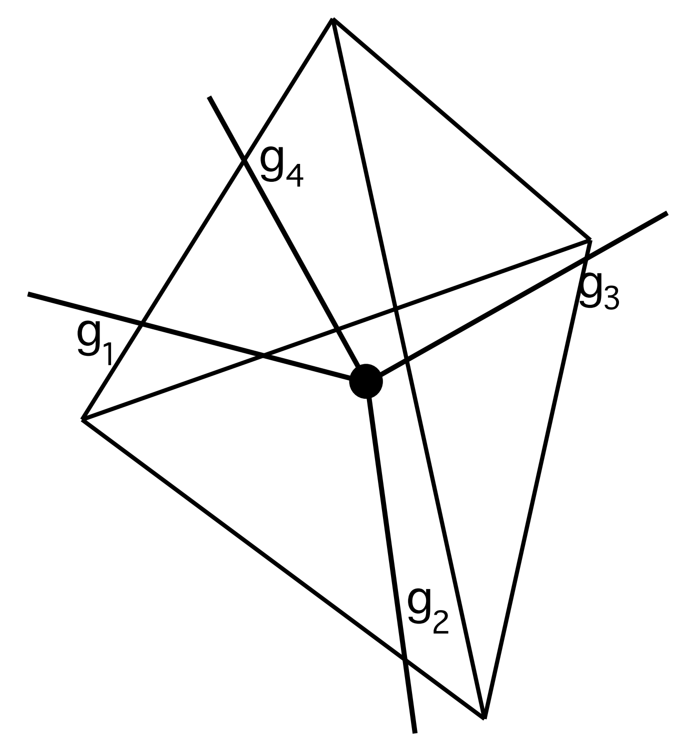
phase space:

$$(\mathcal{T}^* SO(3,1))^4 \simeq (\mathfrak{so}(3,1) \times SO(3,1))^4 \supset (\mathfrak{so}(3) \times SO(3))^4 \simeq (\mathcal{T}^* SO(3))^4$$

equivalent formulation in terms of irreps of G



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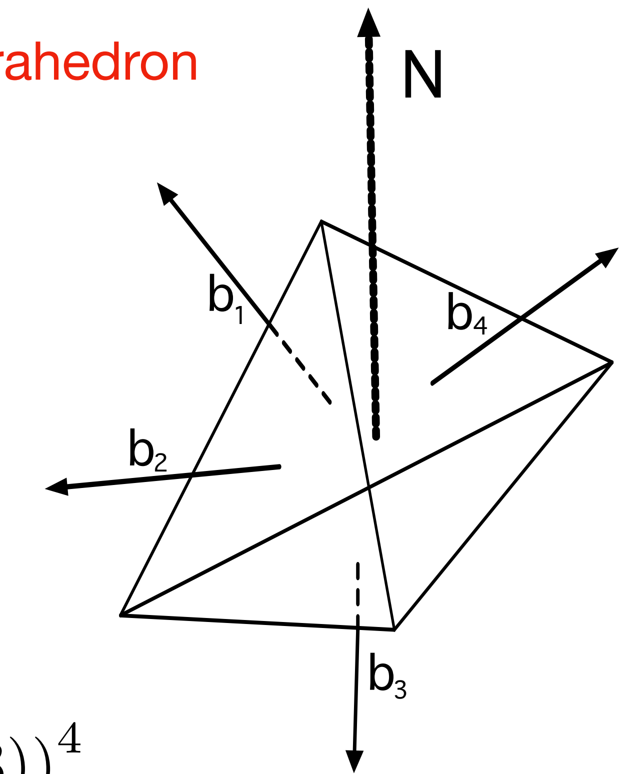

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phase space: + constraints

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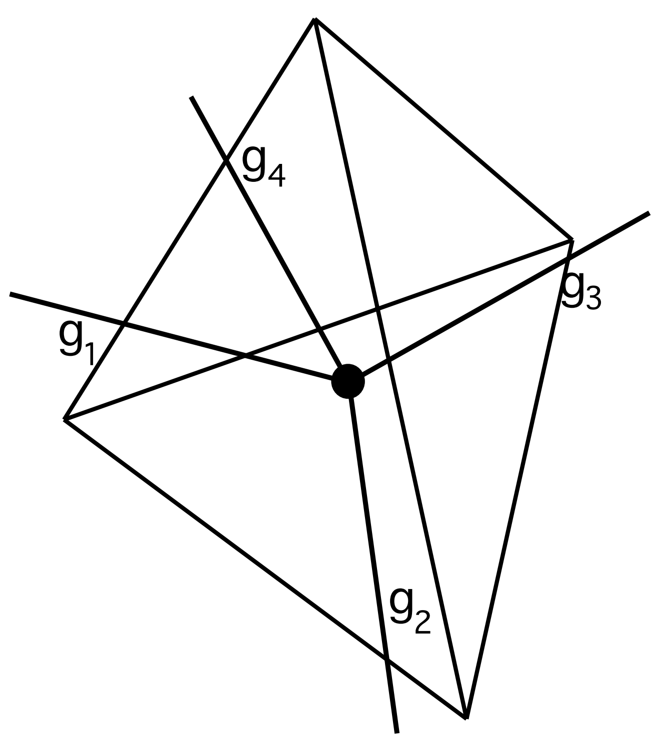

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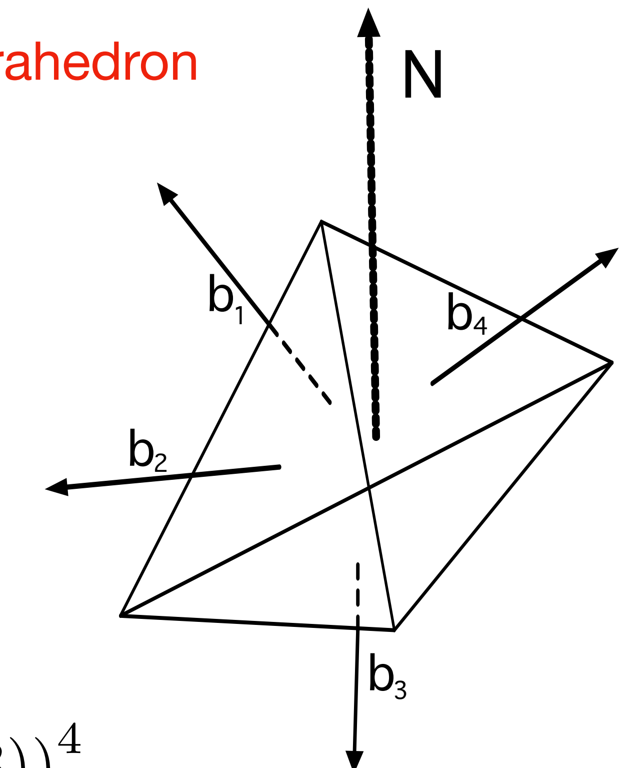

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Quantum geometry in group-theoretic variables

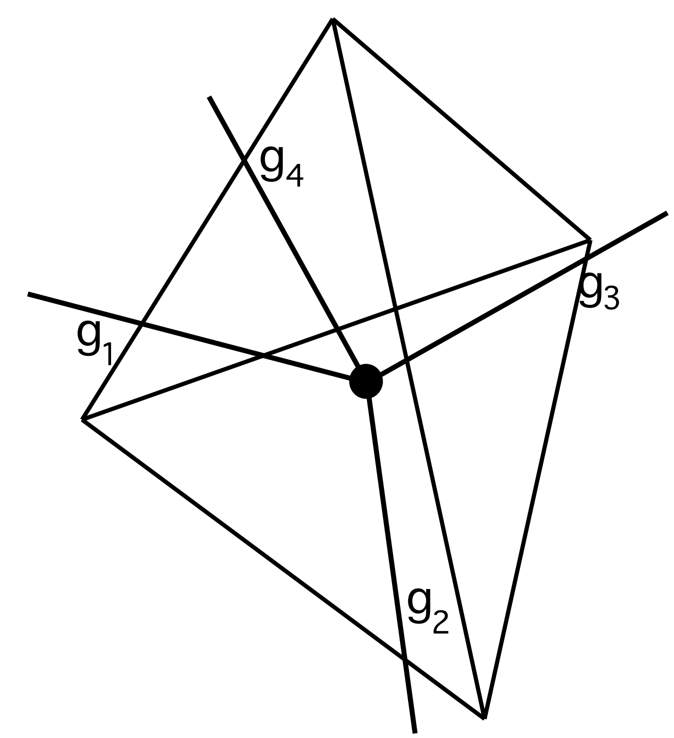
Hilbert space

$$\mathcal{H}_v \;=\; L^2\left(G^d; d\mu_{Haar}\right)$$

+ constraints on states

equivalent formulation in terms of irreps of G

spin network vertex



Quantum space as a many-body system

DO, ‘13

Many-body Hilbert space for “quantum space”: Fock space

$$\mathcal{F}(\mathcal{H}_v) = \bigoplus_{V=0}^{\infty} sym\left\{\left(\mathcal{H}_v^{(1)} \otimes \mathcal{H}_v^{(2)} \otimes \cdots \otimes \mathcal{H}_v^{(V)}\right)\right\}$$

Fock vacuum: “no-space” state | 0 >

Second-quantised representation for (simplicial) geometric operators

$$\left[\hat{\varphi}(\vec{g})\,,\,\hat{\varphi}^\dagger(\vec{g}')\right] = \mathbb{I}_G(\vec{g},\vec{g}') \qquad [\hat{\varphi}(\vec{g})\,,\,\hat{\varphi}(\vec{g}')] = \left[\hat{\varphi}^\dagger(\vec{g})\,,\,\hat{\varphi}^\dagger(\vec{g}')\right] = 0$$

$$\rightarrow \quad \widehat{\mathcal{O}_{n,m}}\left(\hat{\varphi},\hat{\varphi}^\dagger\right) = \int [d\vec{g}_i][d\vec{g}'_j]\;\widehat{\varphi}^\dagger(\vec{g}_1)..\widehat{\varphi}^\dagger(\vec{g}_m)\mathcal{O}_{n,m}\left(\vec{g}_1,..,\vec{g}_m,\vec{g}'_1,..,\vec{g}'_n\right)\widehat{\varphi}(\vec{g}'_1)..\widehat{\varphi}(\vec{g}'_n)$$

e.g. total space volume (extensive quantity):

$$\hat{V}_{tot} = \int [dg_i][dg'_j]\hat{\varphi}^\dagger(g_i)\,V(g_i,g'_j)\,\hat{\varphi}(g'_j) = \sum_{J_i}\hat{\varphi}^\dagger(J_i)\,V(J_i)\,\hat{\varphi}(J_j)$$

volume of single tetrahedron (from simplicial geometry)

Group field theories

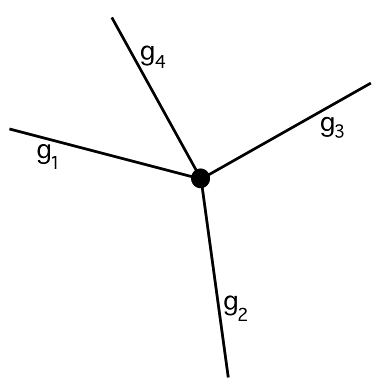
a QFT for the building blocks of (quantum) space

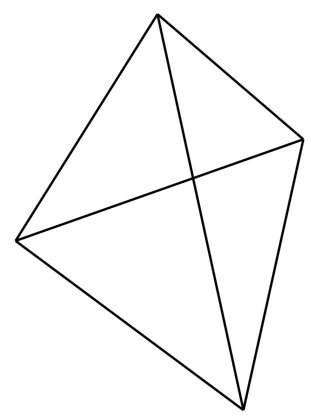
Fock vacuum: “no-space” (“emptiest”) state | 0 >

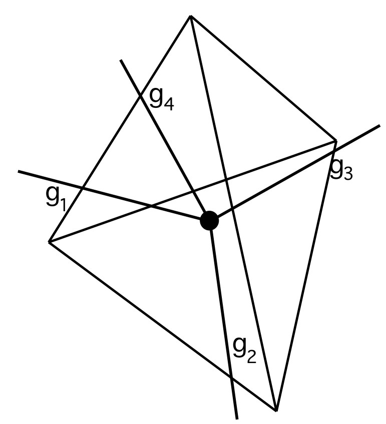
(d=4)

single field “quantum”: spin network vertex or tetrahedron
(“building block of space”)

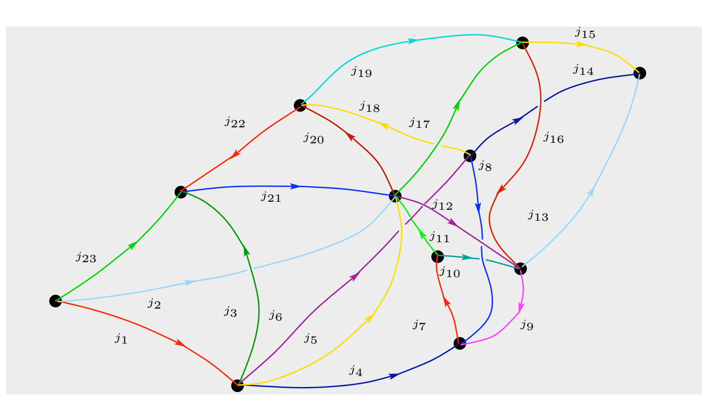
$$\varphi(g_1, g_2, g_3, g_4) \leftrightarrow \varphi(B_1, B_2, B_3, B_4) \to \mathbb{C}$$

