Rotating black holes in higher order gravity theories

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Based on work with
Marco Crisostomi, Ruth Gregory and Nikos Stergioulas
arXiv:1903.05519
Recent Developments in Strings and Gravity

GR is a unique metric theory in 4 dimensions

- Theoretical consistency: In $D = 4$ dimensions, consider $\mathcal{L} = \mathcal{L}(\mathcal{M}, g, \nabla g, \nabla\nabla g)$ where ∇ is a Levi-Civita connection. Then Lovelock's theorem in $D = 4$ states that GR with cosmological constant is the unique metric theory emerging from,

$$S_{(4)} = \int_{\mathcal{M}} d^4x \sqrt{-g^{(4)}} \left[-2\Lambda + R + \alpha \hat{G}\right]$$

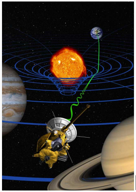
giving,

  - Equations of motion of 2^{nd}-order (Ostrogradski no-go theorem 1850!)
  - given by a symmetric two-tensor, $G_{\mu\nu} + \Lambda g_{\mu\nu}$
  - and admitting Bianchi identities.

GR is the unique massless-tensorial 4 dimensional theory of gravity.

Observational data

- Experimental consistency:
  -Excellent agreement with solar system tests and strong gravity tests on binary pulsars
  -Recent data from the EHT compatible with GR for a supermassive black hole
  -Observational breakthrough GW170817: Non local, $40Mpc$ and strong gravity test from a coalescing binary of neutron stars. $c_T = 1 \pm 10^{-15}$

Time delay of light  Planetary tajectories 

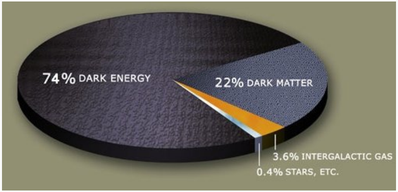
 Neutron star binary

Q: What is the matter content of the Universe today?

Assuming homogeinity-isotropy and GR

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

cosmological and astrophysical observations dictate the matter content of the Universe today:



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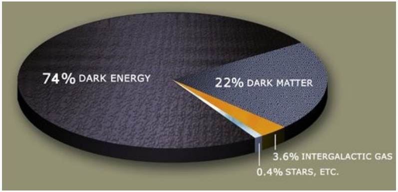

A: -Only a 4% of matter has been discovered in the laboratory. We hope to see more at LHC. But even then...

If we assume only ordinary sources of matter (DM included) there is disagreement between local, astrophysical and cosmological data.

Simplest way out: Assume a tiny cosmological constant $\rho_\Lambda = \frac{\Lambda_{obs}}{8\pi G} = (10^{-3} eV)^4$, ie modify Einstein's equation by,

$$G_{\mu\nu} + \Lambda_{obs} g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- Cosmological constant introduces $\sqrt{\Lambda}$ and generates a cosmological horizon
- $\sqrt{\Lambda}$ is as tiny as the inverse size of the Universe today, $r_0 = H_0^{-1}$
- Note that $\frac{\text{Solar system scales}}{\text{Cosmological Scales}} \sim \frac{10 \text{ A.U.}}{H_0^{-1}} = 10^{-14}$
- Theoretically the cosmological constant should be huge!
- Cosmological dark sector at the IR and UV considerations lead to modification of gravity

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Universe is accelerating $\rightarrow$ Enter the cosmological constant

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Modification of Gravity : Scalar-tensor theories

- are the simplest modification of gravity with one additional degree of freedom
- Admit a uniqueness theorem due to Horndeski 1973 and extended to DHOST/EST theories [Langlois et.al] [Crisostomi et.al.]
- Contain or are limits of other modified gravity theories.
- Include terms that can screen classically a big cosmological constant or give self accelerating solutions.
- GW constraints on dark energy solutions [Creminelli, Vernizzi, Ezquiaga, Zumalacaregui,...] albeit strong coupling issues [DeRham, Melville]
- We concentrate on $c_T = 1$ theories DHOST/EST [Crisostomi, Koyama, Langlois, Noui, Vernizzi,..]
- The theory under scrutiny, $c_T = 1$ has unique characteristics. It is far closer to GR than any version of Hordenski
- Our aim is to find rotating black holes

