Asymptotic Safety is Not Good Enough!

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Corfu Summer Institute, 2019, 31 August -11 September

Weyl Gravity: Finite Derivative Theories

$$S = \int \sqrt{-g} d^4x \left[M_p^2 R + \alpha C^2 \right]$$

Weyl term does not introduce singularities

$$S = \int \sqrt{-g} d^4x \left[R + \alpha R_{\mu\nu} R^{\mu\nu} + \beta R^2 \right]$$

$$\Pi(k) = \frac{1}{k^2}\left(\mathcal{P}^2 - \frac{\mathcal{P}_s^0}{2}\right) - \frac{\mathcal{P}^2}{k^2 - m_2^2} + \frac{1}{2}\frac{\mathcal{P}_s^0}{k^2 - m_0^2}$$

$m_2 = -\left(\frac{1}{2}\alpha\right)^{-1}$ and $m_0 = (\alpha + \beta)^{-1}$

If $\alpha = 0$, Asymptotic safety

The Weyl ghost mass goes to infinity.
This is not Asymptotically free theory, this has cosmological & blackhole singularities

Quadratic Curvature Gravity is renormalizable,
but contains "Ghosts": Vacuum is Unstable

Utiyama (1961), De Witt (1961), Stelle (1977)
t'Hooft, Veltman (1974)

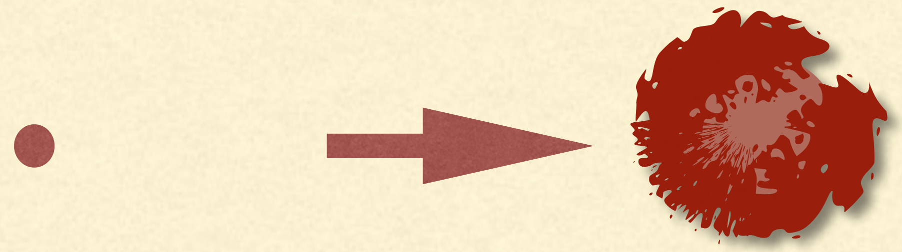
Note on Singularity

Finite derivative theory always has a point support

$$x^n \delta^n(x) = (-1)^n n! \delta(x)$$

Infinite derivatives acting on a delta source does not have any point support

$$e^{\alpha \nabla_x^2} \delta(x) = \frac{1}{\sqrt{2\pi}} \int dk e^{-\alpha k^2} e^{ik\cdot x} = \frac{1}{\sqrt{2\alpha}} e^{-x^2/4\alpha}$$



A point becomes a blob

Non-locality is the key for any for of Quantum Gravity

hep-th/0508194, JCAP (2006), 1804.08195 [gr-qc]

Most general action of gravity in 4d

$$S = \int d^4x\sqrt{-g}\,\left[R + R_{abcd}\mathcal{O}^{abcd}_{efgh}R^{efgh} + R_{\cdots}O^{\cdots}_{\cdots}R^{\cdots}O^{\cdots}_{\cdots}R^{\cdots} + \cdots\right]$$

All possible terms allowed by diffeomorphism symmetry!

Unknown Infinite Functions of Covariant Derivatives

Let us study up to the quadratic curvature part …

1) We can show that it is ghost free

2) We can also show that the gravitational interaction weakens sufficiently not to form a singularity

3) Gravity becomes asymptotically free

Biswas, Mzumdar, Siegel, JCAP (2005),
Biswas, Gerwick, Koivisto, Mazumdar, Phys. Rev. Lett. (2012) (gr-qc/1110.5249)

Perturbative Unitarity in Infinite Derivative Gravity

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} + R\mathcal{F}_1\left(\frac{\Box}{M^2}\right)R + R_{\mu\nu}\mathcal{F}_2\left(\frac{\Box}{M^2}\right)R^{\mu\nu} + R_{\mu\nu\lambda\sigma}\mathcal{F}_3\left(\frac{\Box}{M^2}\right)R^{\mu\nu\lambda\sigma}\right]$$

$$2\mathcal{F}_1 + \mathcal{F}_2 + 2\mathcal{F}_3 = 0 \qquad a(\Box) = 1 - \frac{1}{2}\mathcal{F}_2(\Box)\frac{\Box}{M_s^2} - 2\mathcal{F}_3(\Box)\frac{\Box}{M_s^2}$$

$$\Pi(k^2) = \frac{1}{a(k^2)}\left[\frac{P^{(2)}}{k^2} - \frac{P^0}{2k^2}\right]$$

Demand no extra poles other than massless graviton's, means:

$$a(k^2) = e^{\gamma(k^2)}$$

Entire Function

Simplest choice: $a(k^2) = e^{k^2/M_s^2}$

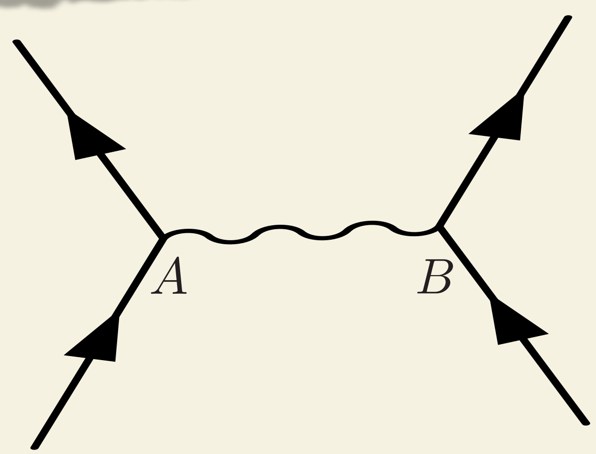
1110.5249 [gr-qc], PRL (2012)

Infinite derivative Gravity action around Minkowski

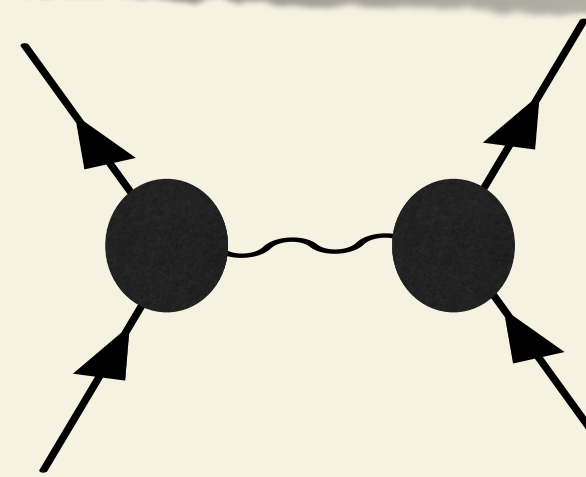
With the help of the earlier constraints:

$$S=\int d^4x\sqrt{-g}\left[M_p^2\frac{R}{2}+R\left[\frac{e^{-\Box/M_s^2}-1}{\Box}\right]R-2R_{\mu\nu}\left[\frac{e^{-\Box/M_s^2}-1}{\Box}\right]R^{\mu\nu}\right]$$

$k < M_s$



$k \geq M_s$



$$\Pi(k^2)=\frac{1}{a(k^2)}\left[\frac{P^{(2)}}{k^2}-\frac{P^0}{2k^2}\right] \qquad a(k^2)=e^{k^2/M_s^2}$$

Massless Graviton, massless spin-2 and spin-0 components propagate

Biswas, AM, Siegel (2006) JCAP, Biswas, Gerwick, Koivisto, AM (2012) Phy. Rev. Lett.

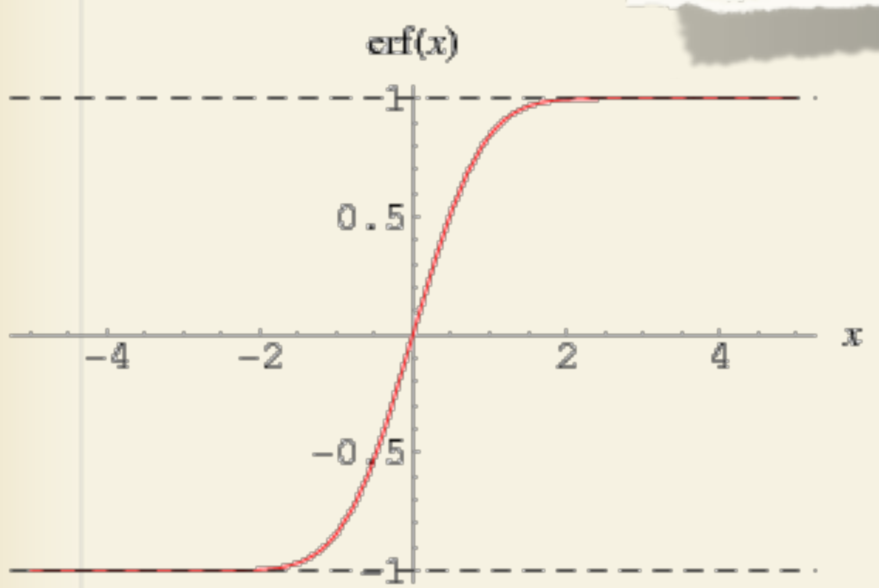
Ghost Free & Singularity Free Gravity

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} + R \left[\frac{e^{\frac{-\Box}{M^2}} - 1}{\Box} \right] R - 2R_{\mu\nu} \left[\frac{e^{-\frac{\Box}{M^2}} - 1}{\Box} \right] R^{\mu\nu} \right]$$

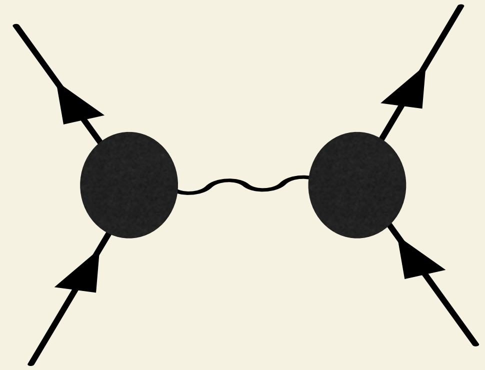
$$ds^2 = -(1 - 2\Phi)dt^2 + (1 + 2\Psi)dr^2$$

