Perturbative schemes and singularity-resolution mechanisms In Vasiliev's higher-spin gravity

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Motivations

- Higher-spin gravity is a system of intermediate complexity between the full String Theory and SUGRA + higher-derivative corrections:
  - contains infinitely many gauge fields of all spins on equal footing
  - expected to contain non-local interactions (of physical spin-s fields)

- Remarkable that one can still control many features of the theory, essentially due to the infinite-dimensional symmetry + compact form of the non-linear eqs.

  [non-linear HS eqs. as a deformation of a classical algebra of differential forms → *master fields* , containing fields of all spins (and descendants)]

- As locality is not imposed, some issues (allowed field redefinitions, large vs small gauge transformations, ...) seem out of reach, at least by standard means.

- There are however indications that much insight is to be gained about current open questions by addressing them within the natural framework of the eqs. – in terms of their natural variables and observables.

Motivations

- Some guiding principle may come from the study of exact solutions. Surprisingly, making use of the large gauge symmetry, constructing solutions spaces is easier than one may think. Recently found many potentially interesting solutions (e.g., 4D HS black-hole-like solutions) and an effective scheme for superposing fluctuations.

- However, their interpretation requires a better understanding of observables and global issues:

  - enlarging the set of relevant classical observables and assess their physical meaning: proper HS/stringy generalization of geometry?
  - understanding how to impose boundary conditions on master fields on NC space ;
  - distinguishing small/large gauge transformations
  - master fields are functions of NC variables, so changes of orderings are a delicate issue. To what extent can one consider changes of ordering admissible?

Motivations

- The natural language and mathematical tools that the eqs. suggest in general blur the identification of the typical ingredients of standard field theories. The language of component fields is in general not the most appropriate one to address the previous questions.

- Learning to address global issues with the variables and observables that are most natural to the eqs. would be important to understand HS geometry, as suggested from the study of exact solutions.

  [note that the concepts of the standard Riemannian geometry lose meaning as they are not HS invariant]

- Interesting to assess the status of HS gravity wrt GR and stringy completion:

  Find and study the analogues of problematic solutions of GR, such as black holes and cosmologies, and see if the coupling with HS fields solves singularities already at the classical level.

Kinematics

- Master-fields living on *correspondence space*, locally $\mathcal{X}$ x $\mathcal{Z}$ x $\mathcal{Y}$:

$$U = dx^\mu U_\mu(Y,Z|x) \longrightarrow \text{gauge fields of all spins + auxiliary}$$
$$\Phi = \Phi(Y,Z|x) \longrightarrow \text{Weyl tensors and their derivatives} \to \text{local dof}$$
$$S = dz^\alpha S_\alpha(Y,Z|x) + d\bar{z}^{\dot\alpha}\bar{S}_{\dot\alpha}(Y,Z|x) \longrightarrow \text{Z-space connection, no extra local dof}$$

- Commuting oscillators $Y_{\underline{\alpha}} = (y_\alpha, \bar{y}_{\dot\alpha})$, $Z_{\underline{\alpha}} = (z_\alpha, -\bar{z}_{\dot\alpha})$ → $\mathfrak{sp}$(4,R) quartets

$$[Y_{\underline{\alpha}}, Y_{\underline{\beta}}]_\star = 2iC_{\underline{\alpha\beta}} = 2i\begin{pmatrix} \varepsilon_{\alpha\beta} & 0 \\ 0 & \varepsilon_{\dot\alpha\dot\beta}\end{pmatrix} , \qquad [Z_{\underline{\alpha}}, Z_{\underline{\beta}}]_\star = -2iC_{\underline{\alpha\beta}} , \quad [Y_{\underline{\alpha}}, Z_{\underline{\beta}}]_\star = 0$$

- Star-product:

$$F(Y,Z)\star G(Y,Z) = \int_{\mathcal{R}} \frac{d^4U d^4V}{(2\pi)^4} e^{iV^{\underline{\alpha}}U_{\underline{\alpha}}} F(Y+U, Z+U)\, G(Y+V, Z-V)$$

