Saving Weyl from Einstein with Stueckelberg

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Based on: arXiv:1904.06596, 1812.08613 (JHEP), 1906.11572 (JHEP)

- Plan of the talk:

  - Introduction
  - Scale invariance beyond SM; global, local, gauged, quantum ...
  - Gauged scale invariance $\Rightarrow$ Weyl conformal geometry & gravity
  - How to break it: From Weyl quadratic gravity to Einstein-Proca action via Stueckelberg
  - Mass scale generation is a geometric transition from Weyl to Riemannian geometry
  - Coupling to matter and Weyl inflation

- Krisis? what krisis? *

- SM and Brount-Englert-Higgs mechanism confirmed @ LHC [official goal...]. Best model so far.

- So what's the problem?

  - pre-LHC: SUSY @ TeV: best candidate theory; stable hierarchy $m_h \ll M_P$.
  - reality check @ LHC: No sign of TeV-susy. May still exist at $\gg$ TeV scale.
  - SUSY preserves $m_h \ll M_P$; but what is the origin of M_P?

- A new symmetry? scale invariance (SI)

  - discrete = self-affinity, in Nature: fern; neurons network; fractals, cond-mat.
  - global SI. e.g.: SM with higgs $m_\phi = 0$ has scale invariance. [Bardeen 1995]
  - local SI

In this talk $\Rightarrow$

  - gauged SI = Weyl gauge symmetry $\to$ Weyl gravity: v. little studied [Weyl 1919]
  - quantum scale invariance (QSI): global, local. [Englert, Gastmans, Truffin 1976]
  - scales (m_h, M_P) from fields vev's... (if no QSI: also dim transmutation)

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* Greek: $\kappa\rho\iota\sigma\eta\mu o s$= at a critical/turning point [or decisive point of disease progression]

(1). Global scale invariance $x'_\mu = \rho\, x_\mu;\quad \phi'(\rho\, x) = \rho^{-1}\,\phi(x),$ forbids $\int d^4x\, m^2\phi^2$ [Bardeen]

- no dim-ful coupling; all scales (M_{Planck},...) from vev's; e.g. SM with $m_h = 0$
- SI usually broken by quantum corrections; can avoid this: $\mu \to \sigma$ (dilaton)
- QSI up to 3 loops: in flat space protects a hierarchy of fields vev's $\phi \ll \sigma$, but:
- not true if gravity present $\beta_\lambda \sim \lambda(..) + \xi(...)$ from $\xi h^2 R$. Tuning h selfcoupling λ
- Global symmetries broken by black hole physics.

Shaposhnikov 2008; D.G., Z. Lalak, P. Olszewski, 1612.09120, 1712.06024; Monin, Gretsch 2015

(2). Local scale invariance: $g'_{\mu\nu} = \Omega(x)^2 g_{\mu\nu},\quad \phi' = \frac{1}{\Omega(x)}\phi$ [t'Hooft 1104.4543, 1410.6675]

[Steinhardt Turok 1307.1848; Englert 1976]

(3). Gauged scale invariance $g'_{\mu\nu} = \Omega(x)^2 g_{\mu\nu},\quad \phi' = \frac{1}{\Omega(x)}\phi,\quad \omega'_\mu = \omega_\mu - \partial_\mu \ln \Omega(x)$

- also called Weyl gauge symmetry. Weyl gravity/conformal geometry (1918).
- Physics invariant under largest possible symmetry, including calibration.
- Non-metricity $\nabla_\mu\, g_{\alpha\beta} = -\omega_\mu\, g_{\alpha\beta}$. Einstein critique (1919). Little studied.

(2). Local scale invariance:

[t'Hooft 1104.4543; 1410.6675; Bars, Steinhardt, Turok 1307.1848]

invariance under $\hat{g}_{\mu\nu} = \Omega(x)^2 \, g_{\mu\nu}, \quad \hat{\sigma} = \dfrac{\sigma}{\Omega(x)} \quad (a)$

want to generate spontaneously: $L_{EH} = -\frac{1}{2}\sqrt{g}\, M_P^2 \; R \;\Rightarrow\; L_1 = -\sqrt{g}\,\frac{1}{2}\Big[\frac{1}{6}\sigma^2\, R + g^{\mu\nu}\,\partial_\mu\sigma\,\partial_\nu\sigma\Big]$ (inv.)

