

Fakeons and quantum gravity:
classical limit,
FLRW metric
and massive gravitons

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- The idea of fake particle ("fakeon") allows us to make sense of higher-derivative theories (under certain assumptions)
- It can also be applied to non-higher-derivative theories
- It amounts to a novel quantization prescription (alternative to the Feynman $i\epsilon$)
- It leads to an essentially unique theory of quantum gravity

Summary of the talk :

- Brief introduction to the theory
- Physical implications and predictions
- Classical limit
- Non-perturbative aspects of the classical limit
- FLRW solution
- Massive gravitons

- The fakeon simulates a physical particle in many situations, but does not belong to the physical spectrum of asymptotic states
- It must be introduced and later projected away (like the Faddeev-Popov ghosts), to have unitarity
- The fakeon prescription/projection does not follow from a gauge principle
- It leads to the violation of microcausality

The problem of quantum gravity is to make it renormalizable and unitary at the same time

$$\frac{1}{(p^2 - m_1^2)(p^2 - m_2^2)} = \frac{1}{m_1^2 - m_2^2} \left[\frac{1}{p^2 - m_1^2} - \frac{1}{p^2 - m_2^2} \right]$$

Higher derivatives lead to renormalizability,

but violate unitarity $S^\dagger S = \underline{1}$

$$\sum_{|n\rangle} \langle a | S^\dagger | n \rangle \langle n | S | b \rangle = \langle a | b \rangle \quad \text{Unless...}$$

Propagator: $\pm \frac{1}{p^2 - m^2}$

Feynman: $\pm \frac{1}{p^2 - m^2 + i\epsilon}$ $\pm \delta(p^2 - m^2)$

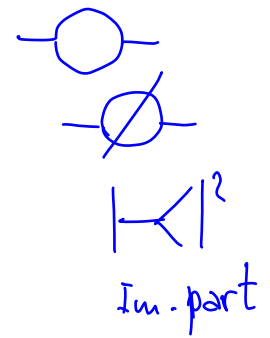
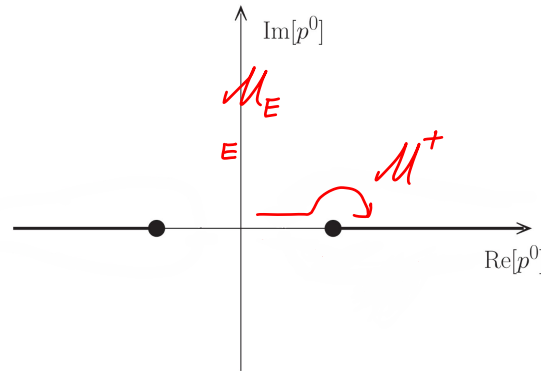
Feynman: 1) $\pm \frac{p^2 - m^2}{(p^2 - m^2)^2}$

2) $\pm \frac{p^2 - m^2}{(p^2 - m^2)^2 + \epsilon^4}$ $(= 0 \text{ on shell !})$
 $\rightarrow 0 \cdot \delta(p^2 - m^2)$

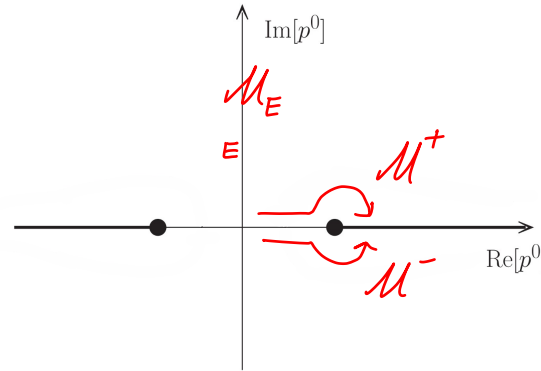
3) follow a certain procedure in loop diagrams

Beyond tree level:

Feynman
analytic continuation



fakeon



Result: average continuation

$$\frac{1}{2} (M^+ + M^-)$$

The so-defined theory
IS UNITARY

Quantum gravity

Consider the (renormalizable) higher-derivative action

$$S_{\text{QG}} = -\frac{1}{2\kappa^2} \int \sqrt{-g} \left[2\Lambda_C + \zeta R + \alpha \left(R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2 \right) - \frac{\xi}{6} R^2 \right] + S_{\text{m}}(g, \Phi)$$

$$\zeta, \alpha, \xi > 0$$

↑
Standard
Model

If quantized as usual, this theory has a ghost

Solution: quantize the would-be ghost as a fermion

D. Anselmi, On the quantum field theory of the gravitational interactions, J. High Energy Phys. 06 (2017) 086, 17A3 Renormalization.com and arXiv: 1704.07728 [hep-th].