- Inner kleinian operator κ:

$$\kappa = e^{iy^\alpha z_\alpha} , \qquad \kappa\star f(z,y) = f(-z,-y)\star\kappa , \qquad \kappa\star\kappa = 1$$
$$\kappa = \kappa_y\star\kappa_z , \quad \kappa_y\star f(z,y) = f(z,-y)\star\kappa_y , \quad \kappa_y\star\kappa_y = 1 ,$$
$$\kappa_y = 2\pi\delta^2(y) = 2\pi\delta(y_1)\delta(y_2)$$

4D bosonic Vasiliev equations

- Full equations:

$$\boxed{\begin{aligned} dU + U \star U &= 0 \\ d\Phi + U \star \Phi - \Phi \star \pi(U) &= 0 \\ dS_\alpha + [U, S_\alpha]_\star &= 0 \\ S_\alpha \star \Phi + \Phi \star \pi(S_\alpha) &= 0 \\ [S_\alpha, S_\beta]_\star &= -2i\epsilon_{\alpha\beta}(1 - b\Phi \star \kappa) \\ [S_\alpha, \bar{S}_{\dot\beta}]_\star &= 0 \, , \end{aligned}}$$

(Vasiliev, '92)

$$\boxed{S_\alpha = z_\alpha - 2iV_\alpha}$$

$$[S_\alpha, f(Z,Y)] = [z_\alpha, f] - 2i[V_\alpha, f]_\star \propto \frac{\partial f}{\partial z^\alpha} + [V_\alpha, f]_\star$$

- *Z*-oscillators → auxiliary, non-commutative coordinates. Equations fix the evolution along *Z* in such a way that it gives rise to consistent interactions to all orders among physical fields, contained in the (*Z*-independent) initial conditions

$$W(x,Y) := U_{|Z=0} \, , \qquad C(x,Y) = \Phi_{|Z=0} \, .$$

- 1st order differential eqs impose a relation between spacetime and twistor space behaviour of their solutions → the physical information can be encoded to a great extent in the twistor-space dependence.

AdS vacuum solution

$$\Phi \;=\; \Phi^{(0)} \;=\; 0\,,$$

$$S_\alpha \;=\; S^{(0)}_\alpha \;=\; z_\alpha\,, \qquad S_{\dot\alpha} \;=\; S^{(0)}_{\dot\alpha} \;=\; \bar z_{\dot\alpha}\,,$$

$$U \;=\; U^{(0)} \;=\; \Omega \;=\; \tfrac{1}{4i}\left(\omega^{(0)\alpha\beta}y_\alpha y_\beta + \bar\omega^{(0)\dot\alpha\dot\beta}\bar y_{\dot\alpha}\bar y_{\dot\beta} + 2e^{(0)\alpha\dot\beta}y_\alpha \bar y_{\dot\beta}\right)$$

$$e^{\alpha\dot\beta}_{(0)} \;=\; -\frac{dx^a(\sigma_a)^{\alpha\dot\beta}}{1-x^2}\,, \qquad \omega^{\alpha\beta}_{(0)} \;=\; \frac{x^a dx^b(\sigma_{ab})^{\alpha\beta}}{1-x^2}$$

$$\longrightarrow \quad \boxed{ds^2_{(0)} \;=\; \frac{4dx^2}{(1-x^2)^2}}$$

- $U^{(0)}$ is a flat connection, can be represented via a gauge function $L(x|Y)$ = AdS_4 coset element

$$U^{(0)} \;=\; \Omega \;=\; L^{-1}\star dL$$

$$L(x;y,\bar y) \;=\; e_\star^{i\tilde x^\mu(x)\delta^a_\mu P_a} \quad : \quad \mathcal{R}^{3,1} \;\longrightarrow\; \frac{SO(3,2)}{SO(3,1)}$$