a) - has a ghost (σ)!

b) - Fake conformal symmetry? vanishing current [Jackiw, Pi 2015],

c) - Going to Einstein frame: d.o.f. number not conserved! $\Rightarrow$ something is missing.... if $\langle\sigma\rangle$ somehow fixed, is this really spontaneous breaking? does not look no.

we want to avoid these problems....$\Rightarrow$ gauged scale invariance

(3). Gauged scale invariance natural in Weyl geometry: defined by equivalent sets: $(g_{\mu\nu}, \omega_\mu)$

- invariance under: $\hat g_{\mu\nu}(x) = \Omega(x)^2\, g_{\mu\nu}(x), \quad \hat\sigma(x) = \dfrac{\sigma(x)}{\Omega(x)}, \quad \hat\omega_\mu(x) = \omega_\mu(x) - \partial_\mu \ln\Omega(x)^2 \quad (b)$

- Weyl geometry: $\tilde\nabla_\mu\, g_{\alpha\beta} = -\,\omega_\mu\, g_{\alpha\beta};$ with $\tilde\Gamma^\rho_{\mu\nu} = \Gamma^\rho_{\mu\nu} + (1/2)\Big[\delta^\rho_\mu\,\omega_\nu + \delta^\rho_\nu\,\omega_\mu - g_{\mu\nu}\,\omega^\rho\Big]$ inv of (b)

$\Gamma^\rho_{\mu\nu}$ = Levi-Civita; and $\nabla_\mu\, g_{\alpha\beta} = 0$ (∇_μ with $\Gamma^\rho_{\mu\nu}$)

$$\tilde R = R - 3\,D_\mu\omega^\mu - \frac{3}{2}\,\omega^\mu\omega_\mu. \quad\Rightarrow\quad \hat{\tilde R} = \frac{\tilde R}{\Omega^2}, \qquad \tilde D_\mu\sigma = (\partial_\mu - 1/2\,\omega_\mu)\,\sigma \;\Rightarrow\; \hat{\tilde D}_\mu\hat\sigma = \frac{\tilde D_\mu\sigma}{\Omega}$$

$\Rightarrow$ if $\omega_\mu \to 0$: $\tilde\Gamma^\rho_{\mu\nu} \to \Gamma^\rho_{\mu\nu}$, Weyl geometry $\to$ Riemannian; $\tilde R \to R$, Weyl tensor $\tilde C_{\mu\nu\rho\sigma} \to C_{\mu\nu\rho\sigma}$

$\Rightarrow$ All invariants of (b): $\sqrt{g}\,\tilde R^2, \quad \sqrt{g}\,\sigma^2\tilde R, \quad \sqrt{g}\,(\tilde D_\mu\sigma)^2, \quad \sqrt{g}\,F^2_{\mu\nu}, \quad \sqrt{g}\,\tilde C^2_{\mu\nu\alpha\beta}; \qquad F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$

- only kinetic terms! no higher dim ops (no scale to suppress them!)

$\tilde X$ denotes a quantity in the Weyl geometry

X denotes a quantity in Riemannian geometry

- Weyl quadratic gravity (no matter) and Einstein critique:

$$L_0 = \sqrt{g}\,\Big\{\frac{\xi_0}{4!}\,\tilde R^2 - \frac{1}{4}\,F_{\mu\nu}^2\Big\},\qquad \xi_0>0, \qquad \text{original Weyl action (1919)}$$

- Ω is real; $(g_{\mu\nu},\omega_\mu)$, equivalent sets related by "gauge" transformation (b).

- Non-metricity $\nabla_\mu\, g_{\alpha\beta} = \omega_\mu\, g_{\alpha\beta}$; then norm of a vector: $l^2 = l_0^2\,\exp\Big\{\int \omega_\mu dx^\mu\Big\}$

$\Rightarrow$ under parallel transport around closed curve: vector norm (rod length) $\&$ clock rate change!