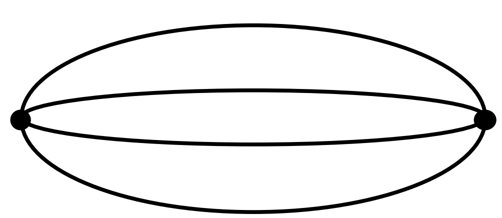


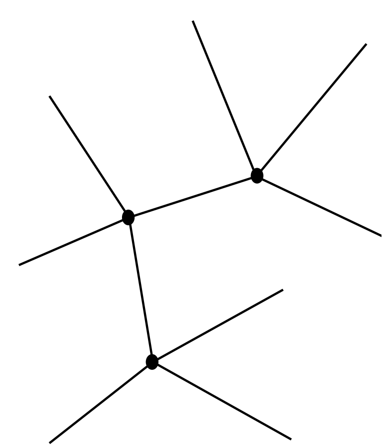


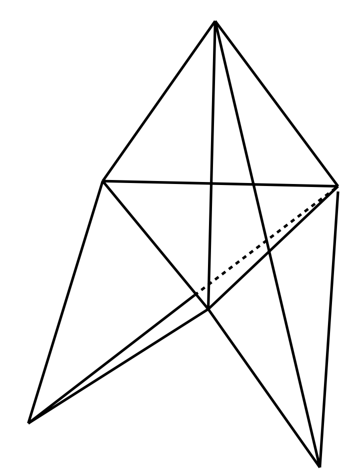


generic quantum state: arbitrary collection of spin network vertices (including glued ones) or tetrahedra (including glued ones)









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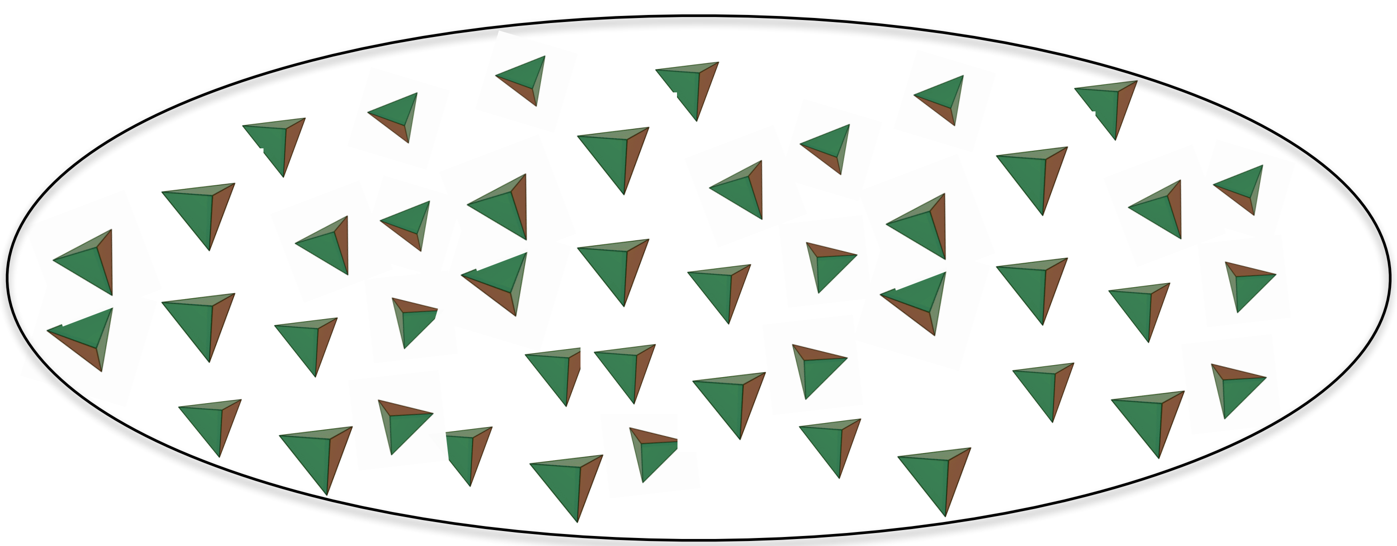
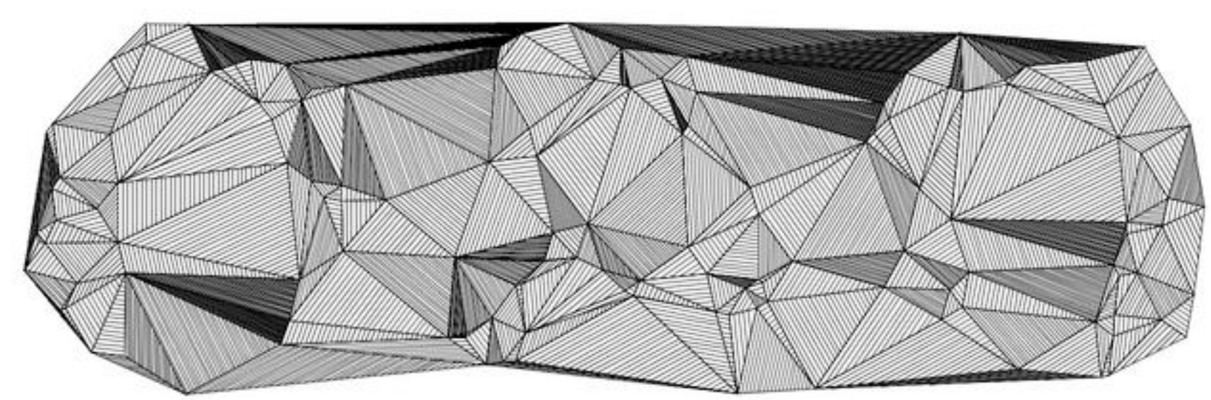
Quantum space as an system of many quantum polyhedra/spin network vertices

generic states not very “spacey” at all - “connected” many-body states a little more “spacey”





Dynamics of quantum space as a group field theory

DO, '09; DO, '14

Dynamics governs gluing processes and formation of extended discrete structures

Interactions processes correspond to (simplicial) complexes in one dimension higher

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details depend on (class of) models

$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

"combinatorial non-locality"
in pairing of field arguments

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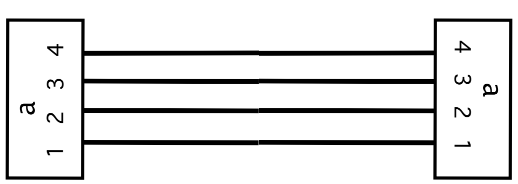
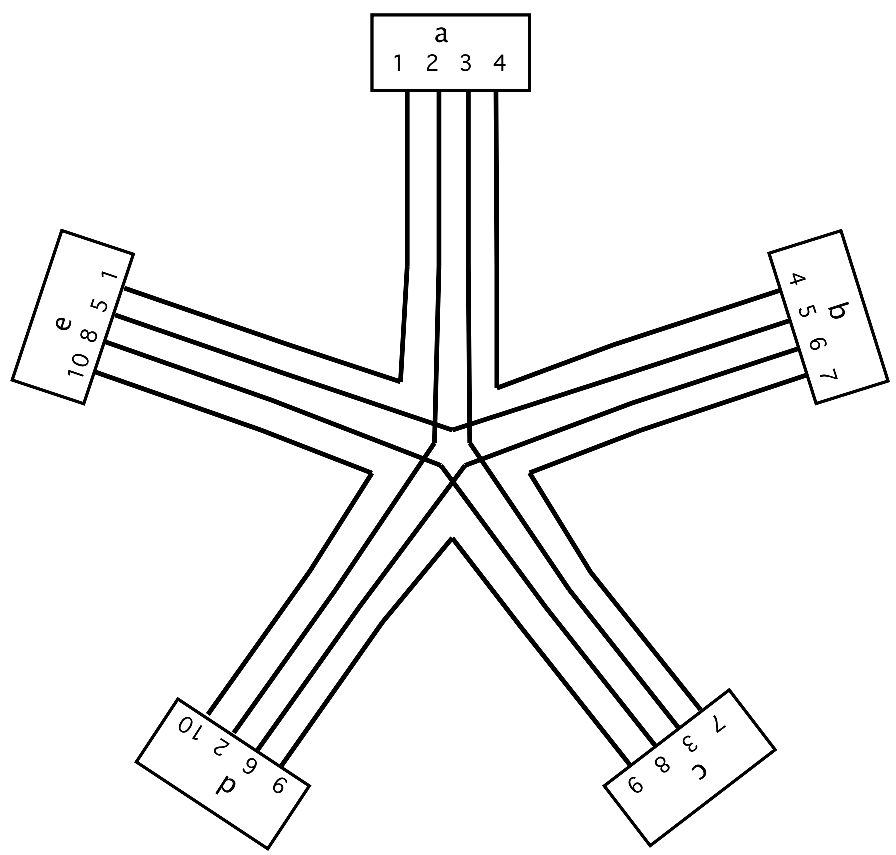
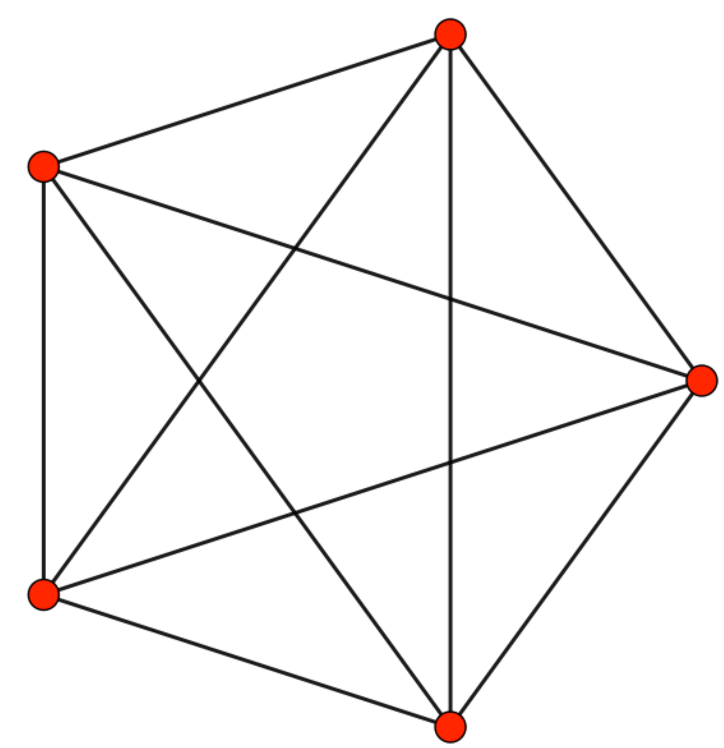
Example: simplicial interactions







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"combinatorial non-locality" in pairing of field arguments

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$$\mathcal{Z} = \int \mathcal{D}\varphi\mathcal{D}\overline{\varphi}\; e^{i\,S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_{\Gamma}\frac{\lambda^{N_\Gamma}}{sym(\Gamma)}\mathcal{A}_\Gamma$$

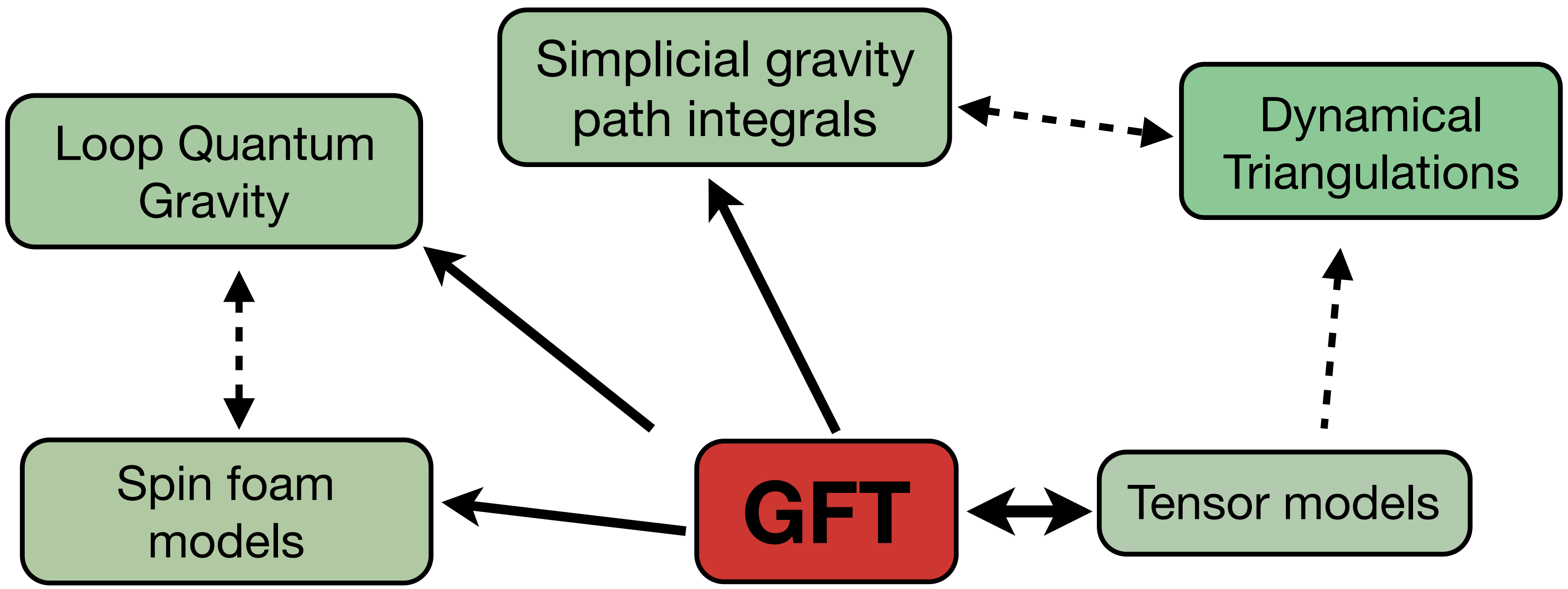
Feynman diagrams = stranded diagrams dual
to cellular complexes of arbitrary topology