$$mM < M_p^2$$

$$\Phi = \Psi = \frac{Gm}{r}\mathbf{erf}\left(\frac{rM}{2}\right)$$



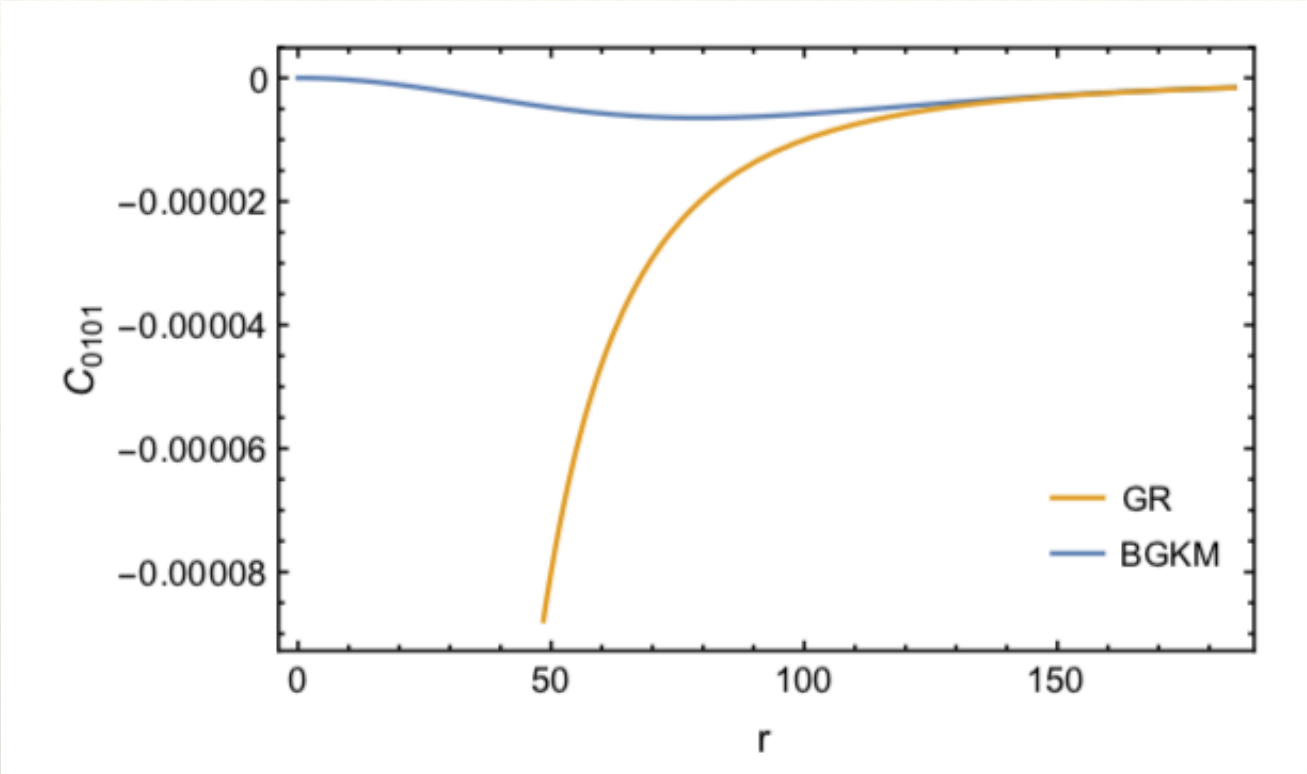
Interaction becomes Non-Local

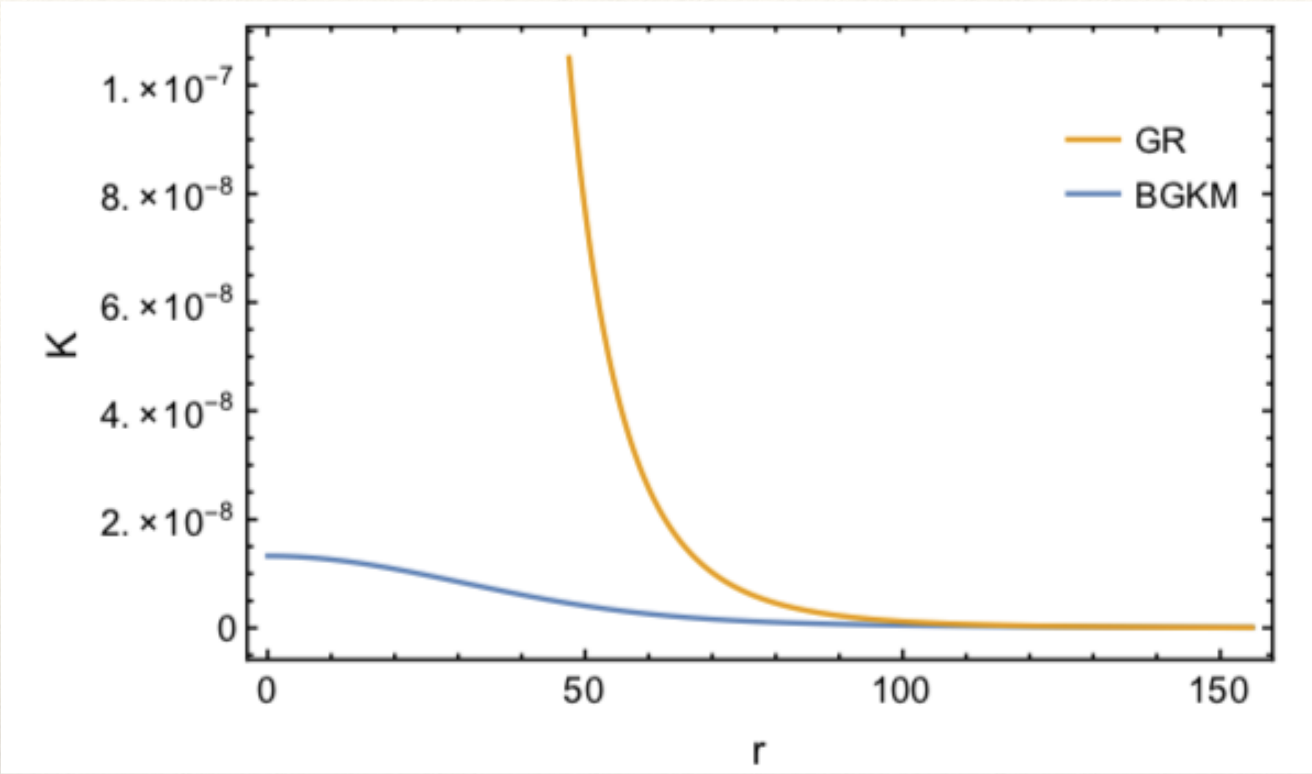


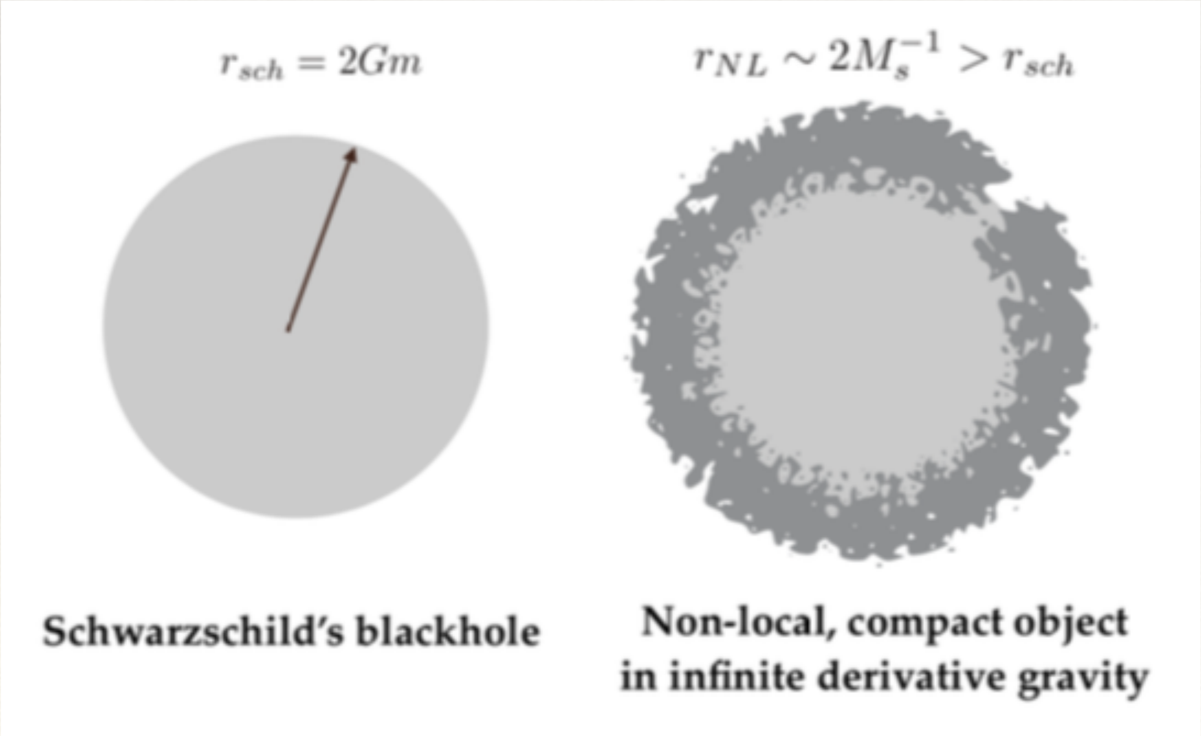
Biswas, Mzumdar, Siegel, JCAP (2005),

Biswas, Gerwick, Koivisto, Mazumdar, Phys. Rev. Lett. (2012) (gr-qc/1110.5249)

Conformally flat: Weyl vanishes, and Kretschmann is finite







Such non-local objects could be BHs provided linear solution is promoted all the way to non-linear level.

[arXiv:1802.00399 [gr-qc]]

Quadratic curvature Infinite derivative Gravity is sufficient

$$S = \int d^4x\sqrt{-g}\left[R + R_{abcd}\mathcal{O}^{abcd}_{efgh}R^{efgh} + R_{\cdots}O^{\cdots}_{\cdots}R^{\cdots}O^{\cdots}_{\cdots}R^{\cdots} + \cdots\right]$$

$$S = \int d^4x\,\sqrt{-g}\left[\frac{R}{2} + R\left[\frac{e^{\frac{-\Box}{M^2}} - 1}{\Box}\right]R - 2R_{\mu\nu}\left[\frac{e^{-\frac{\Box}{M^2}} - 1}{\Box}\right]R^{\mu\nu}\right]$$

Bouncing Universe

Non-singular Blackhole

$$mM < M_p^2 \longrightarrow g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad h_{\mu\nu} < \eta_{\mu\nu}$$

In the entire spacetime manifold

Higher curvature terms, such as cubic, quartic, and …. terms will give even smaller contributions !

Testing Quantum Aspects of Linearized Gravity in a Lab

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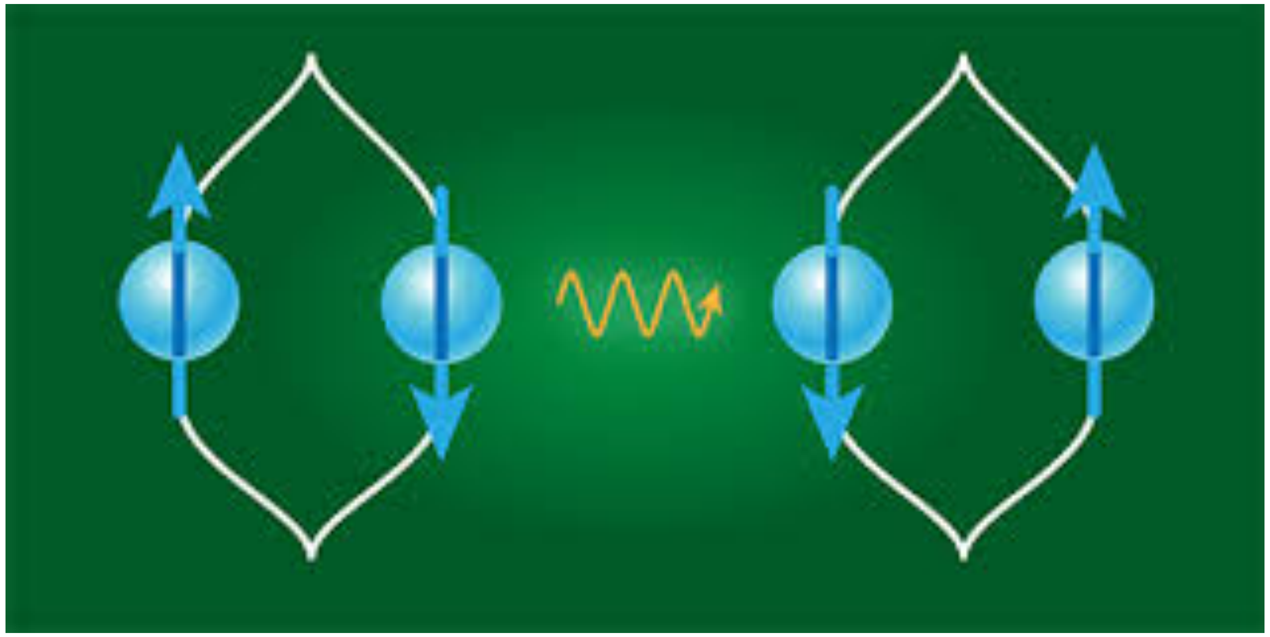


Quantum superpositions of geometries

Bose + AM + Morley + Ulbricht + Toros + Paternostro + Geraci + Barker + Kim + Milburn, Phy. Rev. Lett. [ArXiv: 1707.06050]

Marshman +AM+Bose, [ArXiv: 1907.01568]

See Also: Marletto and Vedral appeared on the same day [1707.06036], Phys. Rev. Lett.

Levels of Excitements …

Can we put a graviton in a quantum superposition?

Can we study coalescing atoms, and see the loss of gravitons (quantized) in a laboratory?

