Local singular structure of multiloop amplitudes

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- Amazing progress in perturbative QCD
- Responsible for a meaningful comparison with LHC measurements
- Theory and experiment are now of a similar accuracy in the most interesting observable
- Experimental precision will surpass theoretical estimates until the end of the high-luminosity LHC programme
- Theory will be the limiting factor
- We need more processes to be evaluated at next-to-next-to-next-to-leading order and beyond
- for scattering processes of five (NNLO) and four

A wish list…

PROCESSS CLASS	EXAMPLES	STATUS	POSSIBLE Phenomenolog y motivated GOAL
$2 \to 1$	H,W,Z,WH,ZH	N3LO	N3LO
$2 \to 2$	jet inclusive, diboson, top-pair, photon-jet, …	NNLO	N3LO
$2 \to 3$	ttH,diphton+jet, WW/ZZ/ZW+jet, top-pair+jet,…	NLO	NNLO

Are we ready for such a leap?

Challenges

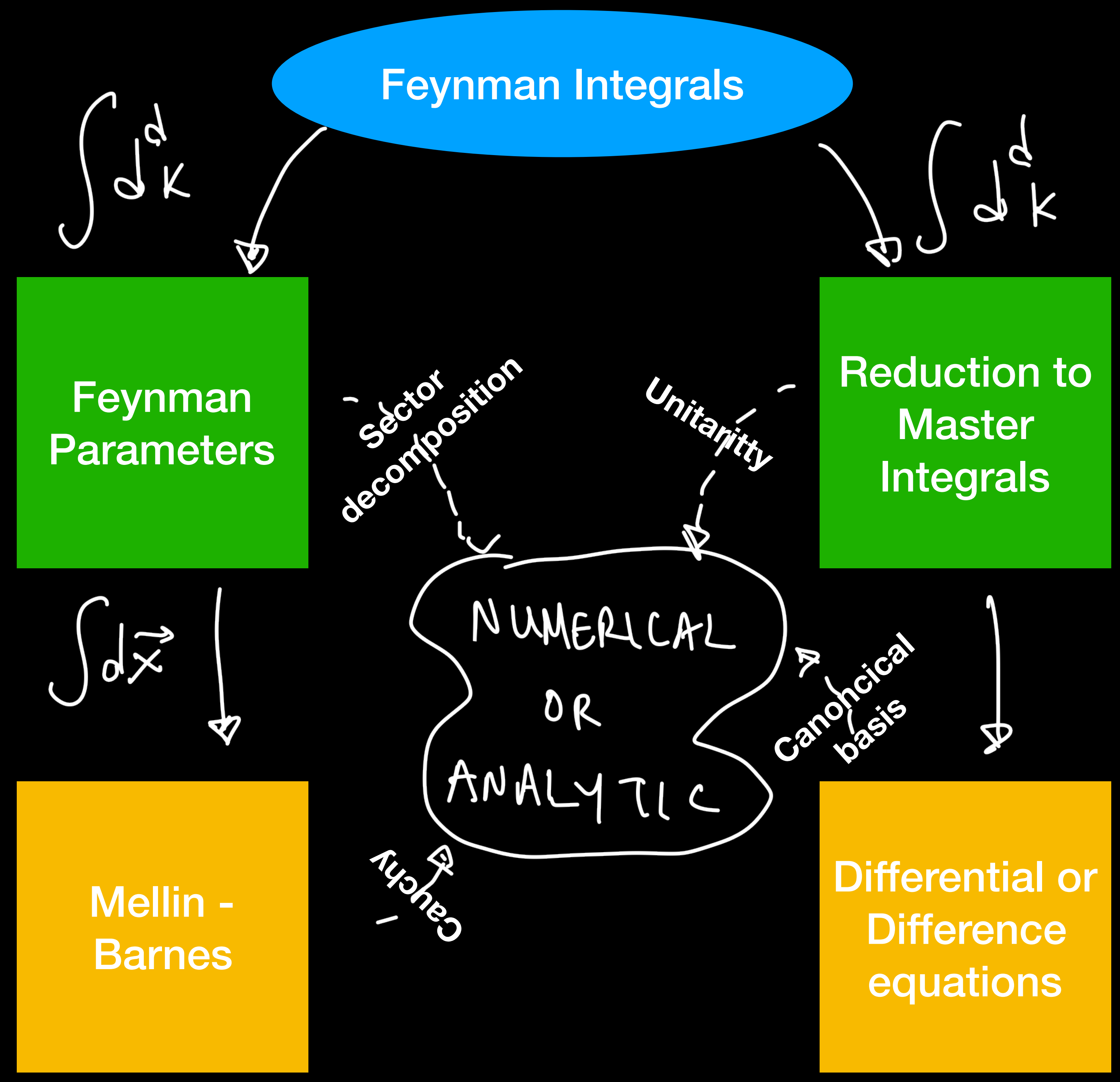
- One big challenge is the proliferation of Feynman diagrams.
- The integrands are simple rational functions of loop-momenta
- But established integration methods for loop amplitudes perform numerous costly operations on the integrands before final integrations.
- These operations are necessitated by the presence of divergences

$$(In\ q\bar{q} \rightarrow Q\bar{Q})$$

Order	Diagrams
tree	1
1-loop	10
2-loop	189
3-loop	134225

(Similar pattern for increasing the number of external legs)

