Dark Matter with an ultralight axion

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Variants of the usual Peccei-Quinn axion theory for the solution of the strong CP problem allow to generate more general axion-like terms in an effective Lagrangean beyond the Standard Model (with a string completion).
One of these extensions involves Stuckelberg axions and (gauged) anomalous abelian symmetries.

Similar interactions are generated by other methods,
for instance by a decoupling of chiral fermions
from the low energy spectrum in an anomaly-free theory.

First realizations of these models involve
a field-theory version of the Green-Schwarz mechanism of
anomaly cancelation (2005).

A similar action can be generated by the decoupling of a fermion
from a high scale.

The fact that this mechanism is "generic" shows that anomaly actions,
which are not unique, may well serve the purpose of describing
the relevant physics at a certain, specific, scale.

We will try to present first a very simple introduction to
Stueckelberg fields, and then moving to more
complex models

Irges, Kiritsis, C.C.
Lazarides, Mariano, Morelli, C.C
Guzzi, Mariano C.C.
Frampton, C.C.

Stueckelberg Models

originated from stacks of branes in special vacua of string theory (orientifold models)

Antoniadis, Kiritsis, Tomaras, Rizos ...

when the gauge symmetry was

$$SU(3)\times SU(2)\times U(1)_a \times U(1)_b \times U(1)_c \;\dots$$

The extra $U(1)$'s were anomalous except for a linear combination which would generate $U(1)_Y$

string compactification axions (Stueckelberg axions)

If we use the "hypercharge" basis

$$SU(3) \times SU(2) \times U(1)_Y \times \underbrace{U(1)_A \times U(1)_B \times \dots U(1)_C}_{U(1)'\text{s}}$$

the $U(1)'$s

when anomalous and in a

Stückelberg phase

Anomaly "cancellation" was encoded into a specific effective action, later studied by Irges, Kiritsis, C.C.

- A specific model with a single extra $U(1)$ (anomalous) was investigated (Irges, Morelli, C.C.)
- Connection with "traditional" axion physics periodic potential, relic densities (Lazarides, Mariano, C.C.)

In those analysis

$$M_s \simeq 2 - 10 \text{ TeV}$$

- models accompanied by (one or more) extra anomalous Z'

  $\Rightarrow$ specific couplings at the LHC

  (M. Guzzi, C.C.)

- These analysis have been revived recently after the observation that dark matter could be extremely light (ultralight axions).

  Ostriker, Witten et al,

  Fuzzy dark matter

Together with Frampton
we have discussed the possibility of using similar actions
but at a far larger M_s scale

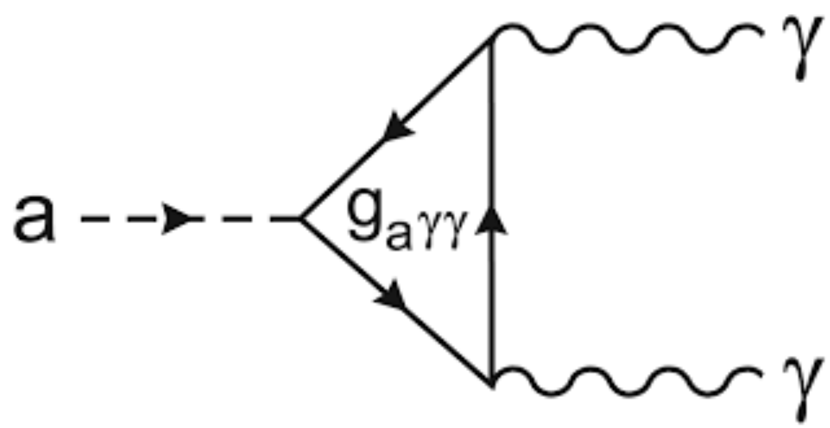
$$M_s \simeq M_{GUT}$$

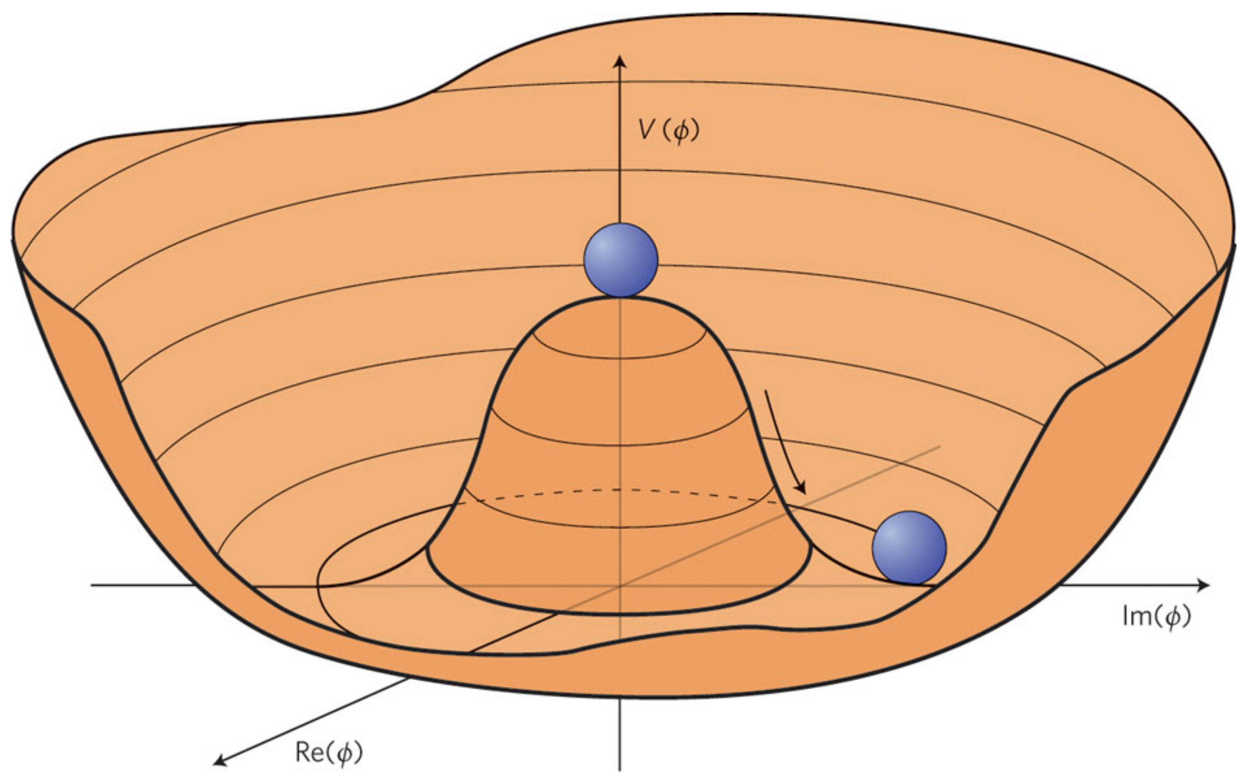
- In general, by choosing a large M_s,
one has a large suppression of the decay rate of
such axions

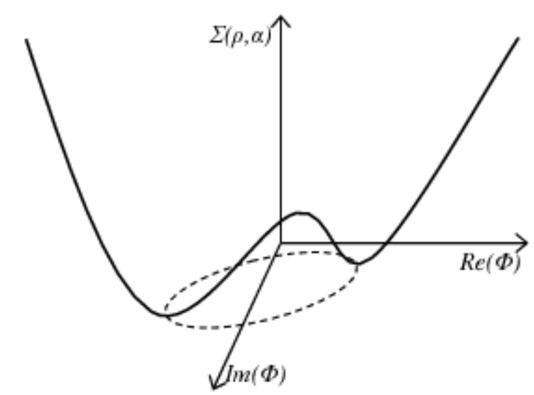
directly coupled to the anomaly.