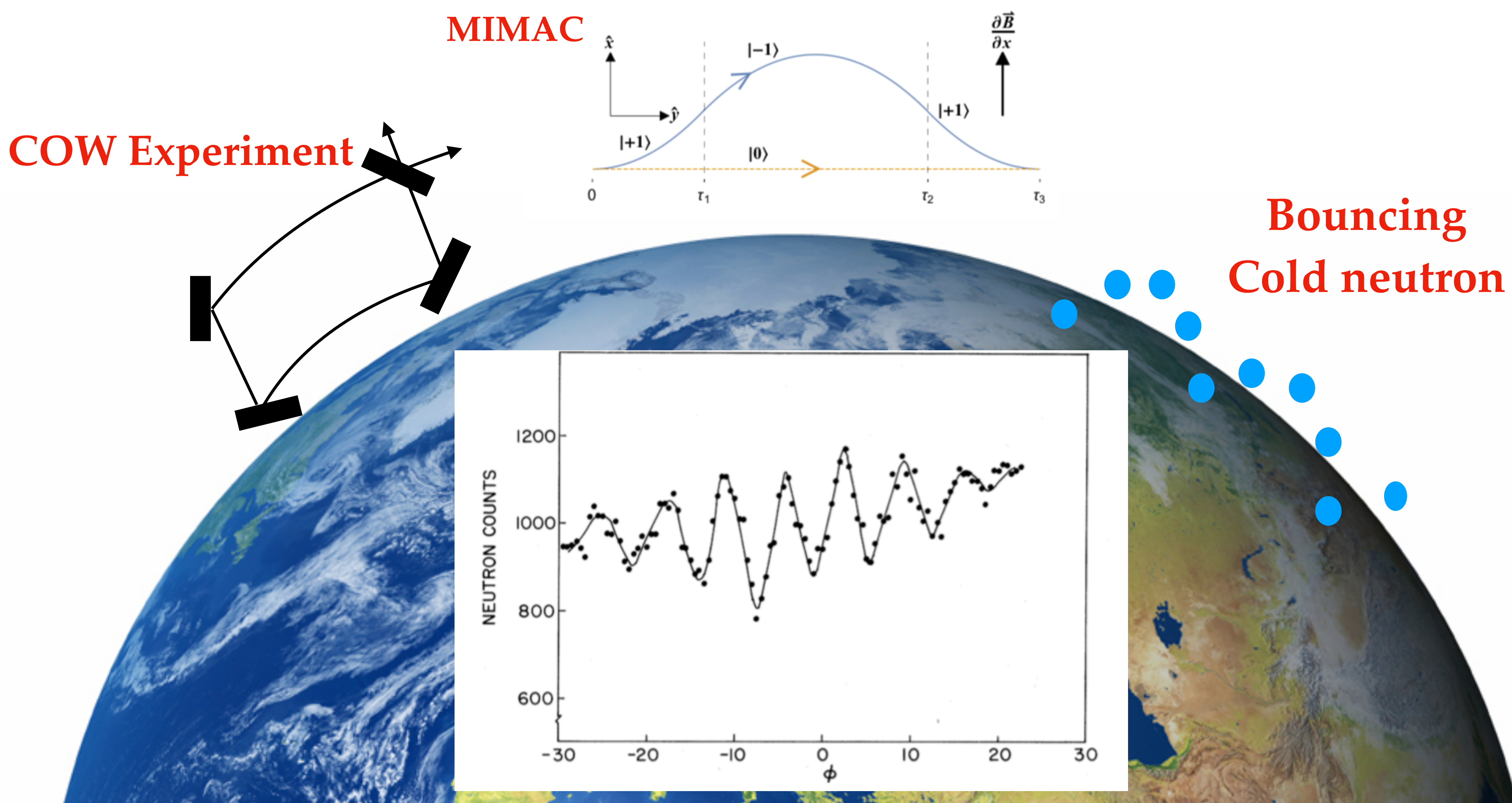
Can we witness quantum entanglement due to gravitons ?

Lively discussions with Markus Aspelmeyers & Anton Zeilinger on these issues in Vienna

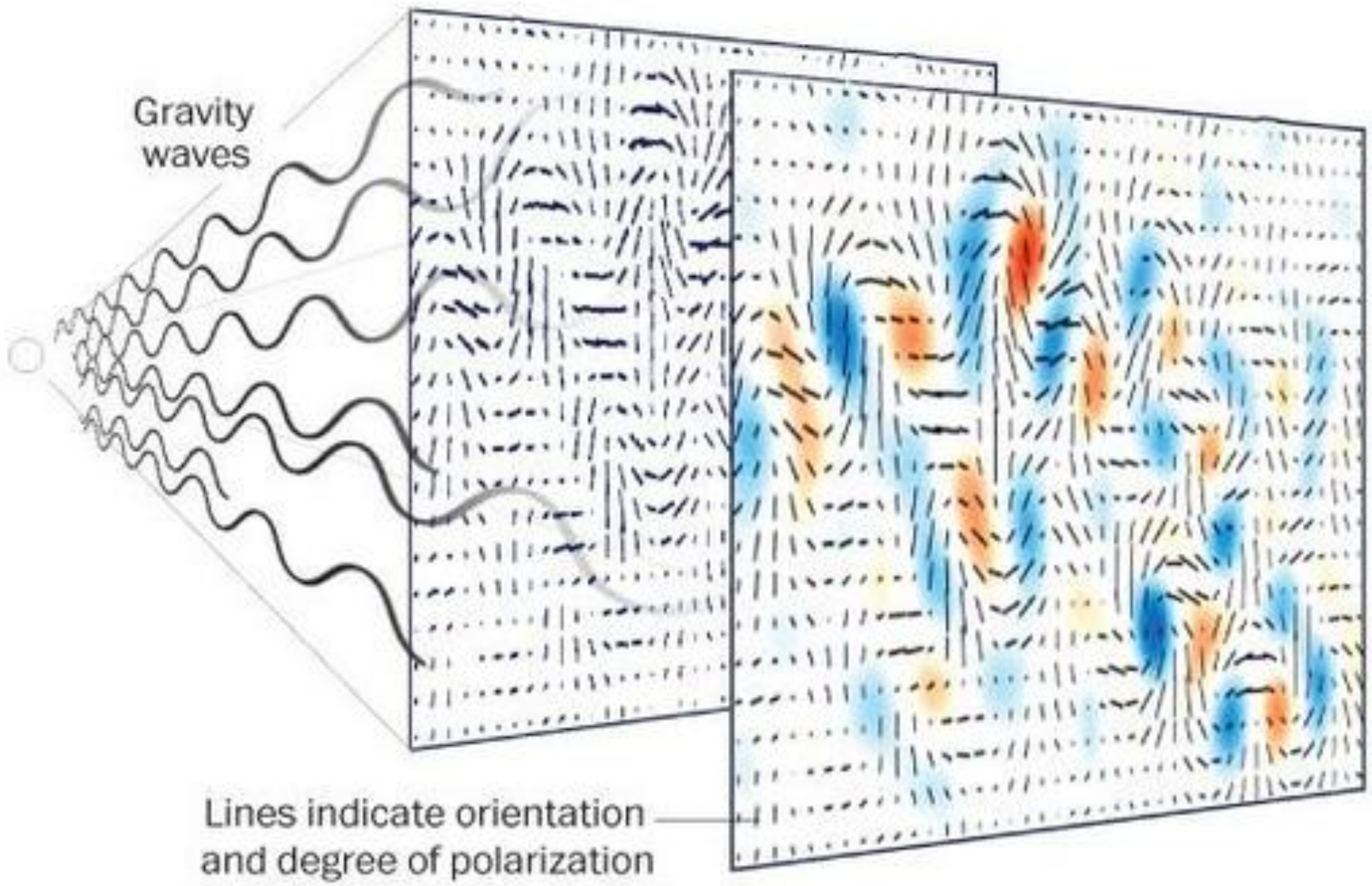
Gravitational Induced Phase is Detectable !

$$\Delta\phi \sim i\frac{S(G,\cdots)}{\hbar}$$

around Earth's gravitational potential



Shaking the Box: Gravitational Waves



Caution!

Positive Detection of a B-mode polarization by BICEP (2013)

But the origin was not found to be primordial in nature.

Initial Conditions; Classical or Quantum?

Mere presence of $\hbar$ is not sufficient to say that gravity is quantum !

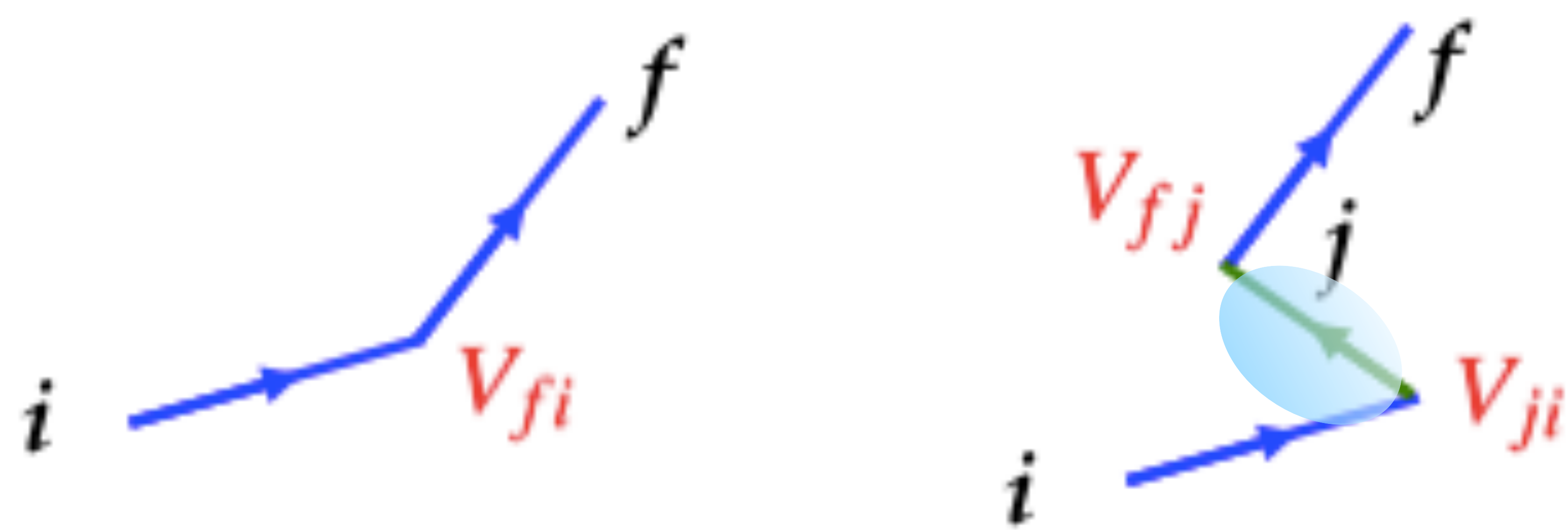
A. Ashoorioon, P. S. Bhupal Dev and A. Mazumdar,
``Implications of purely classical gravity for inflationary tensor modes,''
[arXiv:1211.4678 [hep-th]].

L. M. Krauss and F. Wilczek,
``Using Cosmology to Establish the Quantization of Gravity,''
[arXiv:1309.5343 [hep-th]].

Quantum Scattering

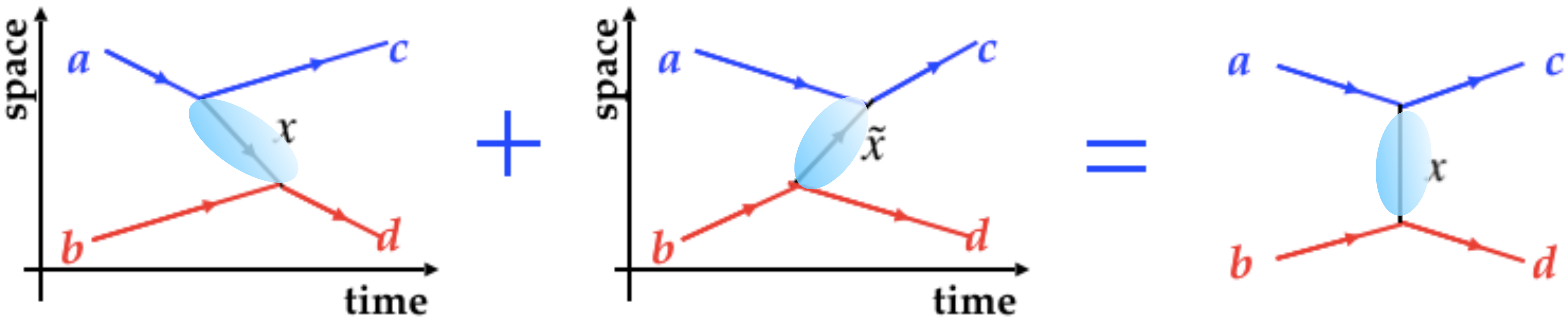
- **"Classical picture"** – particles act as sources for fields which give rise a potential in which other particles scatter – **"action at a distance"**
- **"Quantum Field Theory picture"** – forces arise due to the exchange of virtual particles. No action at a distance + forces between particles now due to particles

$$T_{fi} = \langle f|V|i\rangle + \sum_{j\neq i} \frac{\langle f|V|j\rangle\langle j|V|i\rangle}{E_i - E_j} + \ldots$$



Quantum Mechanics —> Quantum Field Theory

- The sum over all possible time-orderings is represented by a FEYNMAN diagram



- Momentum conserved at vertices
- Energy not conserved at vertices
- Exchanged particle "on mass shell"

$$E_x^2 - |\vec{p}_x|^2 = m_x^2$$

- Momentum AND energy conserved at interaction vertices
- Exchanged particle "off mass shell"

$$E_x^2 - |\vec{p}_x|^2 = q^2 \neq m_x^2$$

VIRTUAL PARTICLE

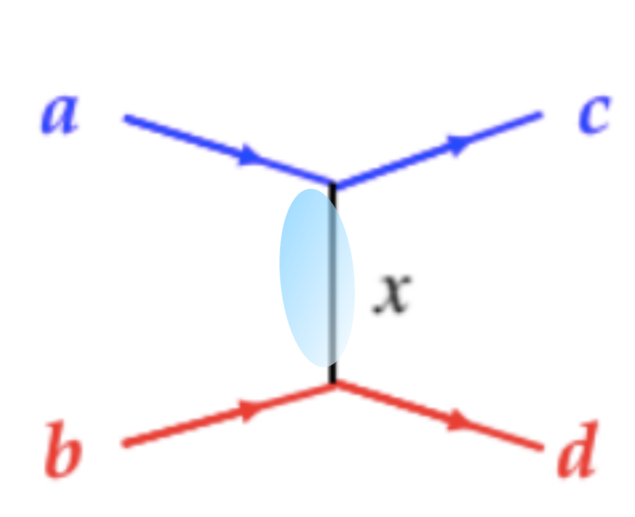
$$\Delta x \Delta p_x \geq \hbar$$

On-shell (Follows Classical Equations of Motion): $E^2 = (pc)^2 + (mc^2)^2$

Off-shell does not follow Classical Equations of Motion

Could we test quantum-ness of a mediator?