Standard perturbative analysis

- The eqs. with at least one component on $\mathcal{Z}$ lend themselves to be integrated (supplemented with gauge choices) with initial conditions *C* (*x*,*Y*) and *W*(*x*,*Y)*

$$
\begin{array}{rcl}
q\Phi + [V,\Phi]_\pi & = & 0 \\
qV + V\star V + \Phi\star J & = & 0 \\
qU + dV + [U,V]_\star & = & 0 \\
dU + U\star U & = & 0 \\
d\Phi + [U,\Phi]_\pi & = & 0
\end{array}
\qquad
\begin{array}{c}
\longrightarrow \\
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\\
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\end{array}
\qquad
\begin{array}{l}
\Phi = C(x,Y) + q^*\left(-[V,\Phi]_\pi\right) \\
V = q\epsilon + q^*\left(-V\star V - \Phi\star J\right) \\
U = W(x,Y) + q^*\left(-[U,V]_\star - dV\right) \\
\\
\\
\end{array}
$$

$$
q := dZ^{\underline{\alpha}}\frac{\partial}{\partial Z^{\underline{\alpha}}}\ , \quad J \ := \ -\frac{i}{4}dz^2\kappa - \text{h.c.}
$$

iteratively in an expansion in curvatures (contained in *C*)

$$
\begin{array}{rcl}
\Phi & = & \displaystyle\sum_{n\geq 1}\Phi^{(n)}\ , \qquad \Phi^{(1)} = C(x,Y) \\
V_\alpha & = & \displaystyle\sum_{n\geq 1}V_\alpha^{(n)} \\
U & = & \displaystyle\sum_{n\geq 0}U^{(n)}\ , \qquad U^{(0)} \ = \ \Omega \ = \ L^{-1}\star dL
\end{array}
$$

Standard perturbative analysis

- The eqs. with at least one component on Z can be integrated to give the Z-dependent fields iteratively in terms of non-linear couplings involving the original dof in $\Phi_{|Z=0}$:

$$\Phi^{(n)} = C^{(n)}(Y) - z^\alpha \sum_{k=1}^{n-1} \int_0^1 dt [V_\alpha^{(n-k)}, \Phi^{(k)}]_{\pi, z\to tz} + \text{h.c.}$$

$$V_\alpha^{(n)} = \partial_\alpha\, \xi^{(n)} + z_\alpha \int_0^1 dt\, t \left(\frac{i}{2} \Phi^{(n)} \star \kappa + \sum_{k=1}^{n-1} V^{(k)\alpha} \star V_\alpha^{(n-k)} \right)_{z\to tz} + \bar{z}^{\dot\beta} \sum_{k=1}^{n-1} \int_0^1 t dt [V_\alpha^{(k)}, \bar{V}_{\dot\beta}^{(n-k)}]_{z\to tz}$$

$$U^{(n)} = W^{(n)}(Y) + \frac{i}{2} z^\alpha \sum_{k=0}^{n-1} \int_0^1 dt [W_\mu^{(k)}, V_\alpha^{(n-k)}]_{\star, t\to tz} - \text{h.c.}$$

On-shell the infinitely many Z-contractions turn into an infinite expansion in derivatives of arbitrarily high order → in a generic frame, one has a non-local, Born-Infeld-like tail at every fixed order in weak fields.

[This depends on the solution scheme for the Z-space eqs.: a different scheme, connected to this one by a non-local field redefinition, cuts the infinite tail to the quasi-local expansion expected via Noether procedure. *(Didenko, Gelfond, Korybut, Vasiliev)* Spacetime nonlocalities are expected beyond cubic level.]

Standard perturbative analysis

- Inserting the perturbative solutions for *U* and *Φ* of the *Z*-space eqs. into the pure spacetime eqs. and setting *Z*=0, one gets

$$
\begin{aligned}
dW &= \mathcal{V}_2(W,C) := W\star W+\mathcal{V}_2^{(1)}(W,W,C)+\mathcal{V}_2^{(2)}(W,W,C,C)+\ldots \\
dC &= \mathcal{V}_1(W,C) := [W,C]_\pi+\mathcal{V}_1^{(2)}(W,C,C)+\ldots
\end{aligned}
$$

  - ➢ writing down vertices gets harder and harder
  - ➢ ambiguities introduced by the Z-dependence resolution scheme at every order.