- The suppression is not necessary in order to have a long-lived axion, (even $M_s \sim 10\,\mathrm{TeV}$ sufficient)
but it allows to associate a far smaller mass to such a particle (χ)

The breaking of the PQ symmetry takes place at a large scale f_a, but The wiggling of the PQ potential Occurs much later, at the QCD phase transition







For a PQ axion a: $m = C/f_a$, while the aFF interaction is also suppressed by : a/f_a FF with f_a = 10^9 GeV

Compared to a Peccei-Quinn axion, the new axion is gauged

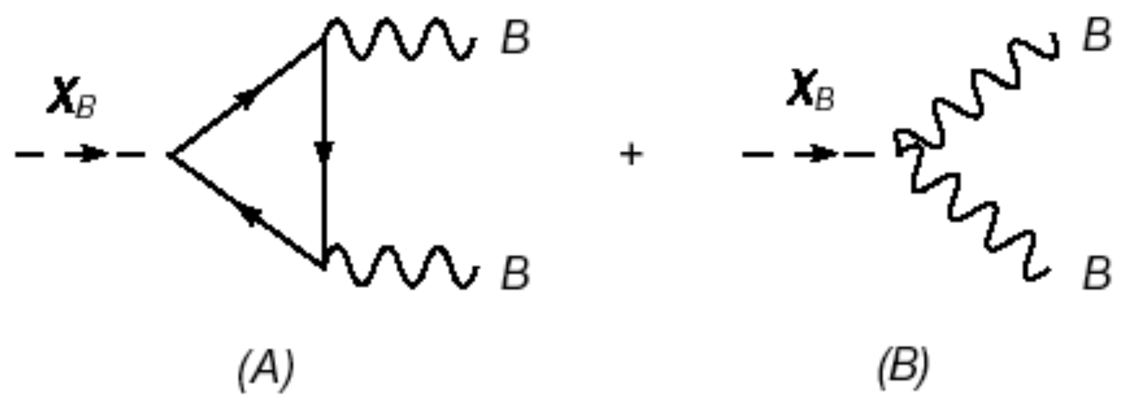
For a PQ axion a: $m = C/f_a$, while the aFF interaction is also suppressed by : a/f_a FF with $f_a = 10^9$ GeV

In the case of these models, the mass of the axion and its gauge interactions are unrelated

the mass is generated by the combination of the Higgs and the Stuckelberg mechanisms combined

The interaction is controlled by the Stuckelberg mass (M_1)

The axion shares the properties of a CP odd scalar



Introduction

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{1}{2} m^2 B_\mu B_\mu$$

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$B_\mu \to B'_\mu = B_\mu + \frac{1}{m} \partial_\mu b(x)$$

$$\mathcal{L} \longrightarrow \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{1}{2} m^2 \left(B_\mu + \frac{1}{m} \partial_\mu b(x) \right)^2$$

gauge symmetry

$$\begin{cases} B_\mu \to B_\mu + \partial_\mu \theta(x) \\ b(x) \to b(x) - m\,\theta(x) \end{cases}$$

$$\left(B_\mu + \frac{1}{m} \partial_\mu \theta(x) \right) \longrightarrow \left(B_\mu + \frac{1}{m} \partial_\mu \theta(x) \right)$$

invariant

Higgs - Stueckelberg (link)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + |D_\mu \Phi|^2 - V(\Phi) \,, \quad D_\mu \Phi = (\partial_\mu + i g_B q_B B_\mu)\Phi$$

$$\Phi = \left(\frac{\rho + v}{\sqrt{2}}\right) e^{i b(x)/v}$$

$$|D_\mu \Phi|^2 = \frac{1}{2}(\partial_\mu \rho)^2 + \frac{1}{2}(\partial_\mu b)^2 + \frac{1}{2} m^2 B_\mu^2 + m B_\mu \partial_\mu b(x)$$

$$+ (\rho, b) \text{ interactions}$$

$$m = g_B q_B v$$

Same structure as Stueckelberg

$$\frac{1}{2} m^2 (B_\mu + \partial_\mu b)^2 = \frac{1}{2} m^2 B^2 + \frac{1}{2}(\partial_\mu b)^2 + m B_\mu \partial_\mu b$$

Combine H + S

$$\mathcal{L} = |(\partial_\mu + i q_B g_B B_\mu)\Phi|^2 + \frac{1}{2}(\partial_\mu b + M_1 B_\mu)^2 - \frac{1}{4} F_B^2 - V(\Phi)$$

$$\Phi = \frac{1}{\sqrt{2}}\left(v + \varphi_1 + i\varphi_2\right)$$

$$\mathcal{L}_S = \frac{1}{2}(\partial_\mu \varphi_1)^2 + \frac{1}{2}(\partial_\mu \varphi_2)^2 + \frac{1}{2}(\partial_\mu b)^2 + \frac{1}{2}\overbrace{\left(M_1^2 + g_B q_B v\right)^2}^{M_B^2} B_\mu B_\mu$$

$$- \frac{1}{2} m_1^2 \varphi_1^2 + B_\mu \partial_\mu \underbrace{\left(M_1 b + g_B q_B v \varphi_2\right)}_{\text{Goldstone}}$$

Goldstone = linear combination of b and φ_2

Higgs-axion mixing

$$\begin{pmatrix} \chi_B \\ G_B \end{pmatrix} = \begin{pmatrix} -\frac{M_1}{M_B} & q_B g_B v \\ q_B g_B v & \frac{M_1}{M_B} \end{pmatrix} \begin{pmatrix} \varphi_2 \\ b \end{pmatrix}$$

$$b = \alpha_1 \chi_B + \alpha_2 G_B$$

✓ physical Higgs $\varphi_1 \equiv h(x)$

✓ a physical CP-odd χ_B (massless)

✓ a Goldstone mode G_B

$$\mathcal{L}_{scalar} = \frac{1}{2}(\partial_\mu \chi)^2 + \frac{1}{2}(\partial_\mu h_1)^2 + \frac{1}{2}(\partial_\mu G_B)^2 + \frac{1}{2} M_B^2 B_\mu^2$$
$$- \frac{1}{2} m_1 h_1^2 \qquad + \dots$$

$$M_B^2 = M_1^2 + (g_B q_B v)^2$$

M_1^2: tree-level Stueckelberg

$(g_B q_B v)^2$: SSB Higgs

$h_1 \equiv \varphi_1$ $\qquad M_B \equiv \sqrt{M_1^2 + (g_B q_B v)^2}$ $\qquad$ N. Irges, Moretti, C.C.

massive Higgs

massless Goldstone

massless Goldstone

$$\mathcal{L}_{scalar} = \frac{1}{2}(\partial_\mu \varphi_1)^2 + \frac{1}{2}(\partial_\mu \varphi_2)^2 + \frac{1}{2}(\partial_\mu b)^2 + \frac{1}{2}\left[M_1^2 + (q_B g_B v)^2\right] B_\mu B_\mu$$
$$- \frac{1}{2} m_1^2 \varphi_1^2 + \underbrace{B_\mu \partial_\mu (M_1 b + v g_B q_B \varphi_2)}_{\text{bilinear mixing}}$$

defines G_B

the Goldstone of $\underline{B_\mu}$,
is a linear combination of $\underline{b}$ and φ_2

$$\Rightarrow \mathcal{L}_{scalar} = \frac{1}{2}(\partial_\mu \chi_B)^2 + \frac{1}{2}(\partial_\mu G_B)^2 + \frac{1}{2}(\partial_\mu h_1)^2 + \frac{1}{2} M_B^2 B_\mu^2$$
$$- \frac{1}{2} m_1^2 h_1^2 + M_B B^\mu \partial_\mu G_B$$

To obtain this expression we have performed a rotation in the CP- odd sector

$$\chi_B = \frac{1}{M_B}\left(-M_1 \varphi_2 + q_B g_B v\, b(x)\right)$$

$$G_B = \frac{1}{M_b}\left(q_B g_B v \varphi_2(x) + M_1\, b(x)\right)$$

SSB in the ordinary φ (Higgs) sector has broken the local shift invariance of the Lagrangian in <u>b(x)</u>

We have a mixing between $\underbrace{b(x) \text{ and } \varphi}$.