What this means?

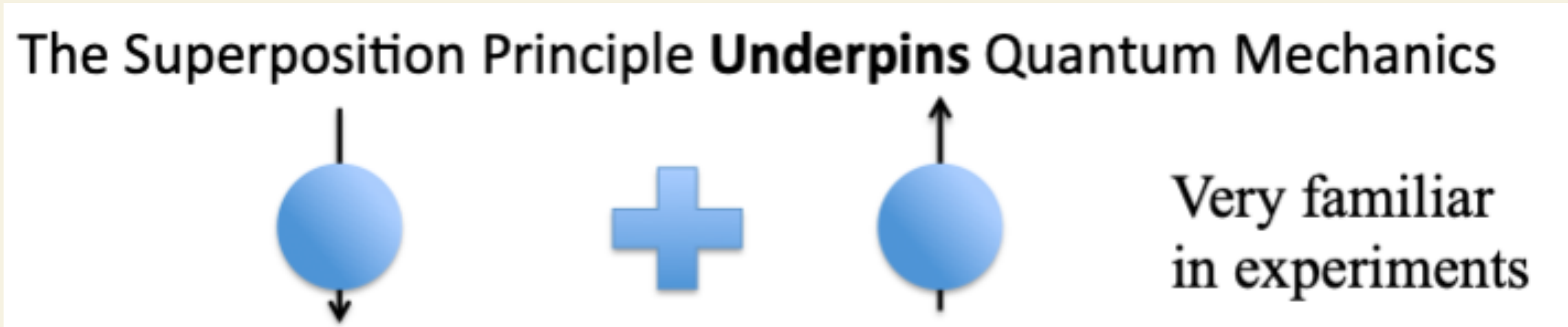


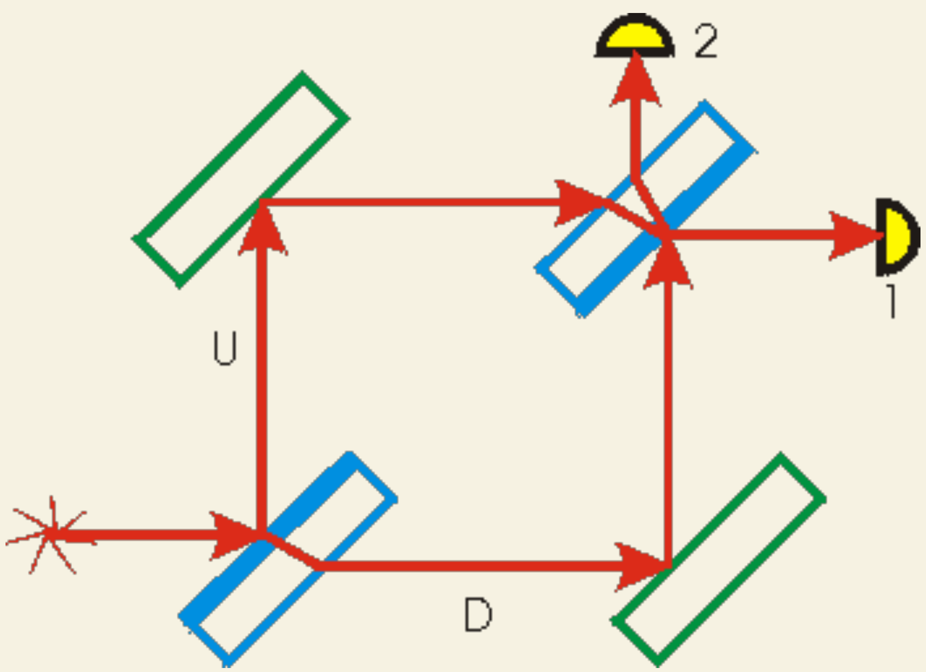
Mediator/interaction is Classical or Quantum?

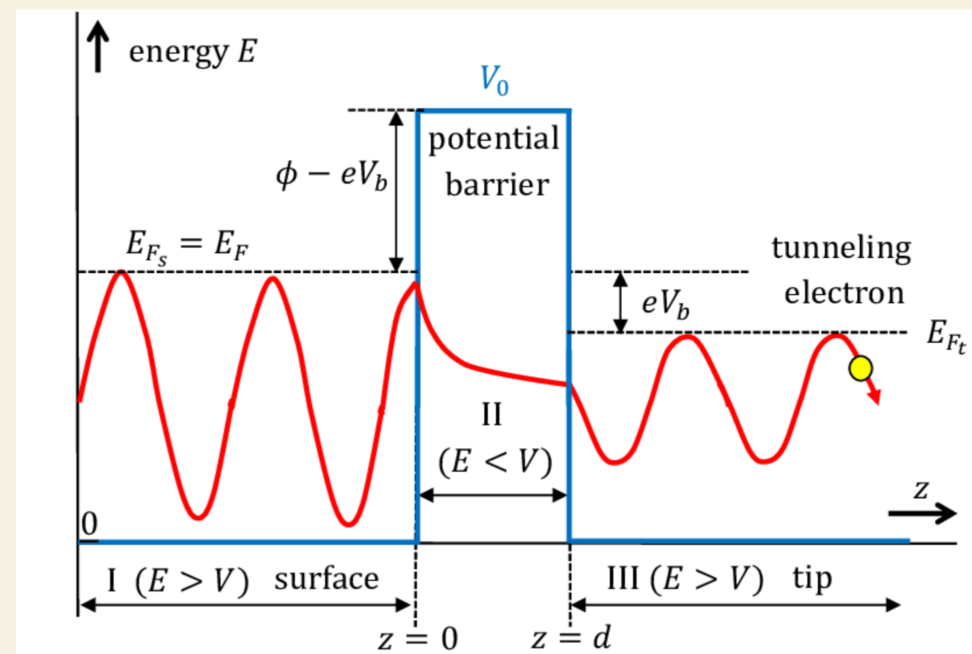
We will not be able to falsify Action at a Distance

Entanglement and Decoherence are two sides of the same coin

Quantum Superposition







Beam Splitter: Essential to Create Superposition of States

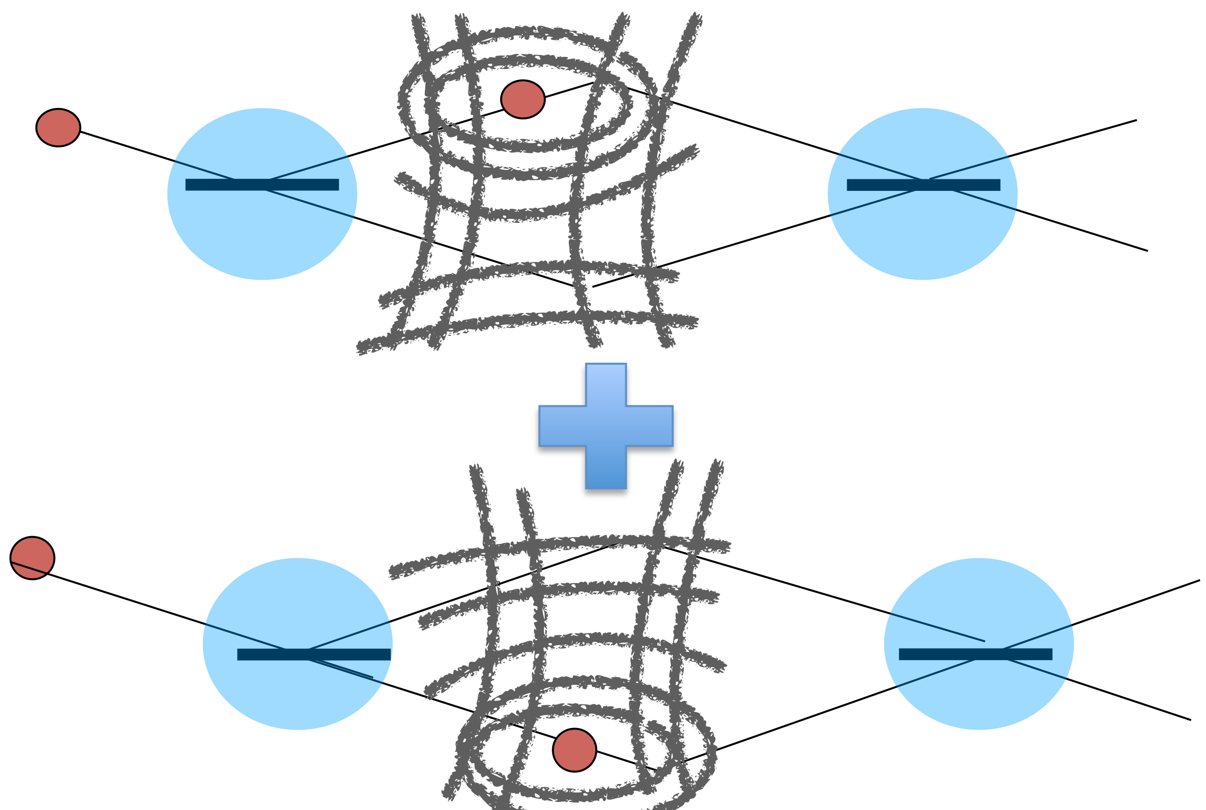
A manifestation of an off-shell process

Superposition of Metric

The Superposition Principle **Underpins** Quantum Mechanics

Very familiar
in experiments

Off-shellness is critical even before creating superposition of macroscopic objects in a lab!!



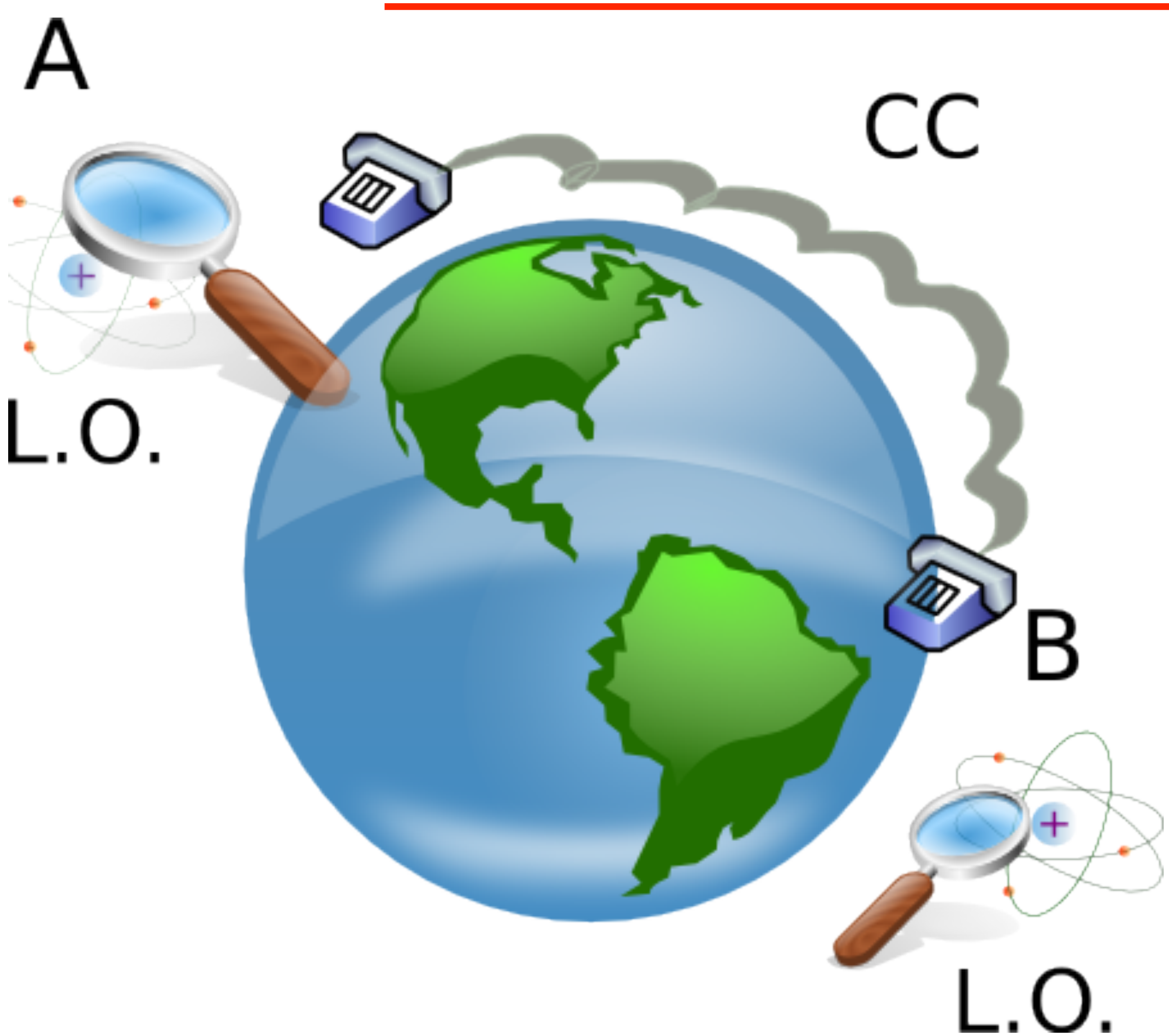
Localization of 2 distinct classical paths

If you *decohere* (kill superpositions) nonclassical features of quantum mechanics go away.
Even old quantum mechanics: the right difference between energy levels obtained only through a superposition of localized states.