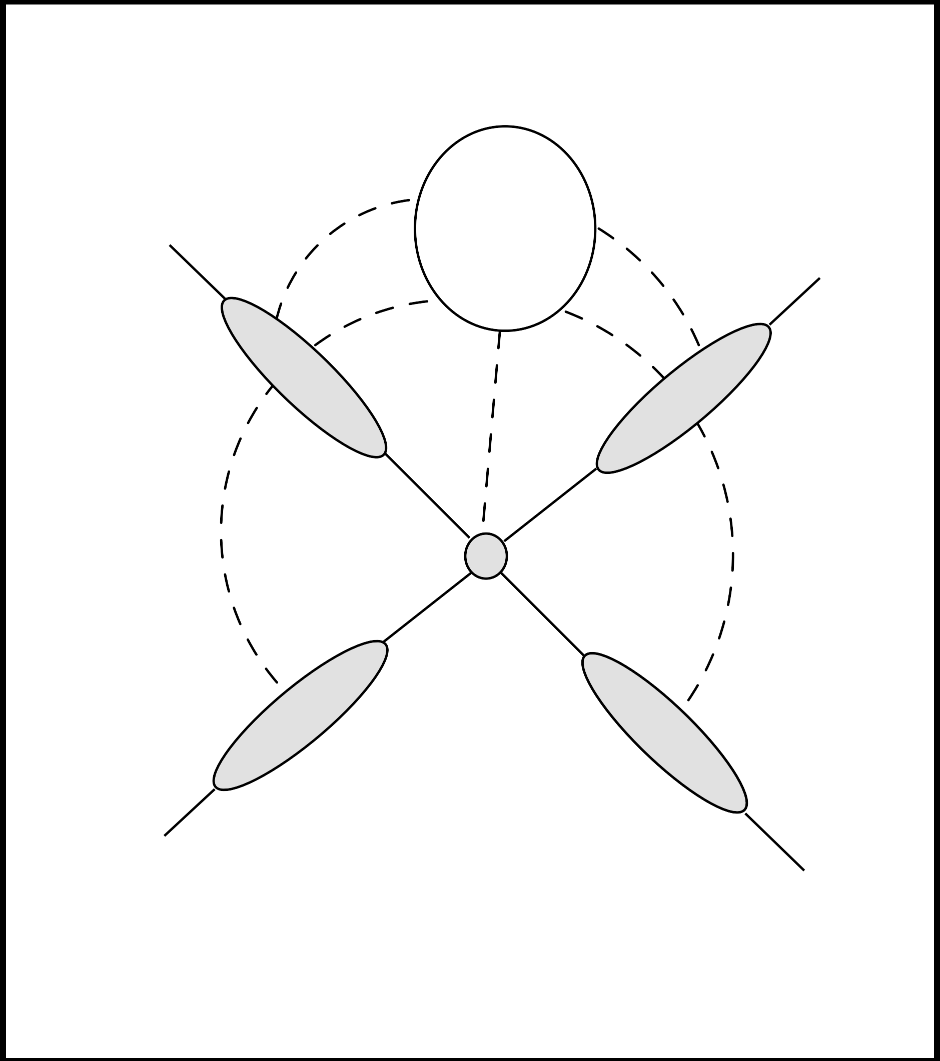
NEED TO THINK OF ALTERNATIVES



Powerful schemes which have lead to impressive breaktroughs.
But, I feel, that we have already achieved most of what is possible with them.

Alternative approach

- Generate amplitudes in momentum space.
- Integrate them directly after subtracting or deforming the integration contour away from singularities.
- The theoretical foundation for this program lies in the proofs of factorization for perturbative QCD *(Collins, Soper, Sterman)*
- For wide-angles and high energy, scattering amplitudes can be separated into short-distance (hard functions) and long-distance factors (jet and soft functions)



Factorization in momentum-space

Basic idea

Amplitude

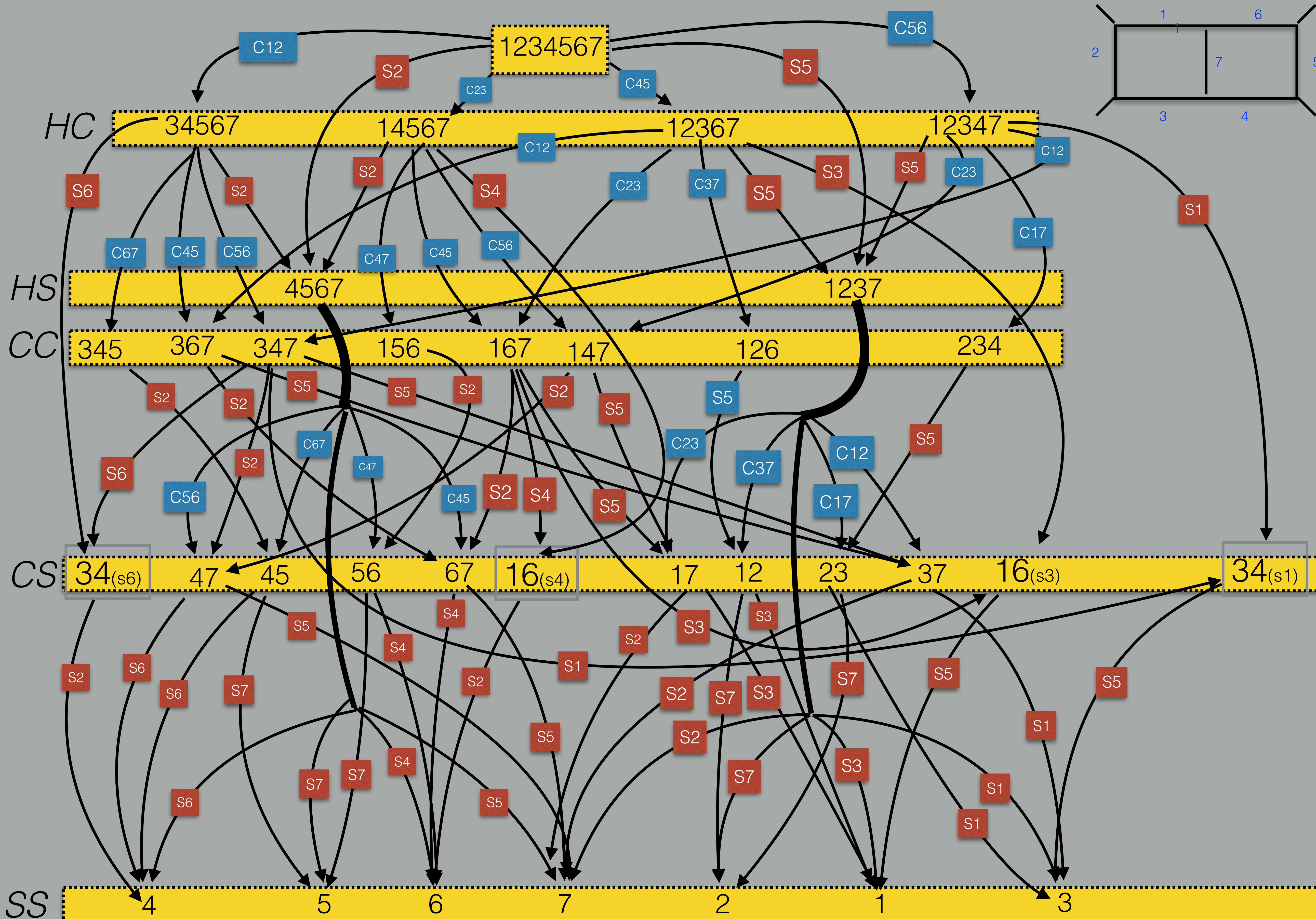
$$G = \int_{-\infty}^{\infty} [dk] \mathscr{I}(k)$$

$$G = \int_C [dk] \left[\mathscr{I}(k) - \mathscr{I}_{approx}(k) \right]$$

Monte-Carlo Integration

$$+ \int_{-\infty}^{\infty} [dk] \mathscr{I}_{approx}(k)$$

Factorization / Analytic Integration or combination with reak-radiation approximations