sum over triangulations/complexes

amplitude for each
triangulation/complex

Dynamics of quantum space as a group field theory

Multiple relations with other QG formalisms



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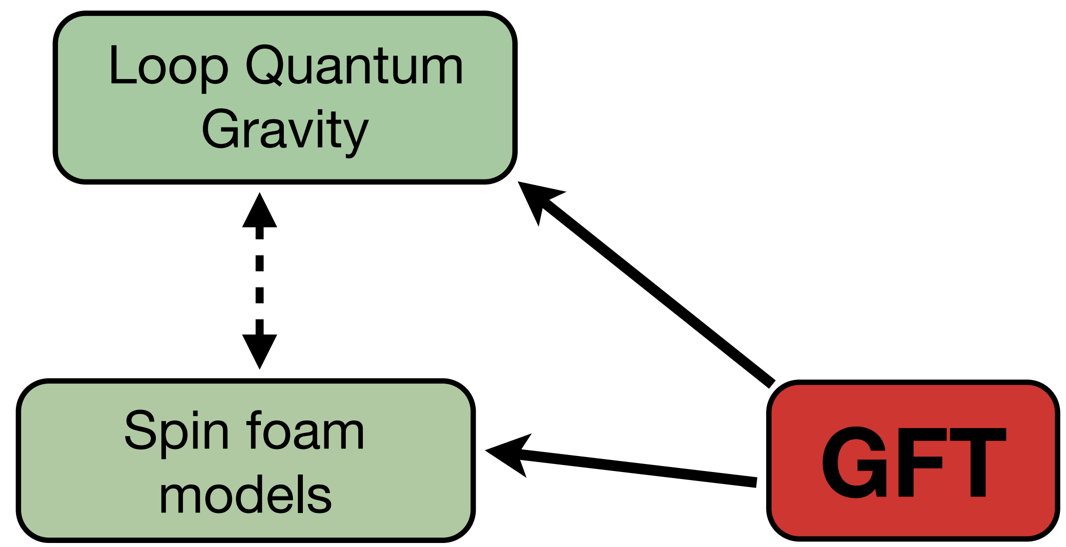
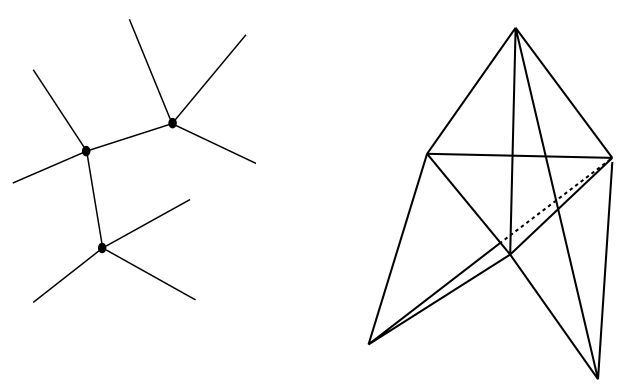


GFT and Loop Quantum Gravity/ Spin foam models



Quantum GFT dofs (quantum states) are same as in LQG (spin networks), organised in different (but similar) Hilbert space

GFT is 2nd quantized reformulation of LQG states and dynamics

Spin foam model = quantum amplitude for spin network evolution

$$Z(\Gamma) = \sum_{\{J\},\{I\}|j,j',i,i'} \prod_f A_f(J,I) \prod_e A_e(J,I) \prod_v A_v(J,I)$$

$$\mathcal{Z} = \int \mathcal{D}\varphi\mathcal{D}\overline{\varphi}\; e^{i\,S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)} \mathcal{A}_\Gamma \qquad Z(\Gamma) \equiv \mathcal{A}_\Gamma$$

Any spin foam amplitude is the Feynman amplitude of a GFT model

Reisenberger, Rovelli, ‘00

GFT states as generalised tensor networks

G. Chirco, DO, M. Zhang, arXiv:1701.01383v2, arXiv:1711.09941 [hep-th]

Quantum states in many-body systems conveniently encoded in tensor networks
= tensors contracted by link maps, associated to graph

Group field theory states are a field-theoretic generalization of random (symmetric) tensor networks

Table B	GFT network	Spin Tensor Network	Tensor Network
node	$\varphi(\vec{g}) \equiv \varphi(g_1, g_2, g_3, g_4)$	$\varphi^{\mathbf{j}}_{\{m\}} \propto \sum_{\{k\}} \hat{\varphi}^{\mathbf{j}}_{\{m\}\{k\}}\, i^{\mathbf{j}\{k\}}$	$T_{\{\mu\}}$
link	$M(g_1^{\dagger} g_\ell g_2)$	M^{j}_{mn}	$M_{\lambda_1\lambda_2}$
sym	$\varphi(h\vec{g}) = \varphi(\vec{g})$	$\prod_s^v D^{j}_{m_s m'_s}(g)\, i^{i}_{m'_1\cdots m'_v} = i^{i}_{m_1\cdots m_v}$	$\prod_s^v U_{\mu_s\mu'_s} T_{\mu'_1\cdots\mu'_v} = T_{\mu_1\cdots\mu_v}$
state	$\left\|\Phi^{g_\ell}_{\Gamma}\right\rangle \equiv \bigotimes_\ell \langle M_{g_\ell}\| \bigotimes_n \|\psi_n\rangle$	$\|\Psi^{\mathbf{ji}}_{\Gamma}\rangle \equiv \bigotimes_\ell \langle M^{j_\ell}\| \bigotimes_n \|\phi_n^{\mathbf{j}_n i_n}\rangle$	$\|\Psi_{\mathcal{N}}\rangle \equiv \bigotimes_\ell^L \langle M_\ell\| \bigotimes_n^N \|T_n\rangle$
indices	$g_i \in G$, $\|g_i\rangle \in \mathbb{H} \simeq L^2[G]$	$m_i \in \mathbb{H}_j$, SU(2) spin-j irrep.	$\mu_i \in \mathbb{Z}_n$, nth cyclic group
dim	∞	$\dim \mathbb{H}_j = 2j+1$	

correspondence even stricter for random tensor models (GFTs stripped down of group data, reduced to combinatorial structures)

$$\varphi(g_1, g_2, g_3) : G^{\times 3} \to \mathbb{C}$$

$$T_{ijk} : \mathbb{Z}_N^{\times 3} \to \mathbb{C}$$

$$T_{ijk} : X^{\times 3} \to \mathbb{C}$$

$$X = 1, 2, \ldots, N$$

Emergence of spacetime from quantum gravity: an example

how does the universe (space, time) “emerge”
from such fundamental constituents?

universe as a “condensate” of the “atoms of space”?

Emergent Spacetime in Quantum Gravity: an example

GFT condensate Cosmology:

S. Gielen, DO, L. Sindoni, G. Calcagni, M. Sakellariadou, E. Wilson-Ewing, A. Pithis, M. De Cesare, ……….

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described by single collective wave function ("wave-function homogeneity")
(depending on homogeneous anisotropic geometric data)

same criterion used in tensor networks context
to define "homogeneous tensor networks"

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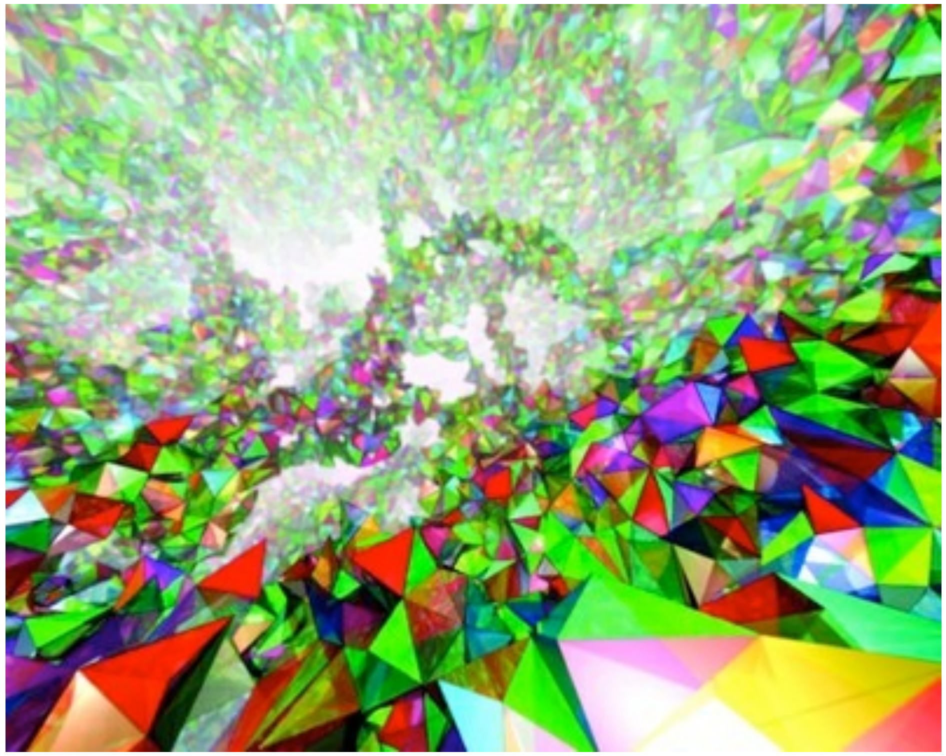
cosmology as QG hydrodynamics!!!

Quantum black holes
in
the Group Field Theory formalism

Building up continuum space and geometry



Goal: extract continuum geometric (gravitational) physics (dynamics) from QG (GFT) models

This means:

- control QG states encoding large numbers of microscopic QG dofs
- identify those with (approximate) continuum geometric interpretation
- characterise their (geometric) properties in terms of observables
- extract their effective dynamics and recast it in GR+QFT form

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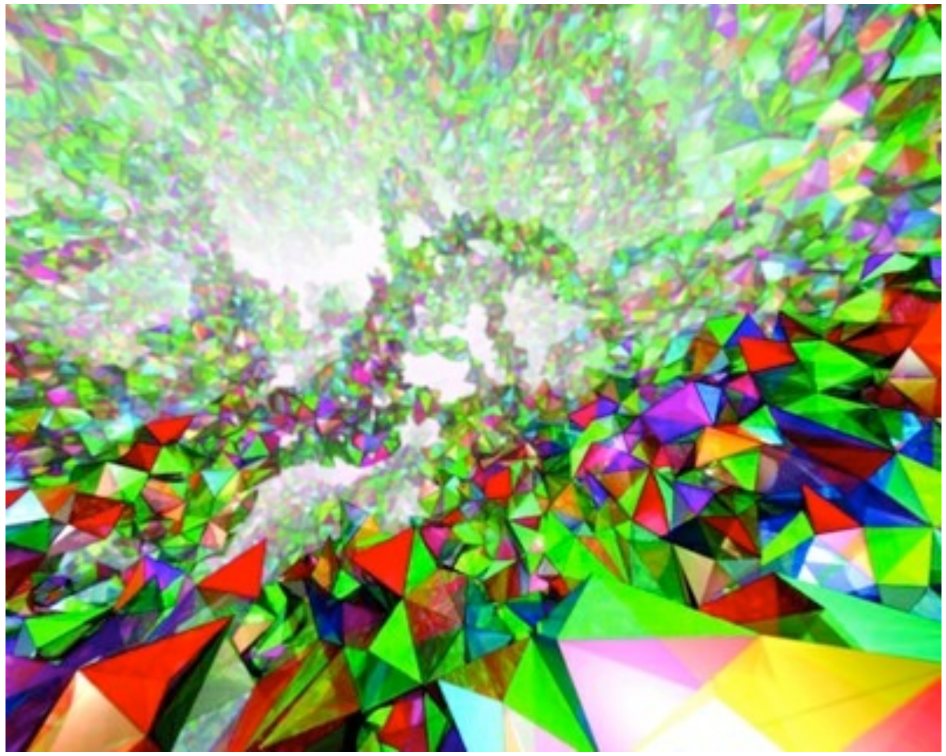

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This requires:

- controlling large graphs/complexes superpositions
- coarse graining of description
- approximations of both states, observables and dynamics

Here: take advantage of QFT formalism/methods (universe as a quantum many-body system!)

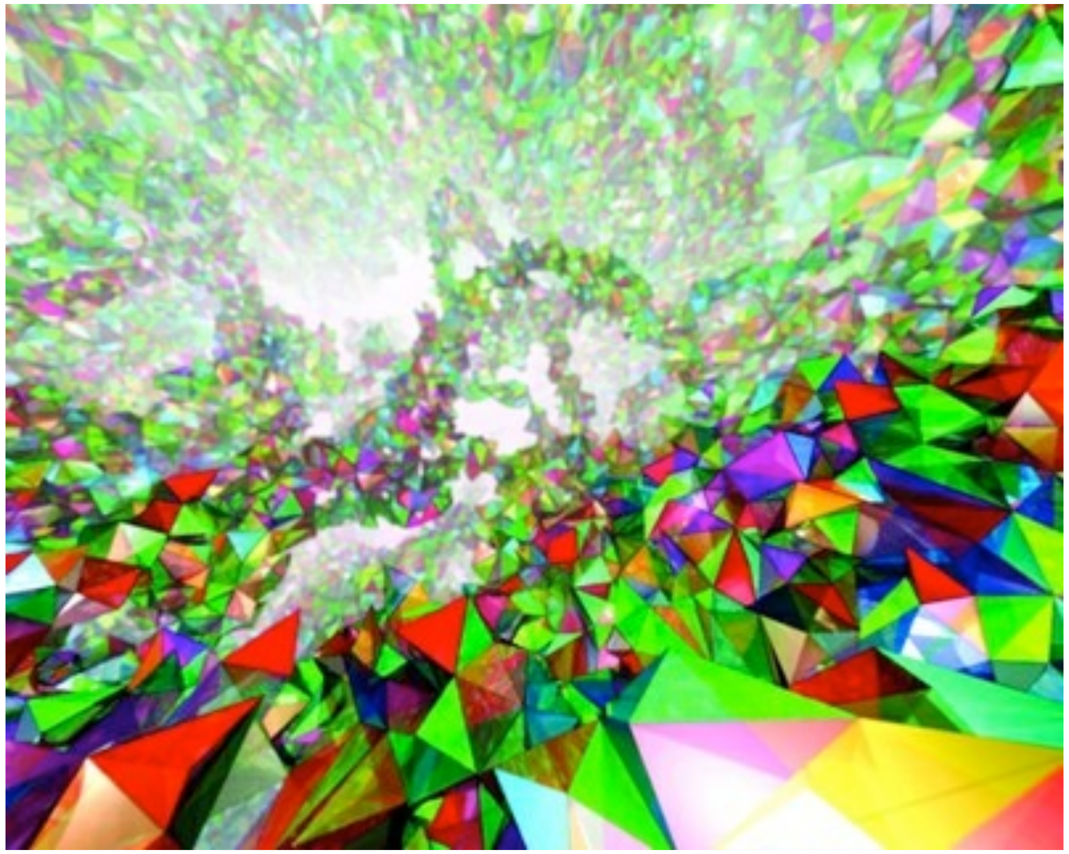
Spherically symmetric geometries and horizons

DO, Pranzetti, Sindoni, '15-'18

!!!Construction work with "building blocks of quantum space"!!!