On-shell the infinitely many Z-contractions turn into an infinite expansion in derivatives of arbitrarily high order → in a generic frame, one has a non-local, Born-Infeld-like tail at every fixed order in weak fields.

- A condition for the interpretation of the *W* and *C* as generating functions of gauge fields and Weyl tensors of all spins is that they be real-analytic in *Y*.

  In the standard perturbative analysis this is ensured by the requirement that all master fields be (formal) polynomials in *Y* and *Z* everywhere on spacetime.

  Interesting solutions will force us to considerably soften this condition.

Exact solutions

- Surprisingly, constructing exact solutions is simpler than one would think!

- Working in the full (x,Y,Z)-space enables one to keep into account all non-linearities in a manageable, algebraic form, and use to one's advantage the formal simpicity of the equations
  (The difficulty one encounters, however, is then at the level of interpretation: what is an admissible gauge? Proper class of functions of NC variables? Physical Interpretation of the solution? Meaning of invariants?...)

- In general, one can use all the traditional methods employed for solving complicated differential equations: using some convenient gauge, imposing symmetries, using an algebraically special Ansatz, separating variables…

- Then one usually selects a physical subspace of the possible solutions encoded by the initial choices via physical requirements/global conditions: finiteness of inner product, finiteness and conservation of asymptotic charges...

Factorized expansion scheme

- A different organization of the perturbative expansion around AdS is much more amenable at solving to *all orders*.

- New perturbative scheme based on two observations:

  1. At first order, the equations for *Φ* are

$$\begin{aligned} q\Phi^{(1)} &= 0 \quad \longrightarrow \quad & \Phi^{(1)} &= C(x,Y) \\ D^{(0)}_{\rm tw}\Phi^{(1)} &= 0 \quad \longrightarrow \quad & C(x,Y) &= L^{-1}\star\Phi'(Y)\star\pi(L) \end{aligned}$$

  2. The source term that triggers the non-linear corrections can be rewritten as

$$\Phi\star\kappa \;=\; \Phi\star\kappa_y\star\kappa_z \;=\; \Psi\star\kappa_z\ , \qquad \Psi \;:=\; \Phi\star\kappa_y$$

→ Organizing the perturbative expansion in powers of Ψ and keeping the Y and Z dependence factorized, one can solve for the Z dependence *universally*.

- Insert

$$\boxed{V_\alpha \;=\; V_\alpha(x,Y,z) \;=\; \sum_{k=1}^{\infty}\Psi^{\star k}\star v^{(k)}_\alpha(z)}$$

in the equation

$$\partial_{[\alpha}V_{\beta]}+V_{[\alpha}\star V_{\beta]} \;=\; -\frac{i}{4}\,\epsilon_{\alpha\beta}\,b\Psi\star\kappa_z$$

Factorized expansion scheme

- First order in Ψ: $$\partial_{[\alpha} v^{(1)}_{\beta]} \;=\; -\frac{i}{4}\,\epsilon_{\alpha\beta}\, b\kappa_z$$

  solved by a distributional z-space element

$$\boxed{v^{(1)\pm} \;\sim\; z^{\pm}\int_{-1}^{1}\frac{d\tau}{(\tau+1)^2}\,e^{i\frac{\tau-1}{\tau+1}z^+z^-} \;\sim\; \frac{1}{z^{\mp}}\lim_{\epsilon\to 0}(1-e^{-\frac{i}{\epsilon}z^+z^-}) \;\sim\; \theta(z^{\pm})\delta(z^{\mp})}$$

$$z^{\pm} := u^{\pm\alpha}z_{\alpha}\;,\quad w_z := z^+z^-\;,\quad [z^-,z^+]_{\star} = -2i \qquad \rightarrow \qquad \lim_{\varepsilon\to 0}\frac{1}{\varepsilon}e^{-i\frac{1}{\varepsilon}z^+z^-} \;=\; \kappa_z$$

with basis spinors $u^{\pm}_{\alpha}$ ($u^{+\alpha}u^{-}_{\alpha} = 1$) entering to achieve an integral realization of a delta function in a Gaussian basis (one could have equally well used a plane wave basis, in which case an auxiliary spinor, the momentum associated to z, would have played that role).