- Einstein critique (1919)*, due to ω_μ. Weyl quadratic gravity abandoned * [review: Scholz 1703.03187]

- only solution: $\omega_\mu\!=\!0$, Weyl integrable geometry = Riemann geometry/Einstein gravity

(London (1927) introduces complex transformations of fields)

- Dirac (1973): new Weyl gravity: linear in $\tilde R$, extra matter ϕ: $\phi^2\tilde R$, [ω_μ still seen as a photon]

- Smolin (1979) et al: in the new Weyl linear gravity, ω_μ can become massive.

$\Rightarrow$ Q: in Weyl quadratic gravity (no matter), can ω_μ be massive, decouple, $\omega_\mu=0$ avoid non-metricity?

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* Weyl: "My work always tried to unite the truth with the beautiful, but when I had to choose one or the other, I usually chose the beautiful"

• Geometric Stueckelberg mechanism: A Weyl gauge invariant action:

$$L_1 = \sqrt{g}\,\Big[-\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}\,(\tilde D_\mu\sigma)^2 + \cdots \Big], \qquad F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$$

$$\tilde D_\mu\sigma \;=\; (\partial_\mu - q/2\,\omega_\mu)\,\sigma = (-q/2)\,\sigma\,\big[\,\omega_\mu - (1/q)\,\partial_\mu \ln\sigma^2\,\big].$$

"gauge fixing" transformation: $\Omega^2 = \dfrac{\sigma^2}{M^2}, \quad \hat g_{\mu\nu} = \Omega^2 g_{\mu\nu} \quad \hat\sigma = \dfrac{1}{\Omega}\,\sigma = M, \quad \hat\omega_\mu = \omega_\mu - \dfrac{1}{q}\,\partial_\mu \ln\Omega^2$

→ Proca action $$L_1 = \sqrt{\hat g}\,\Big[-\frac{1}{4}\,\hat F_{\mu\nu}^2 + \frac{q^2}{8}\,M^2 \hat\omega_\mu\hat\omega^\mu + \cdots \Big].$$

- spontaneous breaking; ω_μ has eaten the scalar $\ln\sigma$ (note $\ln\sigma \to \ln\sigma - \ln\Omega$)

- number of d.o.f. is conserved (=3).

• Weyl quadratic gravity = Einstein gravity + c.c. + massive ω_μ [D.G. arXiv:1812.08613, 1904.06596]

$$L_1 = \sqrt{g}\,\Big\{\frac{\xi_0}{4!}\,\tilde R^2 - \frac{1}{4\,q^2}\,F_{\mu\nu}^2\Big\}, \quad \xi_0>0, \qquad \text{original Weyl action (1918)}$$

$$= \sqrt{g}\,\Big\{\frac{\xi_0}{4!}\big(-2\,\sigma^2\,\tilde R - \sigma^4\big) - \frac{1}{4\,q^2}\,F_{\mu\nu}^2\Big\} \qquad \text{eom: } \sigma^2=-\tilde R; \quad \text{dilaton: } \ln\sigma \to \ln\sigma - \ln\Omega$$

+ Gauss-Bonnet + Weyl-tensor-squared.

• Weyl gauge transf of "gauge fixing":

$$\Omega^2 = \frac{\xi_0\,\sigma^2}{6M_P^2} \Rightarrow \hat g_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \hat\sigma^2 = \frac{6\,M_P^2}{\xi_0}, \quad \hat\omega_\mu = \omega_\mu - \partial_\mu \ln\sigma^2; \quad \text{use } \tilde R = R - 3D_\mu\omega^\mu - \frac{3}{2}\omega_\mu\omega^\mu$$

$$L_1 = \sqrt{\hat g}\,\Big\{-\frac{1}{2}\,M_P^2\,\hat R - \frac{3\,M_P^4}{2\,\xi_0} + \frac{3}{4}\,q^2\,M_P^2\,\hat\omega_\mu\,\hat\omega^\mu - \frac{1}{4}\,\hat F_{\mu\nu}^2\Big\}.$$

⇒ Einstein-Proca action for massive ω_μ which "absorbed" the dilaton $\ln\sigma$ field! mass of $\omega_\mu \propto q\,M_P$.

⇒ Stueckelberg mechanism, massless σ + ω_μ →massive ω_μ. no ghost! # dof=3 conserved!

⇒ massive ω_μ decouples; M_P: emergent scale of Weyl gauge symmetry breaking. ⇒ Weyl gravity viable

⇒ Einstein gravity: broken phase of Weyl gravity!