"piece by piece"

build up a spherically symmetric quantum BH



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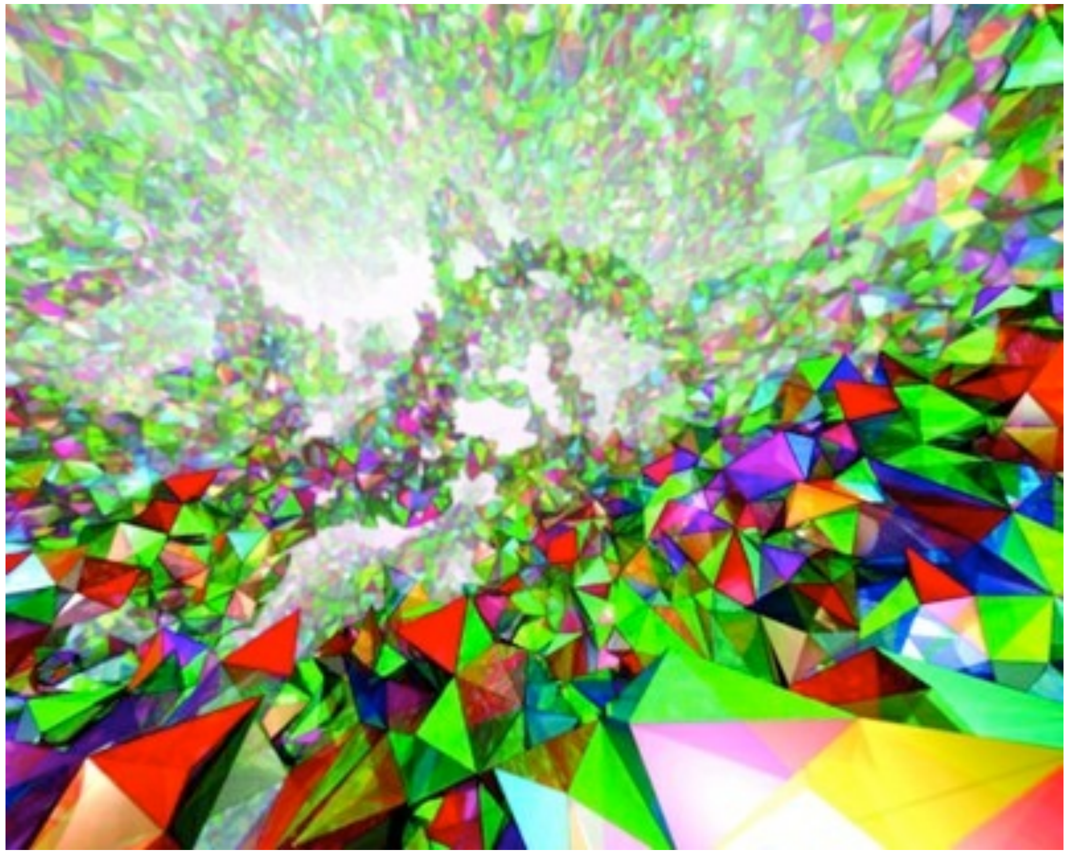

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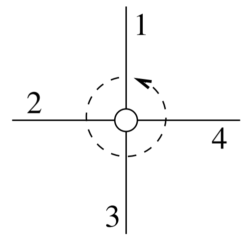
Main result: candidate microscopic quantum states for spherically symmetric horizons + explicit computation of their entropy: area law and holographic properties

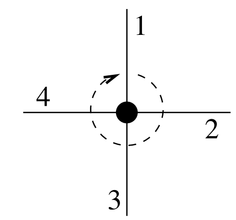
Main limitation: no dynamics, only some (assumed) proxies

Spherically symmetric geometries and horizons

DO, Pranzetti, Sindoni, ’15-‘18

Construction of spherically symmetric GFT states (interplay of random tensor methods and LQG data)

- basic operators: $\hat{\varphi}^\dagger_W(g_1, g_2, g_3, g_4)\,|0\rangle =$ 

$\hat{\varphi}^\dagger_B(g_1, g_2, g_3, g_4)\,|0\rangle =$ 

colored GFT tensors, to control topology, with simplicial geometric variables, to control geometric properties

↓

same criterion used in tensor networks context
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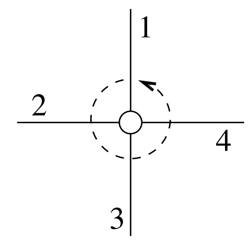
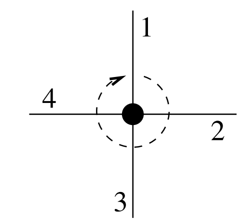
- basic operators: $\hat{\varphi}^\dagger_W(g_1,g_2,g_3,g_4)\,|0\rangle =$ 

$\hat{\varphi}^\dagger_B(g_1,g_2,g_3,g_4)\,|0\rangle =$ 

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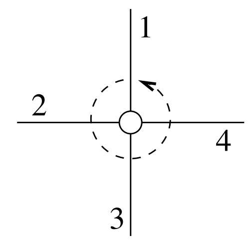
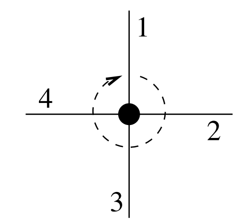
- basic operators: $\hat{\varphi}^{\dagger}_{W}(g_1, g_2, g_3, g_4)\,|0\rangle =$ 

$\hat{\varphi}^{\dagger}_{B}(g_1, g_2, g_3, g_4)\,|0\rangle =$ 

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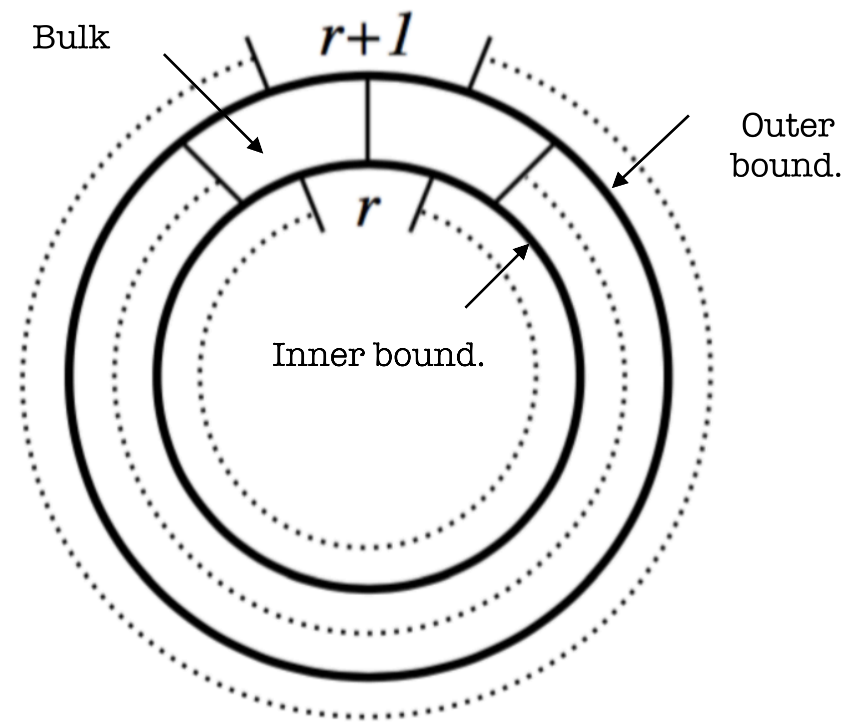
Strategy:

1. Start with a (combinatorially simple) seed state for the desired topology
2. Act iteratively with a refinement operator which is topology-preserving and maintains homogeneity of the vertex wave-functions

Spherically symmetric geometries and horizons

DO, Pranzetti, Sindoni, ’15-‘18

Homogeneous shells



Spherically symmetric geometries and horizons

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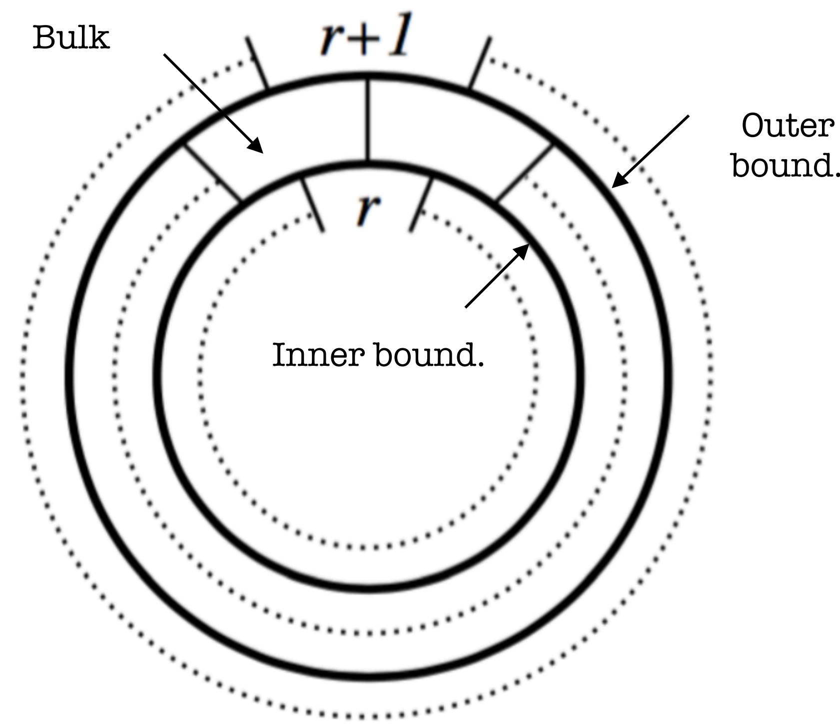

Homogeneous shells

Add labels to vertex wavefunction to identify shell and boundaries

$$\hat{\sigma}_{r,t^v s^v}(h_I^v) = \int \mathrm{d}g_I^v \, \sigma_{r s^v}(h_I^v g_I^v)\, \hat{\varphi}_{t^v}(g_I^v)$$

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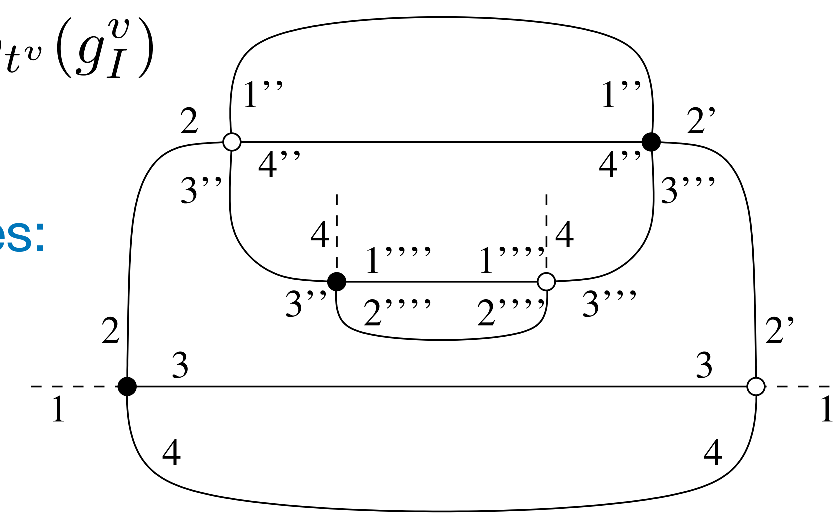
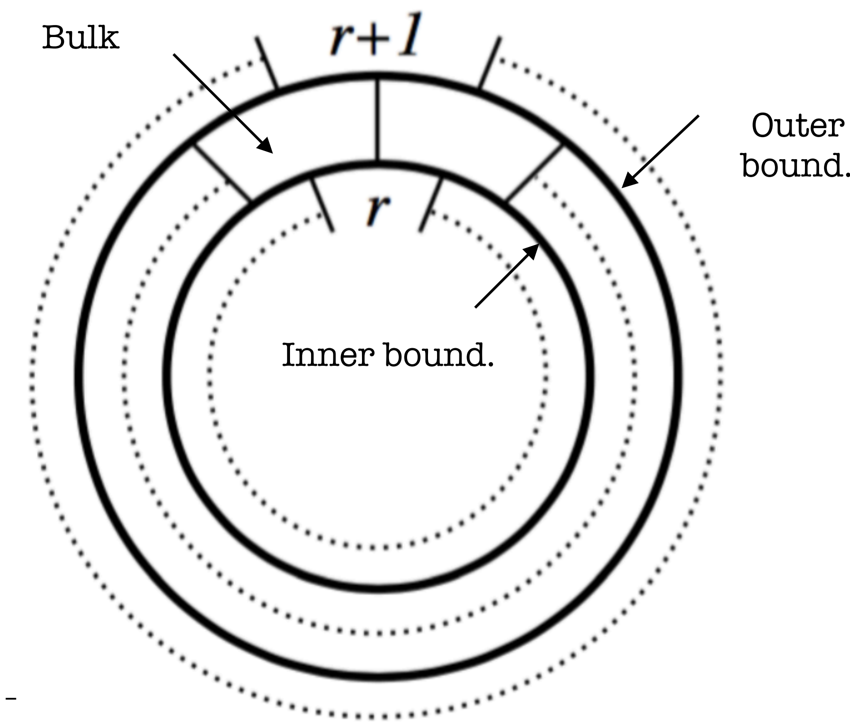
Simple seed state with two boundaries:





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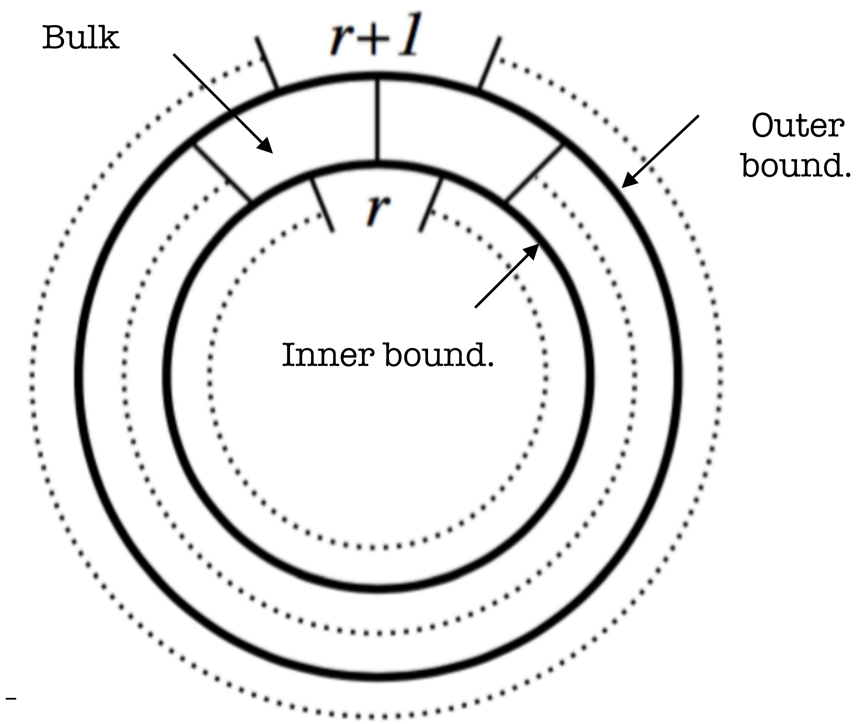

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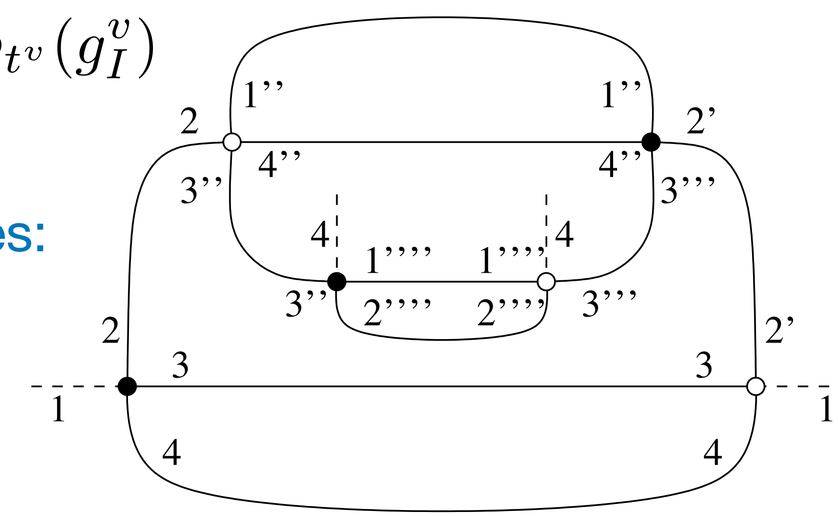
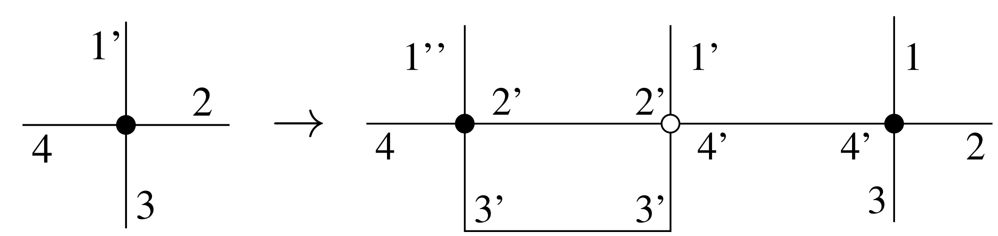


Refinement operator (dipole insertion):

$$\widehat{\mathcal{M}}_{r,B+} \equiv \int \mathrm{d}k_2 \mathrm{d}k_3 \mathrm{d}k_4 \mathrm{d}h_4' \mathrm{d}h_2' \mathrm{d}h_3'$$
$$\hat{\sigma}^\dagger_{r,B+}(e,h_2',h_3',k_4)\hat{\sigma}^\dagger_{r,W+}(e,h_2',h_3',h_4')\hat{\sigma}^\dagger_{r,B+}(e,k_2,k_3,h_4')\hat{\sigma}_{r,B+}(e,k_2,k_3,k_4)$$



Spherically symmetric geometries and horizons

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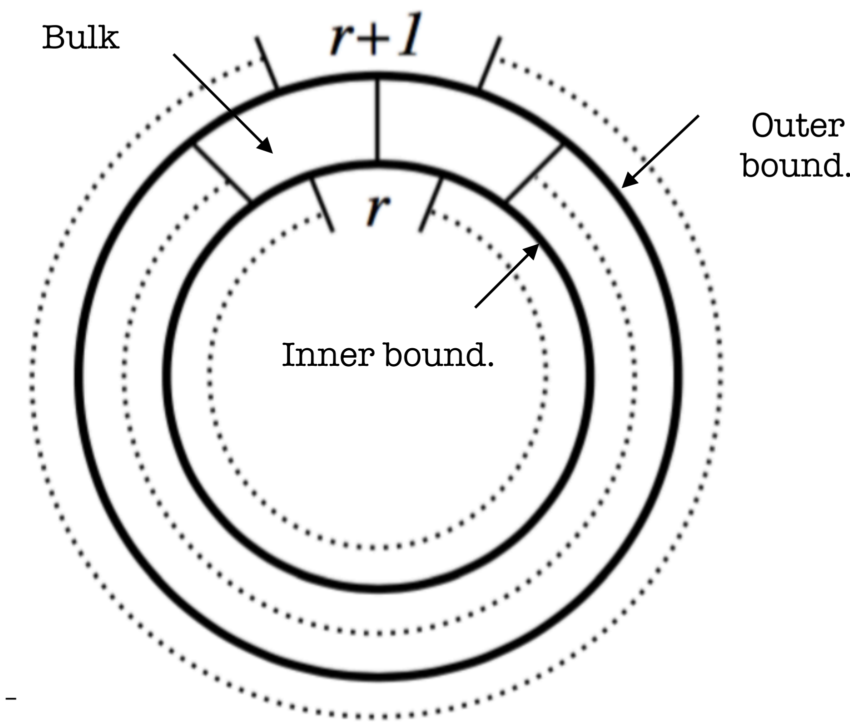

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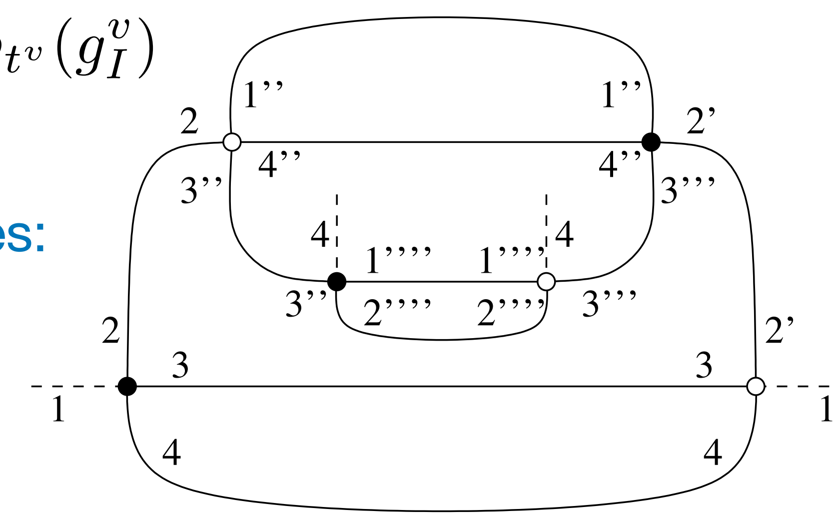

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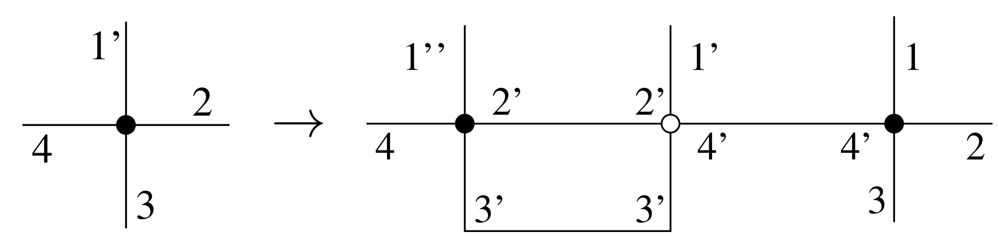


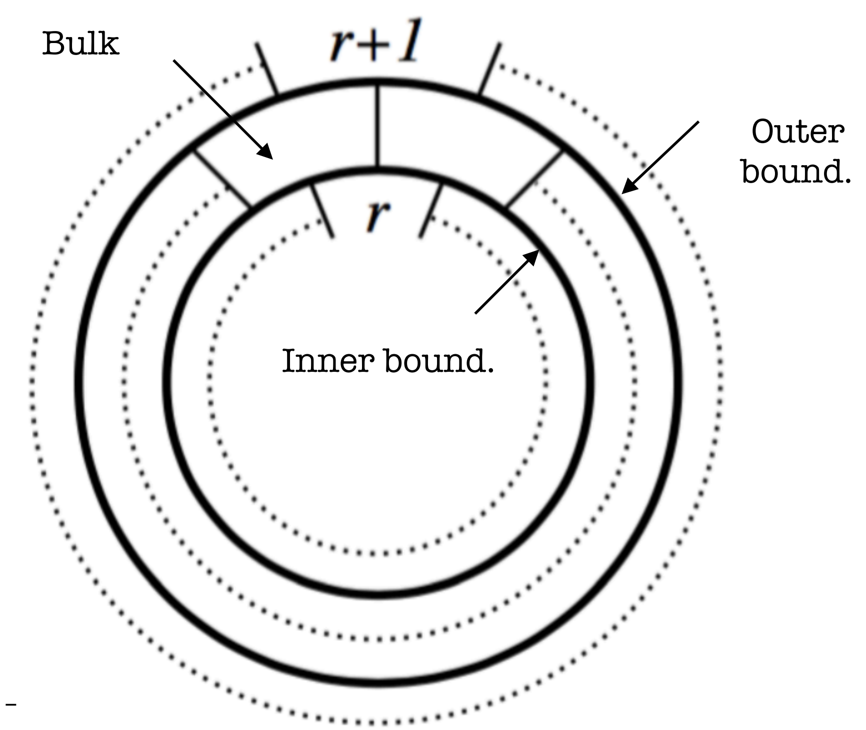
Quantum state for (approximately) continuum homogeneous spherical shell:

(for some high-order polynomial operator function F)

$$|\Psi_r\rangle = F_r(\widehat{\mathcal{M}}_{r,Bs}, \widehat{\mathcal{M}}_{r,Ws})|\tau\rangle$$

Spherically symmetric geometries and horizons

DO, Pranzetti, Sindoni, '15-'18



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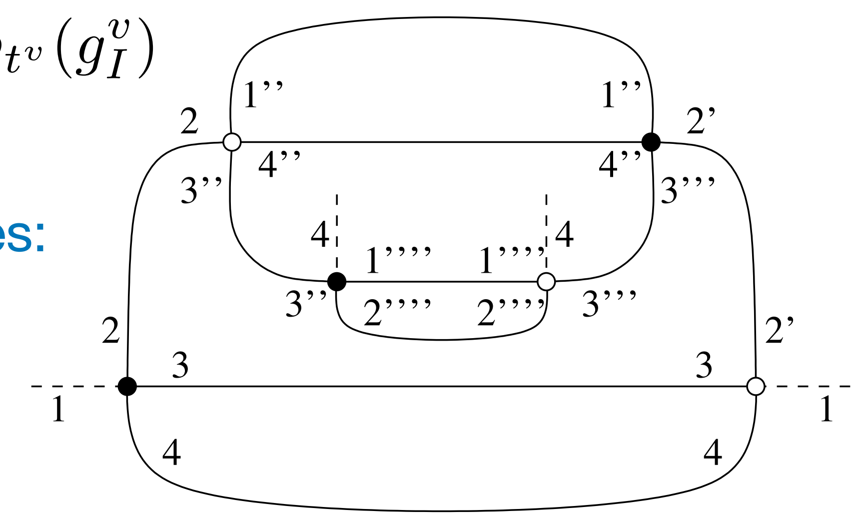

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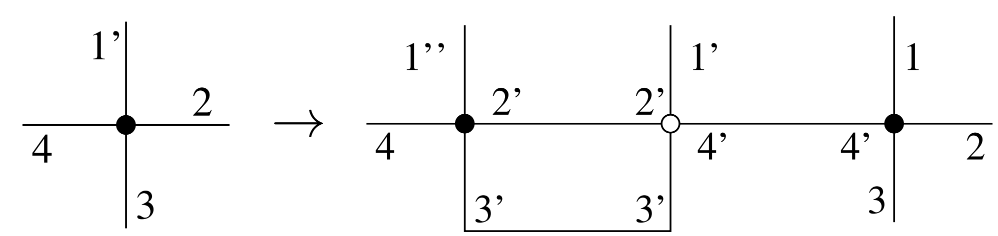