$c_T = 1$ scalar tensor theories and their relation to Horndeski

Shift-symmetric scalar tensor theory $c_T = 1$ minimally coupled to matter is parametrized by K, A_3, G

$$\mathcal{L} = K(X) + G(X)R + A_3(X)\phi^\mu\phi_{\mu\nu}\phi^\nu\Box\phi + A_4\phi^\mu\phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu + A_5(\phi^\mu\phi_{\mu\nu}\phi^\nu)^2\,,$$

- coupling functions depend only on $X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$.
- $K(X) = -\Lambda_{bare} + X + ..$ and the operators A_4, A_5 are fixed with respect to A_3, G
- $c_T = 1$ theories are mapped to Horndeski via a transformation

$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C(X)g_{\mu\nu} + D(X)\nabla_\mu\phi\nabla_\nu\phi$$

  for given functions C and D.
- One can start with a $c_T \neq 1$ Horndeski theory (solution) and map it to a $c_T = 1$ theory (solution) for a specific function D.

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Example of spherical symmetry [Babichev, CC, GEFarèse, Lehébel]

- Example Horndeski,

$$S = \int d^4x \sqrt{-g} \left[\zeta R - 2\Lambda_b - \eta X + \beta G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi\right],$$

- General spherically symmetric solution is known[Babichev, cc], $ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$, $\phi = \phi(t, r)$
- One such solution reads

$$f = h = 1 - \frac{\mu}{r} + \frac{\eta}{3\beta} r^2$$

,

$$\phi = qt \pm \frac{q}{h}\sqrt{1-h}$$

with $\Lambda_{\text{eff}} = -\zeta\eta/\beta$.

- Go to ($c_T = 1$) via a disformal transformation:

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{\beta}{\zeta + \frac{\beta}{2}X} \varphi_\mu \varphi_\nu .$$

- The disformed metric is a black hole essentially because $X = -q^2$ is constant!
- Solution stable in a Λ_b-dependent window-generically we expect self tuning to be spoilt or constrained.

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Going beyond spherical symmetry? [cc, Crisostomi, Gregory, Stergioulas]

Consider an Einstein metric, $R_{\mu\nu} = \Lambda g_{\mu\nu}$ and $X = X_0$ constant. When are such (metric and scalar) solutions to the field equations of $c_T = 1$?

$$A_3(X_0) = 0, \qquad (K_X + 4\Lambda G_X)|_{X_0} = 0$$

where $\Lambda = -K/(2G)|_{X_0}$ (self-tuning condition)

- Any theory parametrized by A_3 having a zero at some value is enough to guarantee a solution.
- The real question though is what $X = \nabla_\mu\phi\nabla^\mu\phi$ is constant really mean?
- Note that if we take $Y_a = \partial_a\phi$ then the derivative of $X = Y_a Y_b g^{ab} = X_0$ is simply $a^b = Y^a\nabla_a Y^b = 0$
- Acceleration zero hence ϕ is related to a geodesic congruence in the given spacetime.
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The example of Carter's solution (de Sitter-Kerr)

- Rotating black hole Einstein metric

$$
\begin{aligned}
ds^2 = &-\frac{\Delta_r}{\Xi^2\rho^2}\left[dt - a\sin^2\theta d\varphi\right]^2 + \rho^2\left(\frac{dr^2}{\Delta_r}+\frac{d\theta^2}{\Delta_\theta}\right)\\
&+\frac{\Delta_\theta \sin^2\theta}{\Xi^2\rho^2}\left[a\,dt - \left(r^2+a^2\right)d\varphi\right]^2 ,
\end{aligned}
$$

$$
\Delta_r = \left(1-\frac{r^2}{\ell^2}\right)\left(r^2+a^2\right) - 2Mr\,, \quad \Xi = 1+\frac{a^2}{\ell^2}\,,
$$

$$
\Delta_\theta = 1+\frac{a^2}{\ell^2}\cos^2\theta\,, \qquad \rho^2 = r^2 + a^2\cos^2\theta\,,
$$

