Adler's induced gravity achieved through Scherk-Schwarz compactification

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Introduction

Several ways to deal with gravity and quantum mechanics:

- Conservative approach is string theory
- Gravity can also be **emergent**
  - Entropic forces [Jacobson, Verlinde]
  - **Adler's induced gravity** [Adler, Sakharov, Zee]

$\longrightarrow$ already employed in the **DGP model** to induce 4D gravity on a brane in 5D [Dvali, Gabadadze, Porrati, 2000]

Adler's induced gravity

- Consider a curved spacetime **without dynamics**
  - $g_{\mu\nu}$ is in the action, not its derivatives
  - The metric is a **background field**, not quantized
- Inject a massive field and integrate it out

  $\longrightarrow$ **The effective action contains gravity**

■ Integrate the massive fields out

$$\mathrm{e}^{iS_{\text{eff}}[g]} = \int \mathrm{d}\{\phi\}\, \mathrm{e}^{iS[\{\phi\},g]}$$

- S_{eff} is a perturbative expansion:

$$S_{\text{eff}} = \int \mathrm{d}^4x\, \sqrt{-g}\Big[\frac{1}{16\pi G_{\text{ind}}}\big(R - 2\Lambda_{\text{ind}}\big)\Big] + \mathcal{O}(\partial_\lambda g_{\mu\nu})^4$$

■ The little trick is to apply $g_{\mu\nu}(y)\frac{\delta}{\delta g_{\mu\nu}(y)}$ on both sides:

$$g_{\mu\nu}(y)\frac{\delta}{\delta g_{\mu\nu}(y)} \int \mathrm{d}^4x\, \sqrt{-g}\Big[\frac{1}{16\pi G_{\text{ind}}}\big(R - 2\Lambda_{\text{ind}}\big)\Big]$$
$$= \frac{\int \mathrm{d}\{\phi\}\, \mathrm{e}^{iS[\{\phi\},g]} g_{\mu\nu}(y)\frac{\delta}{\delta g_{\mu\nu}(y)} S[\{\phi\}, g]}{\int \mathrm{d}\{\phi\}\, \mathrm{e}^{iS[\{\phi\},g]}}$$

■ Identifying the terms gives

$$-\frac{1}{2\pi}\frac{\Lambda_{\mathrm{ind}}}{G_{\mathrm{ind}}} = \langle T(0)\rangle \tag{1}$$

$$\frac{1}{16\pi G_{\mathrm{ind}}} = \frac{-i}{96}\int \mathrm{d}^4x\; x^2\big\langle \mathcal{T}\big(\widetilde{T}(x)\widetilde{T}(0)\big)\big\rangle \tag{2}$$

- T is the trace of the flat-space **stress-energy tensor**
- $\widetilde{T} = T - \langle T\rangle$ is the variation around its background value
- $\mathcal{T}$ is the time-ordering operator

Divergences in 4D

■ Question: **Is $G_{\mathbf{ind}}$ finite?**

• In 4D the calculation is divergent. For instance, a real scalar field has a cosmological constant written as

$$\int \mathrm{d}t \, \mathrm{d}^4 k \; \mathrm{e}^{-(k^2+M^2)t} \propto \int_0^\infty \mathrm{d}t \; t^{-2} \, \mathrm{e}^{-M^2 t}$$

$\Longrightarrow$ **UV divergent**

■ The solution is to use **extra dimensions**

Kaluza-Klein tower

■ [Antoniadis, Benakli, Quiros, 2001]

$\longrightarrow$ Finite Higgs mass **without supersymmetry**

• An **infinite tower** of KK states gives a **finite contribution** to the one-loop radiative correction

• The reason why it works lies in **Poisson resummation**:

$$\mathrm{e}^{-M^2 t} \xrightarrow[\text{KK}]{} \sum_{m} \mathrm{e}^{-\frac{m^2}{R^2} t} \xrightarrow[\text{Poisson}]{} \frac{\sqrt{\pi} R}{\sqrt{t}} \sum_{\widetilde{m}} \mathrm{e}^{-\frac{\pi^2 \widetilde{m}^2 R^2}{t}}$$

