

NEUTRINO MASSES AND BIMETRIC GRAVITY

THE SPECTRAL ACTION IN ACTION

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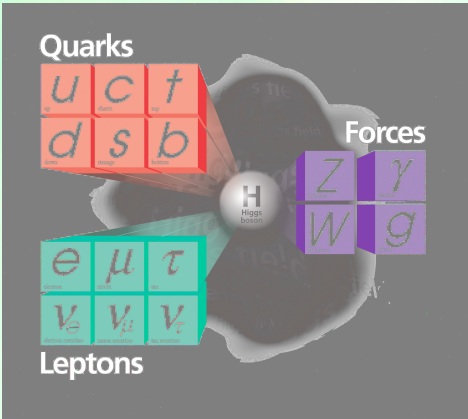
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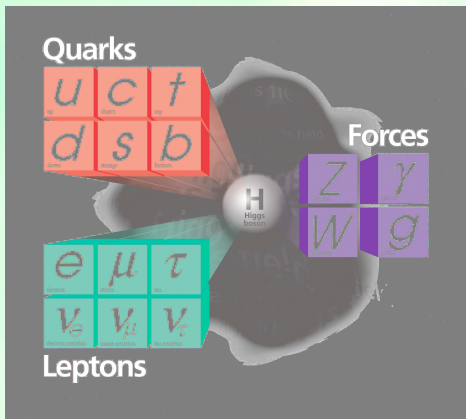
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Humboldt Kolleg
Corfu



PARTICLE INTERACTIONS ARE ENCODED IN GEOMETRY



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„Our imagination is struck only by what is great; but the lover of natural philosophy should reflect equally on little things.”

ALL FUNDAMENTAL INTERACTIONS COME FROM GEOMETRY

GEOMETRY OF THE PARTICLES & SPACETIME

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GEOMETRY OF THE SPACETIME

- 1 the metric is encoded in the Dirac operator (or Laplace operator)
- 2 gauge connections, Higgs, particle masses are *in the Dirac* operator

Spectral triples provide a generalized setup for geometry

DEFINE THE GEOMETRY IN AN ALGEBRAIC WAY [À LA CONNES]

Algebra \mathcal{A} , its (faithful) representation π on a Hilbert space \mathcal{H} , a selfadjoint operator D , satisfying several (reasonable) conditions.

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- 3 guarantee that D is a differential operator
- 4 & ...+ make sure that you can compute something

Spectral action is an effective tool to get the action functional

PROPOSE A FUNCTIONAL ON THE SPACE OF ADMISSIBLE DIRAC OPERATORS

$$\mathcal{S}_{\Lambda, f}(D) = \text{Tr} f\left(\frac{D^2}{\Lambda^2}\right)$$

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By parametrizing the degrees of freedom of the Dirac operator (in the classical case) through metric, torsion and connections (to allow for twisted Dirac operators) we obtain an esymptotic expansion of the action functional as:

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$$\mathcal{S}_{\Lambda}(D) = \Lambda^4 \alpha_4 \int_M \sqrt{g} + \Lambda^2 \alpha_2 \int_M \sqrt{g} (R(g) + \beta |T|^2) + \alpha_0 \int_M \sqrt{g} |F|^2 + \dots$$

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Physics *requires* that the full action is the action for bosonic fields (fields that are part of the Dirac operator) as well as the fermionic fields. Yet the action for fermions is of different type:

$$S_\Phi = \langle \Psi | D(g, A) | \Psi \rangle ,$$

SPECTRAL ACTION CAN BE APPLIED TO FERMIONS

HOW TO MAKE FERMIONIC ACTION SPECTRAL ?

We propose (for the simplest Euclidean model) the cutoff fermionic action

$$S_{g,\Lambda} = \langle \Psi | |g_\Lambda(D_A)| \Psi \rangle ,$$

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and nonminimal coupling of gravity (through scalar curvature) to fermions.

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THE YANG-MILLS SYSTEM

$$\begin{aligned} \Lambda^4 \int_M \sqrt{g} (\text{tr} P_\Psi) &= \Lambda^4 N \int_M \sqrt{g} \langle \Psi(x) | | \Psi(x) \rangle, \\ \Lambda^3 \int_M \sqrt{g} \langle \Psi(x) | D_A | \Psi(x) \rangle, \\ \Lambda^2 \int_M \sqrt{g} \langle \Psi(x) | \left(\frac{R}{12} + F \right) | \Psi(x) \rangle, \end{aligned}$$

FERMIONIC SPECTRAL ACTION FOR NONCOMMUTATIVE SM

THE STANDARD MODEL AS A SPECTRAL GEOMETRY

The discrete Dirac operator written in the basis of fermions, taken in the order $f_R^u, f_R^d, f_L^u, f_L^d$ (each taken multiple times for N generations) is

$$D_F = \begin{pmatrix} & & \gamma_u^* & \\ & & & \gamma_d^* \\ \gamma_u & & & \\ & \gamma_d & & \end{pmatrix},$$

and the fluctuated discrete Dirac operator $D_F(H)$ is:

$$D_F = \begin{pmatrix} & & \gamma_u^* H^0 & \gamma_u^* H^- \\ & & -\gamma_d^* H^- & \gamma_d^* H^0 \\ \gamma_u \overline{H^0} & -\gamma_d H^- & & \\ \gamma_u \overline{H^-} & \gamma_d H^0 & & \end{pmatrix},$$

where $H = H^0 + H^- j$ denotes a quaternionic field (Higgs doublet).