- Black hole parameters are $a, M, \Lambda = 3/l^2$ which describe a black hole with an inner, outer event and cosmological horizon for $\Lambda > 0$.
- To evaluate the HJ potential for geodesics we need to know the inverse metric and solve a first order differential equation.
- Does there exist a HJ potential which is well defined in the black hole spacetime?
- This would be the scalar field in the black hole solution

The example of Carter's solution (de Sitter-Kerr)

- The Hamilton Jacobi potential reads [Carter],

$$\mathcal{S} = -E\,t + L_z\varphi + S(r,\theta)\,,$$

since ∂_t and ∂_ϕ are Killing vectors and is separable $S(r,\theta) = S_r(r) + S_\theta(\theta)$!

$$S_r = \pm \int \frac{\sqrt{R}}{\Delta_r}dr\,, \qquad S_\theta = \pm \int \frac{\sqrt{\Theta}}{\Delta_\theta}d\theta\,,$$

$$\begin{aligned} R &= \Xi^2\left[E\left(r^2+a^2\right) - a\,L_z\right]^2 \\ &- \Delta_r\left[\mathcal{Q} + \Xi^2\left(a\,E - L_z\right)^2 + m^2r^2\right]\,, \qquad (1) \\ \Theta &= -\Xi^2\sin^2\theta\left(a\,E - \frac{L_z}{\sin^2\theta}\right)^2 \\ &+ \Delta_\theta\left[\mathcal{Q} + \Xi^2\left(a\,E - L_z\right)^2 - m^2a^2\cos^2\theta\right]\,. \qquad (2) \end{aligned}$$

- Note we have $E, m, L_z, \mathcal{Q}$ parametrising the Energy at infinity, rest mass, angular momentum and Carter's separation constant.
- We want to identify $\phi = \mathcal{S}$
- ϕ (unlike $\mathcal{S}$) needs to be well defined in all the permitted domain of the coordinates. We clearly need that Θ and R are positive functions.
- Regularity : $L_z = 0$ and $\mathcal{Q} + \Xi^2a^2E^2 = m^2a^2$,

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$$\begin{aligned} R &= \Xi^2\left[E\left(r^2+a^2\right) - a\,L_z\right]^2 \\ &- \Delta_r\left[\mathcal{Q} + \Xi^2\left(a\,E - L_z\right)^2 + m^2r^2\right]\,, \qquad (1) \\ \Theta &= -\Xi^2\sin^2\theta\left(a\,E - \frac{L_z}{\sin^2\theta}\right)^2 \\ &+ \Delta_\theta\left[\mathcal{Q} + \Xi^2\left(a\,E - L_z\right)^2 - m^2a^2\cos^2\theta\right]\,. \qquad (2) \end{aligned}$$

- Note we have $E, m, L_z, \mathcal{Q}$ parametrising the Energy at infinity, rest mass, angular momentum and Carter's separation constant.
- We want to identify $\phi = \mathcal{S}$
- ϕ (unlike $\mathcal{S}$) needs to be well defined in all the permitted domain of the coordinates. We clearly need that Θ and R are positive functions.
- Regularity : $L_z = 0$ and $\mathcal{Q} + \Xi^2a^2E^2 = m^2a^2$,

Rotating black hole

- We have,

$$\phi(t,r,\theta) = -E\,t + \phi_r + \phi_\theta\,,$$

where,

$$\phi_r = \pm \int \frac{\sqrt{R}}{\Delta_r} dr\,, \qquad \phi_\theta = \pm \int \frac{\sqrt{\Theta}}{\Delta_\theta} d\theta\,,$$

$$\Theta = a^2 m^2 \sin^2\theta \left(\Delta_\theta - \eta^2\right)\,, R = m^2(r^2+a^2)\left(\eta^2(r^2+a^2) - \Delta_r\right)$$

where we define $\eta = \frac{\Xi E}{m} \in [\eta_c, 1]$: $\eta < 1$ for $\Theta > 0$ and $\eta > \eta_c$ for $R > 0$