Eliminate the higher derivatives by means of extra fields.

$$S_{\text{QG}}(g, \phi, \chi, \Phi) = S_{\text{H}}(g) + S_{\chi}(g, \chi) + S_{\phi}(\tilde{g}, \phi) + S_{\text{m}}(\tilde{g}e^{\kappa\phi}, \Phi)$$

where $\tilde{g}_{\mu\nu} = g_{\mu\nu} + 2\chi_{\mu\nu}$

$$S_{\text{H}}(g) = -\frac{\zeta}{2\kappa^2} \int \sqrt{-g} R, \quad S_{\phi}(g, \phi) = \frac{3\zeta}{4} \int \sqrt{-g} \left[\nabla_{\mu}\phi\nabla^{\mu}\phi - \frac{m_{\phi}^2}{\kappa^2} (1 - e^{\kappa\phi})^2 \right]$$

$$S_{\chi}(g, \chi) = S_{\text{H}}(\tilde{g}) - S_{\text{H}}(g) - 2 \int \chi_{\mu\nu} \frac{\delta S_{\text{H}}(\tilde{g})}{\delta g_{\mu\nu}} + \frac{\zeta^2}{2\alpha\kappa^2} \int \sqrt{-g} (\chi_{\mu\nu}\chi^{\mu\nu} - \chi^2) \Big|_{g \rightarrow \tilde{g}}.$$

Now, the $\chi_{\mu\nu}$ action has the wrong overall sign:

$$S_\chi(g, \chi) = -\frac{\zeta}{\kappa^2} S_{\text{PF}}(g, \chi, m_\chi^2) - \frac{\zeta}{2\kappa^2} \int \sqrt{-g} R^{\mu\nu} (\chi\chi_{\mu\nu} - 2\chi_{\mu\rho}\chi_\nu^\rho) + S_\chi^{(>2)}(g, \chi)$$

where S_{PF} is the covariantized Pauli-Fierz action

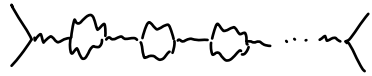
This means that $\chi_{\mu\nu}$ MUST be quantized as a fakeon. This way, we have both renormalizability and unitarity.

Instead, ϕ can be quantized either as a fakeon or as a physical particle

Graviton multiplet: $\{h_{\mu\nu}, \phi, \chi_{\mu\nu}\}$

$$g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}$$

fluctuation of the metric



Fakeon width:

$$\Gamma_x = -\alpha_x C m_x$$

$\Gamma_x < 0$: causality is violated by $\chi_{\mu\nu}$

massive scalar

spin-2
fakeon of mass m_x

$$m_x = \frac{M_{\text{pl}}}{\sqrt{\alpha}}$$

$$M_\phi = \frac{M_{\text{pl}}}{\sqrt{\alpha}}$$

$$C = \frac{N_s + 6N_f + 12N_v}{120}$$

$$\alpha_x = \left(\frac{m_x}{M_{\text{pl}}}\right)^2$$

$$\frac{i}{E - m + i\frac{\Gamma}{2}} \rightarrow \underbrace{\text{sgn}(t) \theta(\Gamma t)}_{\text{III}} \exp\left(-imt - \frac{\Gamma t}{2}\right)$$

$$\Gamma_{\text{BW}}(t)$$

$$\varphi(x) = \int d^4 y \Gamma_{\text{BW}}(x-y) J(y)$$

$\Gamma < 0$:

$$\varphi(t) = - \int_t^{\infty} dt' e^{-im(t-t') - \frac{\Gamma}{2}(t-t')} J(t')$$

The violation is short-range $\Delta t \approx \frac{1}{m}, \frac{1}{|\Gamma|}$

PROJECTION

$$Z_{\text{pr}}(J) = \int [d\varphi d\chi] \exp \left(iS(\varphi, \chi) + i \int J\varphi \right) = \exp (iW_{\text{pr}}(J))$$