1234567
HC
34567
14567
12367
12347
HS
4567
1237
CC
345
367
347
156
167
147
126
234
CS
34(s6)
47
45
56
67
16(s4)
17
12
23
37
16(s3)
34(s1)
SS
4
5
6
7
2
1
3

Nested subtractions at 2-loops

- Order of subtractions:
  - double-soft
  - soft-collinear
  - double-collinear
  - single-soft
  - single-collinear

- Approximations in singular regions do not need to be strict limits!

- Good approximations should not introduce ultraviolet divergences

- Good approximations should be easy to integrate exactly.

Nested subtractions
Feynman diagrams vs Feynman amplitudes

Feynman amplitudes are <u>simpler</u> than Feynman diagrams.

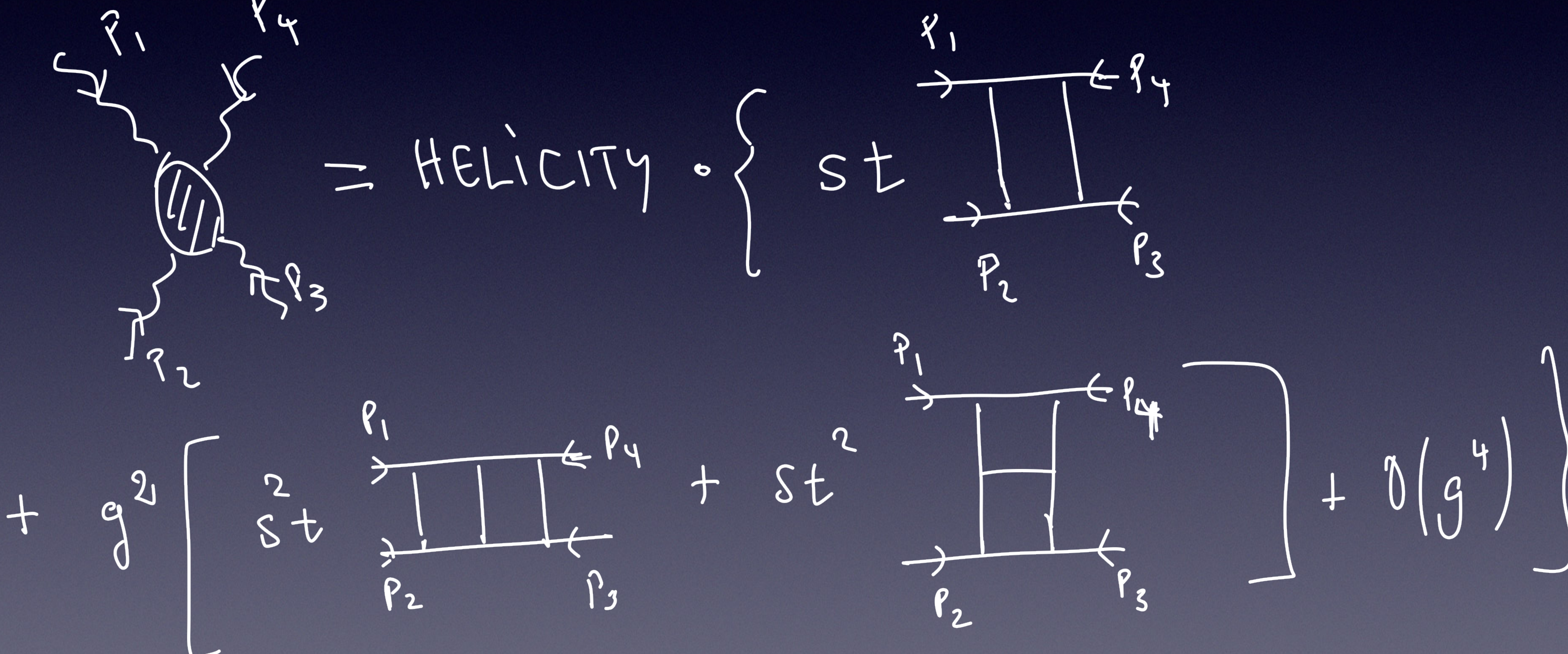
FACTORISATION ?

$$\text{Amplitude} = \underbrace{\text{Soft} \cdot \left(\prod_i \text{Jet}_i \right)}_{\text{SINGULARITIES}} \cdot \underbrace{\text{Hard}}_{\text{FINITE}}$$

HOLDS! AFTER INTEGRATION ! LOCALLY ?

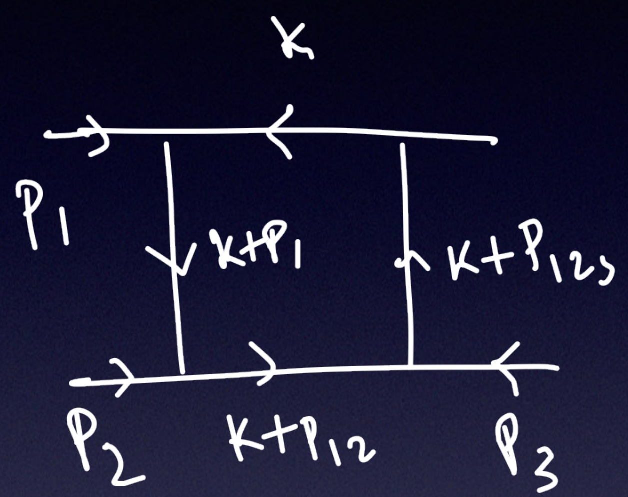
Example: N=4 superymmetric Yang-Mills theory at leading colour

LEADING - COLOUR



+ sub-leading colour.