KK is not enough

■ Without supersymmetry, there are **two problems** for the **induced gravity**:

- The **cosmological constant** is infinite
- The **mode $\widetilde{m} = 0$** is divergent for the gravitational constant

■ **Supersymmetry** can fix it but it needs to be spontaneously **broken**

$\Longrightarrow$ **Scherk-Schwarz compactification**

Scherk-Schwarz mechanism

A **Scherk-Schwarz mechanism** is a KK dimensional reduction together with a spontaneous symmetry breaking

- If there is a symmetry with charges Q in $4+1$ dim, we may impose Q-dependent **boundary conditions**

$$\begin{aligned} & \Phi(x^\mu, x^4 + 2\pi R_4) = \mathrm{e}^{2i\pi Q}\Phi(x^\mu, x^4) \\ \Longrightarrow \quad & \Phi(x^\mu, x^4) = \sum_{m_4} \Phi_{m_4}(x^\mu)\,\mathrm{e}^{i\frac{m_4+Q}{R_4}x^4} \\ \Longrightarrow \quad & M^2 = \left(\frac{m_4+Q}{R_4}\right)^2 \end{aligned}$$

- If $Q = \frac{F}{2}$ (F is the fermion number) then supersymmetry is broken

$\longrightarrow \; m_{3/2} = \frac{1}{2R_4}$

Summary of the calculation

■ Consider n compactified circles of radii R_i

■ Impose **$\vec{Q}$-dependent boundary conditions**

• $\vec{Q}$ is non-zero for all fields because we want to integrate them out (**no massless mode**)

• Q_j is different for bosons and fermions ($\boldsymbol{Q_F - Q_B = 1/2}$)

■ Calculate the contribution of a **real scalar field** ϕ, a **vector field** A_μ and a **Weyl fermion** ψ

■ Choose a field content such that the supersymmetric theory (when $Q_B = Q_F$) yields

$$\Lambda_{\text{ind}}\big|_{\text{susy}} = 0 \quad \text{and} \quad \frac{1}{G_{\text{ind}}}\Big|_{\text{susy}} = 0$$

Real scalar field

$$T\Big|_{\phi} = -\sum_{m} \left[\partial_\mu \phi_m \partial^\mu \phi_m + 2M_m^2 \phi_m^2\right]$$

■ The induced constants are

$$-\frac{1}{2\pi}\frac{\Lambda}{G}\Big|_{\phi} = i\sum_{m}\delta(0) - \frac{2\Gamma\left(2+\frac{n}{2}\right)}{\pi^{4+\frac{n}{2}}}\sum_{\widetilde{m}} \frac{\prod R_i}{\left(\widetilde{m}_i^2 R_i^2\right)^{2+\frac{n}{2}}}\,\mathrm{e}^{2i\pi Q_B\cdot\widetilde{m}}$$

$$\frac{1}{16\pi G}\Big|_{\phi} = \frac{28}{3}C(n)\sum_{\widetilde{m}} \frac{\prod R_i}{\left(\widetilde{m}_i^2 R_i^2\right)^{1+\frac{n}{2}}}\,\mathrm{e}^{2i\pi Q_B\cdot\widetilde{m}}$$

- The δ-function is an infinite constant that will disappear when supersymmetry is introduced

- $C(n)$ is a strictly positive number defined for $n \geq 2$ (n is the number of compactified dimensions)

- As explained earlier, the sum is divergent because of the mode $\widetilde{m} = 0$