⇒ note: $\sigma \to \langle\sigma\rangle \sim M_P$, dynamically in FRW universe [arXiv:1801.07676] see talk by Graham Ross

• Weyl quadratic gravity = Einstein gravity + c.c. + massive ω_μ [DG arXiv:1812.08613, 1904.06596]

$$L_1 = \sqrt{g}\,\Big\{ \frac{\xi_0}{4!}\,\tilde R^2 - \frac{1}{4q^2}\,F_{\mu\nu}^2\Big\}, \quad \xi_0>0, \qquad \text{original Weyl action (1918)}$$

$$= \sqrt{g}\,\Big\{ \frac{\xi_0}{4!}\big(-2\,\sigma^2\,\tilde R - \sigma^4\big) - \frac{1}{4q^2}\,F_{\mu\nu}^2\Big\} \qquad \text{eom: } \sigma^2=-\tilde R; \quad \text{dilaton: } \ln\sigma \to \ln\sigma - \ln\Omega$$

First go to Riemannian picture $\tilde R = R - 3\,D_\mu\omega^\mu - 3/2\;\omega_\mu\omega^\mu$

$$L_1 = \sqrt{g}\Big\{-\frac{\xi_0}{2}\Big[\frac{1}{6}\sigma^2\,R + (\partial_\mu\sigma)^2\Big] - \frac{\xi_0}{4!}\sigma^4 + \frac{q^2}{8}\,\xi_0\,\sigma^2\,\big(\omega_\mu - \partial_\mu\ln\sigma^2\big)^2 - \frac{1}{4}\,F_{\mu\nu}^2\Big\}.$$

• gauge transf (b) "gauge fixing!" $\quad \Omega^2 = \dfrac{\xi_0\,\sigma^2}{6M_p^2} \Rightarrow \hat g_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \hat\sigma^2 = \dfrac{6\,M_P^2}{\xi_0}, \quad \hat\omega_\mu = \omega_\mu - \partial_\mu\ln\sigma^2$

$$L_1 = \sqrt{\hat g}\Big\{-\frac{1}{2}\,M_P^2\,\hat R - \frac{3\,M_P^4}{2\xi_0} + \frac{3}{4}\,q^2\,M^2\hat\omega_\mu\,\hat\omega^\mu - \frac{1}{4}\,\hat F_{\mu\nu}^2\Big\}.$$

⇒ recovered Einstein-Proca action; no ghost! # dof conserved! on-metricity: $m_\omega^2 \sim q^2 M_P^2$ v. large!

- conserved current J_μ, non-trivial as long as ω_μ dynamical: $\partial^\alpha(F_{\alpha\mu}\sqrt{g}) + \underbrace{\frac{1}{2}\sqrt{g}\,\xi_0\,\sigma\,\Big[\partial_\mu - \frac{q}{2}\,\omega_\mu\Big]\,\sigma}_{=J_\mu} = 0$

• Weyl quadratic gravity = Einstein gravity + c.c. + massive ω_μ [DG arXiv:1812.08613, 1904.06596]

- The above result remains true in the case of most general Weyl quadratic gravity which is $L_1 + L_C$:

$$L_1 = \sqrt{g}\,\Big\{\frac{\xi_0}{4!}\,\tilde R^2 - \frac{1}{4q^2}\,F_{\mu\nu}^2\Big\}$$

$$L_C = \frac{\sqrt{g}}{\eta}\tilde C_{\mu\nu\rho\sigma}\,\tilde C^{\mu\nu\rho\sigma} = \frac{\sqrt{g}}{\eta}\Big[C_{\mu\nu\rho\sigma}\,C^{\mu\nu\rho\sigma} + \frac{3}{2}\,F_{\mu\nu}^2\Big]$$

Weyl geometry Riemannian geometry

- This is because Weyl tensor $\tilde C_{\mu\nu\rho\sigma}$, $C_{\mu\nu\rho\sigma}$ are each invariant under the previous transformations.

• After Weyl gauge symmetry breaking ω_μ massive, decouples. Weyl geometry $\Rightarrow$ Riemannian

$\Rightarrow$ Mass scale generation is a geometric transition from Weyl to Riemannian geometry!