- Higher orders: $$\partial_{[\alpha} v^{(k)}_{\beta]} + \frac{1}{2}\sum_{p+q=k}\left[v^{(p)}_{\alpha}, v^{(q)}_{\beta}\right]_{\star} \;=\; 0\;,\quad k\geq 2$$

$$\rightarrow \qquad V_{\alpha} \;=\; \sum_{k\geq 1} v^{(k)}_{\alpha}\star\Psi^{\star k} \;=\; \int_{-1}^{1}\frac{d\tau}{(\tau+1)^2}\,{}_1F_{1\star}(1/2;2;b\log\tau^2\Psi)\star z_{\alpha}\,e^{i\frac{\tau-1}{\tau+1}w_z}$$

Factorized expansion scheme

- Fixing gauges, one can show that a solution of the form

$$\boxed{\begin{array}{rcl} U &=& U^{(0)} \;=\; \Omega(x,Y) \;=\; L^{-1}\star dL \\ \Phi &=& C(x,Y) \;=\; L^{-1}\star\Phi'(Y)\star\pi(L) \\ V_\alpha &=& V_\alpha(Y,z) \;=\; \displaystyle\sum_{k=1}^{\infty}\Psi^{\star k}\star v_\alpha^{(k)}(z) \end{array}}$$

  is actually *exact*.

(C.I., P. Sundell, '17;
D.De Filippi, C.I., P. Sundell, '19)

- A large space of interesting solutions (including HS black holes, HSbh + massless scalar, FLRW-like solutions,...) has been constructed this way.

- While the ordinary perturbative analysis is organized in powers of $\Phi\star\kappa$ and normal order, this can be considered an expansion in Ψ and Weyl order (no contractions between *Y* and *Z*).

- Different solutions are singled out by different basis functions (or distributions) of *Y* on which one expands *C* (i.e., Ψ) .

Comments and observations

- The factorized expansion encodes a (formal) solution space in which Φ is first-order exact, and the Z-dependence is solved in a universal way
  → gives a systematic procedure to non-linearly deform solutions of the KG and Bargmann-Wigner eqs. into solutions of the full Vasiliev eqs.

- Actual solutions must satisfy:
  1. The star-products $(\Psi)^{\star k}$ (and $(\Psi)^{\star k} \star v(z)$) must be finite → conditions on the fiber algebra $\mathcal{A}(Y)$
  2. Observables should be finite (e.g., well-defined inner product)

  In the case that all $\Psi^{\star k}$ can be expanded over a common basis of functions, one can actually write down the full solution in closed form immediately.

- Further constraints placed by requiring the solution to correspond to an asymptotic configuration of Fronsdal fields (over AdS) → V_α should be at least real-analytic in Z.

Particle and HS black-hole modes

- Which solutions of the linearized equations can be dressed into full ones? Which linear sectors can *simultaneously* be dressed into full sectors of the moduli space?

- Factorized scheme already used to nonlinearly deform *massless scalar* modes + spherically symmetric *HS black holes*.

- Massless particle modes build up unitary $\mathfrak{so}(3,2)$ LW modules. In D=4 there are two unitary scalars, distinguished by Neumann/Dirichlet b.c., with ground states

$$\varphi_{(1,0)} \sim \frac{e^{-it}}{(1+r^2)^{1/2}} \,, \qquad \varphi_{(2,0)} \sim \frac{e^{-2it}}{1+r^2}$$

- Type-D, static scalar consists of the solution singular in the origin $\varphi_{(0,0)} \sim \dfrac{1}{r}$
  Generalization to arbitrary spin: type-D spin-s Weyl tensors of the form

$$\boxed{\Phi_{\alpha(2s)} \sim \frac{M}{r^{s+1}} \, (u^+ u^-)^s_{\alpha(2s)}}$$

(Didenko, Vasiliev)

- The spin-2 element coincides with the full AdS-Schwarzschild Weyl tensor.

This follows from the Kerr-Schild property of bhs in gravity: they solve both the linearized and the nonlinear eqs. In gravity, the above are local hallmark of bhs.