How do we know that the Gravity is Quantum?

$$G_{\mu\nu} = \kappa^2 \langle T_{\mu\nu} \rangle$$

Local Operations & Classical Communication (LOCC)



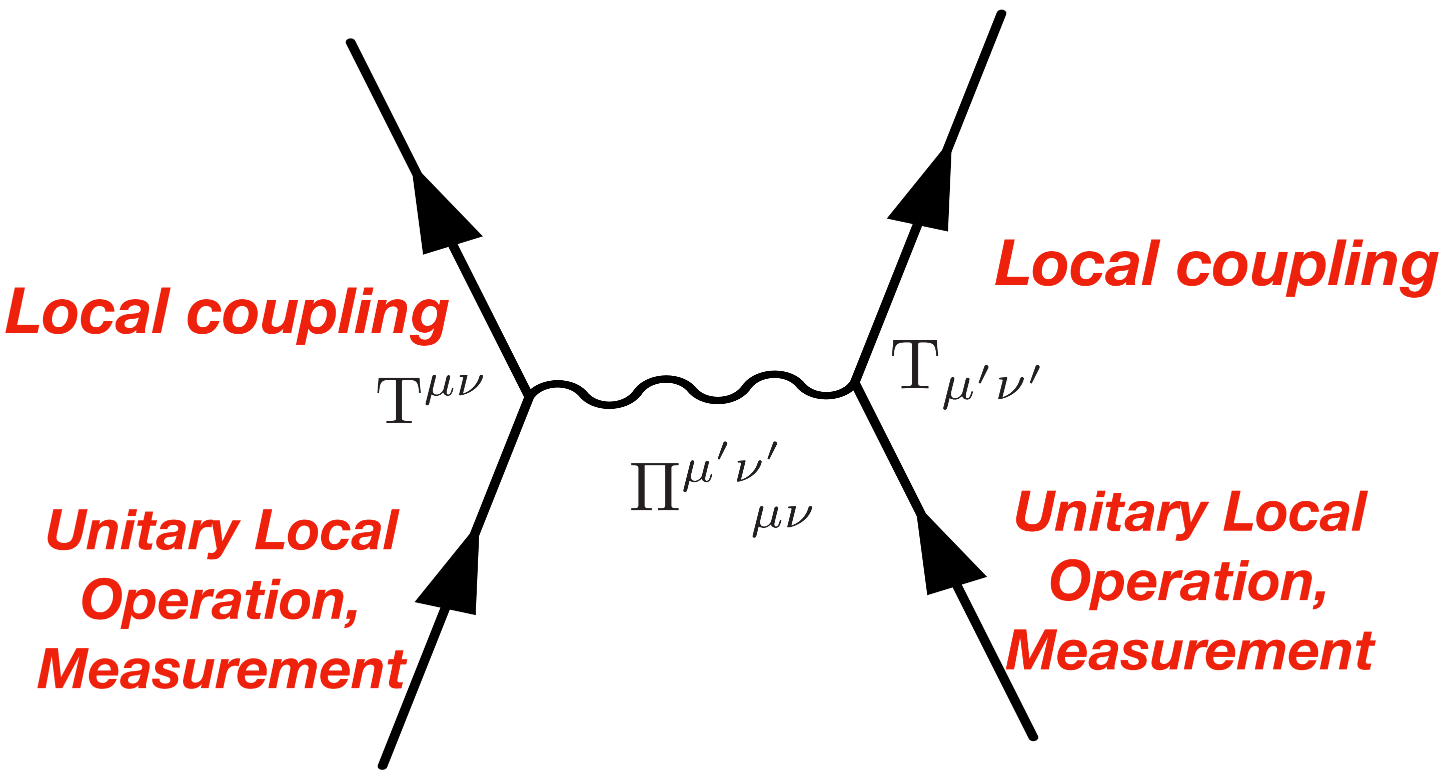
- It is impossible to generate/ increase entanglement between A and B by local operations and classical communications

LOCC keeps Separable state remains Separable (Cannot create entanglement)

Bennett, et.al, (1996)
Review by Plenio+Virmani (2006)

Quantum-ness of a mediator

Graviton as an Off-shell/Virtual mediator



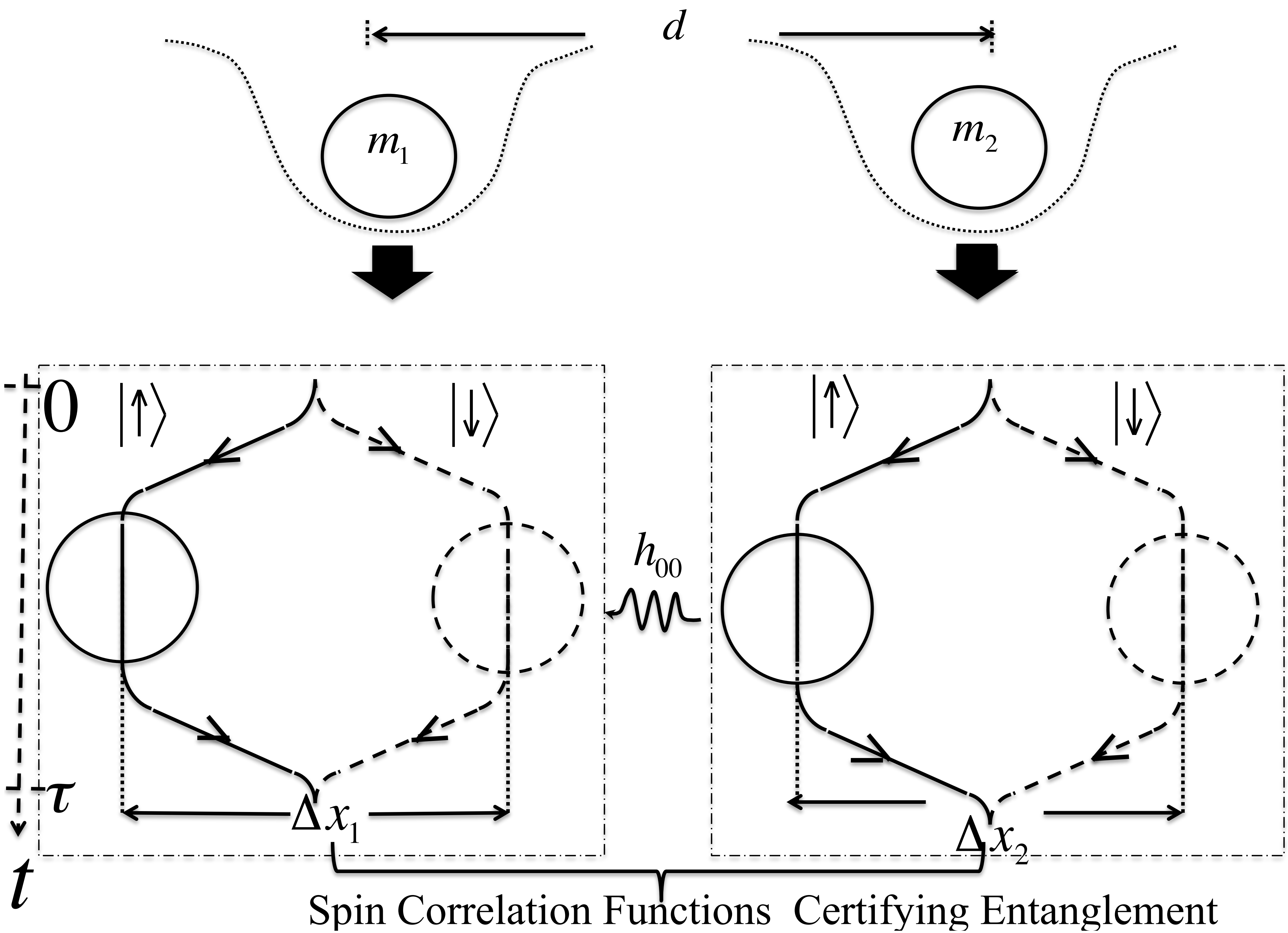
Virtual communication or Quantum communication via off shell mediator

$$\Pi(k^2) \sim \frac{P^{(2)}}{k^2} - \frac{P^{(0)}}{2k^2} \dashrightarrow V \sim \frac{1}{r}$$

Graviton propagator in terms of spin projection operators in 4d, Minkowski space time

P. Van Nieuwenhuizen, Nucl. Phys. B60, 478-492 (1973)
T. Biswas, T. Koivisto and A. Mazumdar,
``Nonlocal theories of gravity: the flat space propagator,''
arXiv:1302.0532 [gr-qc]

2 Free Falling Superposed masses



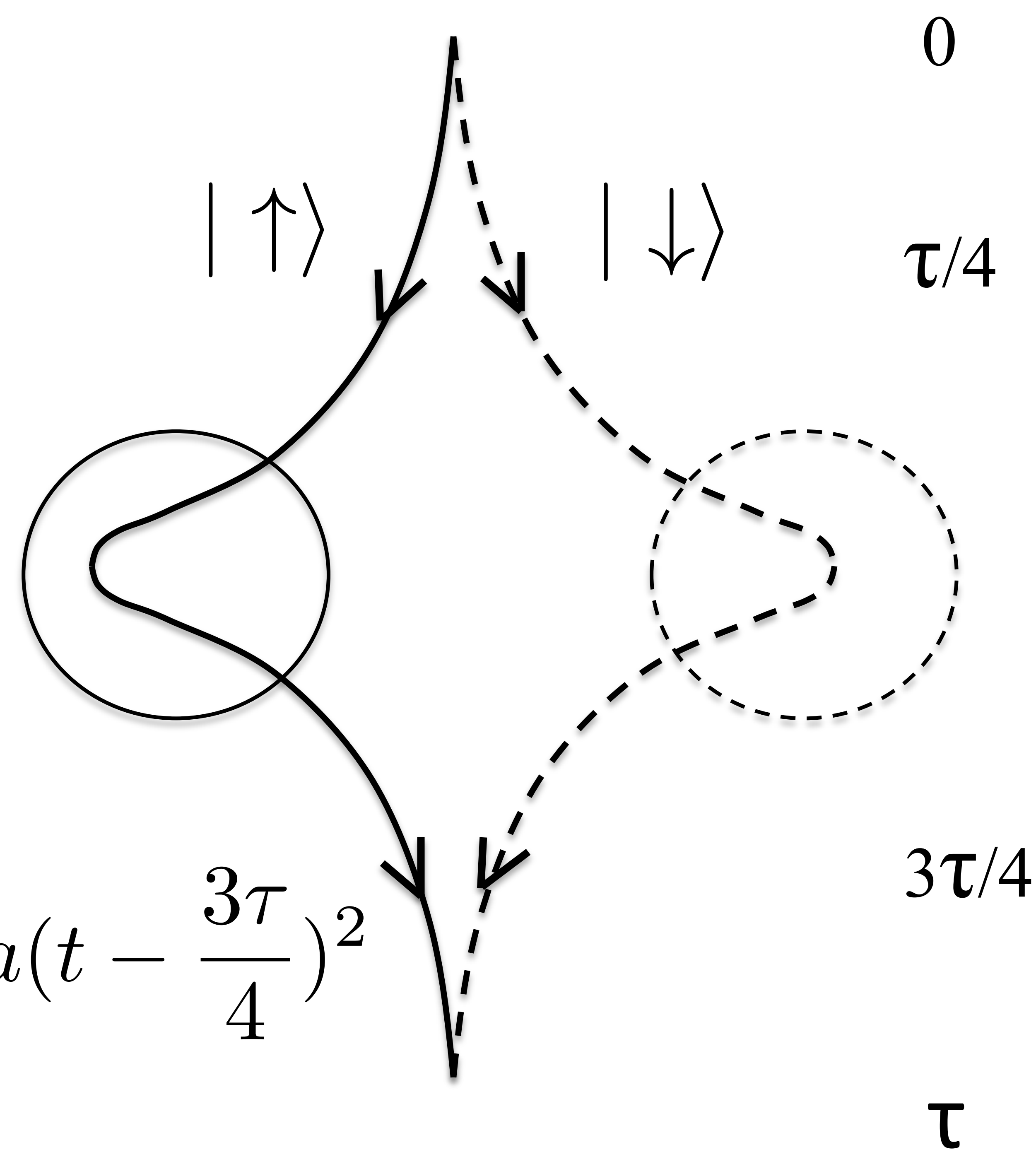
How can we increase the scale of the superposition?