FERMIONIC SPECTRAL ACTION FOR NONCOMMUTATIVE SM

THE PAULI-TYPE TERMS FOR THE SM GEOMETRY

Taking the square of the $D_F(H)$ at the Higgs vacuum expectation value we obtain:

$$(D_F(H_V))^2 = \begin{pmatrix} \Upsilon_e^* \Upsilon_e |H_V|^2 & \\ & \Upsilon_e^* \Upsilon_e |H_V|^2 \end{pmatrix},$$

which similarly as in the Dirac masses case can only add small corrections to the already nonvanishing mass of charged leptons.

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BUT FOR SOME NONSCALAR FUNCTIONS...

For any nonscalar function

$$f_\Lambda^\tau(x) = \tau f_\Lambda(x) \tau,$$

where $\tau = \sigma^2$ (Pauli matrix) the term $f_\Lambda^\tau(D^2)$ is certainly gauge covariant and, consequently, the terms in the fermionic spectral action remain gauge invariant.

FERMIONIC SPECTRAL ACTION FOR NONCOMMUTATIVE SM

EXPLICIT COMPUTATIONS LEADING TO WEINBERG TERM

Then the terms in the fermionic spectral action, that arise from $\text{Tr}(P_\Psi f_\Lambda^\tau(\mathcal{D}^2))$ in the next-to-leading order, could be explicitly rewritten as:

$$\Lambda^2(\Upsilon_e \Upsilon_e^*) \left[(\overline{\nu}_L, \overline{e}_L) \begin{pmatrix} H^0 \\ -H^- \end{pmatrix} \right] \left[\begin{pmatrix} \overline{H^0} \\ -\overline{H^-} \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \right] + \text{h.c.},$$

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- 1 We obtain the Weinberg term (sometimes called Weinberg operator), which is used to describe effective mechanism of neutrino mass generation,
- 2 After the Higgs field gets its vacuum expectation value, a neutrino mass is generated, depending on the scale Λ - of the correct range.
- 3 Corrections for other leptons are negligible.

LOOK AGAIN AT THE BOSONIC ACTION AND GRAVITY

THE SIMPLEST NONCOMMUTATIVE MODEL IS LEFT-RIGHT:

Consider the algebra $\mathcal{A} = C^\infty(M) \otimes (\mathbb{C} \oplus \mathbb{C})$, represented on $L^2(S) \otimes \mathbb{C}^2$.

$$D_o = D \otimes \text{id} + \gamma \otimes D_F, \quad D_F = \begin{pmatrix} 0 & \phi \\ \phi^* & 0 \end{pmatrix}$$

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BUT THIS IS NOT THE MOST GENERAL DIRAC OPERATOR !

Let us consider instead:

$$D = \begin{pmatrix} D_1 & \gamma\phi \\ \gamma\phi^* & D_2 \end{pmatrix},$$

with two independent metrics.

$$D^2 = \begin{pmatrix} D_1^2 + \phi\phi^* & \gamma(D_1\phi - \phi D_2) \\ -\gamma(D_1\phi^* - \phi^* D_2) & D_2^2 + \phi\phi^* \end{pmatrix},$$

WE COMPUTE THE SPECTRAL ACTION FOR THE FULL TWO-SHEETED SPACETIME MODEL

ONLY IN THE FRLW ANSATZ

$$D = \gamma^0 \partial_t + \frac{1}{a(t)} (\gamma^1 \partial_1 + \gamma^2 \partial_2 + \gamma^3 \partial_3) + \gamma^0 \frac{3\dot{a}(t)}{2a(t)},$$

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WE USE PSEUDODIFFERENTIAL CALCULUS AND WODZICKI RESIDUE

Taking:

$$D = \gamma^0 (\partial_t + H(t)) + A(t) D_3 + \gamma F(t),$$

where

$$H(t) = \begin{pmatrix} H_1(t) & 0 \\ 0 & H_2(t) \end{pmatrix}, \quad A(t) = \begin{pmatrix} \frac{1}{a_1(t)} & 0 \\ 0 & \frac{1}{a_2(t)} \end{pmatrix}, \quad F(t) = \begin{pmatrix} 0 & \phi(t) \\ \phi^*(t) & 0 \end{pmatrix}$$

THE ROBERTSON-WALKER GEOMETRY FOR THE TWO-SHEETED MODEL IS EXACTLY COMPUTABLE

THE ACTION (FIRST TWO TERMS) IS:

$$\begin{aligned} S = 2\pi^2 \int dt & \left(\lambda^4 (a_1(t)^3 + a_2(t)^3) \right. \\ & - \lambda^2 c (\dot{a}_1(t)^2 a_1(t) + \dot{a}_2(t)^2 a_2(t)) - \lambda^2 c |\Phi|^2 (a_1(t)^3 + a_2(t)^3) \\ & \left. + \lambda^2 c |\Phi|^2 \left(\frac{(a_1(t) - a_2(t))^2}{(a_1(t) + a_2(t))} (a_1(t)^2 + a_1(t)a_2(t) + a_2(t)^2) \right) \right) + \mathcal{L}_m, \end{aligned}$$

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THE BIMETRIC GRAVITY MODELS ARE CLOSELY RELATED TO NONCOMMUTATIVE GEOMETRY

The additional term is an *interaction* between the two metrics that is similar to the terms in the bimetric theory of gravity.

MODIFIED COSMOLOGY MODELS (SIMPLIFIED)

THE EMPTY UNIVERSE

$$r(t) \sim c_1 e^{-\sqrt{\frac{\Lambda}{24} + \frac{1}{4}\sqrt{6\Lambda + 8\alpha}}} + c_2 e^{-\sqrt{\frac{\Lambda}{24} - \frac{1}{4}\sqrt{6\Lambda + 8\alpha}}}.$$

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THE EMPTY UNIVERSE

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MATTER DOMINATED UNIVERSE

$$r(t) \sim t^{-\frac{1}{3}} \sin\left(\frac{1}{2}\sqrt{-2(\Lambda + \alpha)t}\right).$$

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