Collinear singularities of one-loop amplitude



Collinear-limit: $k = -x_1 p_1$

$$k^2 \to x_1^2 p_1^2 = 0$$

$$(k+p_1)^2 \to (1-x_1)^2 p_2^2 = 0$$

$$(k+p_1+p_2)^2 \to \left((1-x_1)p_1+p_2\right)^2 = s(1-x_1)$$

$$(k+p_1+p_2+p_3)^2 \to (-x_1 p_1 + p_2 + p_3)^2 = t\, x_1$$

$$\frac{st}{k^2 (k+p_1)^2 (k+p_{12})^2 (k+p_{123})^2} \to \frac{1}{k^2 (k+p_1)^2} \, \frac{1}{x_1 (1-x_1)}$$

Collinear factorization

$$1 + g^2 \frac{st}{k^2 (k+p_1)^2 (k+p_{12})^2 (k+p_{123})^2} = \overbrace{\left[1 + g^2 \frac{1}{k^2 (k+p_1)^2} \frac{1}{x_1 (1-x_1)} \right]}^{\text{JET}_1} \cdot$$

C_1

$$\cdot \overbrace{\left[1 + g^2 \frac{1}{(k+p_1)^2 (k+p_{12})^2} \frac{1}{x_2 (1-x_2)} \right]}^{\text{JET}_2} \cdot \overbrace{\left[1 + g^2 \frac{1}{(k+p_{12})^2 (k+p_{123})^2} \frac{1}{x_3 (1-x_3)} \right]}^{\text{JET}_3}$$

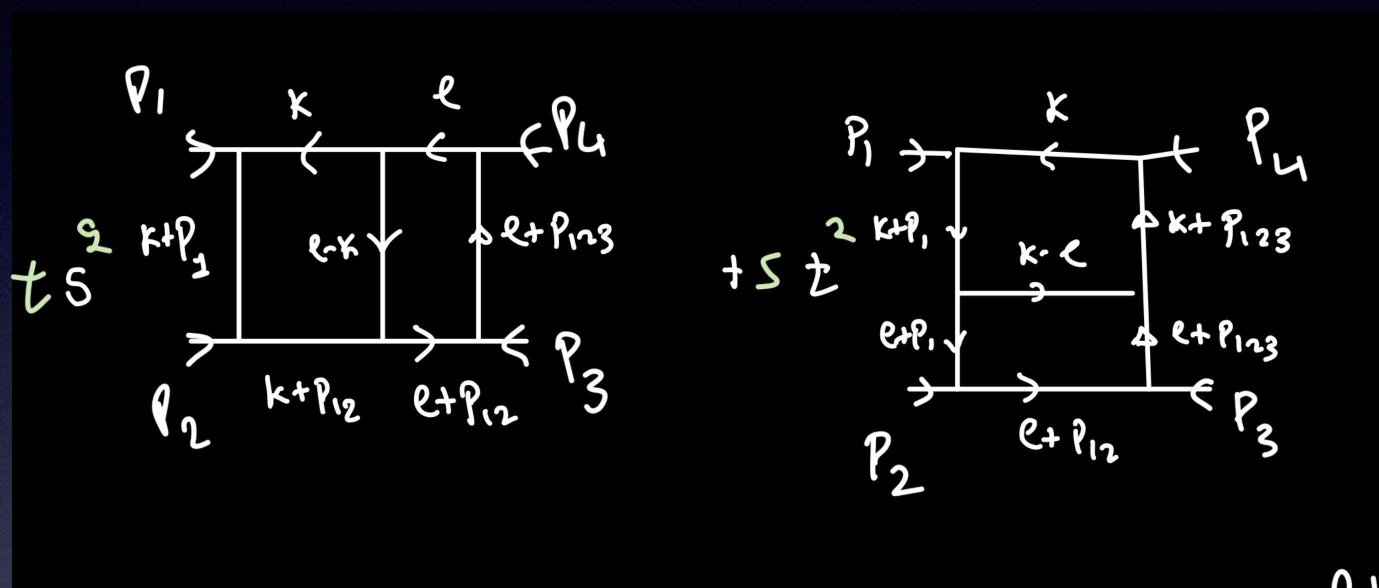
C_2 C_3

$$\cdot \overbrace{\left[1 + g^2 \frac{1}{(k+p_{123})^2 \, k^2} \frac{1}{x_4 (1-x_4)} \right]}^{\text{JET}_4} \cdot$$

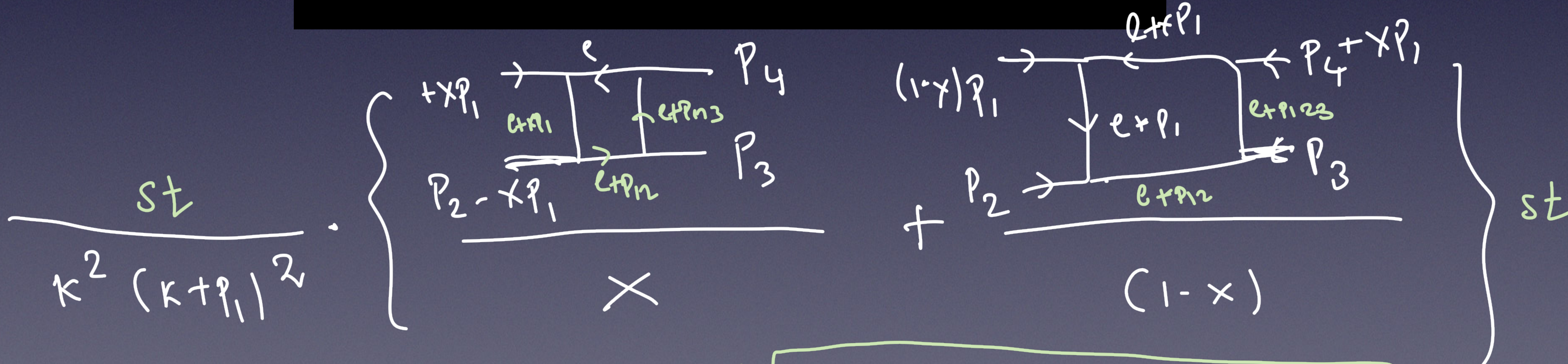
C_4

$$\underbrace{\left\{ 1 + \frac{g^2}{k^2 (k+p_1)^2 (k+p_{12})^2 (k+p_{123})^2} - C_1 - C_2 - C_3 - C_4 \right\}}_{\text{HARD: LOCALLY FINITE !INTEGRABLE!}} + O(g^4)$$