Free particle in an inhomogeneous magnetic field (acceleration $+a$ or $-a$)

$$x_\sigma(t,j) = x_j(0) \pm \frac{1}{2}at^2$$

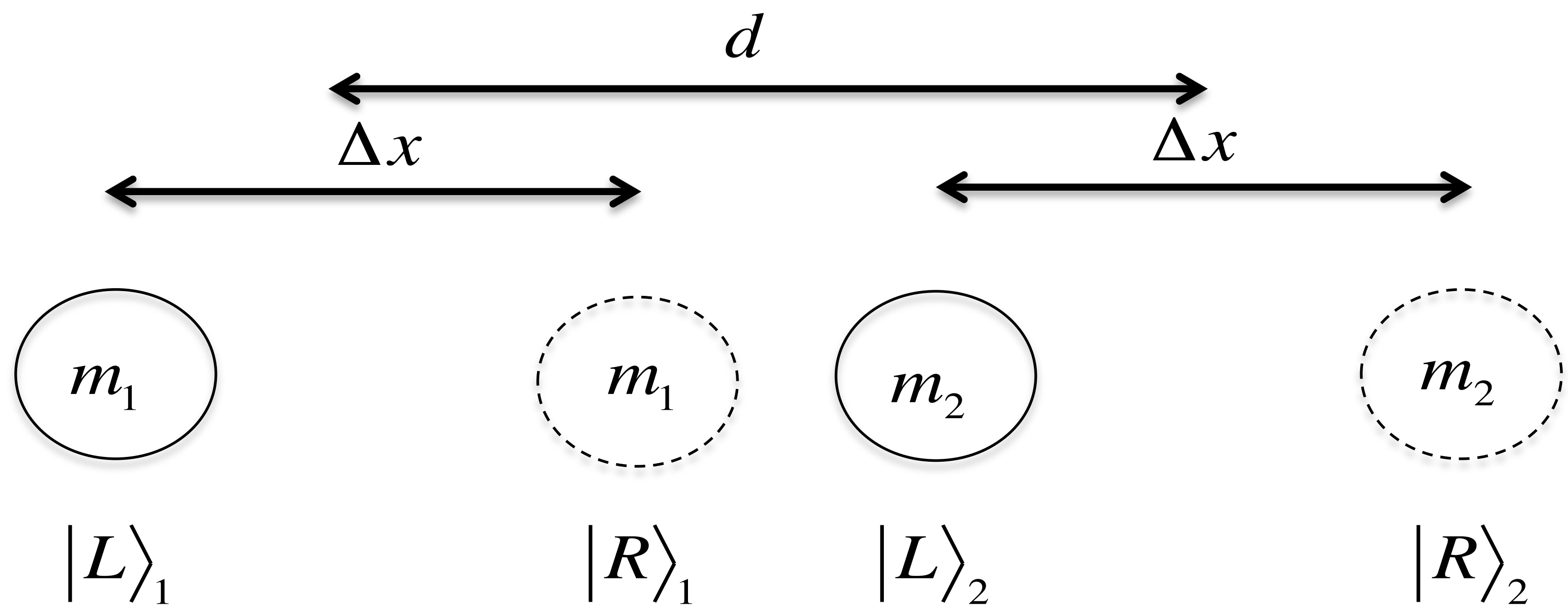
$$= \frac{a\tau}{4}(t - \frac{\tau}{4}) \mp \frac{1}{2}a(t - \frac{\tau}{4})^2$$

$$= \frac{1}{2}a(\frac{\tau}{4})^2 \mp \frac{a\tau}{4}(t - \frac{3\tau}{4}) \pm \frac{1}{2}a(t - \frac{3\tau}{4})^2$$



100 micron separation for 1 sec

- M. Scala, M. S. Kim, G. W. Morley, P. F. Barker, S. Bose, PRL. **111**, 180403 (2013)



$$
\begin{aligned}
|\Psi(t=0)\rangle_{12} &= \frac{1}{\sqrt{2}}(|L\rangle_1 + |R\rangle_1)\frac{1}{\sqrt{2}}(|L\rangle_2 + |R\rangle_2) \\
&= \frac{1}{2}(|L\rangle_1|L\rangle_2 + |L\rangle_1|R\rangle_2 + |R\rangle_1|L\rangle_2 + |R\rangle_1|R\rangle_2) \\
\rightarrow |\Psi(t=\tau)\rangle_{12} &= \frac{1}{2}(e^{i\phi_{LL}}|L\rangle_1|L\rangle_2 + e^{i\phi_{LR}}|L\rangle_1|R\rangle_2 \\
&\quad + e^{i\phi_{RL}}|R\rangle_1|L\rangle_2 + e^{i\phi_{RR}}|R\rangle_1|R\rangle_2),
\end{aligned}
$$

where

$$
\phi_{RL} \sim \frac{Gm_1m_2\tau}{\hbar(d-\Delta x)}, \phi_{LR} \sim \frac{Gm_1m_2\tau}{\hbar(d+\Delta x)},
$$

$$
\phi_{LL} = \phi_{RR} \sim \frac{Gm_1m_2\tau}{\hbar d}
$$

Maximum Entanglement

Step 4: Witness spin entangled state:

$$|\Psi(t = t_{\text{End}})\rangle_{12} = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_1 \frac{1}{\sqrt{2}}(|\uparrow\rangle_2 + e^{i\Delta\phi_{LR}}|\downarrow\rangle_2) + |\downarrow\rangle_1 \frac{1}{\sqrt{2}}(e^{i\Delta\phi_{RL}}|\uparrow\rangle_2 + |\downarrow\rangle_2)\}|C\rangle_1|C\rangle_2$$

through the correlations:

$$\mathcal{W} = |\langle\sigma_x^{(1)} \otimes \sigma_z^{(2)}\rangle - \langle\sigma_y^{(1)} \otimes \sigma_z^{(2)}\rangle|$$

we have

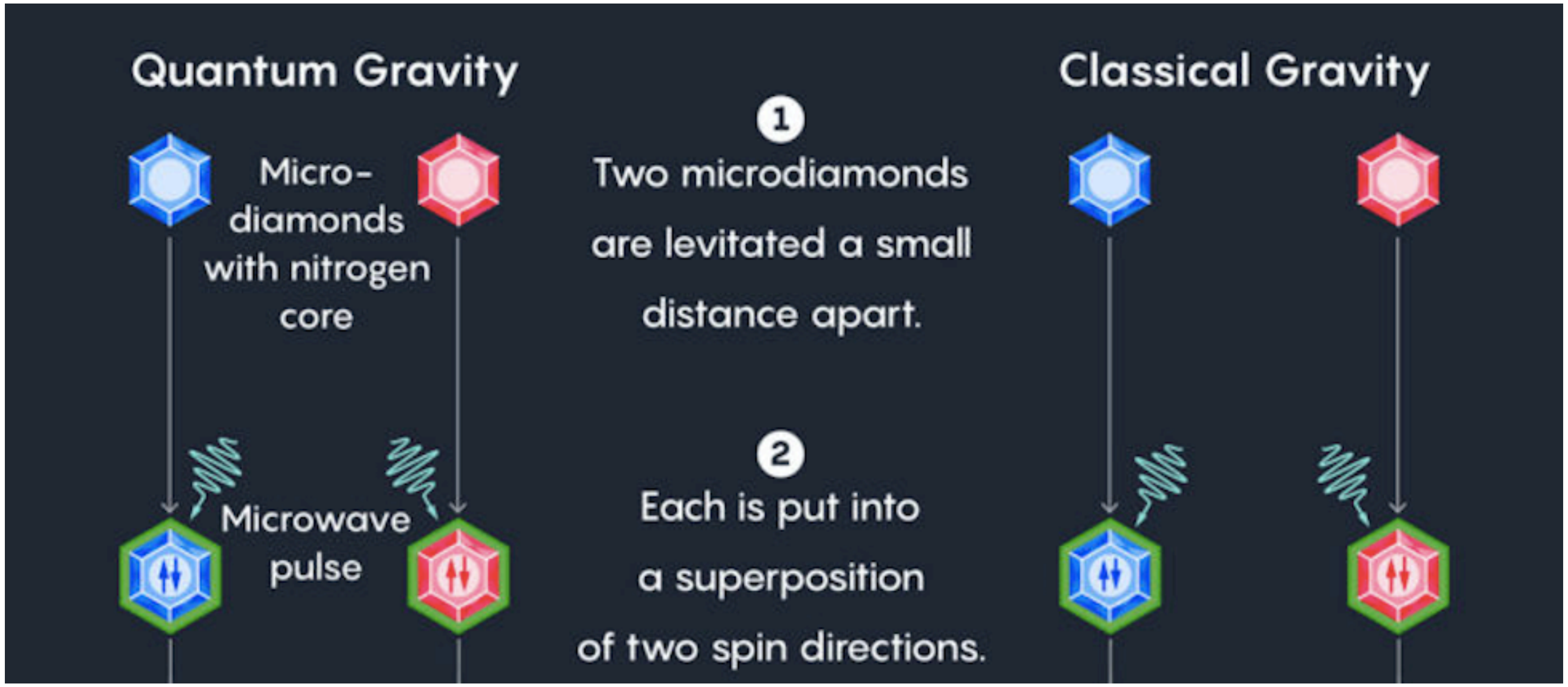
$$\Delta\phi_{RL} \sim \frac{Gm_1m_2\tau}{\hbar(d - \Delta x)} >> \Delta\phi_{LR}, \Delta\phi_{LL}, \Delta\phi_{RR}$$

$$\Delta\phi_{LR} + \Delta\phi_{RL} \sim \mathcal{O}(1)$$

For mass ~ 10^(-14) kg (microspheres), separation at closest approach of the masses ~ 200 microns (to prevent Casimir interaction), **time ~ 1 seconds**, gives:
Scale of superposition ~ 100 microns, **Delta phi_{RL} ~ 1**

Planck's Constant fights Newton's Constant!

Experimental Protocol



$10^{-14} Kg$ Radius : 100nm Frequency of harmonic potential : 0.1MHz

Temperature : mK

Neutralising e.m. charges

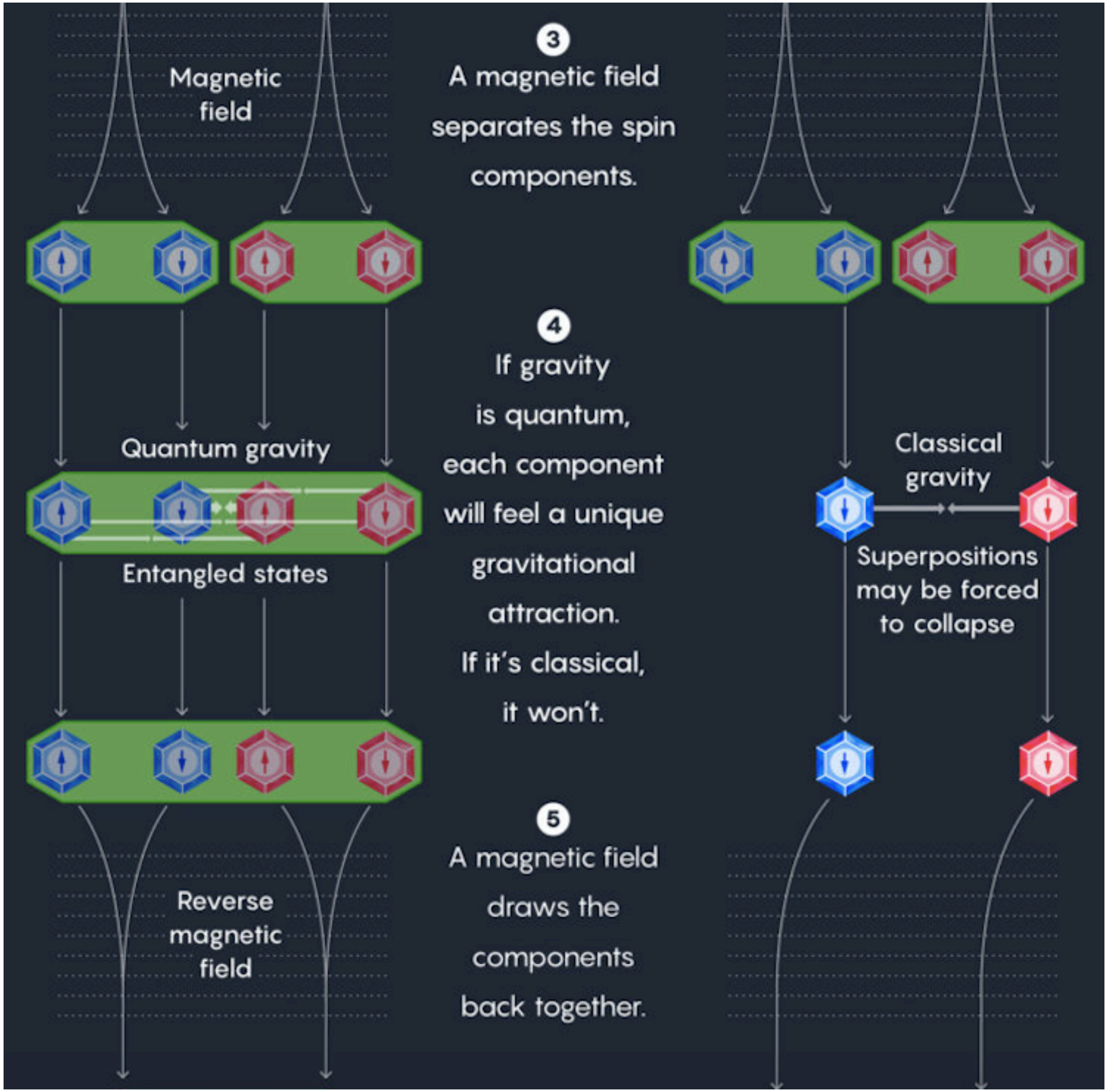
A magnetic field gradient of ~ 10^6 T/m and a time τacc ~ 500 m/s^2,
Δx ~ 250μm, d-Δx ~200μm

T. Krisnanda, M. Zuppardo, M. Paternostro, T. Paterek, arXiv:1607.01140. Superconducting sphere with half a micrometer separation (magnetically levitating)

C. Wan, M. Scala, G. W. Morley, ATM. A. Rahman, H. Ulbricht, J. Bateman, P. F. Barker, S. Bose, and M. S. Kim, Phys. Rev. Lett. 117, 143003 (2016); M. Frimmer, K.Luszcz, S. Ferreiro, V. Jain, E.Hebestreit, and L.Novotny, Phys. Rev. A95, 061801 (2017).