Vector field

$$T\Big|_{A_\mu} = -\sum_m M_m^2 A_{m,\mu} A_m^\mu$$

- The induced constants are

$$-\frac{1}{2\pi}\frac{\Lambda}{G}\Big|_{A_\mu} = i\sum_m \delta(0) - \frac{3 \times 2\Gamma\left(2+\frac{n}{2}\right)}{\pi^{4+\frac{n}{2}}} \sum_{\widetilde{m}} \frac{\prod R_i}{\left(\widetilde{m}_i^2 R_i^2\right)^{2+\frac{n}{2}}} \, \mathrm{e}^{2i\pi Q_B \cdot \widetilde{m}}$$

$$\frac{1}{16\pi G}\Big|_{A_\mu} = 4C(n) \sum_{\widetilde{m}} \frac{\prod R_i}{\left(\widetilde{m}_i^2 R_i^2\right)^{1+\frac{n}{2}}} \, \mathrm{e}^{2i\pi Q_B \cdot \widetilde{m}}$$

• The propagators are taken in the unitarity gauge in order to eliminate the ghosts

• The vector field counts as **three bosonic degrees of freedom** for the cosmological constant (apart from the irrelevant constant)

Weyl fermion

$$T\Big|_\psi = \sum_m \left[\frac{3i}{2}\partial_\mu\bar{\psi}_m\bar{\sigma}^\mu\psi_m - 2M_m\psi_m^2 + \text{c.c.}\right]$$

■ The induced constants are

$$-\frac{1}{2\pi}\frac{\Lambda}{G}\Big|_\psi = -6i\sum_m \delta(0) - \frac{(-2)\times 2\Gamma\left(2+\frac{n}{2}\right)}{\pi^{4+\frac{n}{2}}}\sum_{\widetilde{m}}\frac{\prod R_i}{\left(\widetilde{m}_i^2 R_i^2\right)^{2+\frac{n}{2}}}\,\mathrm{e}^{2i\pi Q_F\cdot\widetilde{m}}$$

$$\frac{1}{16\pi G}\Big|_\psi = -\frac{32}{3}C(n)\sum_{\widetilde{m}}\frac{\prod R_i}{\left(\widetilde{m}_i^2 R_i^2\right)^{1+\frac{n}{2}}}\,\mathrm{e}^{2i\pi Q_F\cdot\widetilde{m}}$$

• The Weyl fermion is **twice the opposite** of a real scalar field for the cosmological constant

• It gives a **negative contribution** to the gravitational constant $\Longrightarrow$ hope for cancellation in susy regime

Spectrum

■ The conditions are

$$\Lambda_{\text{ind}}\big|_{\text{susy}} = 0 \implies n_\phi + 3n_A = 2n_\psi$$
$$\frac{1}{G_{\text{ind}}}\Big|_{\text{susy}} = 0 \implies 28n_\phi + 12n_A = 32n_\psi$$

• The solution is $(n_\phi, n_\psi, n_A) = (3,3,1)k \in \mathbb{N} \longrightarrow$ for example, **one chiral multiplet** and **one massive vector multiplet** are good enough

■ After breaking of supersymmetry, the induced constants read

$$-\frac{1}{2\pi}\frac{\Lambda}{G} = -n_\psi \frac{4\Gamma\left(2+\frac{n}{2}\right)}{\pi^{4+\frac{n}{2}}} \sum_{\widetilde{m}} \frac{\prod R_i}{\left(\widetilde{m}_i^2 R_i^2\right)^{2+\frac{n}{2}}}\, \mathrm{e}^{2i\pi Q_B\cdot\widetilde{m}} \left(1-(-1)^{\widetilde{m}_j}\right)$$
$$\frac{1}{16\pi G} = n_\psi \frac{32}{3} C(n) \sum_{\widetilde{m}} \frac{\prod R_i}{\left(\widetilde{m}_i^2 R_i^2\right)^{1+\frac{n}{2}}}\, \mathrm{e}^{2i\pi Q_B\cdot\widetilde{m}} \left(1-(-1)^{\widetilde{m}_j}\right)$$

Discussion

■ We found a class of models such that the induced constants are convergent $\longrightarrow$ it requires extra dimensions and susy breaking

■ **Why $n \geq 2$?** It could be an artefact caused by the perturbative expansion.

■ Is it valid **for all higher derivative terms**?

■ Is it valid when **interactions** are introduced?

• Higher loops will appear $\Longrightarrow$ is it convergent even at order R (Ricci scalar)?