NO SOURCE J_χ FOR χ

Projection = integrating out the fakeons with the fakeon prescription

At the level of generating functionals: $\Gamma(\varphi, \chi)$

$\varphi =$ physical fields

$\chi =$ fakeons

Solve $\delta\Gamma(\varphi, \chi)/\delta\chi = 0$ by means of the fakeon prescription

Let $\langle \chi \rangle$ denote the solution

Projected functionals: $\Gamma_{\text{pr}}(\varphi) = \Gamma(\varphi, \langle \chi \rangle)$

Classical limit

The action

$$\mathcal{S}_{\text{QG}}(g, \phi, \chi, \Phi) = S_{\text{H}}(g) + S_{\chi}(g, \chi) + S_{\phi}(\tilde{g}, \phi) + S_{\text{m}}(\tilde{g}e^{\kappa\phi}, \Phi)$$

is NOT the classical limit, because it is unprojected

Unprojected field equations :

$$g_{\mu\nu} : \quad R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = \frac{\kappa^2}{\zeta} [e^{3\kappa\phi} fT_{\text{m}}^{\mu\nu}(\tilde{g}e^{\kappa\phi}, \Phi) + fT_{\phi}^{\mu\nu}(\tilde{g}, \phi) + T_{\chi}^{\mu\nu}(g, \chi)]$$

$$\phi : \quad -\frac{1}{\sqrt{-\tilde{g}}} \partial_{\mu} (\sqrt{-\tilde{g}} \tilde{g}^{\mu\nu} \partial_{\nu} \phi) - \frac{m_{\phi}^2}{\kappa} (e^{\kappa\phi} - 1) e^{\kappa\phi} = \frac{\kappa e^{3\kappa\phi}}{3\zeta} T_{\text{m}}^{\mu\nu}(\tilde{g}e^{\kappa\phi}, \Phi) \tilde{g}_{\mu\nu},$$

$$\chi_{\mu\nu} : \quad \frac{1}{\sqrt{-g}} \frac{\delta S_{\chi}(g, \chi)}{\delta \chi_{\mu\nu}} = e^{3\kappa\phi} fT_{\text{m}}^{\mu\nu}(\tilde{g}e^{\kappa\phi}, \Phi) + fT_{\phi}^{\mu\nu}(\tilde{g}, \phi),$$

At the tree level, the subtleties about integration paths and average continuations are not important,

so we can take

$$\frac{p^2 - m^2}{(p^2 - m^2)^2 + \mathcal{E}^4} = \mathcal{P} \frac{1}{p^2 - m^2}$$

Use it to solve the fakeon equations

The projected classical action is then

$$\mathcal{S}_{\text{QG}}(g, \phi, \Phi) = S_{\text{H}}(g) + S_{\chi}(g, \langle \chi \rangle) + S_{\phi}(\bar{g}, \phi) + S_{\text{m}}(\bar{g} e^{\kappa \phi}, \Phi)$$

where $\langle \chi \rangle$ is the solution

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + 2\langle \chi_{\mu\nu} \rangle$$

Example : $\mathcal{L}_{\text{HD}} = \frac{m}{2}(\dot{x}^2 - \tau^2 \ddot{x}^2) + x F_{\text{ext}}(t)$

$$m \frac{d^2}{dt^2} \left(1 + \tau^2 \frac{d^2}{dt^2} \right) x = F_{\text{ext}}$$

invert with $\mathcal{P} \frac{1}{1 + \tau^2 \frac{d^2}{dt^2}}$

The projected equation is

$$m\ddot{x} = \int_{-\infty}^{\infty} du \frac{\sin(|u|/\tau)}{2\tau} F_{\text{ext}}(t - u)$$

→ violation of microcausality

~~$$\vec{F} = m\vec{a}$$~~

$$\langle \vec{F} \rangle = m\vec{a} \quad !!$$

But in general the projection is defined perturbatively
(since it comes from quantum gravity, which is defined perturbatively)

→ The classical equations are defined perturbatively : one may have to face asymptotic series and nonperturbative effects
(just to write the equations)

The FLRW metric

D. Anselmi, Fakeons and the classification of quantum gravity: the FLRW metric,
J. High Energy Phys. 04 (2019) 61, arXiv:1901.09273 [gr-qc]