Massless particle modes

- Massless particle modes build up unitary $\mathfrak{so}(3,2)$ LW modules.

Unfolded Weyl 0-form equations, i.e., reformulation of the Bargmann-Wigner eqs. via a covariant constancy condition on the twisted adjoint module,

$$C(x|Y) \ = \ L^{-1}(x,Y) \star C'(Y) \star \pi(L)(x,Y)$$

show that particle modes can be encoded into specific algebraic elements: operators on singleton Fock space, *non-polynomial* functions of Y with definite eigenvalues under the Cartan subalgebra (*E*,*J*) of $\mathfrak{so}(3,2)$,

$$C'(Y) \ \in \ \mathcal{M} \ = \ \bigoplus_{\mathbf{n},\mathbf{m}} \mathbf{C} \otimes P_{\mathbf{n}|\mathbf{m}}$$

$$P_{\mathbf{n}|\mathbf{n}'} \star P_{\mathbf{m}|\mathbf{m}'} \ = \ \delta_{\mathbf{n}',\mathbf{m}} P_{\mathbf{n}|\mathbf{m}'} \ , \qquad P_{\mathbf{n}|\mathbf{m}'} \ \sim \ |\mathbf{n}><\mathbf{m}| \ , \qquad \mathbf{n}, \ \mathbf{m} \equiv (n_1,n_2),(m_1,m_2)$$

$$E \star P_{\mathbf{n}|\mathbf{m}} \ = \ \frac{n_1+n_2}{2} P_{\mathbf{n}|\mathbf{m}} \ , \qquad J \star P_{\mathbf{n}|\mathbf{m}} \ = \ \frac{n_2-n_1}{2} P_{\mathbf{n}|\mathbf{m}}$$

$$[E, P_{\mathbf{n}|\mathbf{m}}]_\pi \ = \ \frac{n_1+n_2+m_1+m_2}{2} P_{\mathbf{n}|\mathbf{m}} \ , \qquad [J, P_{\mathbf{n}|\mathbf{m}}]_\pi \ = \ \frac{n_2-n_1+m_1-m_2}{2} P_{\mathbf{n}|\mathbf{m}}$$

- Modules built by solving LW conditions $[L^{--}_r, P_{n|m}]_\pi = 0$ and then acting with L^+_r.

- This offers a simple way of solving for all the AdS-massless particle modes.

Massless scalar particle modes

- For example, the rotationally-invariant scalar field modes are encoded by projectors $\mathcal{P}_n = \mathcal{P}_n(E) = |n/2,n/2\rangle\langle n/2,n/2|$,

$$\mathcal{P}_n(E) \;=\; 2\mathcal{N}_n\, e^{-4E} L^{(1)}_{n-1}(8E) \;=\; \mathcal{N}_n \oint_{C(\varepsilon)} \frac{d\eta}{2\pi i} \left(\frac{\eta+1}{\eta-1}\right)^n e^{-4\eta E}.$$

- Indeed, using the simple AdS gauge function L we reconstruct exactly the Breitenlohner-Freedman scalar modes,

$$C'(Y) \;=\; C'_{s=0}(Y) \;=\; \sum_n \widetilde{\nu}_n \mathcal{P}_n(E)\;, \qquad (\widetilde{\nu}_n)^* \;=\; \widetilde{\nu}_{-n}$$

$$C_{s=0}(x|Y) \;=\; L^{-1}(x) \star C'_{s=0} \star \pi(L)(x) \;=\; (1-x^2)\sum_n \widetilde{\nu}_n \mathcal{O}_n \frac{e^{iy^\alpha M_\alpha{}^{\dot\beta}(x,\eta)\bar{y}_{\dot\beta}}}{1-2i\eta x_0+\eta^2 x^2}$$

$$\mathcal{O}_n \;:=\; \mathcal{N}_n \oint_{C(\varepsilon)} \frac{d\eta}{2\pi i}\left(\frac{\eta+1}{\eta-1}\right)^n$$