If such a mixing is not present, the Stueckelberg field remains unphysical (a gauge artifact)

$$U = \begin{pmatrix} -\cos\theta_B & \sin\theta_B \\ \sin\theta_B & \cos\theta_B \end{pmatrix}$$

with $\theta_B = \arccos(M_1/M_B) = \arcsin(q_B g_B v/M_B)$. The axion b can be expressed as linear combination of the rotated fields χ_B,G_B as

$$b = \alpha_1 \chi_B + \alpha_2 G_B = \frac{q_B g_B v}{M_B}\chi_B + \frac{M_1}{M_B} G_B, \tag{41}$$

$$\mathcal{L}_{gf} = -\frac{1}{2}\mathcal{G}_B^2$$

$$\mathcal{G}_B = \frac{1}{\sqrt{\xi_B}}\left(\partial \cdot B - \xi_B M_B G_B\right),$$

$$
\begin{aligned}
\mathcal{L}_B \;=\; & \frac{1}{2}(\partial_\mu\chi)^2 - \frac{1}{2\xi_B}(\partial\cdot B)^2 + \frac{1}{2}(\partial_\mu G_B)^2 + \frac{1}{2}(\partial_\mu h_1)^2 - \frac{1}{2}m_1^2 h_1^2 + \frac{1}{2}M_B^2 B_\mu^2 - 4\frac{v g_B^2}{M_B} B_\mu G_B \partial^\mu h_1 \\
& - \frac{4\lambda v^4 g_B^4}{M_B^4} G_B^4 + \frac{8 v^2 g_B^4}{M_B^2}(B_\mu)^2 G_B^2 + \frac{8\lambda M_1 v^3 g_B^3}{M_B^4}\chi_B G_B^3 - \frac{8 M_1 v g_B^3}{M_B^2}(B_\mu)^2 \chi_B G_B \\
& - \frac{4 g_B^2 \lambda v^3}{M_B^2} G_B^2 h_1 + 4 g_B^2 (B_\mu)^2 h_1 v + 2\frac{g_B^2 M_1^2}{M_B^2}(B_\mu)^2\chi^2 + 2 g_B^2 (B_\mu)^2 h_1^2 \frac{v g_B^2}{M_B} B_\mu h_1 \partial^\mu G_B \\
& + \frac{2\lambda M_1 v g_B}{M_B^2}\chi G_B h_1^2 + \frac{2 g_B \lambda M_1^3 v}{M_B^4} G_B \chi^3 + \frac{4 g_B \lambda M_1 v^2}{M_B^2} G_B h_1 \chi_B - \frac{2 g_B M_1}{M_B} B^\mu \partial_\mu \chi h_1 \\
& - \frac{\lambda M_1^4}{4 M_B^4}\chi^4 + \frac{2 g_B M_1}{M_B} B^\mu \partial_\mu h_1 \chi_B - \frac{1}{4}\lambda h_1^4 - \lambda v h_1^3 + \frac{3\lambda M_1^4}{2 M_B^4}\chi^2 G_B^2 - \frac{3\lambda M_1^2}{2 M_B^2}\chi^2 G_B^2 \\
& - \frac{1}{2}\lambda h_1^2 G_B^2 - \frac{1}{2} M_B^2 \xi_B G_B^2 - \frac{\lambda M_1^2}{2 M_B^2}\chi^2 h_1^2 + \frac{\lambda M_1^2}{2 M_B^2} G_B^2 h_1^2 - \frac{\lambda M_1^2 v}{M_B^2}\chi^2 h_1
\end{aligned}
\tag{47}
$$

at this stage nothing special. We are just describing a model in which the scalar CP odd sector has been extended with a real pseudoscalar that contributes to SSB thanks to its mixing with the ordinary Higgs

The Higgs-Stueckelberg mixing potential

$$V = \mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2 \qquad \mu^2 < 0$$

$$V_{periodic} = b_1 \big(\underbrace{\varphi\, e^{-i q_B g_B b/M_1}} \big) + \lambda_1 \left(\varphi\, e^{-i q_B g_B \frac{b}{M_1}} \right)^2 + 2\lambda_2 (\varphi^* \varphi) \left(\varphi\, e^{-i g_B q_B b/M_1} \right) + h.c.$$

we need to write down all the dimension 4 operators allowed by the symmetry

Under the $U(1)_B$ symmetry (the Stueckelberg is changed under $U(1)_B$)

$$\begin{cases} \varphi \to e^{+i g_B q_B \theta(x)} \varphi \\ b \to b - M_1 \theta(x) \\ B_\mu \to B_\mu + \partial_\mu \theta(x) \end{cases}$$

invariant

$$\alpha \equiv \varphi(x)\, e^{-i g_B q_B \frac{b(x)}{M_1}} \longrightarrow \left(\varphi(x)\, e^{i g_B q_B \theta(x)} \right) e^{-i g_B q_B \frac{1}{M_1}(b - M_1 \theta(x))} \equiv \alpha$$

Notice that the potential is periodic

We are going to see an example in the context of a $\underbrace{SM \times U(1)'}$ model

when SM is the Standard Model

$$\bar{V}_{total}(\varphi, b) = V + V_{periodic}$$

the CP-odd sector gives at quadratic level

$$(\varphi_2, b)\, \mathcal{M}_2 \begin{pmatrix} \varphi_2 \\ b \end{pmatrix}, \quad \mathcal{M}_2 = -\frac{1}{2} C_\chi v^2 \begin{pmatrix} 1 & -v\, q_B g_B / M_1 \\ -\frac{v\, q_B g_B}{M_1} & \frac{v^2 q_B^2 g_B^2}{M_1^2} \end{pmatrix}$$

$$m_\chi^2 = -\frac{1}{2} C_\chi v^2 \left[1 + \frac{q_B^2 g_B^2 v^2}{M_1^2} \right]$$

$$C_\chi = 4\left(\frac{b_1}{v^3} + \frac{4\lambda_1}{v^2} + \frac{2\lambda_2}{v} \right)$$

$$\mathcal{L}_0 = \left|\left(\partial_\mu + i g_B q_B B_\mu\right)\Phi\right|^2 - \frac{1}{4}F_A^2 - \frac{1}{4}F_B^2$$

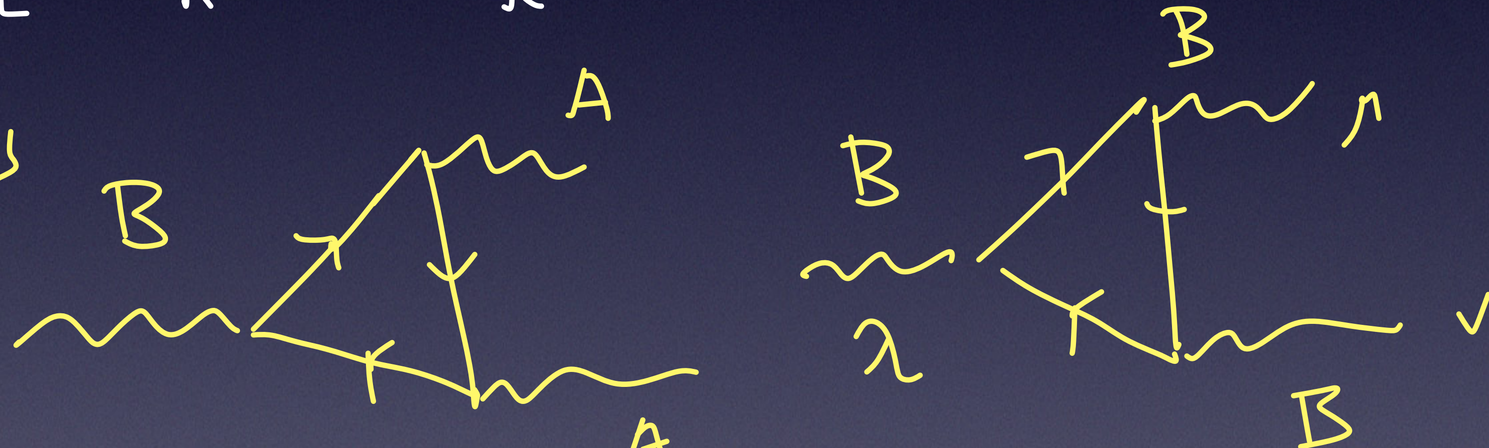
$$+ \frac{1}{2}\left(\partial_\mu b + M_1 B_\mu\right)^2 - \lambda\left(|\phi|^2 - v^2/2\right)^2 + V_{\text{periodic}}$$

$$+ \bar{\psi}\, i\gamma^\mu\left(\partial_\mu + ieA_\mu + i g_B \gamma_5 B_\mu\right)\psi$$

$$-\lambda_1 \bar{\psi}_L \phi \psi_R - \lambda_1 \bar{\psi}_R \phi^* \psi_L$$

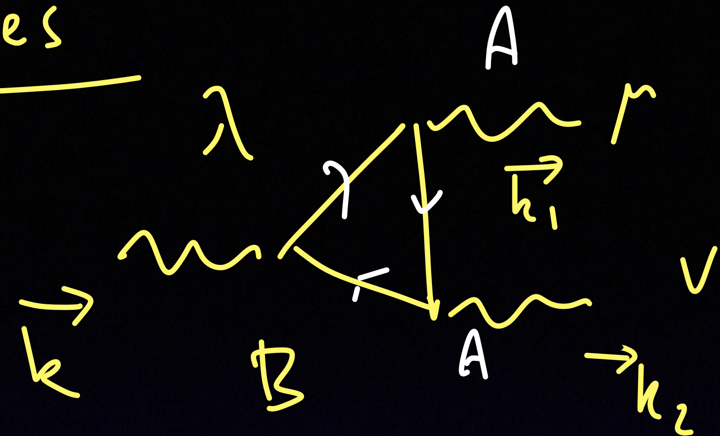
ψ = Dirac fermion

Anomalies



and consider the 1PI effective action

Anomalies



B^3 anomaly

$$\begin{cases} k^\lambda \Delta^{\lambda\mu\nu} = a_1 \, \varepsilon^{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta \\ k_1^\mu \Delta^{\lambda\mu\nu} = a_2 \, \varepsilon^{\lambda\mu\alpha\beta} k_2^\alpha k_1^\beta \\ k_2^\nu \Delta^{\lambda\mu\nu} = a_3 \, \varepsilon^{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta \end{cases}$$

