# Single-field inflation in models with a $R^2$ term

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# Minimal Inflation

- Inflation solves the **horizon** and **flatness** problems.
- When treated quantum-mechanically, it can also provide a mechanism for the generation of the perturbations that have resulted in the anisotropies observed in the CMB.

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left[ \frac{R}{2} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] , \qquad (M_{\mathrm{Pl}}^2 \equiv 1)$$

Friedmann equations:

$$\left( \frac{\dot{a}}{a} \right)^2 \equiv H^2 = \frac{1}{3} \left[ \frac{\dot{\phi}^2}{2} + V \right]$$

$$\dot{H} = -\frac{1}{2} \dot{\phi}^2$$

Klein-Gordon equation:

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0$$

# Slow-roll Approximation (HSRPs)

Slow-roll approximation:

$$V(\phi) \gg \dot{\phi}^2\,, \qquad |\ddot{\phi}| \ll |3H\dot{\phi}|,\, |V'|$$

First **Hubble slow-roll parameter** (HSRP)

$$\epsilon_H = -\frac{\dot{H}}{H^2} = \frac{3\dot{\phi}^2}{\dot{\phi}^2 + 2V}\,, \quad \frac{\ddot{a}}{a} = H^2(1-\epsilon_H)$$

Inflation ends **exactly** when $\epsilon_H = 1$.

Second HSRP

$$\eta_H = -\frac{\ddot{\phi}}{H\dot{\phi}}$$

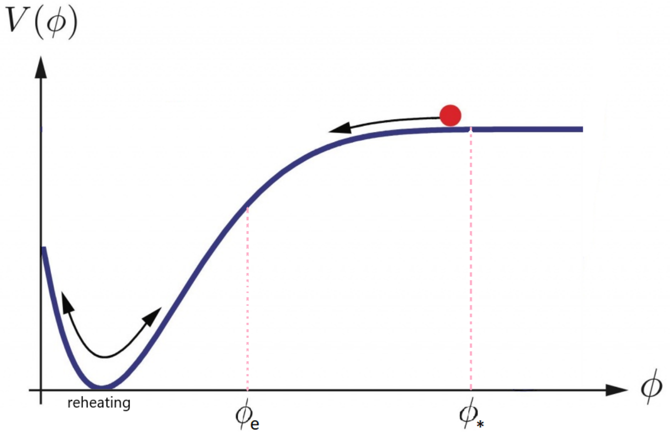
Friedmann and Klein-Gordon equations become

$$H^2 \approx \frac{1}{3}V(\phi)\,, \quad \dot{\phi} \approx -\frac{V'}{3H}\,.$$

# Slow-roll Approximation (PSRPs)

The shape of the potential is encoded in the **potential slow-roll parameters**

$$\epsilon_V = \frac{1}{2}\left(\frac{V'}{V}\right)^2 , \qquad \eta_V = \frac{V''}{V}$$



# Number of $e$-folds and Inflationary Observables

The HSPRs can be related to the PSRPs through the Friedmann and Klein-Gordon equations. One finds Liddle *et al.* astro-ph/9408015

$$\epsilon_V = \epsilon_H \left(\frac{3-\eta_H}{3-\epsilon_H}\right)^2 , \qquad \eta_V = \sqrt{2\epsilon_H}\frac{\eta'_H}{3-\epsilon_H} + \left(\frac{3-\eta_H}{3-\epsilon_H}\right)(\epsilon_H + \eta_H)$$

Taylor expansion (1st order)

$$\epsilon_H \simeq \epsilon_V \, , \qquad \eta_H \simeq \eta_V - \epsilon_V$$

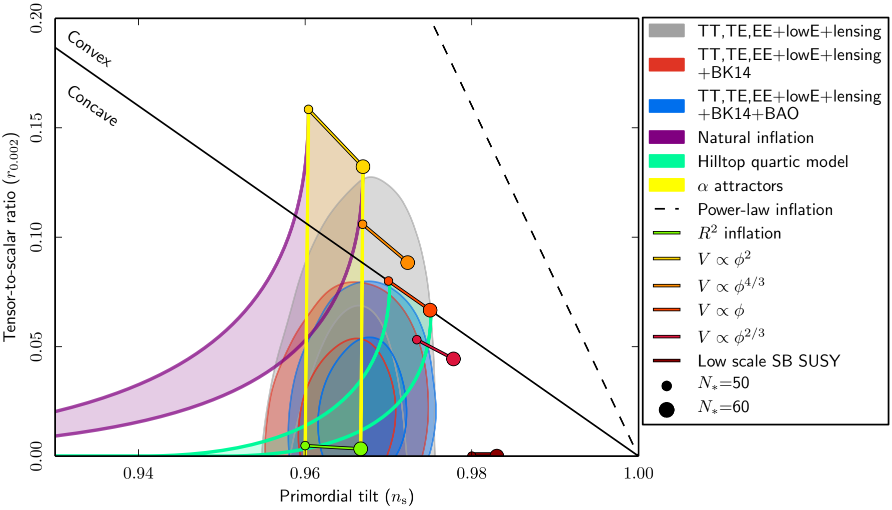
Scalar spectral index and tensor-to-scalar ratio