- For instance, the LW element n=1 ($C' = 4e^{-4E}$) gives rise to the ground state of the $\mathcal{D}(1,0)$ scalar, as expected:

$$\boxed{4\widetilde{\nu}_1 \frac{1-x^2}{1-2ix_0+x^2} \;\sim\; \widetilde{\nu}_1 \frac{e^{-it}}{(1+r^2)^{1/2}}}$$

(C.I., P. Sundell)

Twisted projectors and HSBH

- Star-multiplication with κ_y induces a change of sign of the right *E*-eigenvalue, so the *twisted projectors*

$$\boxed{\widetilde{\mathcal{P}}_n \;:=\; \mathcal{P}_n \star \kappa_y \;\simeq\; |n/2;0\rangle\langle -n/2;0| \;\in\; \mathcal{D}_0 \otimes \widetilde{\mathcal{D}}_0^*}$$

(C.I., P. Sundell)

realize, via twisted-adjoint action, states with zero energy, *static* → *soliton*-like solutions (*y*-space Fourier duals of particle states).

- Such states are outside the class of formal polynomials → delta functions of (shifted) *Y* oscillators,

$$\widetilde{\mathcal{P}}_n \;:=\; \mathcal{P}_n \star \kappa_y \;=\; 2\pi\, \mathcal{O}_n\, \delta^2(y - i\eta\sigma_0\bar{y})$$

- Yet, they are well-behaved under star product, and in fact form a star product subalgebra together with particle state projectors,

$$\boxed{\begin{array}{llll} \mathcal{P}_n \star \mathcal{P}_m = \delta_{nm}\mathcal{P}_n \;, & \widetilde{\mathcal{P}}_n \star \widetilde{\mathcal{P}}_m = \delta_{n,-m}\mathcal{P}_n \\ \mathcal{P}_n \star \widetilde{\mathcal{P}}_m = \delta_{nm}\widetilde{\mathcal{P}}_n \;, & \widetilde{\mathcal{P}}_n \star \mathcal{P}_m = \delta_{n,-m}\widetilde{\mathcal{P}}_n \end{array}}$$

Twisted projectors and HSBH

- Indeed, dressing with the gauge function the spacetime behaviour of individual fields shows that the twisted projectors are the fibre, local data of spherically-symmetric HS black holes!

  If *C'*(*Y*) is expanded in twisted projectors,

$$C'(Y) \;=\; C'_{\rm bh}(Y) \;=\; \sum_n \nu_n \widetilde{\mathcal{P}}_n \;, \qquad \nu_n \;=\; i^n \mu_n$$

  the Weyl 0-form becomes a gaussian in Y, for generic spacetime point

$$C_{\rm bh}(x|Y) \;=\; L^{-1}\star C'_{\rm bh}\star\pi(L) \;=\; \sum_n \nu_n\, \mathcal{O}_n\, \frac{i}{\eta r}\exp\left(-\frac{1}{2\eta}y\varkappa^{-1}y + iy\varkappa^{-1}v\bar{y} + \frac{\eta}{2}\bar{y}\bar{\varkappa}^{-1}\bar{y}\right)$$

  Coeffs. of the *Y*-expansion → a tower of type-D Weyl tensors of all spins (+ derivatives):

$$\boxed{C^{(n)}_{\alpha(2s)} \;\sim\; \frac{i^{n-1}\mu_n}{r^{s+1}}\,(u^+u^-)^s_{\alpha(2s)}}$$

Curvature singularities

- Each individual Weyl tensor has a curvature singularity in r=0. At the master-field level, this converts into the statement that in the $r \to 0$ limit the Weyl zero-form becomes *a delta function in Y*,

$$\boxed{\Phi_{\rm bh}(x|Y) \;\sim\; \mathscr{O}_n\,\frac{1}{\eta r}\,\exp i\,\frac{\widetilde{y}^+\widetilde{y}^-}{r} \qquad \xrightarrow[r\to 0]{} \qquad \mathscr{O}_n 2\pi\delta^2(\hat{y})}$$

$$\widetilde{y}_\alpha \;:=\; y_\alpha - i\eta(v\bar{y})_\alpha\ , \qquad\qquad \hat{y} \;:=\; \widetilde{y}_\alpha|_{r=0} \;=\; y_\alpha - i\eta(\sigma_0\bar{y})_\alpha$$

- HS symmetry forces such static solutions of all spins to appear together in a infinite-dim. multiplet, packed into the *Y* expansion of the Weyl zero-form. At this level the spacetime singularities have a more readable meaning: *r* appears as the parameter of a delta sequence in *Y*, so effectively unfolding trades the spacetime singularities for a distributional behaviour in *Y*.