$a_1 = -\frac{i}{8\pi^2}$

$a_2 = -\frac{i}{8\pi^2}$

$a_3 = -\frac{i}{4\pi^2}$

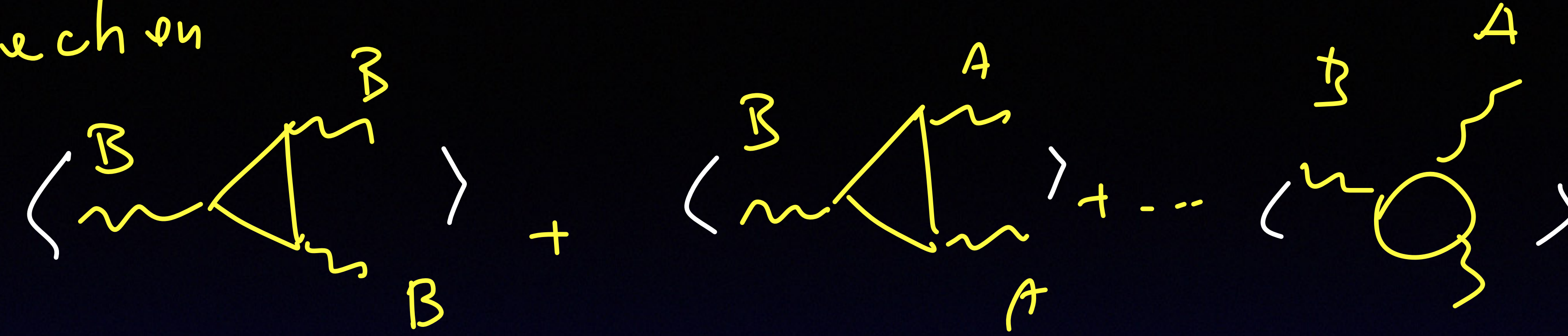
notice that $\boxed{a_1 + a_2 + a_3 = c_n}$ is fixed independent of the regularization

In coordinate space

$$\begin{cases} \frac{\partial}{\partial x^\mu} T^{\lambda\mu\nu} = i \, a_1 \, \varepsilon^{\lambda\nu\alpha\beta} \frac{\partial}{\partial x^\alpha} \frac{\partial}{\partial x^\beta} \left(\delta^4(x-z) \, \delta^4(y-t) \right) \\ \vdots \qquad\qquad\qquad \vdots \end{cases}$$

Now compute the variation of the 1PI effective action

$$S = S_0 +$$



these are the only anomalous contributions

example

$$\langle BAA \rangle \equiv \int d^4x\, d^4y\, d^4z\; T^{\lambda\mu\nu}(z,x,y)\, B^\lambda(z)\, A^\mu(x)\, A^\nu(y)$$

perform a gauge transformation

$$\delta B_\mu = \partial_\mu \theta_B$$

$$\delta_B \langle BAA \rangle = \int d^4x\, d^4y\, d^4z\; T^{\lambda\mu\nu}(z,x,y)\, \underbrace{\delta B^\lambda(z)}_{\partial_z \theta_B(z)}\, A^\mu(x)\, A^\nu(y)$$

integrate by parts

$$= i\,\frac{a_3}{4} \int d^4x\; \theta_B(x)\, F^A_{\mu\nu}\, F^A_{\rho\sigma}\, \epsilon^{\mu\nu\rho\sigma}$$

the idea, in order to restore gauge symmetry is to introduce an interaction counterterm (or compensator)

$$\frac{b(x)}{M_1} F^A_{\mu\nu} F^A_{\rho\sigma} \varepsilon^{\mu\nu\rho\sigma}$$

Since the anomaly is on B_μ, we need a pseudoscalar $b(x)$

$$\delta B_\mu = \partial_\mu \theta_B(x)$$

$$b \to b - M_1 \theta_B(x) \, , \quad \boxed{\delta\left(\frac{b(x)}{M} F \wedge F\right) = \theta_B F_A \tilde{F}_A}$$

then

$$\delta \langle BAA \rangle + \delta \int \left(\frac{b(x)}{M_1} F_A \wedge F_A \right) = 0$$

since $\delta \langle BAA \rangle \simeq \int dx \, \theta F_A \tilde{F}_A$ and $\delta b = M_1 \theta_B(x) \Rightarrow$

$$\mathcal{L} \;=\; -\frac{1}{12}H^{\mu\nu\rho}H_{\mu\nu\rho} - \frac{1}{4g^2}F^{\mu\nu}F_{\mu\nu} + \frac{M}{4}\,\epsilon^{\mu\nu\rho\sigma}A_{\mu\nu}\;F_{\rho\sigma},$$

$$H_{\mu\nu\rho} = \partial_\mu A_{\nu\rho} + \partial_\rho A_{\mu\nu} + \partial_\nu A_{\rho\mu}, \qquad F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$\mathcal{L}_0 = -\frac{1}{12}H^{\mu\nu\rho}H_{\mu\nu\rho} - \frac{1}{4g^2}F^{\mu\nu}\;F_{\mu\nu} - \frac{M}{6}\,\epsilon^{\mu\nu\rho\sigma}H_{\mu\nu\rho}\;B_\sigma + \frac{1}{6}\,b(x)\,\epsilon^{\mu\nu\rho\sigma}\partial_\mu H_{\nu\rho\sigma}.$$

$$H^{\mu\nu\rho} = -\epsilon^{\mu\nu\rho\sigma}\,(MB_\sigma - \partial_\sigma b)\,.$$

Stueckelberg forms from effective string int.

$$\mathcal{L}_A \;=\; -\frac{1}{4g^2}\;F^{\mu\nu}\;F_{\mu\nu} - \frac{1}{2}\,(MB_\sigma - \partial_\sigma b)^2$$

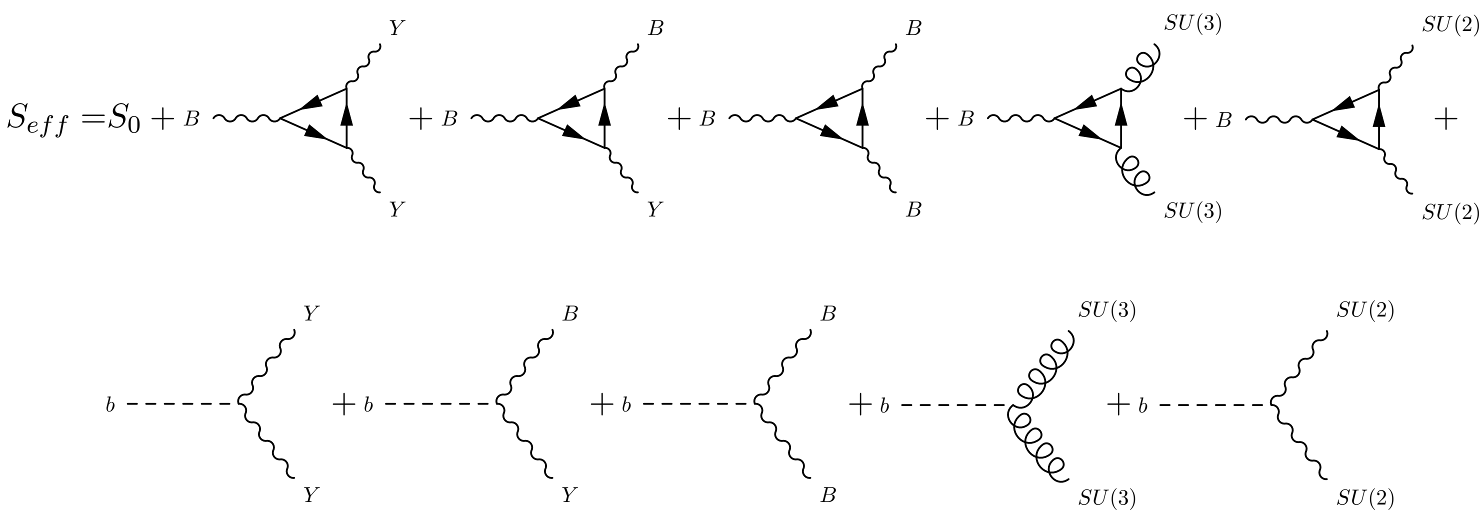
The Standard Model with 1 extra anomalous U(1) and an axion

f	Q	u_R	d_R	L	e_R
q^B	q^B_Q	$q^B_{u_R}$	$q^B_{d_R}$	q^B_L	$q^B_{e_R}$

f	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_B$
Q	3	2	$1/6$	q^B_Q
u_R	3	1	$2/3$	$q^B_Q + q^B_u$
d_R	3	1	$-1/3$	$q^B_Q - q^B_d$
L	1	2	$-1/2$	q^B_L
e_R	1	1	-1	$q^B_L - q^B_d$
H_u	1	2	$1/2$	q^B_u
H_d	1	2	$1/2$	q^B_d