$$n_S = 1 - 4\epsilon_H + 2\eta_H \simeq 1 - 6\epsilon_V + 2\eta_V \, , \qquad r = 16\epsilon_H \simeq 16\epsilon_V$$

Number of e-folds

$$N(\phi) = \int_t^{t_{\rm end}} H \mathrm{d}t = \int_{\phi_{\rm end}}^{\phi} \frac{\mathrm{d}\phi}{\sqrt{2\epsilon_H}} \approx \int_{\phi_{\rm end}}^{\phi} \frac{\mathrm{d}\phi}{\sqrt{2\epsilon_V}} \sim 50 - 60$$

# Planck 2018 Results




Planck Collaboration 1807.06211

# Starobinsky Inflation

A simple extension of the Einstein-Hilbert action (Starobinsky, 1980):

$$S_{\rm Star.} = \frac{M_{\rm Pl}^2}{2} \int {\rm d}^4x\sqrt{-g}\left(R + \frac{1}{6m^2}R^2\right) ,$$

which belongs to the general class of $F(R)$ theories

$$S_F = \frac{M_{\rm Pl}^2}{2} \int {\rm d}^4x\sqrt{-g}\, F(R) \quad \rightarrow \quad S[g_{\mu\nu},\chi] = \frac{M_{\rm Pl}^2}{2} \int d^4x\sqrt{-g}\, \left[F'(\chi)(R-\chi) + F(\chi)\right]$$

After a Weyl rescaling of the metric $g_{\mu\nu}$ and a field redefinition

$$S[g_{\mu\nu},\varphi] = \frac{M_{\rm Pl}^2}{2} \int {\rm d}^4x\sqrt{-g}R - \int {\rm d}^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + V(\varphi)\right] ,$$

where

$$V = \left(\frac{M_{\rm Pl}^2}{2}\right)\frac{\chi F'(\chi) - F(\chi)}{F'(\chi)^2} , \quad F'(\chi) = \exp\left(\sqrt{\frac{2}{3}}\varphi/M_{\rm Pl}\right) , \quad \varphi = \frac{\sqrt{3}M_{\rm Pl}}{\sqrt{2}}\ln F'(\chi) .$$

For the $(R + R^2)$ model

$$V(\varphi) = \frac{3}{4}M_{\rm Pl}^2 m^2\left[1 - \exp\left(-\sqrt{\frac{2}{3}}\varphi/M_{\rm Pl}\right)\right]^2 .$$

# Non-minimal Coleman-Weinberg inflation with an $R^2$ term

A simple extension of the non-minimal Coleman-Weinberg inflationary model (1810.12884):

$$S^J = \int \mathrm{d}^4x\sqrt{-\bar{g}}\left[\frac{\xi\phi^2}{2}\bar{R} + \frac{\alpha}{2}\bar{R}^2 - \frac{1}{2}\bar{\nabla}^\mu\phi\bar{\nabla}_\mu\phi - \frac{\lambda_\phi}{4}\phi^4\right],$$

Introducing an auxiliary scalar field $\chi$:

$$S^J = \int \mathrm{d}^4x\sqrt{-\bar{g}}\left[\frac{1}{2}\left(\xi\phi^2 + \alpha\chi^2\right)\bar{R} - \frac{\alpha}{8}\chi^4 - \frac{1}{2}\bar{\nabla}^\mu\phi\bar{\nabla}_\mu\phi - \frac{\lambda_\phi}{4}\phi^4\right].$$

After a Weyl rescaling of the metric

$$g_{\mu\nu} = \Omega^2\bar{g}_{\mu\nu}\,, \qquad \Omega^2 = \left(\alpha\chi^2 + \xi\phi^2\right)/M_{\mathrm{Pl}}^2 \equiv \frac{\zeta^2}{6M_{\mathrm{Pl}}^2}\,,$$