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t) d\sigma^2$$

$g_{\mu\nu}, \phi, \cancel{\chi_{\mu\nu}}$
↑ fakeon

$$d\sigma^2 = \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$$

Unprojected equations $\left(\frac{\nabla}{\sqrt{-g}} \sim R + \frac{1}{m_\phi^2} R^2 \right)$

$$\Sigma \left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right) = \frac{4\pi G}{3} (\rho - 3p), \quad \Upsilon \left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} - \frac{k}{a^2} \right) = -4\pi G (\rho + p),$$

where

$$\Sigma = 1 + \frac{1}{m_\phi^2} \left(3 \frac{\dot{a}}{a} + \frac{d}{dt} \right) \frac{d}{dt}, \quad \Upsilon = \Sigma + \frac{2}{m_\phi^2} \left[\frac{k}{a^2} + 3 \frac{d}{dt} \left(\frac{\dot{a}}{a} \right) \right].$$

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G}{3} \tilde{\rho},$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = -8\pi G \tilde{p}.$$

where

$$\tilde{\rho} = \frac{1}{4} \langle \rho - 3p \rangle_{\Sigma} + \frac{3}{4} \langle \rho + p \rangle_{\Upsilon}$$

$$\tilde{p} = \frac{1}{4} \langle \rho + p \rangle_{\Upsilon} - \frac{1}{4} \langle \rho - 3p \rangle_{\Sigma}$$

$$\langle A \rangle_X \equiv \frac{1}{2} \left[\frac{1}{X} \Big|_{\text{rit}} + \frac{1}{X} \Big|_{\text{adv}} \right] A$$

The projection can be handled exactly for radiation combined with the vacuum energy:

$$p = \frac{\rho}{3} + p_0 \quad p_0 = \text{constant}$$

Exact solution: $\rho_0 = 3\sigma^2/(8\pi G) \quad \tilde{p} = (\tilde{\rho} - 4\rho_0)/3$

$\sigma, \sigma' = \text{constants}$

$$\rho(t) = \frac{3}{8\pi G} \left(\sigma^2 + \frac{\sigma'^2}{4a^4} \right), \quad \sigma'^2 = \sigma'^2 \left(1 + \frac{4\sigma^2}{m_\phi^2} \right) \quad \tilde{\rho}(t) = \frac{3}{8\pi G} \left(\sigma^2 + \frac{\sigma'^2}{4a^4} \right)$$

$$a(t) = \sqrt{\frac{\sinh(\sigma t)}{\sigma} \left(\sigma' \cosh(\sigma t) - \frac{k}{\sigma} \sinh(\sigma t) \right)}$$

But in general the projection is defined perturbatively
(since it comes from quantum gravity, which is defined perturbatively)

→ The classical equations are defined perturbatively : one may have to face asymptotic series and nonperturbative effects
(just to write the equations)

Example: cosmic dust ($p = 0$) or $p = w\rho$
 $w \neq \frac{1}{3}, -1$

Causality

- The relation between higher-derivatives and the violation of microcausality has been known since the Abraham-Lorentz force of classical electrodynamics

~~runaway solution~~ → violation of microcausality

$$\vec{F} = m\vec{a} - \tau m \dot{\vec{a}} \quad \Rightarrow$$

$$= m \left(1 - \tau \frac{d}{dt} \right) \vec{a}$$

$$m\vec{a} = \langle F \rangle = \frac{1}{\tau} \int_t^{\infty} dt' e^{-(t-t')/\tau} F(t')$$

[J.D. Jackson, Classical electrodynamics, chap. 17]

- In the Lee-Wick models (particular HD QFTs) it was noted right away by Lee and Wick

T.D. Lee and G.C. Wick, Negative metric and the unitarity of the S-matrix, Nucl. Phys. B 9 (1969) 209;

T.D. Lee and G.C. Wick, Finite theory of quantum electrodynamics, Phys. Rev. D 2 (1970) 1033.