The effective action has the structure given by

$$\mathcal{S} = \mathcal{S}_0 + \mathcal{S}_{Yuk} + \mathcal{S}_{an} + \mathcal{S}_{WZ} + \mathcal{S}_{CS}$$



$$
\begin{aligned}
\mathcal{L} = & - \frac{1}{2} tr\, G_{\mu\nu}G^{\mu\nu} - \frac{1}{2} tr\, W_{\mu\nu}W^{\mu\nu} - \frac{1}{4} F^{l}_{\mu\nu} F^{\mu\nu,l} \\
& - |(\partial_\mu + i\frac{g_2}{2}\tau^a W^a_\mu + i q^{(H_u)}_l g_l A^l_\mu) H_u|^2 - |(\partial_\mu + i\frac{g_2}{2}\tau^a W^a_\mu + i q^{(H_d)}_l g_l A^l_\mu) H_d|^2 \\
& + Q^\dagger_{Li}\sigma^\mu \mathcal{D}_\mu Q_{Li} + u^\dagger_{Ri}\overline{\sigma}^\mu \mathcal{D}_\mu u_{Ri} + d^\dagger_{Ri}\overline{\sigma}^\mu \mathcal{D}_\mu d_{Ri} \\
& + L^\dagger_{Li}\sigma^\mu \mathcal{D}_\mu L_{Li} + e^\dagger_{Ri}\overline{\sigma}^\mu \mathcal{D}_\mu e_{Ri} + \nu^\dagger_{Ri}\overline{\sigma}^\mu \mathcal{D}_\mu \nu_{Ri} \\
& + \gamma^u_{ij} H^T_u \tau^2\, (Q^t_{Li}\sigma^2 u_{Rj}) + \gamma^d_{ij} H^\dagger_d\, (Q^t_{Li}\sigma^2 d_{Rj}) + c.c. \\
& + \gamma^e_{ij} H^\dagger_u\, (L^t_{Li}\sigma^2 e_{Rj}) + \gamma^\nu_{ij} H^T_d \tau^2\, (L^t_{Li}\sigma^2 \nu_{Rj}) + c.c. \\
& - \frac{1}{2}\sum (\partial_\mu a^I + g_l \mathcal{M}^I_l A^l_\mu)^2 + E_{lmn}\, \epsilon^{\mu\nu\rho\sigma}\, A^l_\mu\, A^m_\nu\, F^n_{\rho\sigma} \\
+ & \sum_I (D_I\, a^I\, tr\,\{G \wedge G\} + F_I\, a^I\, tr\,\{W \wedge W\} + C_{Imn}\, a^I\, F^m \wedge F^n) \\
+ & V(H_u, H_d, a^I).
\end{aligned}
$$

Generic extension

The gauge symmetry under which this Lagrangian is invariant is

$$
SU(3)_c \times SU(2)_W \times G_1, \qquad G_1 = \prod_{l=1}^{4} U(1)_l.
$$

Gauge kinetic

Stuckeberg mass terms

Chern Simons abelian interactions

$$
SU(3) \times SU(2) \times U(1)_a \times U(1)_b \times U(1)_c \times U(1)_d.
$$

$$a_I, \qquad I = 1, 2, ...n$$

Stuckelberg axions

$$F_I$$

$$H_u \qquad H_d$$

$$E_{lmn}\epsilon^{\mu\nu\rho\sigma} A^l_\mu A^m_\nu F^n_{\rho\sigma}$$

Abelian CS terms

Higgs sector

$$|\mathcal{D}_\mu H_u|^2 + |\mathcal{D}_\mu H_d|^2 + \frac{1}{2}\sum_I (\partial a_I' + M_I A^I)^2$$

$$\begin{aligned}
\mathcal{D}_\mu H_u &= \left(\partial_\mu + \frac{i}{\sqrt{2}} g_2 \left(T^+ W^+ + T^- W^-\right) + \frac{i}{2} g_2 \tau_3 W_{3\mu} + \frac{i}{2} g_Y A_\mu^Y + \frac{i}{2}\sum_I q_u^I g_I\, A_\mu^I\right) H_u \\
\mathcal{D}_\mu H_d &= \left(\partial_\mu + \frac{i}{\sqrt{2}} g_2 \left(T^+ W^+ + T^- W^-\right) + i\frac{g_2}{2} \tau_3 W_{3\mu} + \frac{i}{2} g_Y A_\mu^Y + \frac{i}{2}\sum_I q_d^I g_I\, A_\mu^I\right) H_d
\end{aligned}$$

Typical mass terms for the gauge bosons are generated both from the Higgs and the Stuckleberg contributions

$$\frac{1}{2}\sum_I M_I^2 (A_\mu^I)^2 + \frac{1}{4}(-g_2 W_{3\mu} + g_Y A_\mu^Y + \sum_I q_u^I\, g_I\, A_\mu^I)^2 v_u^2$$
$$+\frac{1}{4}(-g_2 W_{3\mu} + g_Y A_\mu^Y + \sum_I q_d^I\, g_I\, A_\mu^I)^2 v_d^2,$$

There will be bilinear mixings in the broken (electroweak) phase

$$Z^\mu\,\partial_\mu\left\{f_u C^u + f_d C^d + \sum_I g_I M_I O^A_{ZI} a'_I\right\} + \sum_J Z'^{\,\mu}_J\,\partial_\mu\left\{f_{u,J} C^u + f_{d,J} C^d + \sum_I g_I M_I O^A_{Z'_J I} a'_I\right\},$$

We can extract the NG modes by a rotation, identifying 1 single physical axion

$$\begin{pmatrix} \mathrm{Im} H_u^0 \\ \mathrm{Im} H_d^0 \\ . \\ a'_I \\ . \end{pmatrix} = O^\chi \begin{pmatrix} \chi \\ G_1^0 \\ G_2^0 \\ . \\ . \end{pmatrix}$$

The scalar potential has an ordinary 2-Higgs doublet part and an extra contribution

$$V_{PQ} = \sum_{a=u,d}\left(\mu_a^2 H_a^\dagger H_a + \lambda_{aa}(H_a^\dagger H_a)^2\right) - 2\lambda_{ud}(H_u^\dagger H_u)(H_d^\dagger H_d) + 2\lambda'_{ud}|H_u^T \tau_2 H_d|^2$$

$$\begin{aligned} V_{\not P \not Q} = \quad & b\,(H_u^\dagger H_d e^{-i\sum_I (q_u^I - q_d^I)\frac{a'_I}{M_I}}) + \lambda_1 (H_u^\dagger H_d e^{-i\sum_I (q_u^I - q_d^I)\frac{a'_I}{M_I}})^2 \\ + \quad & \lambda_2 (H_u^\dagger H_u)(H_u^\dagger H_d e^{-i\sum_I (q_u^I - q_d^I)\frac{a'_I}{M_I}}) + \lambda_3 (H_d^\dagger H_d)(H_u^\dagger H_d e^{-i\sum_I (q_u^I - q_d^I)\frac{a'_I}{M_I}}) + c.c. \end{aligned}$$