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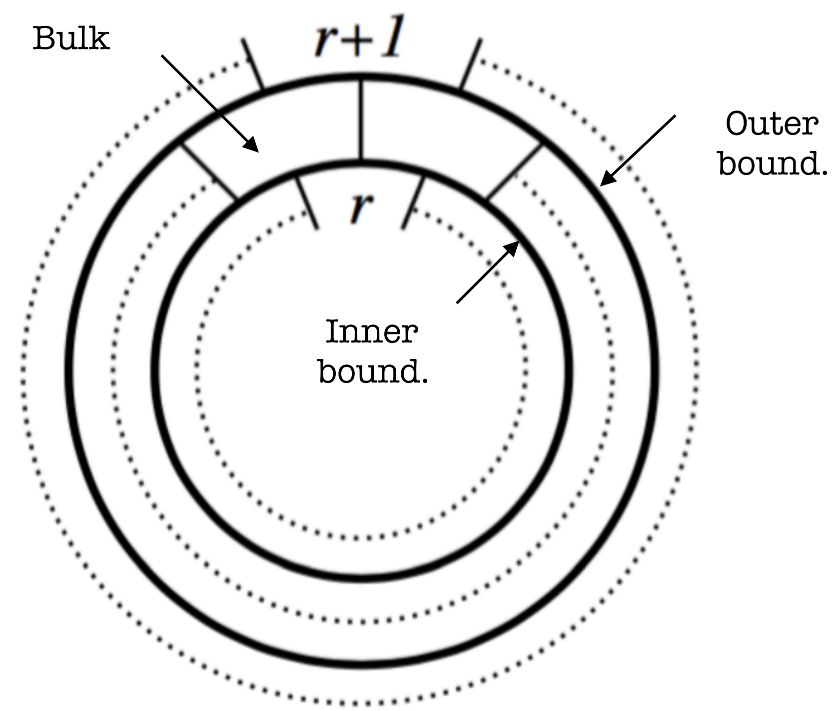
large (infinite) superpositions of arbitrarily complex spin network/simplicial states

Spherically symmetric geometries and horizons

DO, Pranzetti, Sindoni, '15-'18

Spherically symmetric continuum quantum geometry



Quantum (pure) state obtained by product of shell states, sharing boundary data, refined in a coordinated way - large (potentially infinite) superposition of spherically symmetric cellular complexes, from gluing homogeneous shells

Spherically symmetric geometries and horizons

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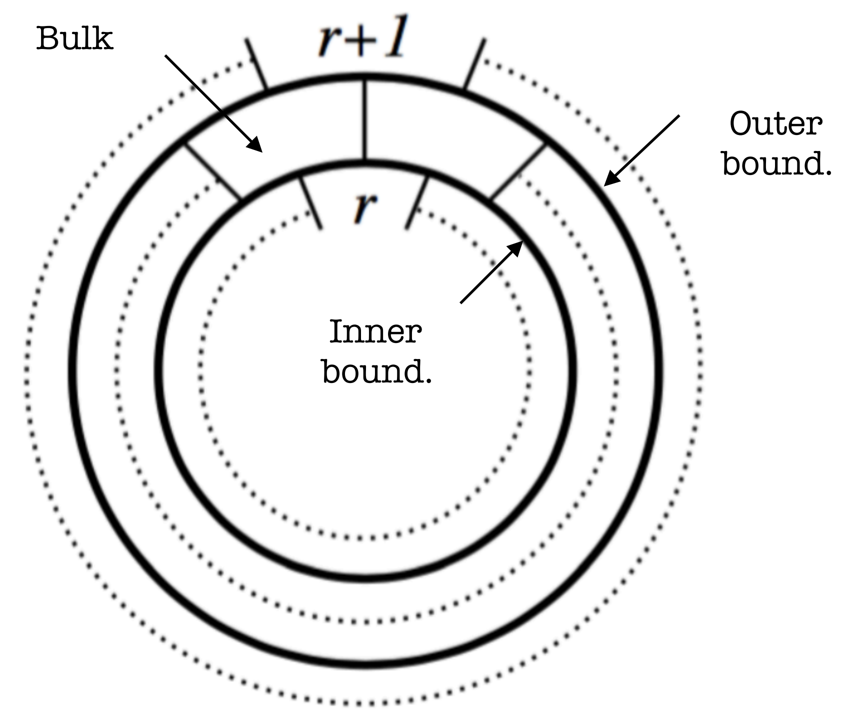

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can define and compute geometric quantities

e.g. area operator (for shell r and boundary s)

$$\hat{\mathbb{A}}_{Jr,s} \equiv \sum \int (dg)^4 \hat{\sigma}^\dagger_{r,ts}(g_I) \sqrt{E^i_J E^j_J \delta_{ij}} \hat{\sigma}_{r,ts}(g_I)$$

where $E^i_J \triangleright f(g_I) := \lim_{\epsilon \to 0} \mathrm{i} \frac{d}{d\epsilon} f(g_1, \dots, e^{-\mathrm{i}\epsilon\tau^i} g_J, \dots, g_4)$

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expectation value of the area for a single dual J-link

number operator : $\widehat{n}_{r,s} = \sum_{t=B,W} \int dh_I \, \hat{\sigma}^\dagger_{r,ts}(h_I) \hat{\sigma}_{r,ts}(h_I)$

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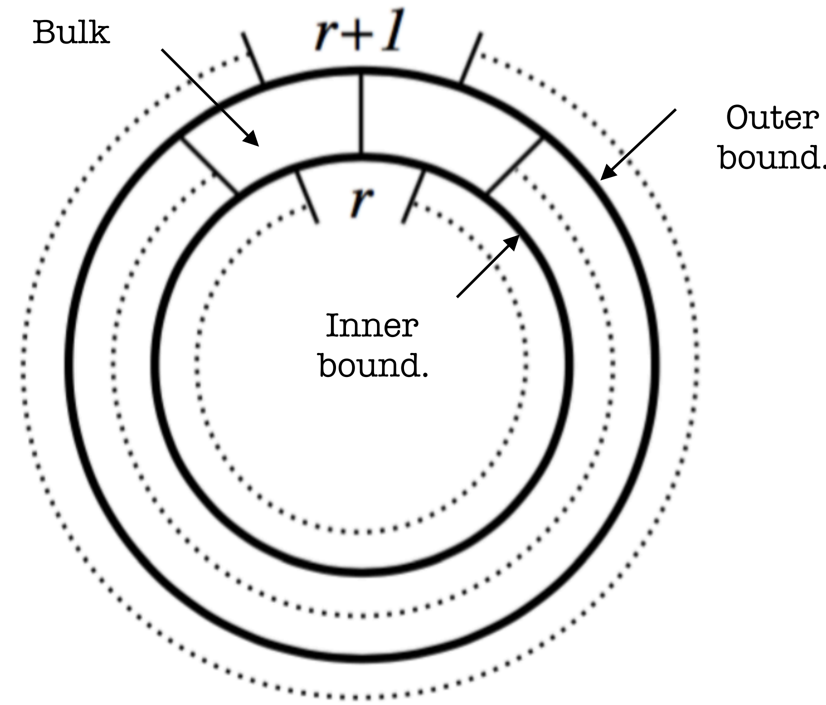

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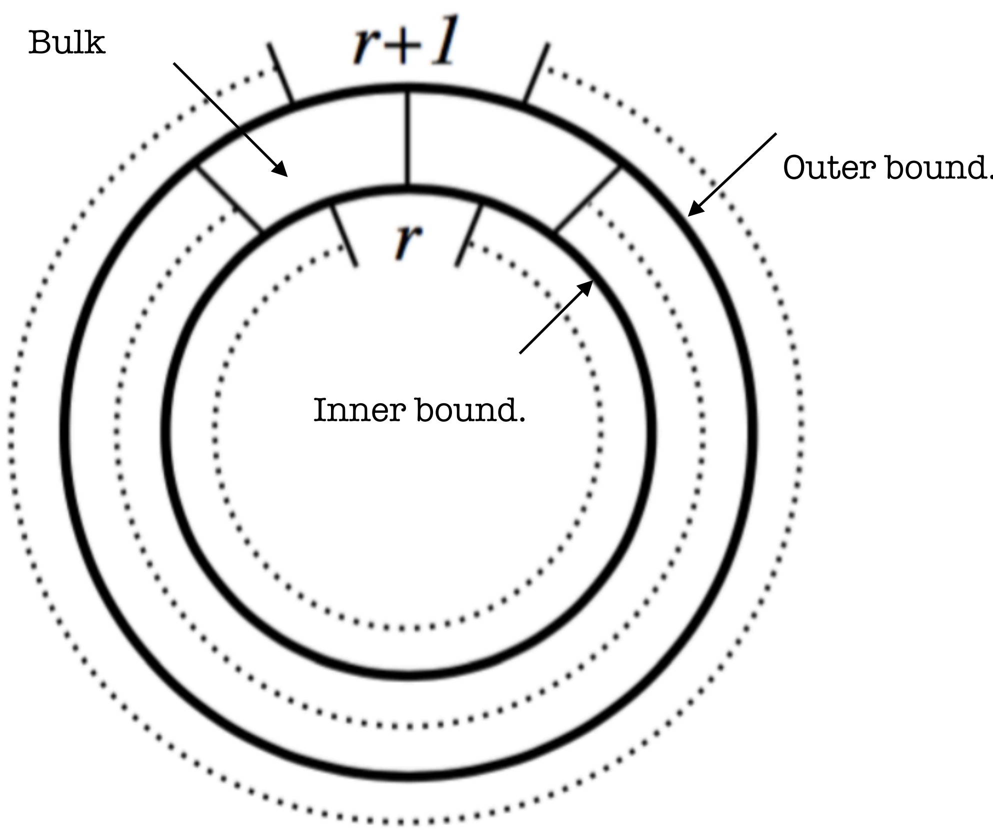
could now impose "horizon (boundary) conditions" …. postpone

Horizon entropy calculation

DO, Pranzetti, Sindoni, ’15-‘18

“weak holographic principle” - horizon density matrix by tracing over all other shells (inside and outside)



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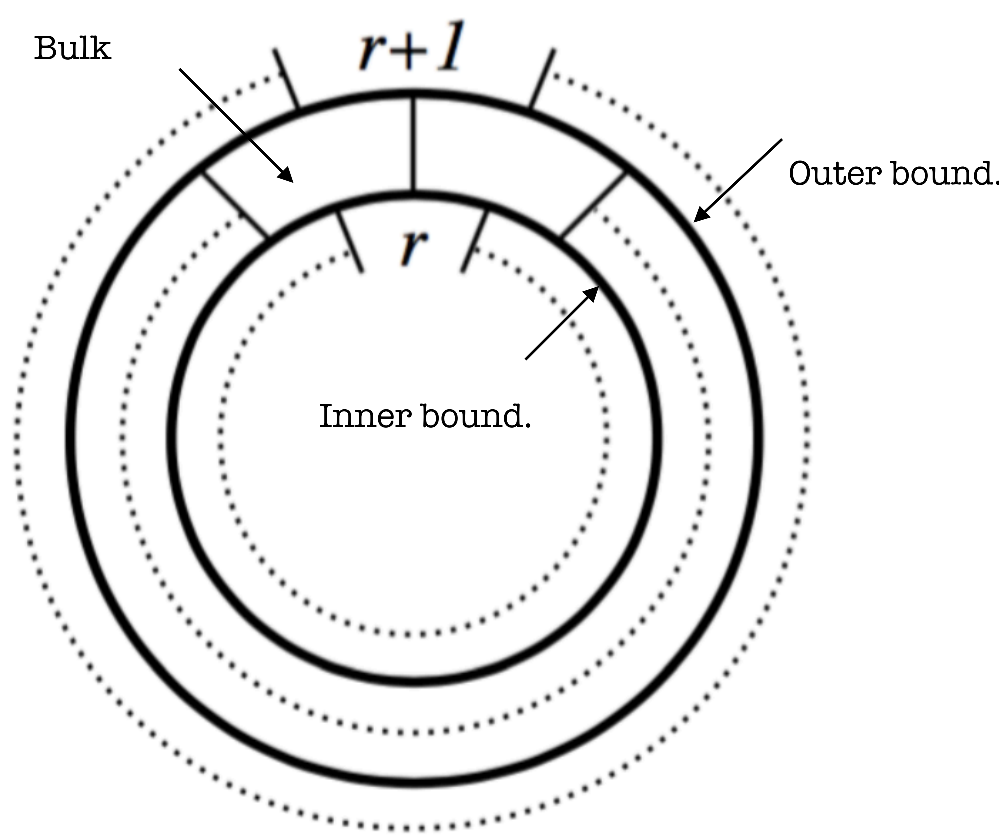