- This is a more tractable problem: a delta function of non-commutative variables can be considered smooth, in the sense that it is well-behaved under star product (and is in fact part of the associative algebra that governs such solutions, so the limit $r \to 0$ is uneventful).

Curvature singularities

- Another way of seeing the smoothness at $r = 0$ at the level of master fields is that a change of ordering prescription (e.g., from Weyl ordering to normal ordering in Y) turns the delta function into a real-analytic function,

$$\delta^2(\hat{y}) \;=: \; e^{-2\hat{y}^+\hat{y}^-} \;:$$

and the gauge-invariant classical observables of the theory are (formally) invariant under change of orderings.

- Vasiliev's HS gravity is then a theory in which HS geometries are described on $\mathcal{X}$ x $\mathcal{Z}$ x $\mathcal{Y}$ via master fields:

  - At generic points on the base manifold, they are real-analytic in fibre coordinates, and the coefficients of their power series expansion in Y are bounded component fields. In weak-field regions, they satisfy Fronsdal's equations.

  - At special surfaces they can approach non-analytic functions (and distributions) but remain well-defined as star product algebra elements. Only their interpretation in terms of component Fronsdal fields breaks down.

Degenerate metrics

- Vasiliev's HSG is formulated by means of unfolded equations, encoding non-trivial dynamics into zero-curvature / covariant constancy conditions.

  - the background is introduced via a gauge function (and the corresponding flat connection)
  - the inverse background vielbein never appears in the eqs. describing fluctuations.

- Possible to construct extensions of gravitational manifold by gluing them across surfaces where the metric degenerates.

- BTZ black hole: the natural geometry resulting from identifications along KVF $K = \partial/\partial\phi$ in the embedding flat spacetime is

$$\mathrm{CMink}_2 \times_\xi S^1_K \ , \qquad ds^2_{\mathrm{EBTZ}} = \frac{4dx^2}{(1-x^2)^2} + \xi^2 d\phi^2$$

$$\xi = \sqrt{M}\frac{1+x^2}{1-x^2} > 0$$

- The singularity in $\xi = 0$ is matched by a singular behaviour (divergent frequency) of fluctuation fields.

Degenerate metrics

- However, in the unfolded approach, one can naturally solve for the flat background connection with a gauge function such that

$$\mathrm{ExtEBTZ} = AdS_2 \times_\xi S^1_K \ , \qquad \frac{4dx^2}{(1-x^2)^2} + \xi^2 d\phi^2 \ , \quad \xi \gtreqless 0$$

i.e., the resulting manifolld is the gluing of two EBTZ across the singularity $\xi = 0$.

- This is not unnatural, since fluctuations, packed into mater fields, indeed experience $\xi = 0$ as a smooth surface in terms of their behaviour as star product elements (and inverse vielbein are never involved in defining the dynamics).

- The resulting manifold is obtained NOT from identifications, but intrinsecally → does not suffer from being non-Hausdorff around $\xi = 0$ in the spinless case.

Conclusions and outlook

- The construction of exact solutions to the Vasiliev equations offers many insights into several open questions and challenges related to HS gravity (allowed class of functions, field redefinitions, role of ordering prescriptions, boundary conditions…).

- Several indications that HS gravity requires to go beyond the standard field theoretic interpretation (at the level of component fields), which only makes sense in special regimes.

- Resolution of classical GR singularities relies not only on its embedding into HS gravity, But also on its implementation via unfolded formulation and master fields.

- Many interesting open questions to investigate:

  - HS bhs or bh microstates?
  - proper formulation of boundary value problem?
  - multi-soliton solutions?
  - HS geometry
  - …