Axionic contributions

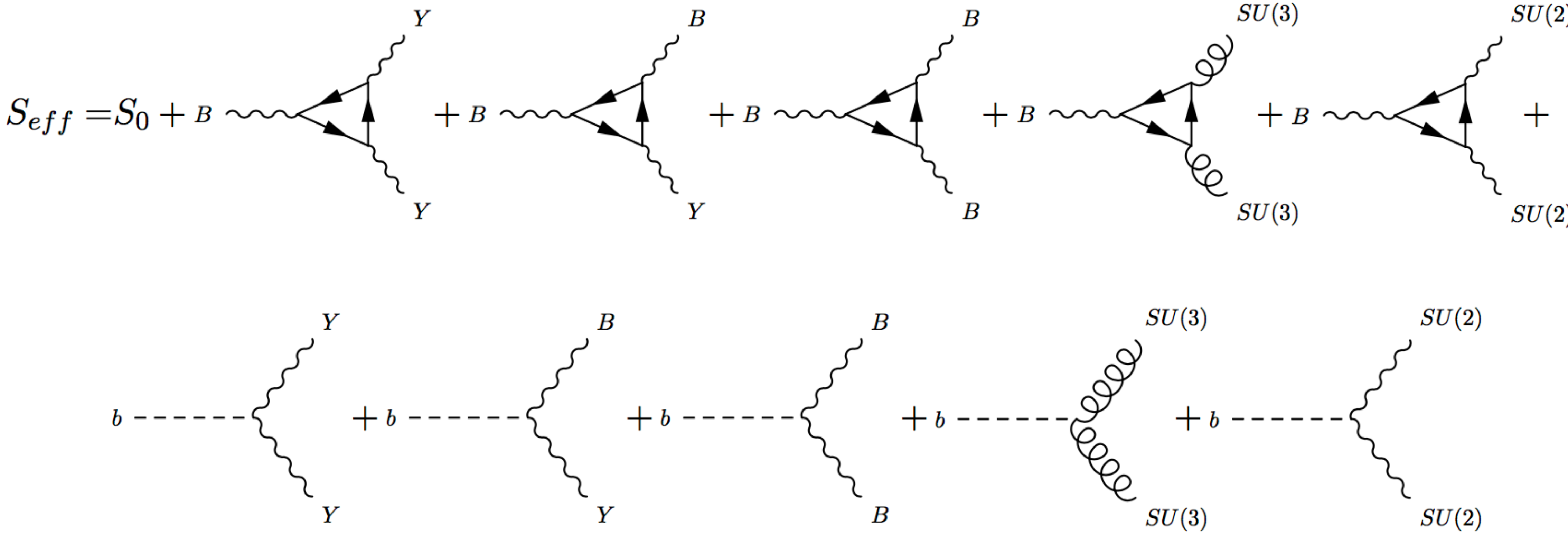
$$\begin{aligned}\mathcal{S}_{WZ} &= C_{BB}\langle b\, F_B \wedge F_B\rangle + C_{YY}\langle b\, F_Y \wedge F_Y\rangle + C_{YB}\langle b\, F_Y \wedge F_B\rangle \\ &\quad + F\langle b\, Tr[F^W \wedge F^W]\rangle + D\langle b\, Tr[F^G \wedge F^G]\rangle,\end{aligned}$$

Abelian/non-abelian Chern Simons terms

$$\begin{aligned}\mathcal{S}_{CS} &= +d_1\langle BY \wedge F_Y\rangle + d_2\langle YB \wedge F_B\rangle \\ &\quad + c_1\langle \epsilon^{\mu\nu\rho\sigma} B_\mu C^{SU(2)}_{\nu\rho\sigma}\rangle + c_2\langle \epsilon^{\mu\nu\rho\sigma} B_\mu C^{SU(3)}_{\nu\rho\sigma}\rangle.\end{aligned}$$

$$\begin{aligned}C^{SU(2)}_{\mu\nu\rho} &= \frac{1}{6}\left[W^i_\mu\left(F^W_{i,\,\nu\rho} + \frac{1}{3}\, g_2\, \varepsilon^{ijk} W^j_\nu W^k_\rho\right) + cyclic\right], \\ C^{SU(3)}_{\mu\nu\rho} &= \frac{1}{6}\left[G^a_\mu\left(F^G_{a,\,\nu\rho} + \frac{1}{3}\, g_3\, f^{abc} G^b_\nu G^c_\rho\right) + cyclic\right].\end{aligned}$$

With a single anomalous U(1) these terms care not essential.



$$
\begin{aligned}
C_{BYY} &= -\frac{1}{6}q_Q^B + \frac{4}{3}q_{u_R}^B + \frac{1}{3}q_{d_R}^B - \frac{1}{2}q_L^B + q_{e_R}^B,\\
C_{YBB} &= -(q_Q^B)^2 + 2(q_{u_r}^B)^2 - (q_{d_R}^B)^2 + (q_L^B)^2 - (q_{e_R}^B)^2,\\
C_{BBB} &= -6(q_Q^B)^3 + 3(q_{u_R}^B)^3 + 3(q_{d_R}^B)^3 - 2(q_L^B)^3 + (q_{e_R}^B)^3,\\
C_{Bgg} &= \frac{1}{2}(-2q_Q^B + q_{d_R}^B + q_{u_R}^B),\\
C_{BWW} &= \frac{1}{2}(-q_L^B - 3q_Q^B).
\end{aligned}
$$

SM x U(1)'

suppressed by the Stueckelberg scale M

$$O^{\chi} = \begin{pmatrix} \frac{v_d}{v} & \frac{v_u}{v} & 0 \\ -\frac{g_B(q_d-q_u)v_d v_u^2}{v\sqrt{g_B^2(q_d-q_u)^2 v_d^2 v_u^2+2M^2v^2}} & \frac{g_B(q_d-q_u)v_d^2 v_u}{v\sqrt{g_B^2(q_d-q_u)^2 v_d^2 v_u^2+2M^2v^2}} & \frac{\sqrt{2}Mv}{\sqrt{g_B^2(q_d-q_u)^2 v_d^2 v_u^2+2M^2v^2}} \\ \frac{\sqrt{2}Mv_u}{\sqrt{g_B^2(q_d-q_u)^2 v_u^2 v_d^2+2M^2v^2}} & -\frac{\sqrt{2}Mv_d}{\sqrt{g_B^2(q_d-q_u)^2 v_u^2 v_d^2+2M^2v^2}} & \frac{g_B(q_d-q_u)v_d v_u}{\sqrt{g_B^2(q_d-q_u)^2 v_u^2 v_d^2+2M^2v^2}} \end{pmatrix}$$

$$b = O^{\chi}_{13}G_0^1 + O^{\chi}_{23}G_0^2 + O^{\chi}_{33}\chi,$$

$$\chi = O^{\chi}_{31}\mathrm{Im}H_d + O^{\chi}_{32}\mathrm{Im}H_u + O^{\chi}_{33}b.$$

it is possible to describe the physical axion
by looking at the phases of the periodic potential

$$H_u^0 = \frac{1}{\sqrt{2}}\left(\sqrt{2}v_u + \rho_u^0(x)\right) e^{i\frac{F_u^0(x)}{\sqrt{2}v_u}}$$

$$H_d^0 = \frac{1}{\sqrt{2}}\left(\sqrt{2}v_d + \rho_d^0(x)\right) e^{i\frac{F_d^0(x)}{\sqrt{2}v_d}},$$

$$\theta(x) \equiv \frac{g_B(q_d-q_u)}{2M}b(x) - \frac{1}{\sqrt{2}v_u}F_u^0(x) + \frac{1}{\sqrt{2}v_d}F_d^0(x),$$

$$\theta(x) \equiv \frac{\chi(x)}{\sigma_\chi},$$

we have a phase that sets the periodicity of the potential

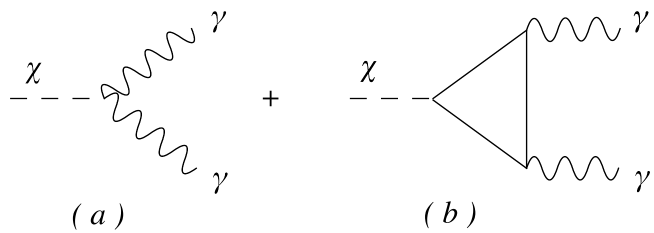
$$\sigma_\chi \equiv \frac{2v_u v_d M}{\sqrt{g_B^2(q_d-q_u)^2 v_d^2 v_u^2 + 2M^2(v_d^2+v_u^2)}}.$$