Does the factorization structure present beyond one-loop



$k = -x\, p_1 \Longrightarrow$



$\Rightarrow$ $\boxed{\text{LEADS TO CONVOLUTION}}$

with $x = \dfrac{-2k\cdot n}{2p_1\cdot n}$ [depends on integration variable]

"Ward-identity"

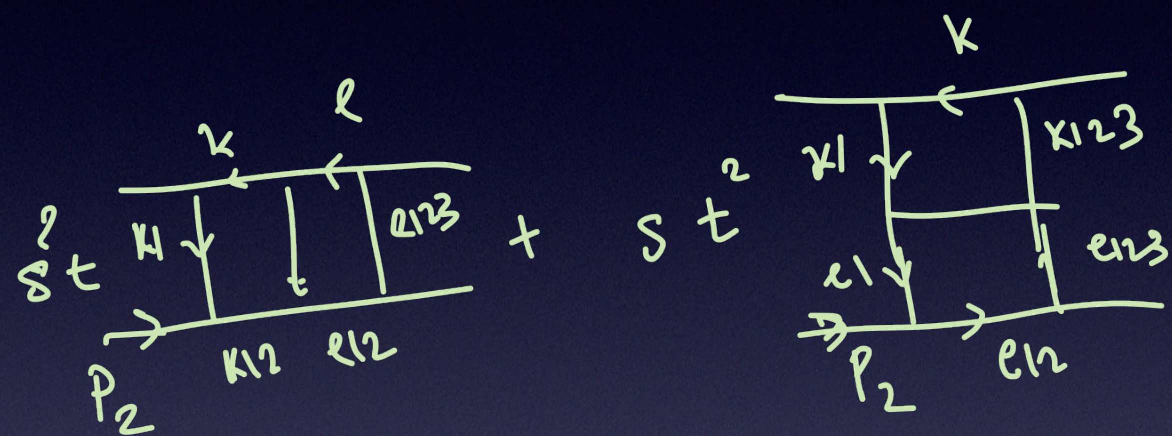
$$k = -x p_1 \Longrightarrow \frac{1}{k^2 (k+p_1)^2} \frac{1}{(l+x p_1)^2} \cdot \left\{ \frac{1}{x} \frac{1}{(l+p_1)^2} + \frac{1}{1-x} \frac{1}{l^2} \right\} \frac{st}{(l+p_{12})^2 (l+p_{123})^2}$$

$$= \underbrace{\frac{1}{k^2 (k+p_1)^2 \, x \, (1-x_1)}}_{\substack{C_1 \\ \text{same "splitting-function"}}} \quad \underbrace{\frac{1}{l^2 (l+p_1)^2 (l+p_{12})^2 (l+p_{123})^2}}_{\text{1-loop amplitude}}$$

CAPTURED BY FACTORISATION

Does factorization hold locally?

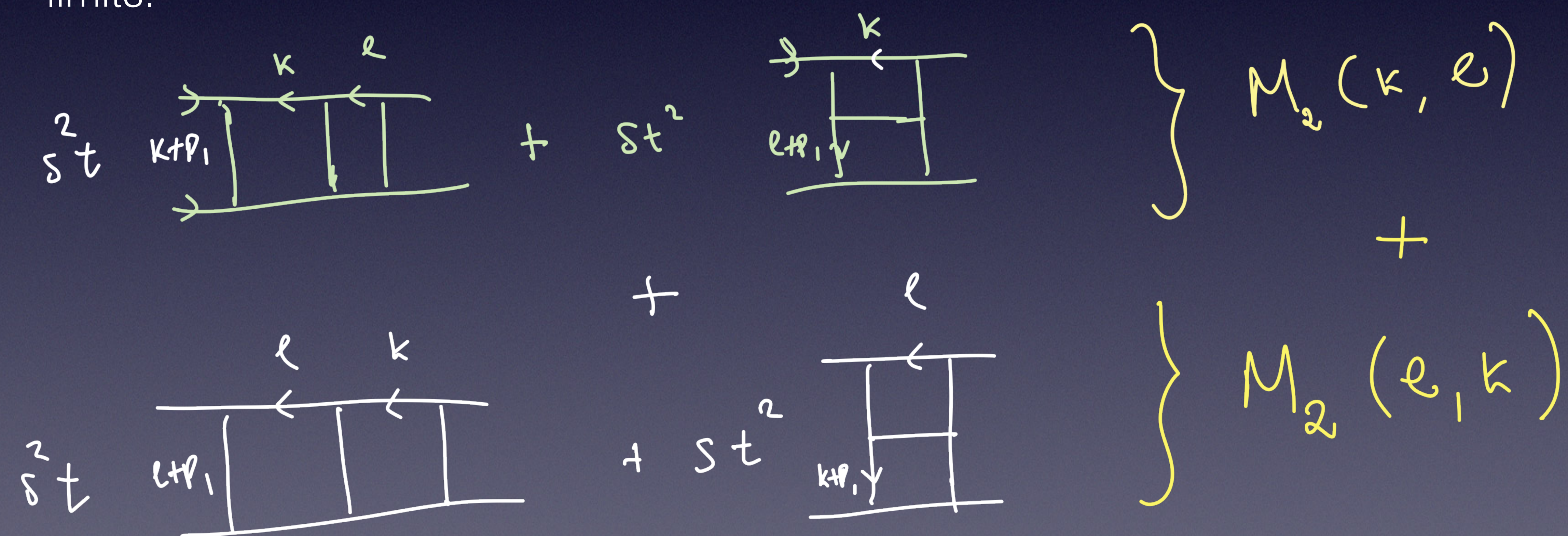
- It appeared so in one of the collinear limits
- But we needed to choose properly the routing of the loop momenta.
- But we have many collinear limits. Does one momentum routing factorize all?
- NO....



For example, in the p_2-collinear limit the routing of the momenta does not permit the application of the "Ward-identity" ?