- Weyl quadratic gravity: a possible renormalizable embedding of Einstein gravity. [K. Stelle 1979]

• Weyl gravity with matter ϕ- scalar field, higgs-like, inflaton, etc

$$L_2 = \sqrt{g}\,\Big\{\frac{\xi_0}{4!}\,\tilde R^2 - \frac{1}{4q^2}\,F_{\mu\nu}^2\Big\} - \frac{\sqrt{g}}{12}\,\xi_1\,\phi^2\,\tilde R + \sqrt{g}\,\Big\{\frac{1}{2}(\tilde D_\mu\phi)^2 - \frac{\lambda}{4!}\phi^4\Big\}, \qquad \tilde R^2 \rightarrow -2\sigma^2\,\tilde R - \sigma^4$$

$$= \sqrt{g}\,\Big\{ -\frac{1}{4!}(\xi_0\,\sigma^2 + \xi_1\,\phi^2)\,\tilde R - \frac{1}{4\,q^2}\,F_{\mu\nu}^2 + \frac{1}{2}(\tilde D_\mu\phi)^2 - \frac{1}{4!}\,(\lambda\,\phi^4 + \xi_0\,\sigma^4)\Big\}, \qquad \rho^2 = \frac{1}{6}\,(\xi_0\sigma^2 + \xi_1\,\phi^2)$$

• Weyl gauge transformation (b): "gauge fixing": $\Omega = \dfrac{\rho^2}{M_P^2}, \quad \hat\rho = M_P, \quad \hat\omega_\mu = \omega_\mu - \ln\rho^2$

Einstein-Proca action for ω_μ :

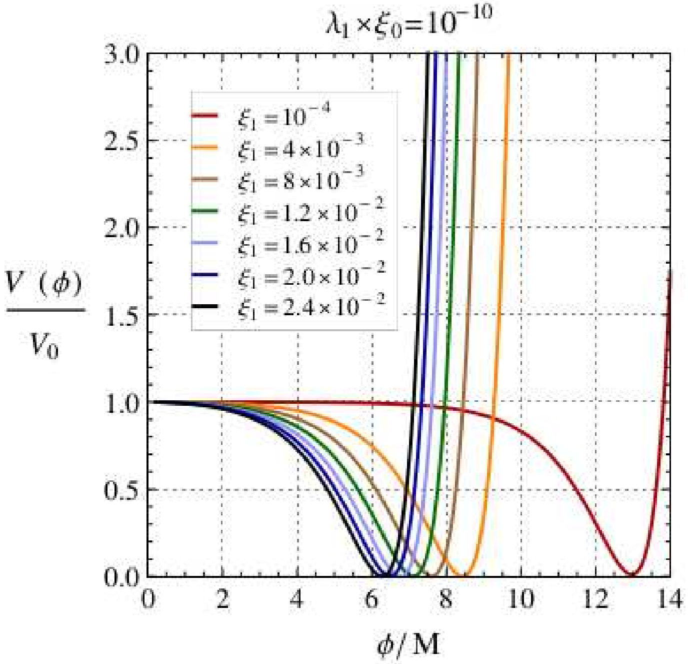
$$L_2 = \sqrt{\hat g}\,\Big\{ -\frac{1}{2}\,M_P^2\,\hat R + \frac{3}{4}M_P^2\,q^2\,\hat\omega_\mu\hat\omega^\mu - \frac{1}{4\,q^2}\hat F_{\mu\nu}^2 + \frac{1}{2}(\hat{\tilde D}_\mu\hat\phi)^2 - V\Big]\Big\},$$