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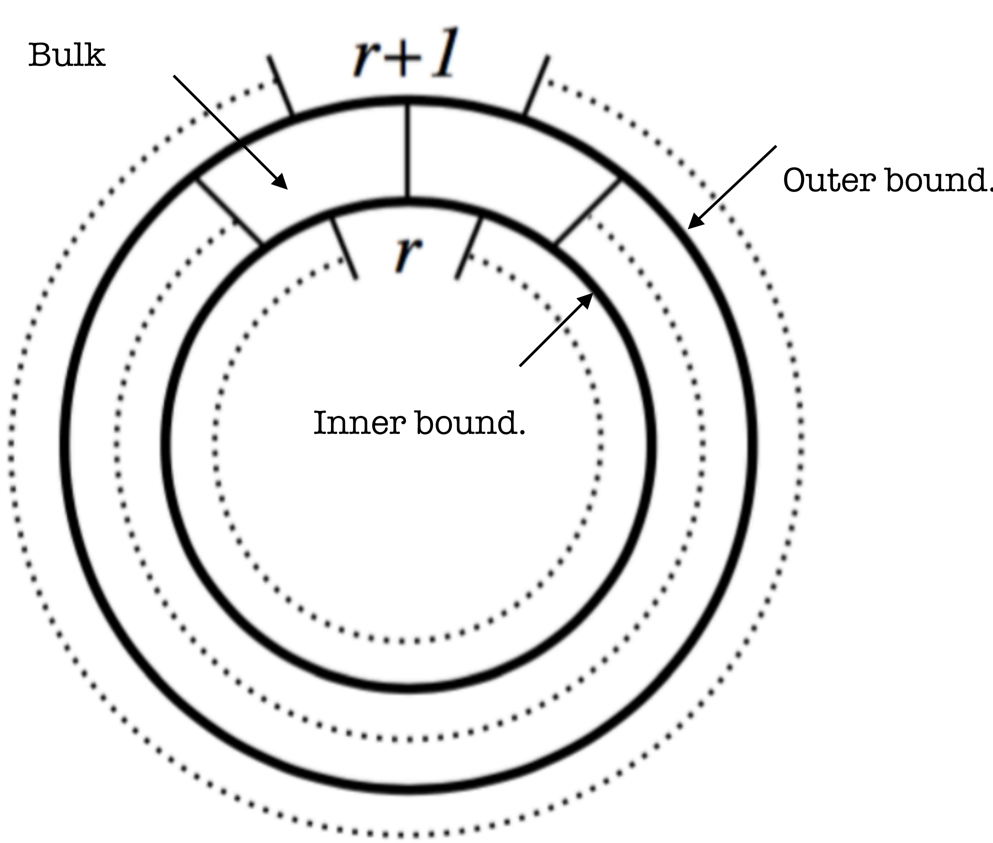

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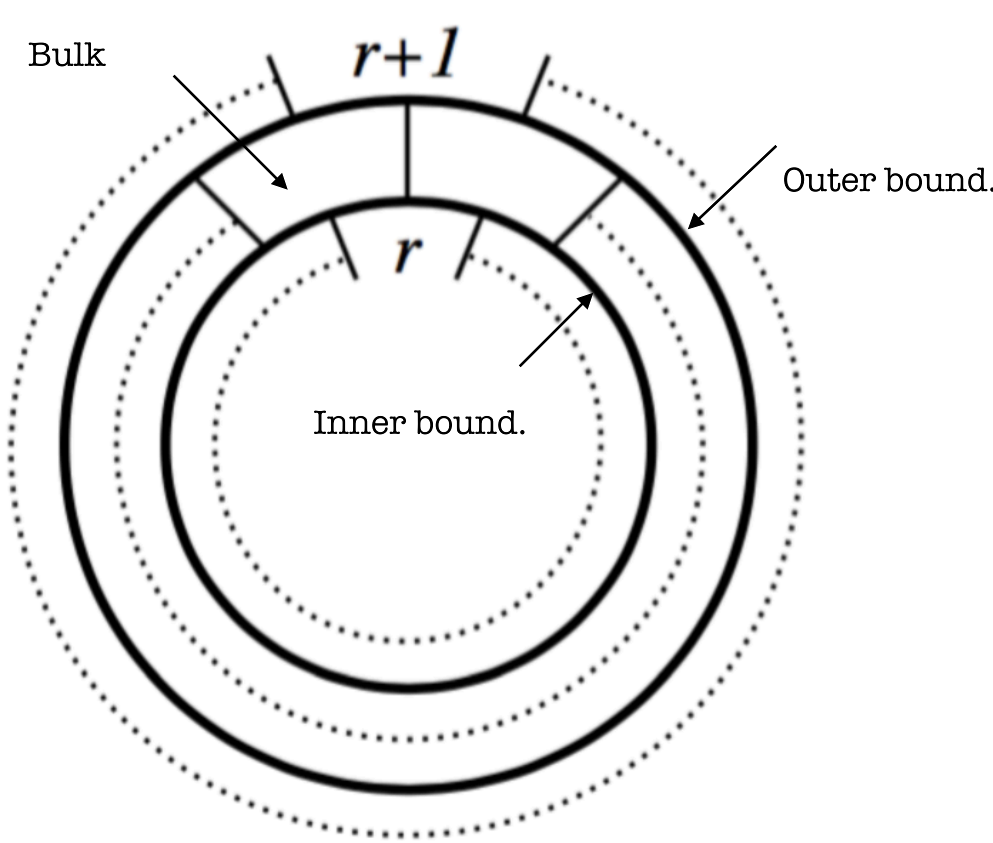

NB: here, “horizon” is simply one reference shell

Horizon entropy calculation

DO, Pranzetti, Sindoni, ’15-‘18

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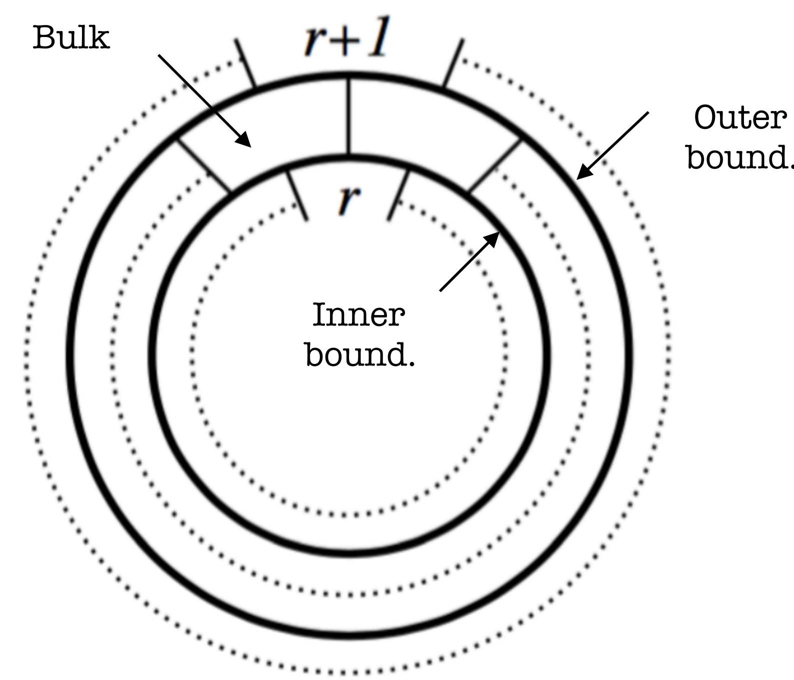

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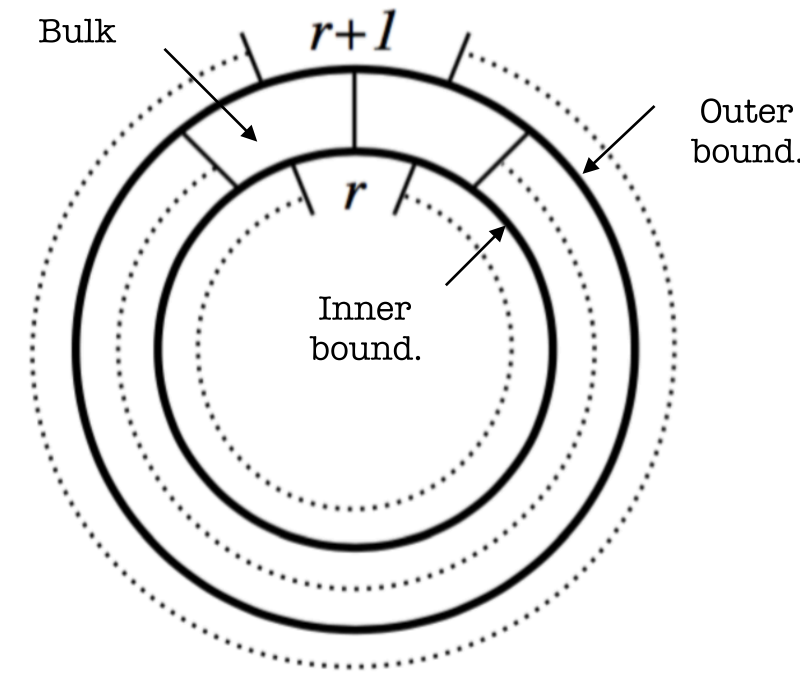
- no use of microscopic quantum dynamics (just trial states)
- large area, small shell volume, small fluctuations (many vertices)
- require maximal entropy ——> "proxy" for "horizon conditions"
- consistency with semiclassical thermodynamics



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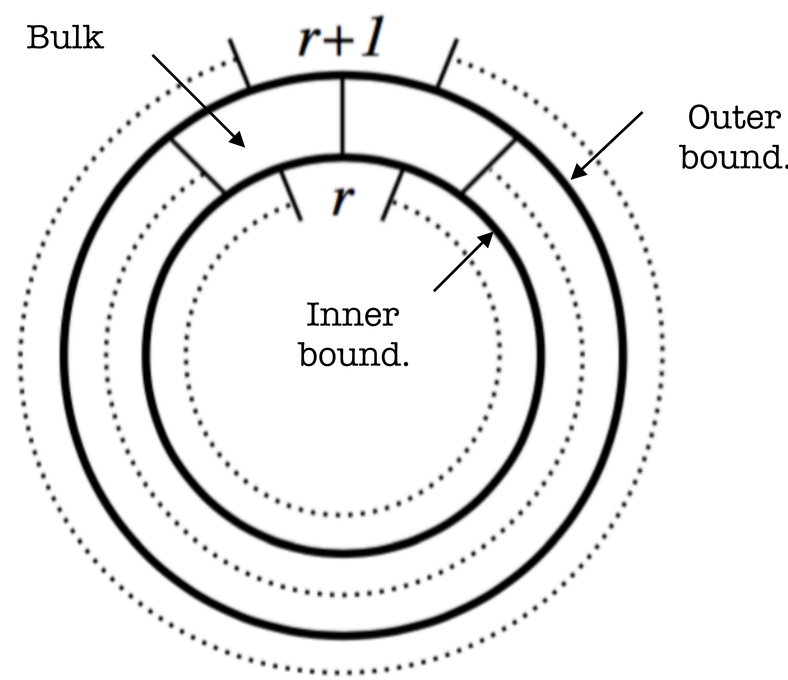

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Graphs can be counted explicitly

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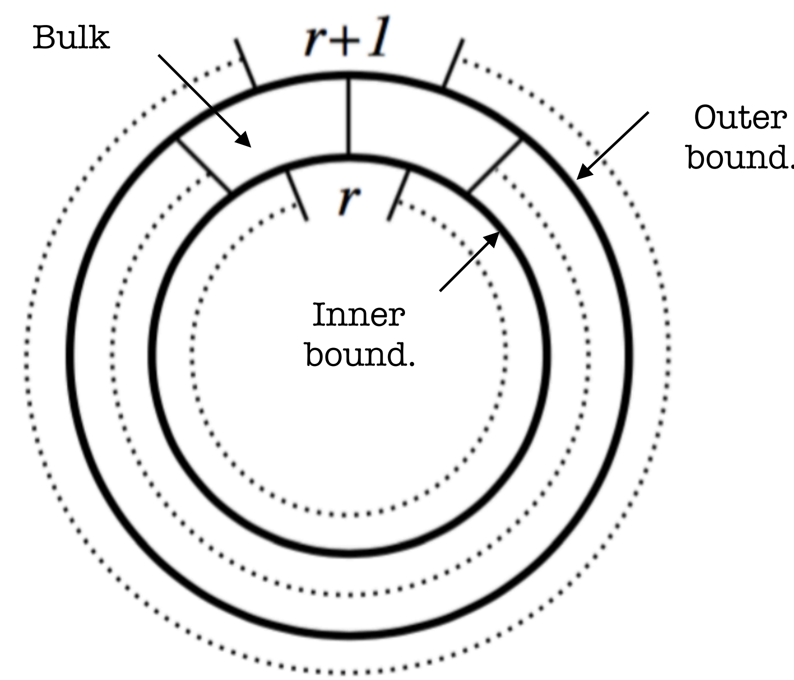

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Graphs can be counted explicitly

degeneracy of the single vertex Hilbert space

number of components independently refined

$$S(n,a) = \log\left(\mathcal{N}(n)\Delta(a)\right) \approx 2\,n\,l\log(2) + \log(\Delta(a)) - \frac{l}{2}\log(n)$$

fixed (large) number n of vertices

expectation value of the area for a single radial link

Horizon entropy calculation

DO, Pranzetti, Sindoni, '15-'18

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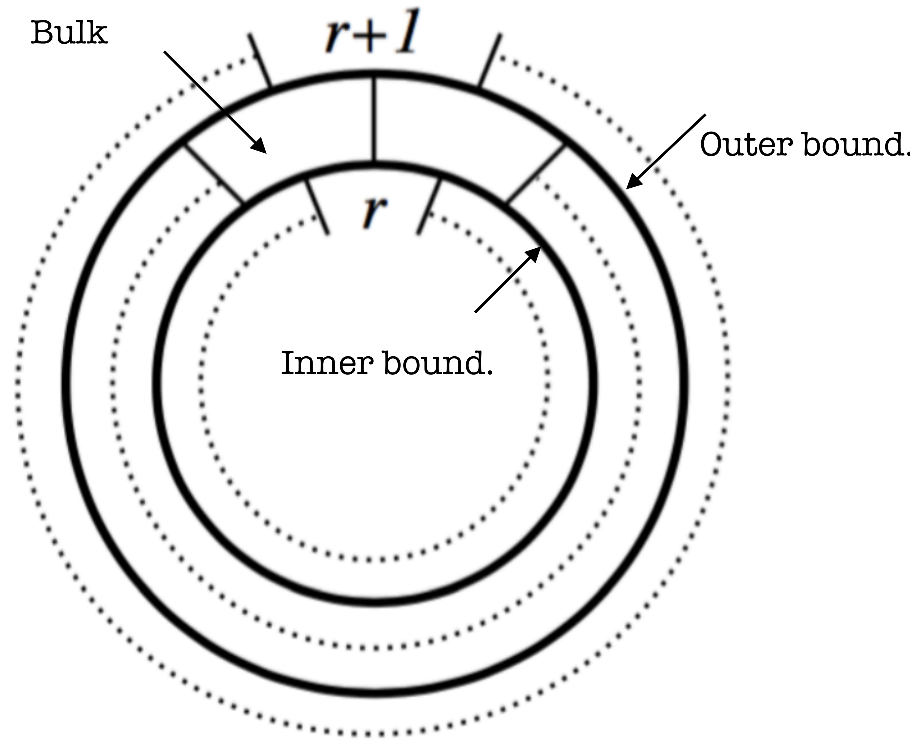