Possible solution

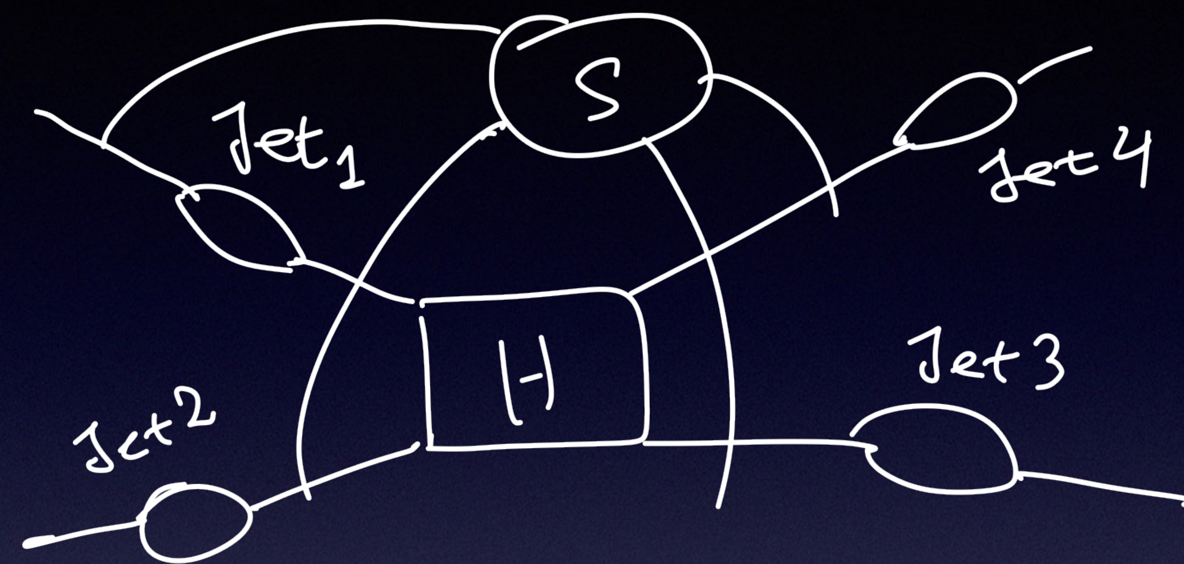
- Consider sums of copies of the amplitude integrand with diverse momentum routings.
- Tune the routings so that the Ward identities can be applied to factorize all limits.



THIS WORKS FOR ALL COLLINEAR LIMITS

Global vs local factorisation

- Amplitudes factorize into universal factors, Soft, Hard and Jet functions.
- This can be shown globally, at the integral level, exploiting Ward identities and the ability to shift separately the momenta in the various singular contributions and apply Ward identities which factorize all singular limits.
- Does this factorization hold at the integrand level, locally?
- If so, we can exploit it to build universal subtractions for all processes. Leading to an automated numerical evaluation of multi-loop amplitudes.
- Ward identities are local…but we are not allowed to shift the momenta.
- Our substitute of shifts: SYMMETRIZATION and perhaps some “sectoring”