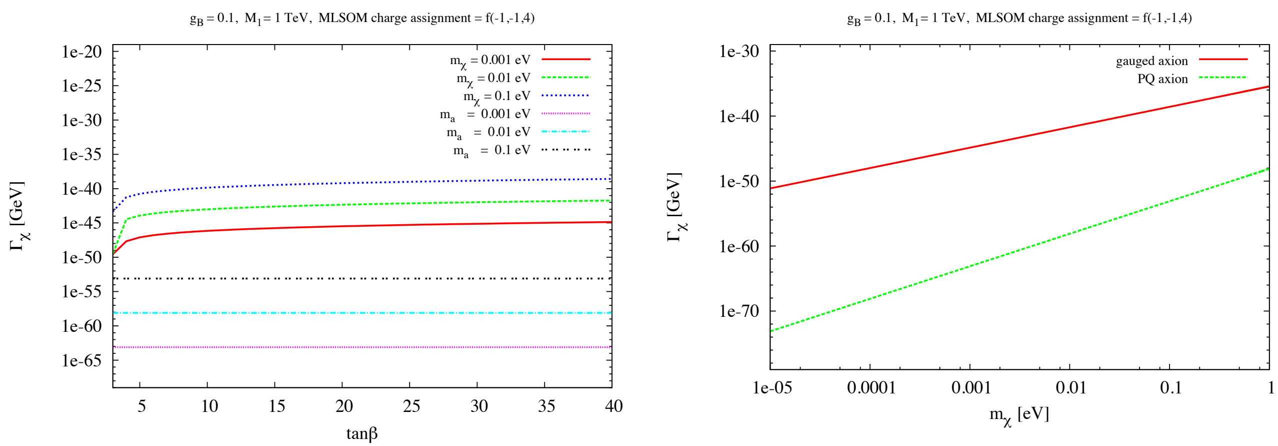
$$V' = 4v_u v_d \left(\lambda_2 v_d^2 + \lambda_3 v_u^2 + \lambda_0\right)\cos\left(\frac{\chi}{\sigma_\chi}\right) + 2\lambda_1 v_u^2 v_d^2 \cos\left(2\frac{\chi}{\sigma_\chi}\right),$$

$$\begin{aligned}
\delta H_u &= -\frac{i}{2} q_u g_B \alpha_B H_u \\
\delta H_d &= -\frac{i}{2} q_d g_B \alpha_B H_d \\
\delta F_0^u &= -\frac{v_u}{\sqrt{2}} q_u g_B \alpha_B \\
\delta F_0^d &= -\frac{v_d}{\sqrt{2}} q_d g_B \alpha_B \\
\delta b &= -M\alpha_B
\end{aligned}$$

combine such gauge transformations

one can check that **chi** is gauge invariant





Total decay rate of the axi-Higgs for several mass values. Here, for the PQ axion, we have chosen $f_a = 10^{10}$ GeV.

$$m_\chi^2 = -\frac{1}{2}\, c_\chi\, v^2 \left[1 + \left(\frac{q_u^B - q_d^B}{M_1}\, \frac{v\, \sin 2\beta}{2}\right)^2\right] = -\frac{1}{2}\, c_\chi\, v^2 \left[1 + \frac{(q_u^B - q_d^B)^2}{M_1^2}\, \frac{v_u^2 v_d^2}{v^2}\right],$$

G. Lazarides, A.Mariano, C.C.

PQ axion. Vacuum misalignment at the QCD phase transition

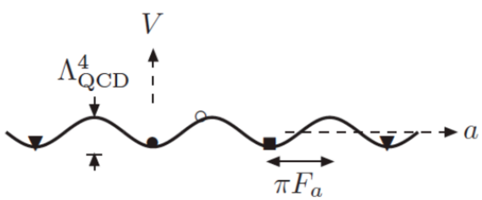
The PQ axion feels the QCD vacuum via the interaction $$\frac{a}{f_a} G\tilde{G}$$

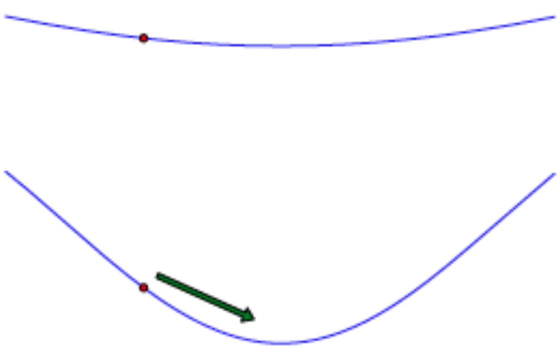
The angle of misalignment is

$$\theta = \frac{a(x)}{f_a}$$

The mass is sizeable

$$10^{-3} - 10^{-4} eV$$





If an axion has charges both under SU(3) and SU(2) we could consider the possibility of sequential misalignments. The dominant misalignment clearly comes from the largest potential

Stepping up with the Stueckelberg scale (M_s)

M_s is an independent variable

- in principle it can be close to the Planck or GUT scales

We have discussed that Stueckelberg-like Lagrangians can be introduced at any scale.

Here we consider an E_6 model

heterotic string $E_8 \times E_8 \to E_6$ on CY compactification

Fermions $\underline{27}$ of E_6 (Anomaly free)

E_6 has a PQ symmetry (Frampton, Kephart)

$$\mathcal{L} = \mathcal{L}_{E_6} + \mathcal{L}_{St} + \mathcal{L}_{anom} + \mathcal{L}_{WZ},$$

$$\mathbf{27}_{X_1} \qquad \mathbf{27}_{X_2} \qquad \mathbf{27}_{X_3},$$

$$\sum_{i=1}^{3} X_i^3 = 0, \qquad \sum_{i=1}^{3} X_i = 0.$$

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$$A_{\mu\nu}^{(1)} \to e^{i\theta} A_{\mu\nu}^{(1)} \qquad A_{\mu\nu}^{(2)} \to e^{i\theta} A_{\mu\nu}^{(2)} \qquad \Psi_\mu \to e^{-(\frac{1}{2} i\theta)} \Psi_\mu.$$

$$
\begin{aligned}
V_p \;=\; & M_{GUT}^2 A^{(1)}_{\mu\nu} \bar{A^{(2)}}^{\mu\nu} e^{-i4\frac{b}{M_S}} + e^{-i8\frac{b}{M_S}} \Big[(h_1\; (A^{(1)}_{\mu\nu} \bar{A^{(2)}}^{\mu\nu})^2 + h_2\; A^{(1)}_{\mu\nu} \bar{A^{(2)}}^{\nu\sigma} A^{(1)}_{\sigma\tau} \bar{A^{(2)}}^{\tau\mu} \\
& + h_3\; d^{\mu\nu\lambda} d_{\xi\eta\lambda} A^{(1)}_{\mu\sigma} A^{(1)}_{\nu\tau} \bar{A^{(2)}}^{\xi\sigma} \bar{A^{(2)}}^{\eta\tau} \\
& + h_4\; d^{\mu\nu\alpha} d^{\sigma\tau\beta} d_{\xi\eta\alpha} d_{\lambda\rho\beta} A^{(1)}_{\mu\sigma} A^{(1)}_{\nu\tau} \bar{A^{(2)}}^{\xi\lambda} \bar{A^{(2)}}^{\eta\rho} \\
& + h_5\; d^{\mu\nu\alpha} d^{\sigma\beta\gamma} d_{\xi\eta\beta} d_{\lambda\alpha\gamma} A^{(1)}_{\mu\sigma} A^{(1)}_{\nu\tau} \bar{A^{(2)}}^{\xi\lambda} \bar{A^{(2)}}^{\eta\tau} \\
& + h_6\; d^{\mu\nu\alpha} d^{\sigma\tau\beta} d_{\alpha\beta\gamma} d^{\gamma\zeta\xi} d_{\xi\eta\zeta} d_{\lambda\rho\chi} A^{(1)}_{\mu\sigma} \bar{A^{(2)}}^{\xi\lambda} A^{(1)}_{\nu\tau} \bar{A^{(2)}}^{\eta\rho} \Big] + h.c.
\end{aligned}
$$

$$
\begin{aligned}
(\mathbf{351}) \;=\; & (1,3^*,3) + (1,3^*,6^*) + (1,6,3) + (3,3,1) + (3,6^*,1) + (3,3,8) + \\
& (3^*,1,3^*) + (3^*,1,6) + (3^*,8,3^*) + (6^*,3,1) + (6,1,3^*) + (8,3^*,3)
\end{aligned}
$$

$$V_p \sim \sum_{j=1}^{12} \lambda_0 M_{\rm GUT}^2 (H_j^{(1)\dagger} H_j^{(2)} e^{-4ig_B\frac{b}{M_S}}) + \sum_{j,k=1}^{12} \Big[\lambda_1 (H_j^{(1)\dagger} H_j^{(2)} e^{-i4g_B\frac{b}{M_S}})^2 + \lambda_2 (H_i^{(1)\dagger} H_i)(H_i^{(1)\dagger} H_j^{(2)} e^{-i4g_B\frac{b}{M_S}})$$
$$+\lambda_3 (H_k^{(2)\dagger} H_k^{(2)})(H_j^{(1)\dagger} H_k^{(2)} e^{-i4g_B\frac{b}{M_S}}) \Big] + \text{h.c.}, \tag{95}$$