H. Pino, J. Prat-Camps, K. Sinha, B. P. Venkatesh, and O. Romero-Isart, arXiv:1603.01553v2

Challenges & Sources of Decoherence



Electronic spins coherent for 1s (in steps 1 and 3), which should be possible for macro-diamond below 77 K

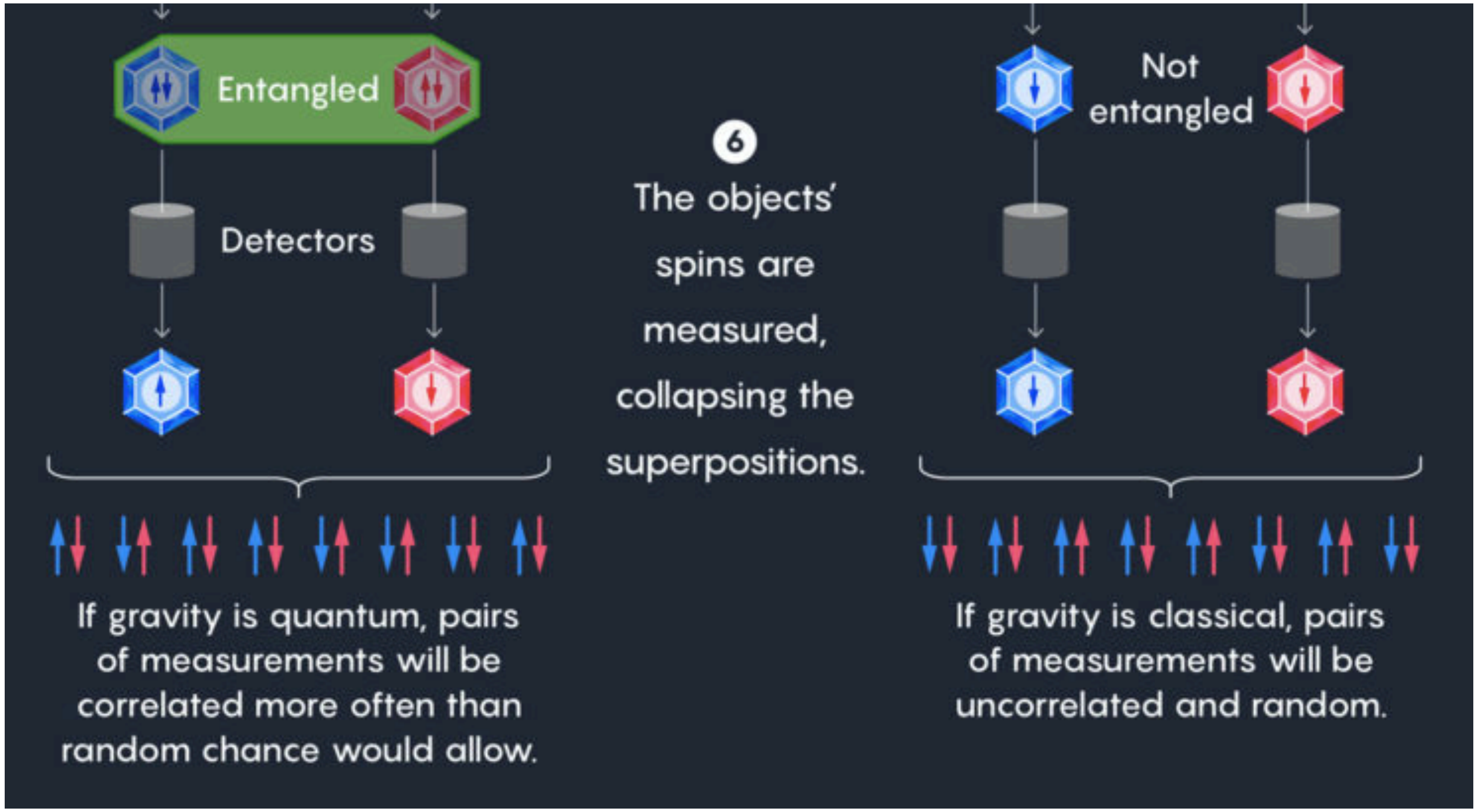
N. Bar-Gill, L.M. Pham, A. Jarmola, D. Budker, R. L. Walsworth, Nature Comm, 4, 1743 (2013),

S. Knowles, D. M. Kara and M. Atatu¨re, Nature Materials 13, 21 (2014),

Kaltenbaek, Aspelmeyer, (2015)

To estimate collisional and thermal decoherence times of the orbital degree of freedom we consider the pressure P = $10^{-15} Pa$ and the temperature 0.15 K. the collisional decoherence time for a superposition size of Δx ~ 250μm is the same order of magnitude as the total microsphere's fall time τ + 2τacc ~ 3.5 s

Measuring Spin Correlation & Establishing the Entanglement



$$\mathcal{W} = |\langle \sigma_x^{(1)} \otimes \sigma_z^{(2)} \rangle - \langle \sigma_y^{(1)} \otimes \sigma_z^{(2)} \rangle|$$

$$\text{If } \mathcal{W} > 1 \implies \mathit{Graviton\ is\ quantum}$$

Basis Dependent Witness, similar to Bell's

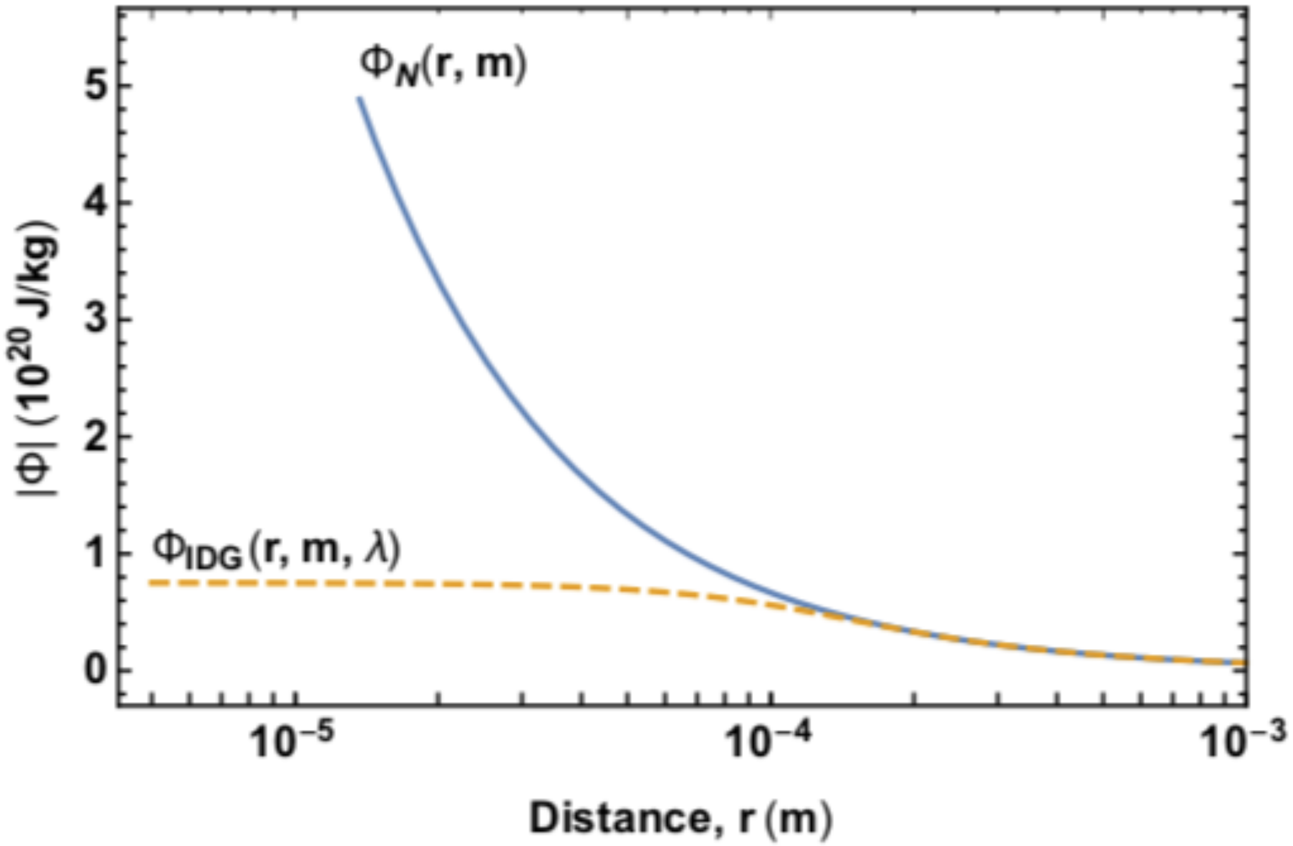
Basis Independent Witness: $S_{\rm A} = -{\rm Tr}_{\rm A}\rho_{\rm A} \log \rho_{\rm A} = S_{\rm B}$

Different theories of gravity will provide different witnesses!

Conformal Gravity?

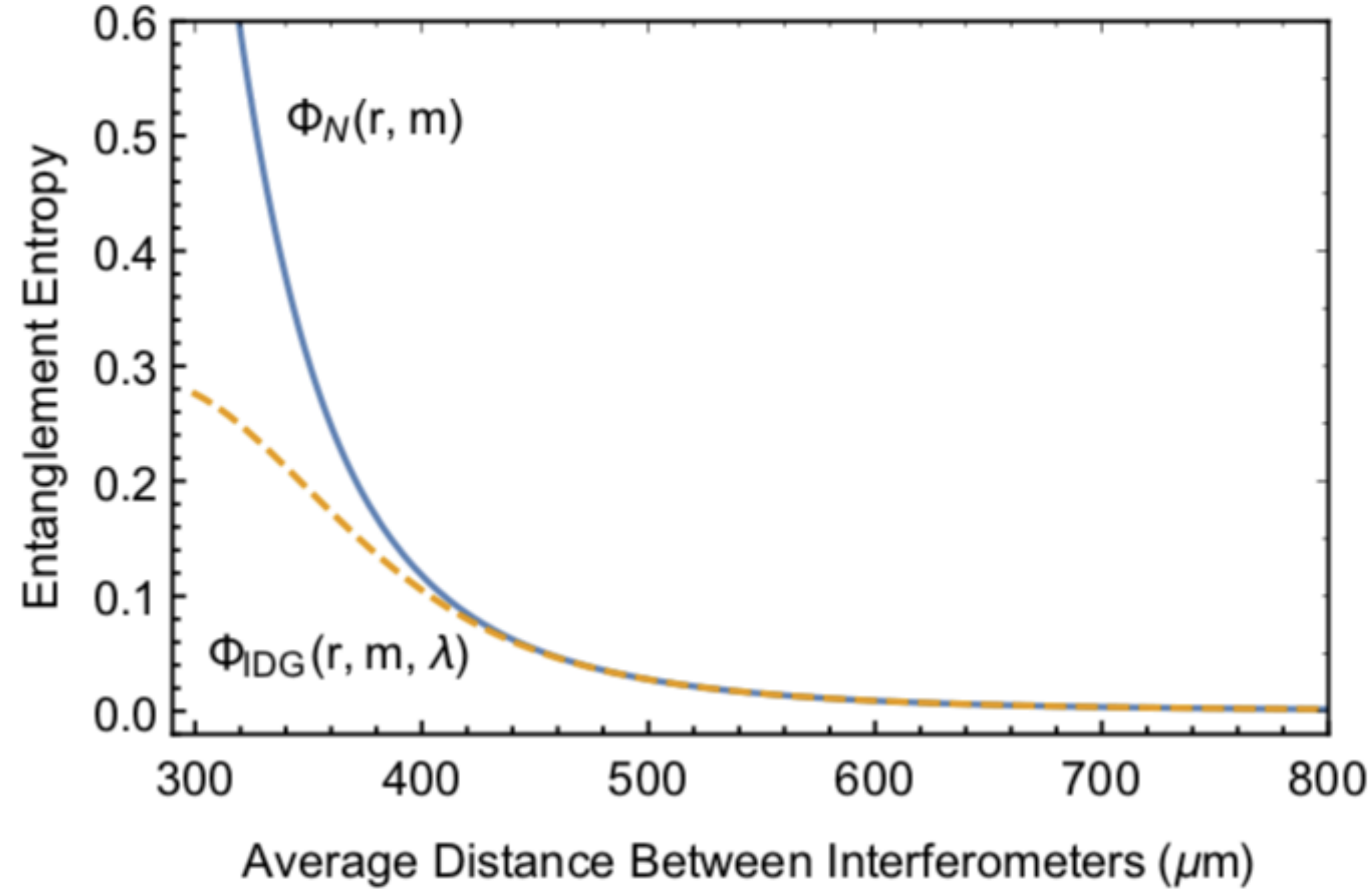
Ghost free, infinite derivative gravity which is free from cosmological and 1/r singularities