- Take $\Lambda = 0$, ie Kerr, we have $\eta = 1$
- The scalar ϕ then has no θ dependance. Coincides with known solution if $a = 0$ ($E = m = q$).
- Solution is regular at the event horizon for one of the branches by going to advanced EF coordinates.
- $v = t + \int dr\, \frac{r^2+a^2}{\Delta_r}, \qquad \bar{\varphi} = \varphi + a \int \frac{dr}{\Delta_r}$
- Solution then is stealth Kerr with a non trivial scalar field which has identical regularity to spacetime

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Rotating black hole

- Scalar reads,
$$\phi(t,r,\theta) = -E\,t + \phi_r + \phi_\theta\,,$$
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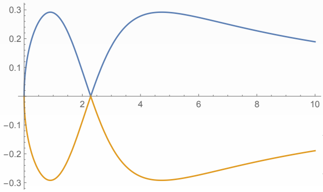
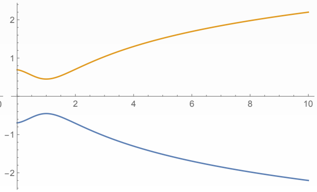
where $\eta = \frac{\Xi E}{m} \in [\eta_c, 1]$.
- η_c is the limiting value of $R > 0$. ie., it is such that R has a double zero at $r_{EH} < r_0 < r_{CH}$
- we have $\eta_c < 1$ and as Λ increases η_c decreases
- We have two branches of solutions. Going to EF coords we see that one chart is regular at the EH while the latter at the CH but none at both.
- $v = t \pm \int dr\, \frac{r^2+a^2}{\Delta_r}$, $\qquad \bar{\varphi} = \varphi \pm a\int \frac{dr}{\Delta_r}$





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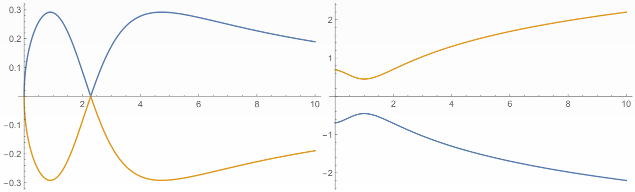
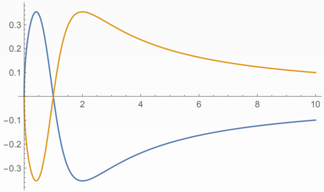


Regular Rotating black hole with $\Lambda \neq 0$

- ,

$$\phi(t,r,\theta) = -E\,t + \phi_r + \phi_\theta\,,$$
$$\phi_r = \pm\int \frac{\sqrt{R}}{\Delta_r}dr\,, \qquad \phi_\theta = \pm\int \frac{\sqrt{\Theta}}{\Delta_\theta}d\theta\,,$$
$$\Theta = a^2 m^2 \sin^2\theta \left(\Delta_\theta - \eta^2\right)\,, R = m^2(r^2+a^2)\left(\eta^2(r^2+a^2) - \Delta_r\right)$$

  where $\eta = \frac{\Xi E}{m} \in [\eta_c, 1]$.
- Fixing $\eta = \eta_c$ the two branches join with C_2 regularity at $r = r_0$.
- Then using both branches ie., $\phi_r = H[r-r_0]\int_{r_0}^r \frac{\sqrt{R}}{\Delta_r} - H[r_0-r]\int_{r_0}^r \frac{\sqrt{R}}{\Delta_r}$, we have a regular scalar field everywhere



Conclusions

- We have obtained a rotating black hole with hair which is everywhere regular
- Although solution is stealth, perturbations defining quasi normal modes and resulting phenomenology will be different [CC, Crisostomi, Langlois, Noui].
- Can obtain any GR vacuum solution with well defined hair in such theories
- One can use this stealth solution to construct numerically other non Kerr solutions by relaxation techniques
- For $c_T = 1$ the only $X = \textit{constant}$ solutions are Einstein spaces. If we expect solutions to have asymptotically X constant then in this theory all solutions are asymptotically Einstein spaces.