$$V_p \sim v_1 v_2 \left(\lambda_2 v_2^2 + \lambda_3 v_1^2 + \overline{\lambda_0} M_{GUT}^2\right) \cos\left(\frac{\chi}{\sigma_\chi}\right) + \lambda_1 v_1^2 v_2^2 \cos\left(2\frac{\chi}{\sigma_\chi}\right),$$

$$m_\chi^2 \sim \frac{2v_1v_2}{\sigma_\chi^2}\left(\bar{\lambda}_0 v_1^2 + \lambda_2 v_2^2 + \lambda_3 v_1^2 + 4\lambda_1 v_1 v_2\right) \approx \lambda v^2$$

$$\sigma_\chi \sim M_{\rm GUT} + \mathcal{O}(M_{\rm GUT}^2/M_{\rm Planck}^2), \qquad m_\chi^2 \sim \lambda_0 M_{\rm GUT}^2,$$

$$\lambda_0 \sim e^{-2\pi/\alpha(M_{\mathrm{GUT}})}, \qquad 1/33 \le \alpha_{GUT} \le 1/32,$$

$$e^{-201} \sim 10^{-91} \le \lambda_0 \le e^{-205} \sim 10^{-88},$$

Stueckelberg models predict
ultralight axions if the Stueckelberg scale is sufficiently large.

In general we face a large (representation-wise)
scalar sector which would be interesting to simplify
in some way

Axions and the Strong CP Problem

Axions have appeared in physics in an attempt to solve the strong CP problem of QCD.

Why is the $\theta G \tilde{G}$ term so small?
Consider an $SU(2)$ gauge theory

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc} A^b_\mu A^c_\nu$$

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] \qquad G_{\mu\nu} = G^a_{\mu\nu} T^a$$

$$A_\mu \to U A_\mu U^{-1} + U \partial_\mu U^{-1}$$

$$G_{\mu\nu} \to U G_{\mu\nu} U^{-1}$$

We look for minima of the Euclidean action

$$S = -\frac{1}{2g^2}\int d^4x Tr G_{\mu\nu} G_{\mu\nu}$$

In a nonabelian theory a vanishing field strength is possible with

$$A_\mu = U\partial_\mu U^{-1}$$

(pure gauge). Solutions of this condition are instanton configurations, characterised by a topological number.

$$-16\pi^2 Q(x) = Tr[G_{\mu\nu}\tilde{G}_{\mu\nu}] = Tr[\epsilon_{\mu\nu\alpha\beta}[2\partial_\mu(A_\nu\partial_\alpha A_\beta + \frac{2}{3}A_\nu A_\alpha A_\beta)]],$$

$$\tilde{G} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}G^{\alpha\beta},\ \ Q(x) = \partial_\mu J_\mu,\ J_\mu = -\frac{1}{8\pi^2}\epsilon_{\mu\nu\alpha\beta}A_\nu(\partial_\alpha A_\beta + \frac{2}{3}A_\alpha A_\beta)$$

For an $SU(3)$ gauge theory such as QCD, similarly, the Lagrangean then allows a total derivative term $\theta G\tilde{G}$ which is a boundary term, but cannot be neglected. For instantons

$$G = \tilde{G}, \qquad \int d^4x G\tilde{G}(x) = 32\pi^2 n,$$

Therefore $\rightarrow$ *There is a dimension-4 operator that we can write down in the Standard Model (SM)*

$$\theta_0 G \tilde{G}$$

(violates Parity and Time reversal, CP is broken)

It is a total derivative term and as such it does not contribute *in perturbation theory*

Adding a total derivative term gives a zero momentum vertex in perturbation theory, but it contributes non-perturbatively

How?

If we consider an instanton (Euclidean) configuration, then the contribution to the path integral is

$$\sim e^{-\mathcal{S}_0} = e^{-\frac{1}{4g^2}\int d^4x FF} = e^{-\frac{8\pi^2}{g^2}}$$

- These configurations, at small coupling, give a negligible contribution
- They are solutions of the classical eq. of motion of QCD, which is scale invariant at classical level
  However, the solution of the equation $G = \tilde{G}$ involves an integration constant, the size of the instanton.
- The solution breaks scale invariance, because of the integration constant, which remains arbitrary.
  It tells us where the energy of the configuration is localized.
  At tree level g is constant, but at 1-loop it runs. Scale invariance is broken by renormalization.

In the functional integral we need to sum over all these configurations.

Small instantons (R)

- $\rightarrow$ large scale $\lambda \sim 1/R$
- $\rightarrow$ small coupling $g(\lambda) \ll 1$
- $\rightarrow$ large suppression in $e^{-\frac{8\pi^2}{g^2(\lambda)}}$. The contribution is perturbative, since g is small, but it is negligible.
  The instanton contribution to the QCD action is dominated by large instantons ($g(\lambda)$ large). Unfortunately the contribution is non-perturbative.

- The running is controlled by the size of the instanton, $g = g(\lambda)$

In the functional integral we need to sum over all these configurations.

Small instantons (R)

- $\rightarrow$ large scale $\lambda \sim 1/R$
- $\rightarrow$ small coupling $g(\lambda) \ll 1$
- $\rightarrow$ large suppression in $e^{-\frac{8\pi^2}{g^2(\lambda)}}$. The contribution is perturbative, since g is small, but it is negligible. The instanton contribution to the QCD action is dominated by large instantons ($g(\lambda)$ large). Unfortunately the contribution is non-perturbative.
- The saddle point approximation is not valid any more since the action is O(1).

The partition function can be written in the form

$$\sim e^{-8\pi^2/g^2(\lambda)-i\theta_0}$$

and summing over instantons/anti instantons

$$\sum_{I\bar{I}} \sim e^{-8\pi^2/g^2(\lambda)} \cos\theta_0$$

θ_0 is not directly observable. One expects the energy density to dependen on θ_0 Notice, however, that QCD has a $U(1)_A$ anomaly, due to fermions. There is an axial symmetry

$$q \to q e^{i\gamma_5\alpha}$$

and the integration measure is not invariant

$$Dq D\bar{q} \to Dq D\bar{q} e^{-\frac{i}{16\pi^2}\alpha\int F\tilde{F} d^4x}$$

Therefore θ_0 is not physical because it can be shifted by a field redefinition

$$\theta_0 \to \theta_0 + 2\alpha$$

But also the quark mass term gets a phase under the chiral transformation

$$\bar{q}_L M q_R + h.c. \rightarrow \bar{q}_L M q_R e^{2i\alpha} + h.c.$$

therefore

$$argM \rightarrow argM + 2\alpha$$

and

$$\theta \equiv \theta_0 - argM$$

is invariant under field redefinitions. *If we have fermions in complex representations of the gauge group, θ_0 is affected by field redefinitions and is not physical*, but θ is physical. This can be generalized to n_f fermions.

$$\theta_0 \rightarrow \theta_0 + 2n_f\alpha, \qquad ArgdetM \rightarrow ArgdetM + 2n_f\alpha$$

$$\theta \equiv \theta_0 - ArgdetM$$

is physical.

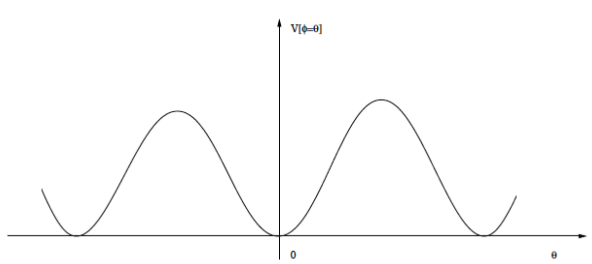
Experimentally θ is very small. We can set this value to zero assuming a cancellation between

- θ_0 (reated to gluon dynamics)
- $ArgDetM$ (related to the electroweak sector, Yukawas and Higgs)

We can easily derive some properties of the vacuum energy as a function of θ.

$$e^{-VE(\theta)} = \left| \int D\Phi e^{-S[\Phi] - \frac{i}{32\pi^2}\theta \int F\tilde{F} d^4x} \right|$$

$$\leq \int D\Phi \left| e^{-S[\Phi] - \frac{i}{32\pi^2}\theta \int F\tilde{F} d^4x} \right| = e^{-VE(\theta=0)}$$



$$E(\theta) \geq E(0)$$

It is also even in θ: $E(\theta) = E(-\theta)$. Periodic of period 2π.