Entanglement Phase Evolution & Entanglement Entropy



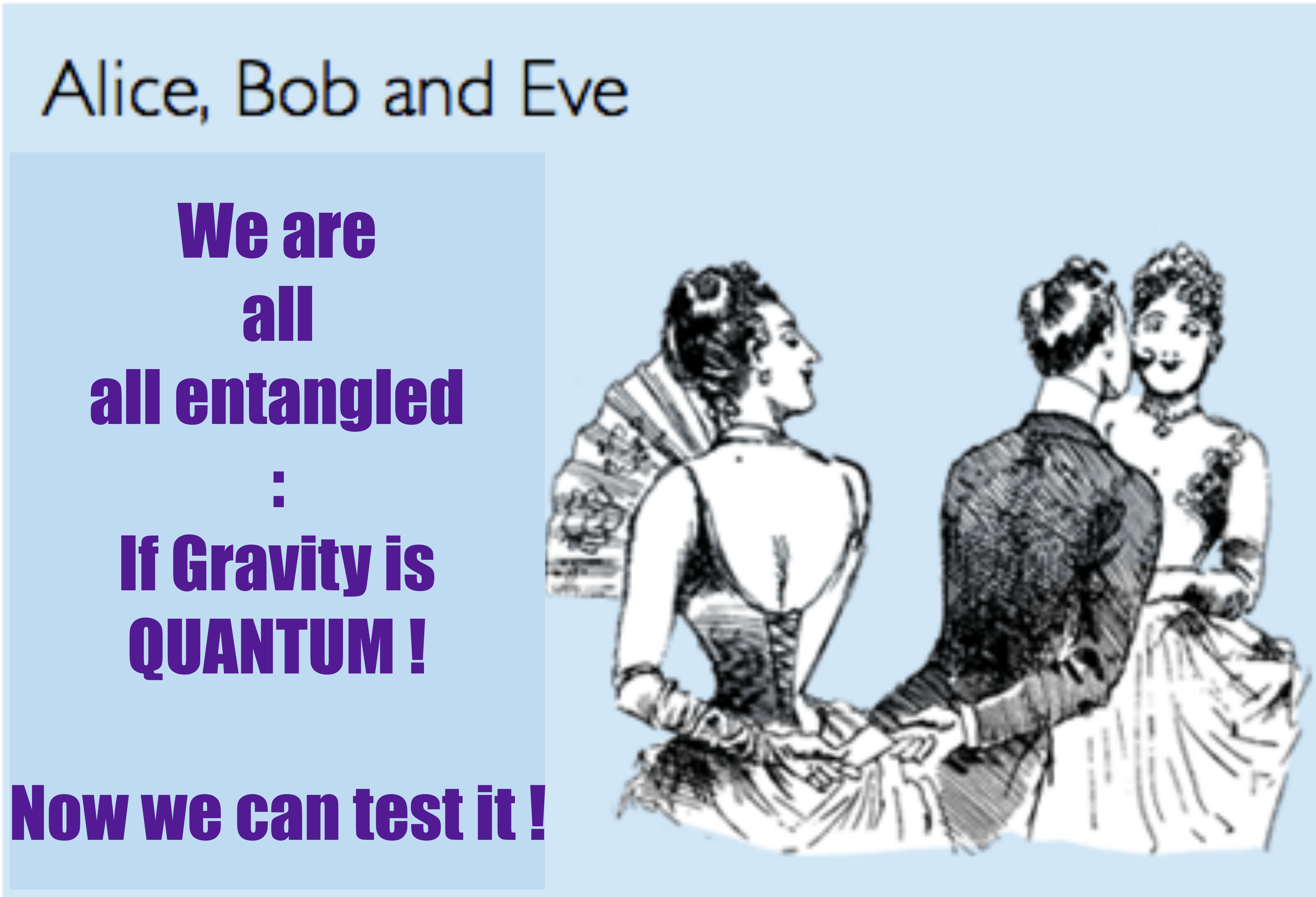
This sheds light on very nature of gravity and entanglement entropy in the bulk

$$\mathcal{S}\left(\hat{\rho}_A\right) = -Tr\left[\hat{\rho}_A \log\left(\hat{\rho}_A\right)\right]$$



1907.01568 [quant-ph]

Conclusion: We can potentially test linearized Quantum Gravity in a Lab !



Bose+AM+Morley+Ulbricht+Toros+Paternostro+Geraci+Barker+Kim+Milburn, PRL (2017) [1707.06050]
Marshman+AM+Bose, [1907.01568]

Extra slides

Locality & Entanglement in Table-Top Testing of the Quantum Nature of Linearized Gravity

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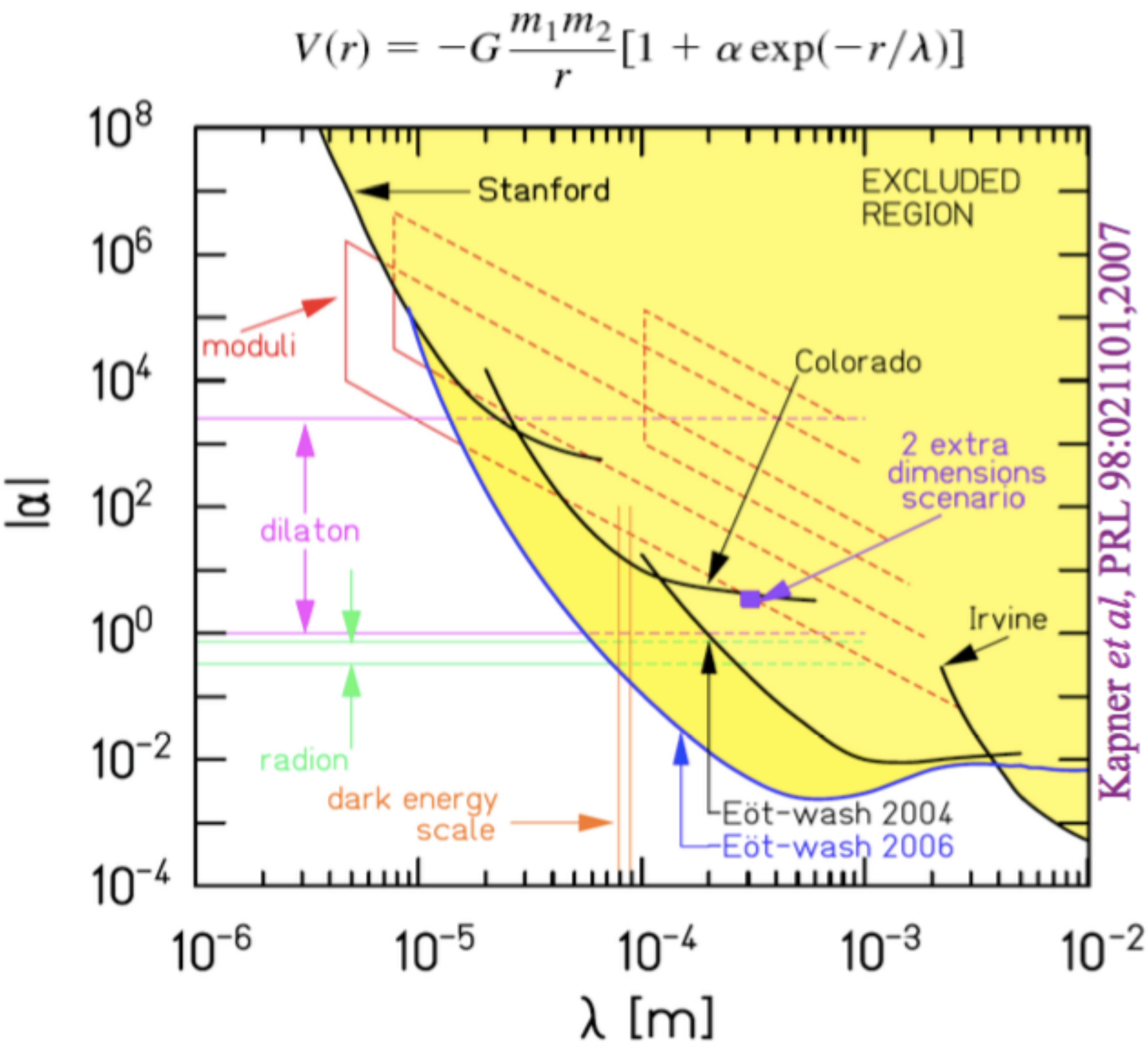
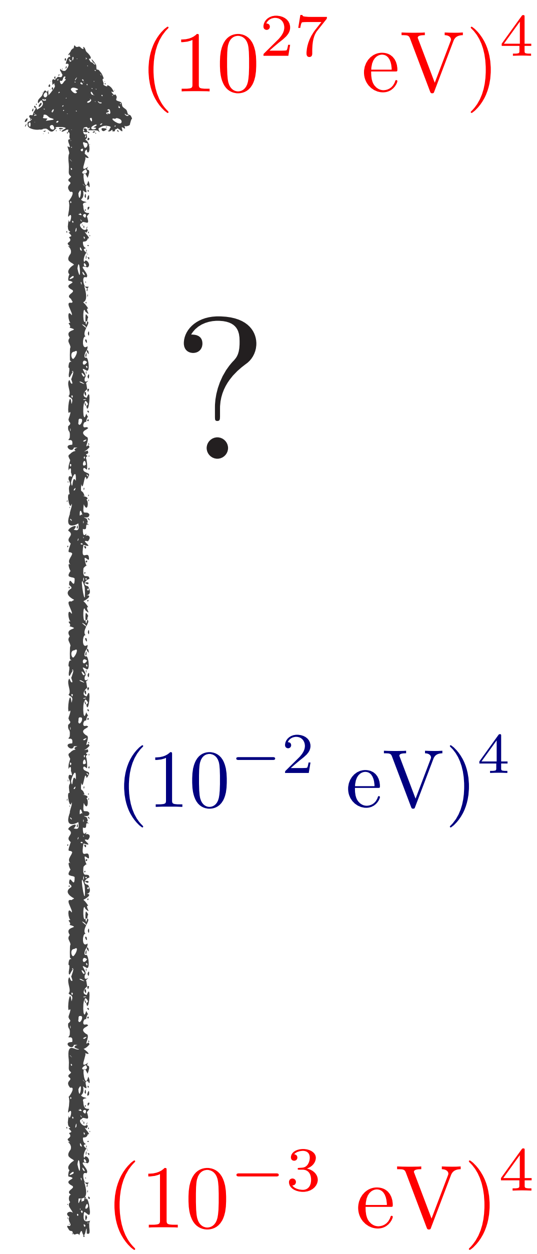
(Dated: July 4, 2019)

This paper highlights the importance of the assumption of locality of physical interactions, and the concomitant necessity of the off-shell propagation of quanta between two non-relativistic test masses in probing the quantum nature of linearized gravity in the laboratory. At the outset, we will argue that observing the quantum nature of a system is not limited to evidencing $O(\hbar)$ corrections to a classical theory: it instead hinges upon verifying tasks that a classical system cannot accomplish, which is the method adopted in the aforementioned tabletop experiments. We explain the background concepts needed from quantum field theory, namely forces arising through the exchange of virtual (off-shell) quanta, as well as the background exploited from quantum information theory, such as Local Operations and Classical Communication (LOCC) and entanglement witnesses. We clarify the key assumption inherent in our evidencing experiment, namely the locality of physical interactions, which is a generic feature of interacting systems of quantum fields around us, and naturally incorporates micro-causality in the description of our experiment. We also present the types of states the matter field must inhabit, putting the experiment on firm relativistic quantum field theoretic grounds. At the end we use a non-local (but not complete action at a distance) theory of gravity to illustrate how our mechanism may still be used to detect the qualitatively quantum nature of a force when the scale of non-locality is finite. We find that the scale of non-locality, including the entanglement entropy production in local/ non-local gravity, may be revealed from the results of our experiment.

1907.01568 [quant-ph]

Gravity is Least Constrained





Dark Energy