The action in the Einstein frame takes the form

$$S^E = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\mathrm{Pl}}^2}{2}R - \frac{6M_{\mathrm{Pl}}^2}{\zeta^2}\left(\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi + \frac{1}{2}\nabla^\mu\zeta\nabla_\mu\zeta\right) - V^{(0)E}(\phi,\zeta)\right],$$

where the tree-level potential becomes

$$V^{(0)E}(\phi,\zeta) = \frac{36M_{\mathrm{Pl}}^4}{\zeta^4}\left[\frac{\lambda_\phi}{4}\phi^4 + \frac{1}{8\alpha}\left(\frac{\zeta^2}{6} - \xi\phi^2\right)^2\right].$$

# Gildener Weinberg approach

Due to the running of the couplings, the tree-level potential is flat at some renormalization scale $\Lambda_{\rm GW}$. Including the one-loop corrections, the effective potential obtains a radial shape along the flat direction and a non-zero VEV is dynamically generated (E. Gildener & S. Weinberg, 1976):

$$\frac{dV^{(0)E}}{d\phi}\Big|_{\phi=v_\phi} = \frac{d\,V^{(0)E}}{d\zeta}\Big|_{\zeta=v_\zeta} = 0\,, \qquad \Rightarrow \qquad v_\phi^2 = \frac{\xi}{6\left(\xi^2+2\alpha\lambda_\phi\right)} v_\zeta^2$$

Mass matrix diagonalization

$$\begin{pmatrix} \phi \\ \zeta \end{pmatrix} = \begin{pmatrix} \cos\omega & -\sin\omega \\ \sin\omega & \cos\omega \end{pmatrix} \begin{pmatrix} s \\ \sigma \end{pmatrix}, \qquad \tan\omega = \frac{v_\zeta}{v_\phi}\,, \qquad v_s = \frac{v_\phi}{\cos\omega} = \frac{v_\zeta}{\sin\omega}$$

The mass eigenvalues are

$$m_s^2 = 0\,, \qquad m_\sigma^2 = \frac{\xi\left(\xi + 12\lambda_\phi\alpha + 6\xi^2\right)}{6\alpha\left(2\lambda_\phi\alpha+\xi^2\right)} M_{\rm Pl}^2$$

One-loop correction

$$V^{(1)} = \frac{m_\sigma^4}{64\pi^2 v_s^4} s^4 \left[\log\left(\frac{s^2}{v_s^2}\right) - \frac{1}{2}\right]\,, \qquad v_s^2 = v_\phi^2 + v_\zeta^2\,, \qquad v_\zeta^2 = 6M_{\rm Pl}^2$$

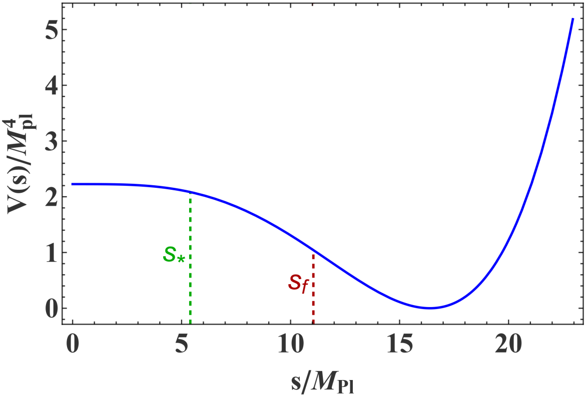
# One-loop potential

We require the vanishing of the one-loop effective potential at the minimum

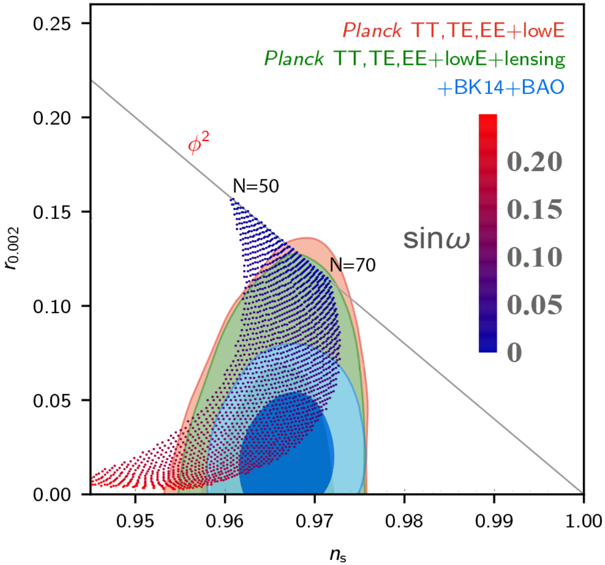
$$V(v_s) \equiv V^{(0)E}(v_s) + V^{(1)}(v_s) = 0$$