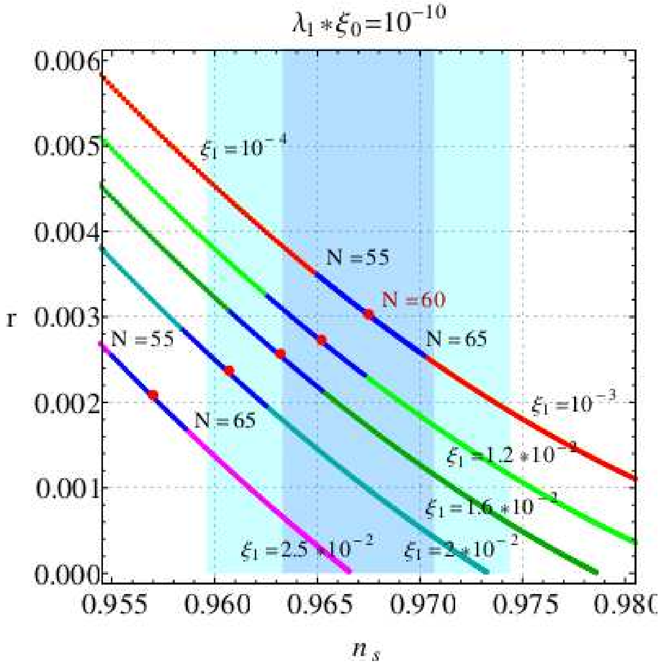
$$V = \frac{3M_P^4}{2\,\xi_0}\Big[1 - \frac{\xi_1\hat\phi^2}{6\,M_P^2}\Big]^2 + \frac{\lambda}{4!}\,\hat\phi^4 + \mathcal{O}\Big(\frac{\phi^2}{M_P^2}\Big), \quad \phi \ll M_P.$$

$\Rightarrow$ No dilaton $\ln\sigma$ left (eaten by ω_μ). Gravitational higgs mechanism. Also $m_\phi^2 = (-\xi_1/\xi_0)\,M_P^2$.

$\Rightarrow$ Fixed point for ξ_0? Quantum corrections?

• Weyl inflation: $V = V_0 \left\{ \left[1 - \xi_1 \sinh^2 \frac{\phi}{M_P\sqrt{6}}\right]^2 + \lambda\xi_0 \sinh^4 \frac{\phi}{M_P\sqrt{6}}\right\}, \quad V_0 = \frac{3}{2}\frac{M_p^4}{\xi_0}; \quad \phi > M_P$: OK!





$$\lambda\xi_0 \ll \xi_1^2 \ll 1: \quad \epsilon = \frac{\xi_1^2}{3}\sinh^2\frac{2\phi}{M_P\sqrt{6}} + \mathcal{O}(\xi_1^3); \qquad \eta = -\frac{2\xi_1}{3}\cosh\frac{2\phi}{M_P\sqrt{6}} + \mathcal{O}(\xi_1^2)$$

$$n_s = 1 + 2\eta - 6\epsilon = 1 - \frac{4}{3}\xi_1 \cosh\frac{2\phi_*}{M_P\sqrt{6}} + \mathcal{O}(\xi_1^2); \quad \Rightarrow \quad r = 3(1-n_s)^2 + \mathcal{O}(\xi_1^2)$$

$\Rightarrow 0.002567 \le r \le 0.00303$ if $n_s = 0.9670 \pm 0.0037$; $n_s \approx 1 - 2/\overline{N}$, $r = 12/\overline{N}^2$, $\overline{N} \approx N + 4.1$; N efolds

r midly smaller than Starobinsky, where: $\overline{N} \to N$; $r_s \approx 0.0031$, $n_s \approx 0.968$ $(N = 60)$

[D.G. arxiv:1906.11572; G. Ross, C. Hill, P, Ferreira, J. Noller 1906.03415] see next talk by Graham Ross.

- Conclusion:

- Weyl quadratic gravity $=$ Einstein-Proca for ω_μ $+$ cc, after Stueckelberg; no ghost, $\#$ dof conserved.
dilaton $\ln\sigma$=spin 0 mode in $\tilde{R}^2$ eaten by Weyl "photon" $m_\omega \sim qM_P$, where $M_P \sim \langle\sigma\rangle$

- Einstein gravity action: "low energy", broken phase of Weyl gauge symmetry. M_P emergent scale!
- field values above M_P are natural...
- Einstein's criticisms avoided since ω_μ=massive. Non-metricity scale (M_ω) bounds are low (TeV).
- Weyl quadratic gravity is a viable theory!

- Masss generation: geometric interpretation: transition from Weyl $\Rightarrow$ Riemann geometry
- Weyl gravity: a renormalizable theory (?) embedding of Einstein gravity.

- Weyl $\tilde{R}^2$ inflation predicts: $0.00257 \leq r \leq 0.00303$ for $N = 60$, n_s= measured 68%CL
(such small r values will soon be tested, LiteBIRD, CMB-S4).