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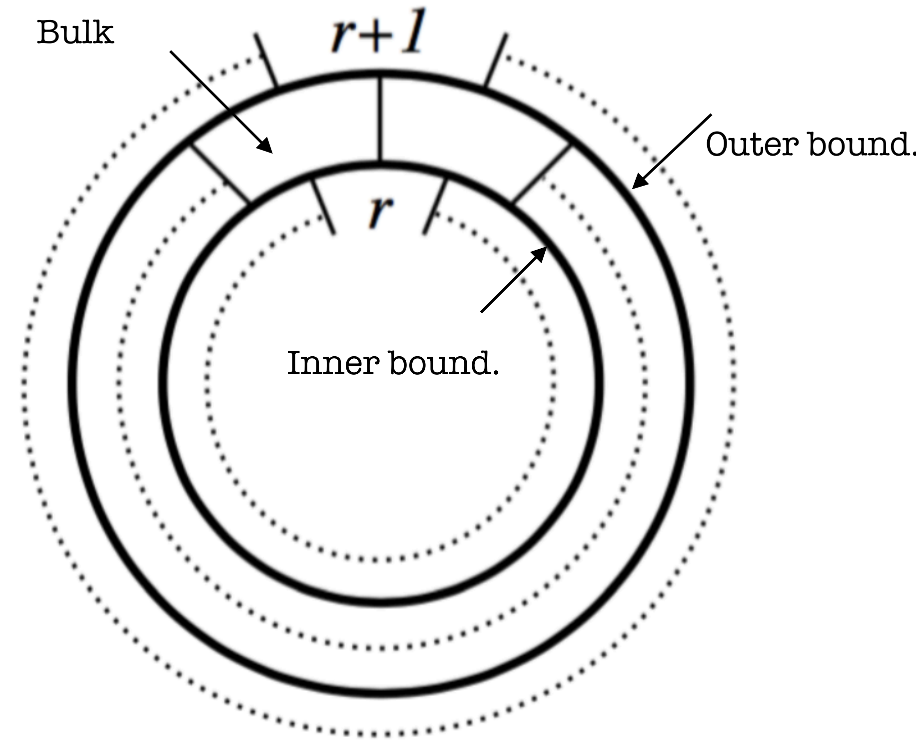

maximize entropy for given total (average) area:

$$\Sigma(n,a,\lambda) = S(n,a) + \lambda(\mathcal{A}_{IH}/\ell_P^2 - 2an)$$

$$\frac{\partial\Sigma}{\partial\lambda} = \frac{\mathcal{A}_{IH}}{\ell_P^2} - 2an = 0$$

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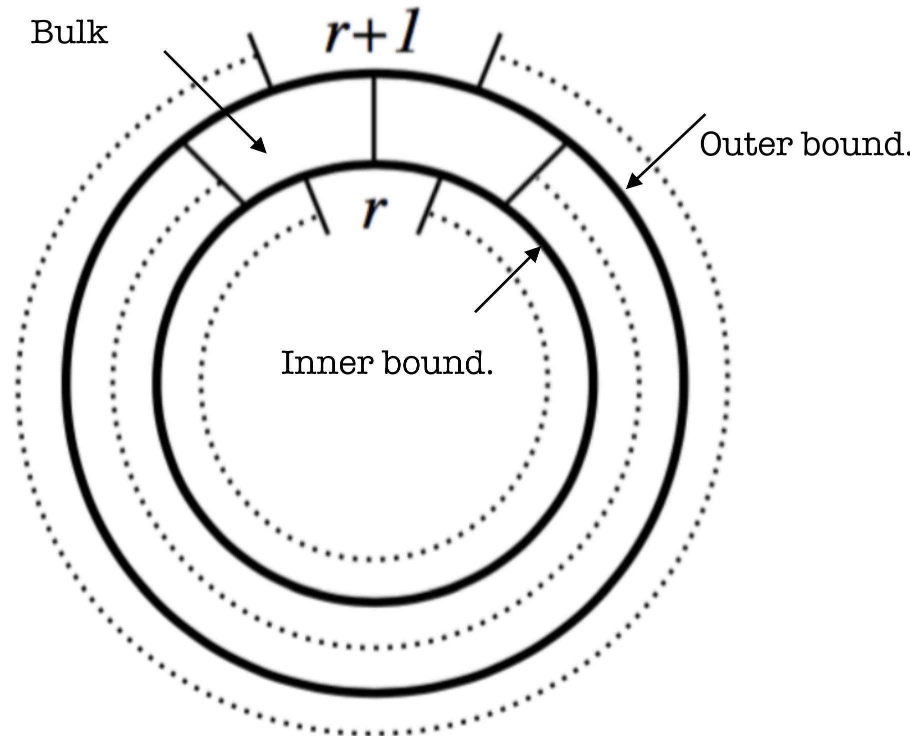

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Area law
+ logarithmic corrections

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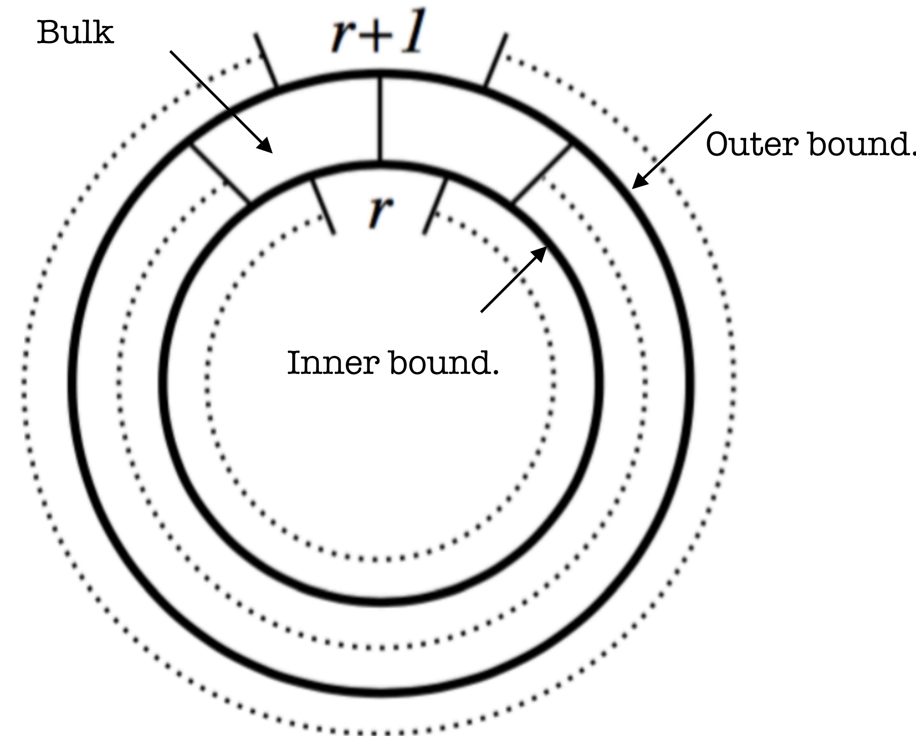

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Area law
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Assuming local (proper distance ℓ)
horizon thermodynamics

with (Unruh) temperature $\beta_U = 2\pi\ell/\ell_P^2$

and energy: $\mathcal{E}_{IH} = \frac{\mathcal{A}_{IH}}{8\pi\ell}$

consistency requires:

$$\beta_U = \frac{2\pi\ell}{\ell_P^2} = \frac{\partial S}{\partial\mathcal{E}_{IH}} = \frac{8\pi\ell}{\ell_P^2}2\lambda \quad\Rightarrow\quad 2\lambda = 1/4$$

Frodden, Ghosh, Perez, '11; Pranzetti, '13

Horizon entropy calculation

DO, Pranzetti, Sindoni, ’15-‘18

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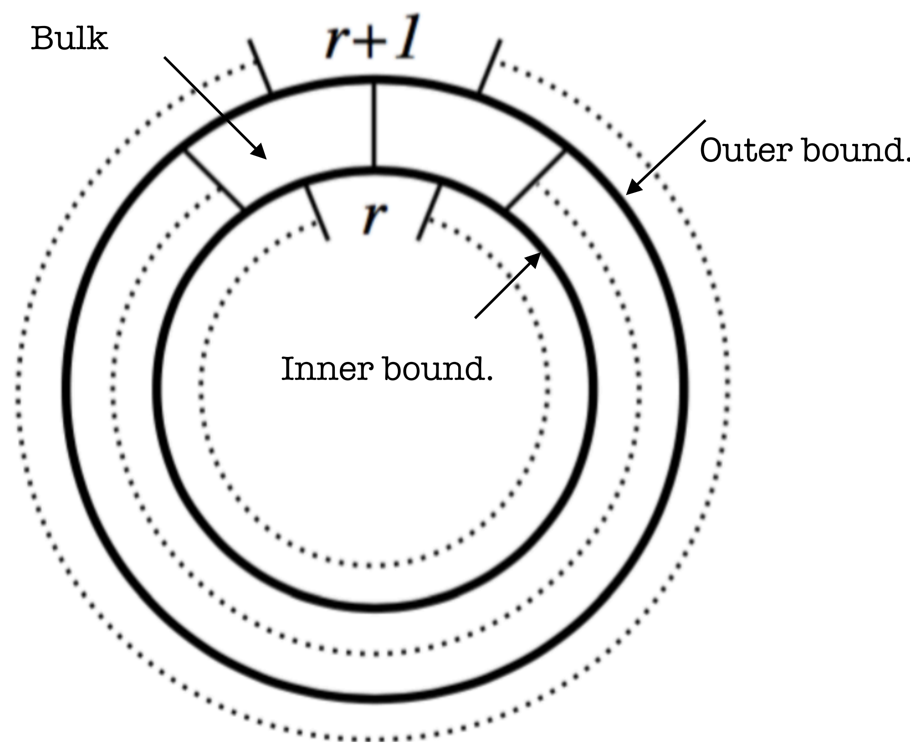

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Frodden, Ghosh, Perez, ’11; Pranzetti, ‘13

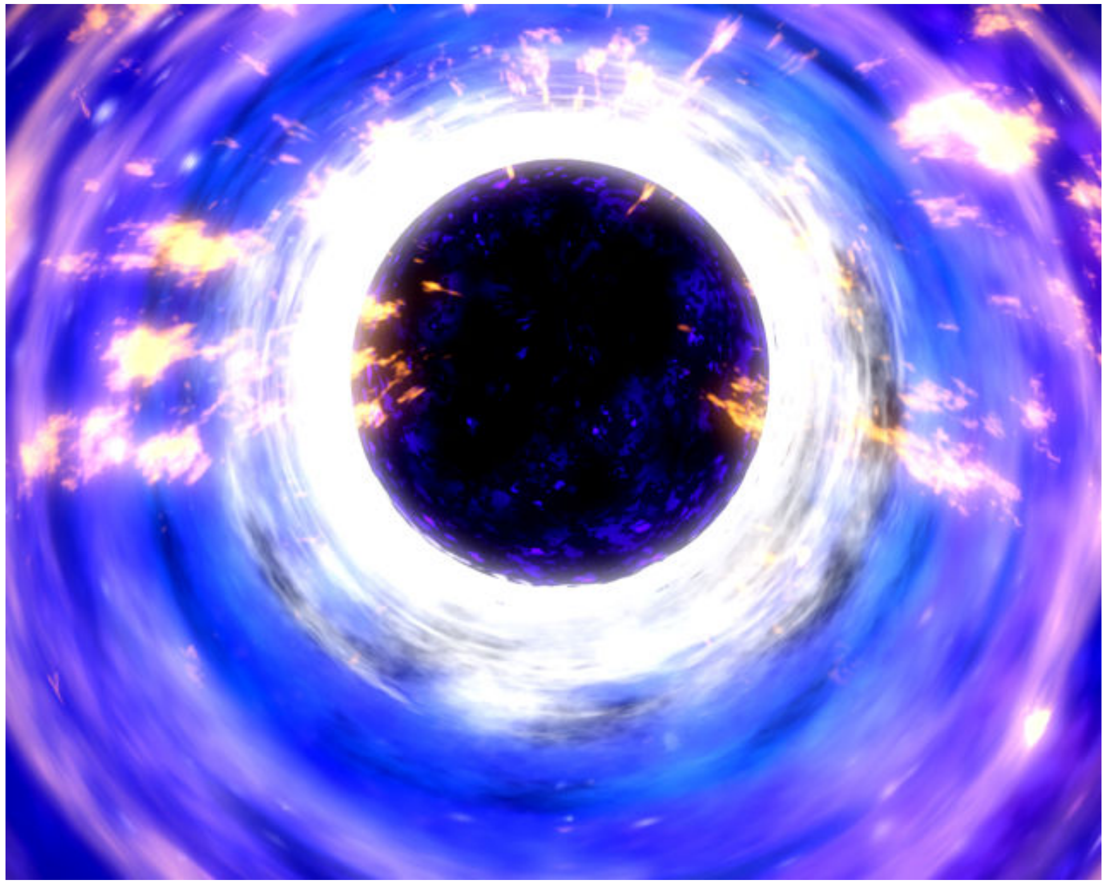
Bekenstein-Hawking entropy formula from full QG!

no dependence on Immirzi parameter

Outlook and lessons

What's next

- transition channels and Hawking radiation
- quantum dynamics
- generalization (matter fields, rotation, other states,…)
- ……….



Preliminary lessons

- (approx.) continuum quantum states for BHs can be studied (via GFT techniques)
- Boltzmann and von Neumann entropy may be related
- holography in subset of state space
- continuum description is approximate/emergent (~ hydro approx)
- horizon structure most likely modified

expect more surprises …..

Thank you for your attention!