The total one-loop effective potential along the flat direction is given by

$$V(s) = \frac{m_\sigma^4}{128\pi^2}\left[\frac{\sin^2\omega}{36 M_{\rm Pl}^4} s^4 \left(2\ln\left[\frac{s^2\sin^2\omega}{6M_{\rm Pl}^2}\right] - 1\right) + 1\right], \qquad m_s^2 = \frac{\sin^2\omega}{48\pi^2}\frac{m_\sigma^4}{M_{\rm Pl}^2}$$



# Inflationary predictions



# Metric vs. Palatini

- In **metric formulation**, the metric is the only dynamical degree of freedom and the connection is always the Levi-Civita

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1+\xi\phi^2}{2}g^{\mu\nu}R_{\mu\nu}\left(g,\partial g,\partial^2 g\right) - \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi)\right)$$

- In **Palatini formulation**, both the metric and the connection are independent dynamical degrees of freedom

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1+\xi\phi^2}{2}g^{\mu\nu}R_{\mu\nu}\left(\Gamma,\partial\Gamma\right) - \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi)\right)$$

# Palatini inflation in models with an $R^2$ term

In 1810.10418 & 1812.00847 we considered (see also Enckell et al.: 1810.05536)

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left\{\frac{1}{2}\left(M_0^2+\xi\phi^2\right)R + \frac{\alpha}{4}R^2 - \frac{1}{2}\left(\nabla\phi\right)^2 - V(\phi)\right\}, \quad R = g^{\mu\nu}R^{\rho}{}_{\mu\rho\nu}\left(\Gamma,\partial\Gamma\right)$$

Introducing an auxiliary scalar $\chi$

$$\mathcal{S} = \int \mathrm{d}^4x\,\sqrt{-g}\left\{\frac{1}{2}\left(M_0^2+\alpha\chi^2+\xi\phi^2\right)R - \frac{1}{2}\left(\nabla\phi\right)^2 - \frac{\alpha}{4}\chi^4 - V(\phi)\right\}.$$

Consider a Weyl rescaling of the metric

$$\bar{g}_{\mu\nu} = \Omega^2(\phi)\,g_{\mu\nu} \qquad \text{with} \qquad \Omega^2(\phi) = \frac{M_0^2+\xi\phi^2+\alpha\chi^2}{M_P^2}.$$

The resulting Einstein frame action is

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-\bar{g}}\left\{\frac{1}{2}M_P^2\,\bar{R} - \frac{1}{2}\frac{\left(\bar{\nabla}\phi\right)^2}{\Omega^2} - \bar{V}\right\}, \quad \bar{V}(\phi,\chi) = \frac{1}{\Omega^4}\left(V(\phi)+\frac{\alpha}{4}\chi^4\right).$$

- No kinetic term has been generated for the field $\chi$ (scalaron in metric formalism)
- EOM of $\chi$ reduces to a constraint
- $\phi$ is the only propagating scalar DOF $\to$ inflaton

# Palatini inflation in models with an $R^2$ term

Varying the action with respect to $\chi$:

$$\delta_\chi S = 0 \rightarrow \chi^2 = \frac{\frac{4V(\phi)}{(M_0^2+\xi\phi^2)} + \frac{(\nabla\phi)^2}{M_P^2}}{\left[1 - \frac{\alpha(\nabla\phi)^2}{M_P^2(M_0^2+\xi\phi^2)}\right]}$$