$$A = S \cdot \left(\prod_i Jet_i \right) \cdot H$$

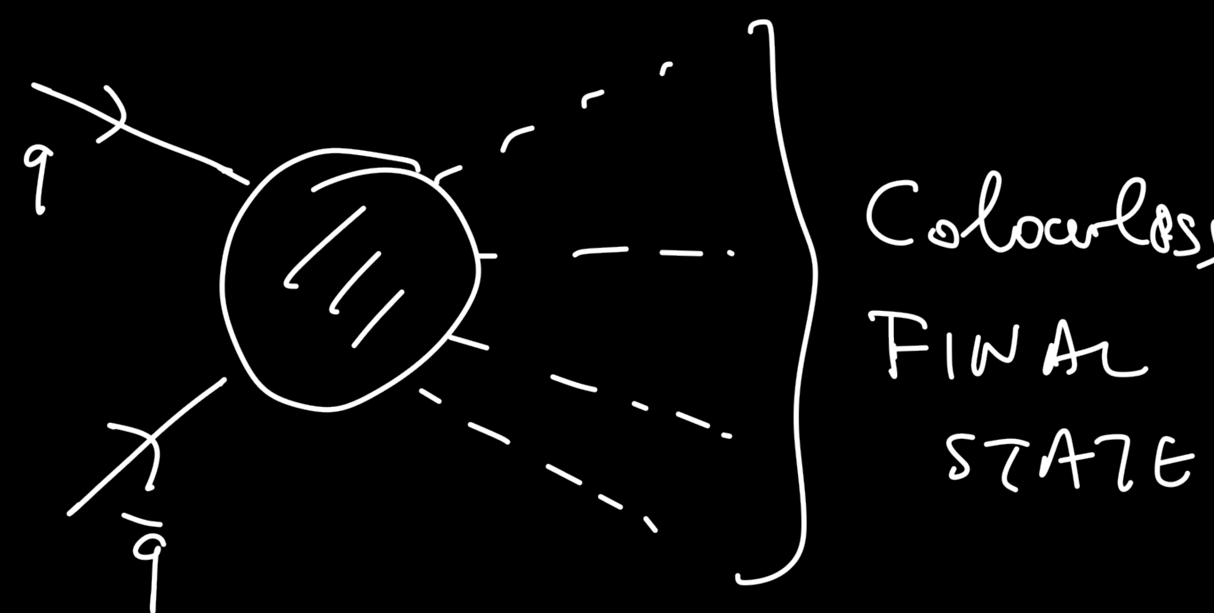
FACTORISATION

$$1 = \frac{C_1}{C_1 + C_2} + \frac{C_2}{C_1 + C_2}$$

SECTORING

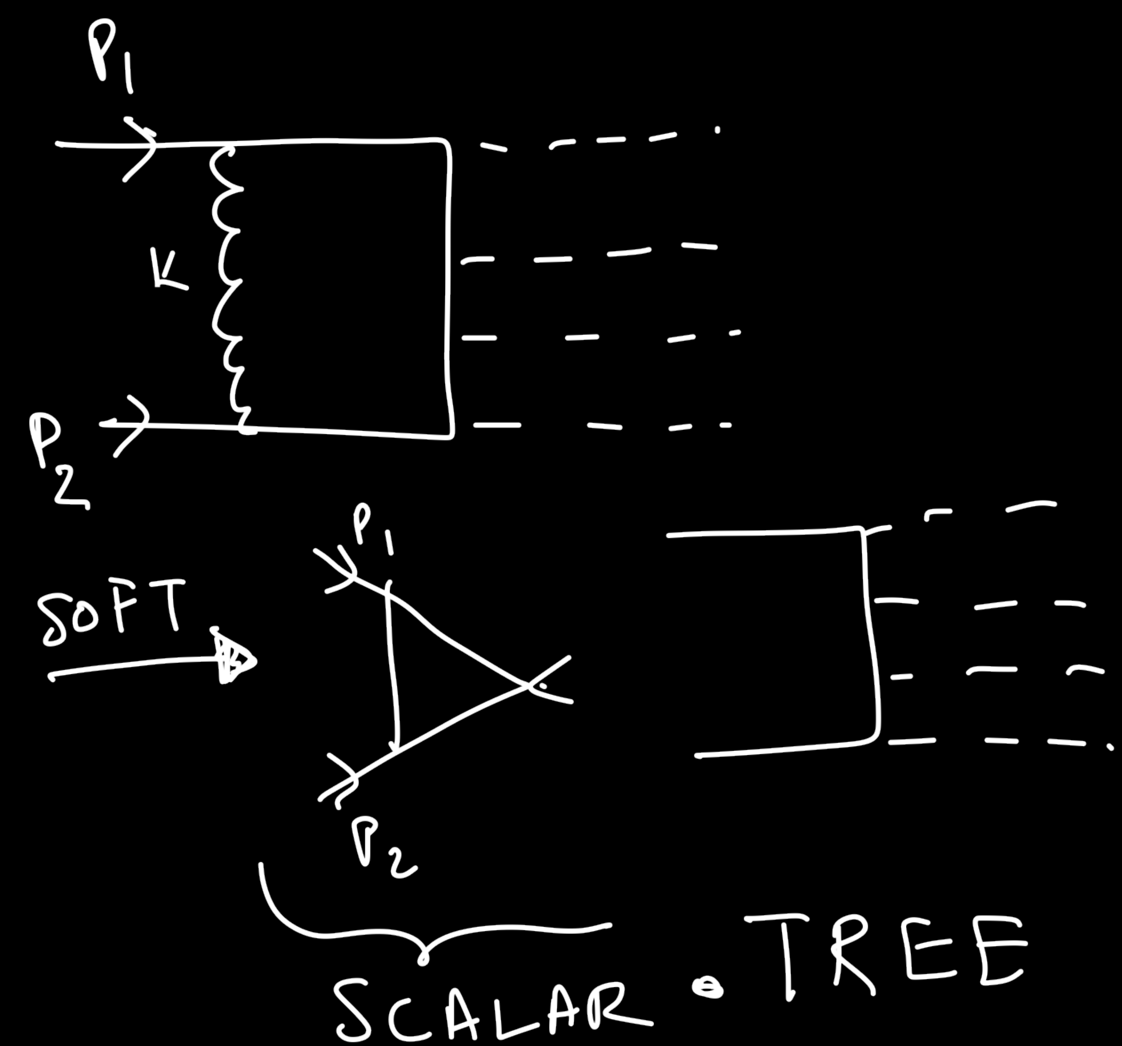
Application to amplitudes

- Consider the process for the production of a heavy colourless final-state from the scattering of a massless quark-antiquark pair.
- This encompasses a large set of processes (multi Z,W, photon production and combinations)
- Easy to verify at one-loop that a simple set of local counterterms exists for all these processes.