- By the way, the fakeons allow us to understand the LW models better and actually complete their formulation, which was plagued with several issues

D. Anselmi and M. Piva, A new formulation of Lee-Wick quantum field theory, J. High Energy Phys. 06 (2017) 066, arXiv:1703.04584

D. Anselmi, Fakeons and Lee-Wick models, J. High Energy Phys. 02 (2018) 141, 18A1 arXiv:1801.00915 [hep-th].

D. Anselmi and M. Piva, Perturbative unitarity in Lee-Wick quantum field theory, Phys. Rev. D 96 (2017) 045009 arXiv:1703.05563

The Feynman-Wheeler electrodynamics could contain the classical massless fakeon with propagator

$$\mathcal{D} \frac{1}{p^2}$$

However, this option violates causality at-large, so Feynman and Wheeler developed an "emitter-absorber theory" to eliminate the massless fakeon and recover causality

J.A. Wheeler and R.P. Feynman, Interaction with the absorber as the mechanism of radiation, Rev. Mod. Phys. 17 (1945) 175;

J.A. Wheeler and R.P. Feynman, Classical electrodynamics in terms of direct inter-particle action, Rev. Mod. Phys. 21 (1949) 425.

Hoft - Veltman, Diagrammar, CERN 73-09, § 6.1

Every diagram, when multiplied by the appropriate source functions and integrated over all x contributes to the S-matrix. The contribution to the T-matrix, defined by

$$S = 1 + iT \quad (6.7)$$

is obtained by multiplying by a factor $-i$. Unitarity of the S-matrix implies an equation for the imaginary part of the so defined T matrix

$$T - T^\dagger = iT^\dagger T. \quad (6.8)$$

The T-matrix, or rather the diagrams, are also constrained by the requirement of causality. As yet nobody has found a definition of causality that corresponds directly to the intuitive notions; instead formulations have been proposed involving the off-mass-shell Green's functions. We will employ the causality requirement in the form proposed by Bogoliubov that has at least some intuitive appeal and is most suitable in connection with a diagrammatic analysis. Roughly speaking Bogoliubov's condition can be put as follows: if a space-time point x_1 is in the future with respect to some other space-time point x_2 , then the diagrams involving x_1 and x_2 can be rewritten in terms of functions that involve positive energy flow from x_2 to x_1 only.

The trouble with this definition is that space-time points cannot be accurately pinpointed with relativistic wave packets corresponding to particles on mass-shell. Therefore this definition cannot be formulated as an S-matrix constraint. It can only be used for the Green's functions.

Other definitions refer to the properties of the fields. In particular there is the proposal of Lehmann, Symanzik and Zimmermann that the fields commute outside the light cone. Defining fields in terms of diagrams, this definition can be shown to reduce to Bogoliubov's definition. The formulation of Bogoliubov causality in terms of cutting rules for diagrams will be given in Section 6.4.

Personal position:

- it is hard to elevate micro causality to a fundamental physical principle
- quantum gravity does not allow it at small distances
- micro causality is probably a blunder, due to our limited insight and poor experimental accuracy

Downgraded to : unusual form of the equations, fuzziness of the input/output relation ...

Massive gravitons?

D. Anselmi, Fakeons, unitarity, massive gravitons and the cosmological constant, arXiv: 1909.04955[hep-th]

Massive gauge fields :

$$\mathcal{L}_{\text{gf}} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} - \frac{1}{2\lambda}\mathcal{G}^a(A)\mathcal{G}^a(A) - \bar{C}^a\mathcal{G}^a(DC) + \frac{m_0^2}{2}A_0^{a2} - \frac{m_\gamma^2}{2}\mathbf{A}^{a2} - \bar{m}^2\bar{C}^a C^a$$

"Special" gauge :

$$\mathcal{G}^a(A) = \lambda\partial_0 A_0 + \nabla \cdot \mathbf{A}.$$

$$\langle A^0(k)A^0(-k) \rangle_0 = -\frac{i}{\lambda E^2 - \mathbf{k}^2 - m_0^2} \Big|_f, \quad \langle A^i(k)A^0(-k) \rangle_0 = 0,$$

$$\langle A^i(k)A^j(-k) \rangle_0 = \frac{i\Pi^{ij}}{E^2 - \mathbf{k}^2 - m_\gamma^2 + i\epsilon} + \frac{i\lambda}{\lambda E^2 - \mathbf{k}^2 - \lambda m_\gamma^2} \Big|_f \frac{k^i k^j}{\mathbf{k}^2},$$

$$\langle C(k)\bar{C}(-k) \rangle_0 = \frac{i}{\lambda E^2 - \mathbf{k}^2 - \bar{m}^2} \Big|_f,$$

$$\Pi^{ij} = \delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}$$

Gravity:

$$\mathcal{L}_{\text{gf}} = -\frac{1}{2\kappa^{d-2}} \sqrt{|g|} R + \frac{1}{4\lambda_1 \kappa^{d-2}} \mathcal{G}_0^2(g) - \frac{1}{4\lambda_2 \kappa^{d-2}} \mathcal{G}_i^2(g) + \bar{C}_0 \mathcal{G}_0(\overline{DC}) - \bar{C}_i \mathcal{G}_i(\overline{DC})$$

$$\mathcal{G}_0(g) = \frac{\lambda}{2} \partial_0 g_{00} + \frac{1}{2} \partial_0 g_{ii} - \partial_i g_{0i},$$

$$\mathcal{G}_i(g) = -\lambda_1 \partial_j g_{ij} + \frac{1}{2} (2\lambda_1 - 1) \partial_i g_{jj} + \lambda \partial_0 g_{0i} - \frac{\lambda}{2} \partial_i g_{00},$$

$$\bar{P}_{\text{phys}} = \frac{1}{E^2 - \mathbf{k}^2 + i\epsilon}, \quad \bar{P}_{\text{f}} = \frac{1}{\lambda E^2 - \mathbf{k}^2} \Big|_{\text{f}}, \quad \bar{P}'_{\text{f}} = \frac{1}{\lambda E^2 - \lambda_1 \mathbf{k}^2} \Big|_{\text{f}},$$

$$\langle C^0 \bar{C}^0 \rangle_0 = -i \bar{P}_{\text{f}}, \quad \langle C^0 \bar{C}^i \rangle_0 = \langle C^i \bar{C}^0 \rangle_0 = 0, \quad \langle C^i \bar{C}^j \rangle_0 = i \bar{P}'_{\text{f}} \Pi^{ij} + i \bar{P}_{\text{f}} \frac{k^i k^j}{\mathbf{k}^2}$$

$$\begin{aligned}
\langle h_{00}h_{00} \rangle_0 &= \frac{d-3}{d-2} i\bar{P}_f, & \langle h_{00}h_{ij} \rangle_0 &= \frac{i\delta_{ij}\bar{P}_f}{d-2}, \\
\langle h_{0i}h_{0j} \rangle_0 &= -\frac{i\lambda_1}{2} \left(\bar{P}'_f \Pi_{ij} + \bar{P}_f \frac{k_i k_j}{\mathbf{k}^2} \right), & \langle h_{00}h_{0i} \rangle_0 &= \langle h_{0i}h_{jk} \rangle_0 = 0, \\
\langle h_{ij}h_{mn} \rangle_0 &= \frac{i\bar{P}_{\text{phys}}}{2} \left(\Pi_{im}\Pi_{jn} + \Pi_{in}\Pi_{jm} - \frac{2}{d-2} \Pi_{ij}\Pi_{mn} \right) - \frac{\lambda}{\mathbf{k}^2} \frac{i\bar{P}_f}{d-2} (\Pi_{ij}k_m k_n + k_i k_j \Pi_{mn}) \\
&\quad + \frac{\lambda i\bar{P}'_f}{2\mathbf{k}^2} (\Pi_{im}k_j k_n + \Pi_{in}k_j k_m + \Pi_{jm}k_i k_n + \Pi_{jn}k_i k_m) + \lambda i\bar{P}_f \frac{d-3}{d-2} \frac{k_i k_j k_m k_n}{(\mathbf{k}^2)^2}.
\end{aligned} \tag{4}$$

If we give the graviton masses, the theory remains unitary and renormalizable

The massless limit is smooth: no $\sqrt{D-2}$ discontinuity

Comments on alternative approaches to the problem of quantum gravity

--- **string theory** is criticized for being nonpredictive. Moreover, its calculations often require very involved mathematics

--- **loop quantum gravity** is even more challenging, because it is at an earlier stage of development

--- **holography** (AdS/CFT correspondence) do not admit a weakly coupled expansion

--- **asymptotic safety** in nonperturbative as well.

None is as close to the standard model as the solution based on the fakeon idea, which is a quantum field theory, admits a perturbative expansion in terms of Feynman diagrams and allows us to make calculations with a comparable effort. It can be coupled to the standard model straightforwardly.

Our solution bests its competitors in calculability, predictivity and falsifiability. It is also rather rigid, because it contains only two new parameters. It could turn out to be the most predictive theory ever.