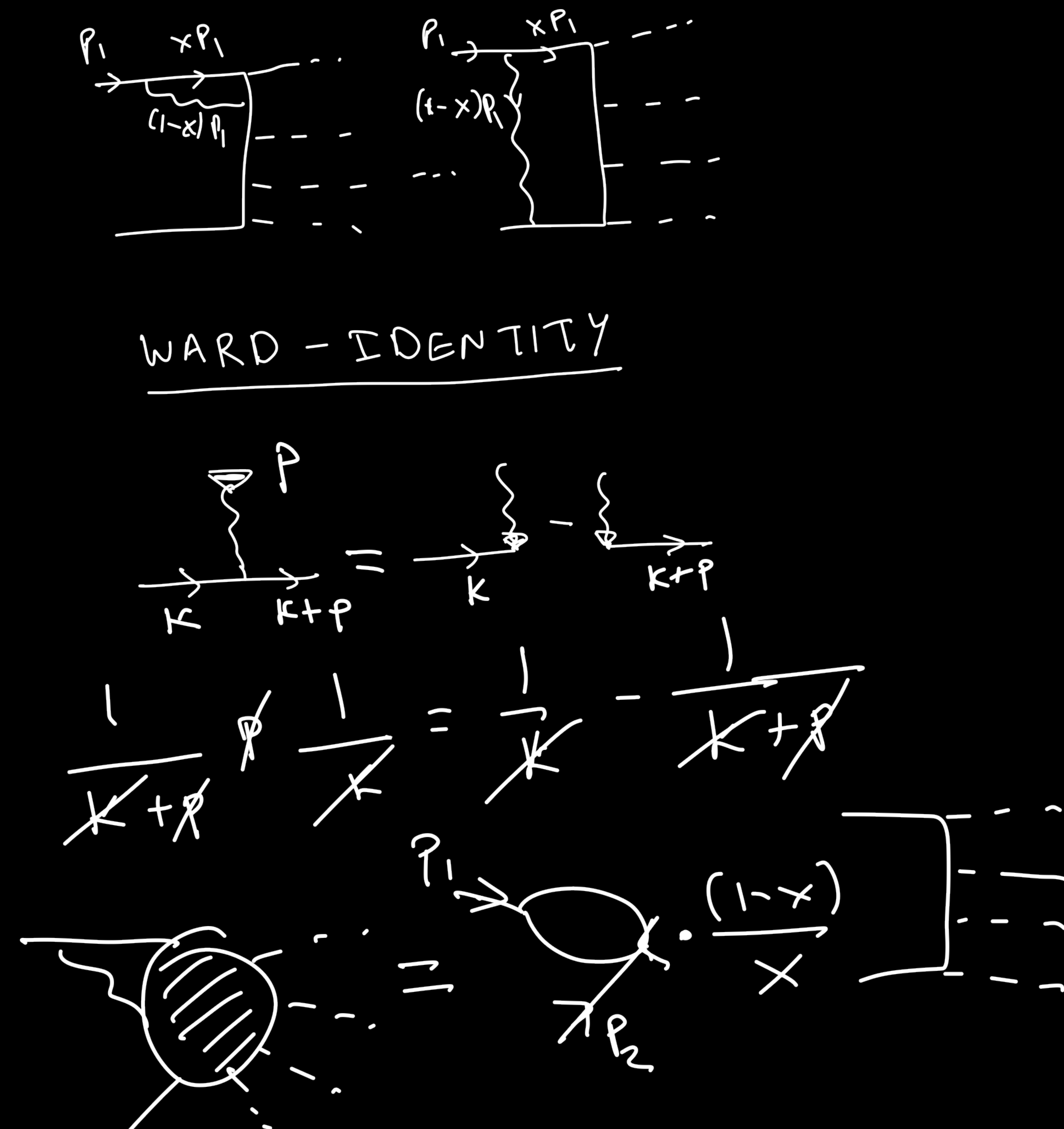
Application to amplitudes

- Per tree-diagram, there is one 1-loop diagram with a soft singularity.
- The soft limit is (up to trivial factors), an one-loop scalar integral times a tree-diagram.



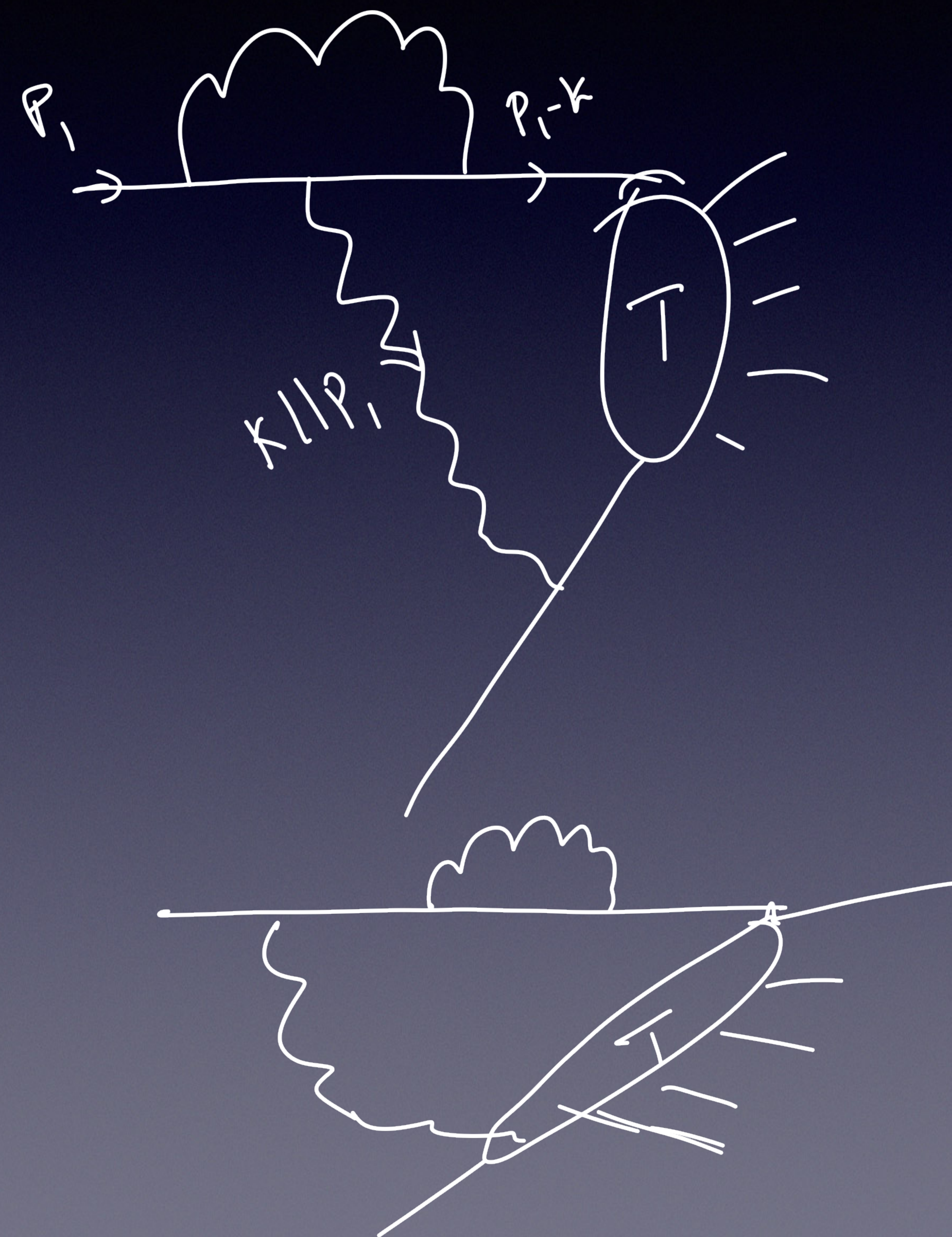
Application to amplitudes

- Many graphs yield collinear divergences.
- Summing over all such graphs, cancellations take place (“Ward”-identity)
- The net-result is factorization of the amplitude in the collinear limit in terms of a splitting-functions and a tree-diagram.



Application to amplitudes

- We could find factorizing universal counterterms for the 2-loop amplitude as well.
- We employed 8 symmetrizations of the two-loop amplitude.
- Tricky regions...collinear emissions from hard and self-energy corrections in propagators adjacent to external legs
- Could be dealt with a sectoring in the counterterms.



Summary/ prospects

- Factorization of infrared divergences has been instrumental for the foundations of QCD.
- It is established at the integral level.
- We are seeing that it can be made manifest at an inner level, the integrand of the amplitudes.
- Can then be turned into a computational tool for complete numerical evaluations of amplitudes...removing soft and collinear singularities locally.
- We have worked it out fully for scattering processes with colorless final states at the LHC.
- Currrently, working out other issues pertinent to the numerical evaluation of the hard remainder, after infrared and ultraviolet subtractions.
- If successful, we can automate the evaluation of 2 to 3 processes at NNLO in the midterm and processes at N3LO in the long term.