Substituting back

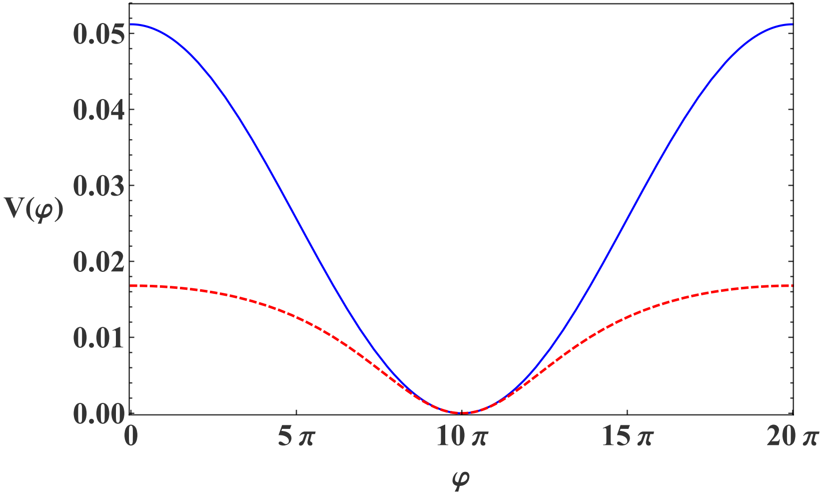
$$\mathcal{S} \approx \int d^4x\sqrt{-g}\left\{\frac{M_P^2}{2}R - \frac{1}{2}\frac{(\nabla\phi)^2}{\Omega_0^2}\left(\frac{1}{1+\frac{4\tilde{\alpha}V}{\Omega_0^4}}\right) - \frac{V}{\Omega_0^4}\left(\frac{1}{1+\frac{4\tilde{\alpha}V}{\Omega_0^4}}\right) + \mathcal{O}\left((\nabla\phi)^4\right)\right\},$$

where $\tilde{\alpha} = \alpha/M_P^4$ and $\Omega_0^2 = \left(M_0^2 + \xi\phi^2\right)/M_P^2$.

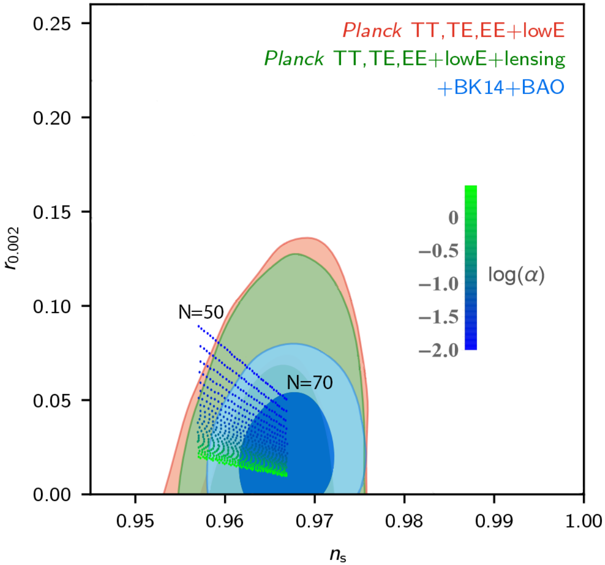
- Regardless of the shape of $V$, the $R^2$ term decreases the height of the effective potential
- For large values of $\phi$ tends to a plateau $M_{\rm P}^4/4\alpha$
- The rate of change of the field is also modified

# Example: Natural Inflation

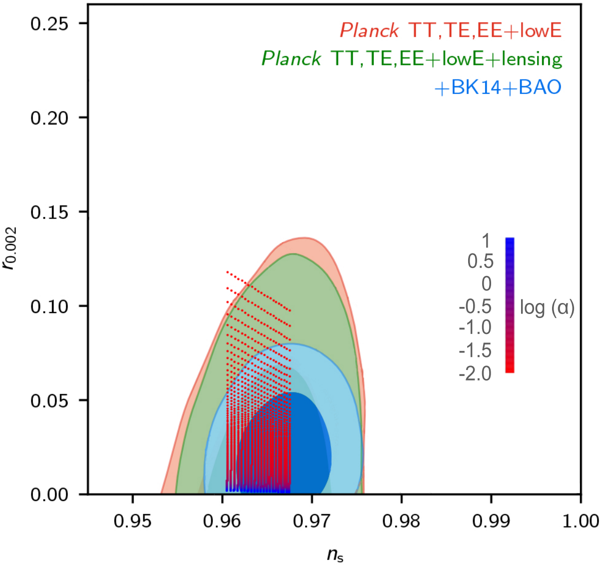
$$V(\phi) = M^4 \left(1 + \cos\left(\frac{\phi}{f}\right)\right)$$



# Example: Natural Inflation



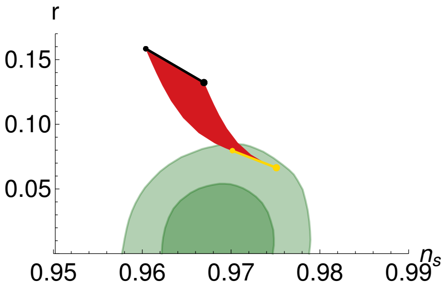
# Example: Quadratic Inflation $2V(\phi) = m^2\phi^2$

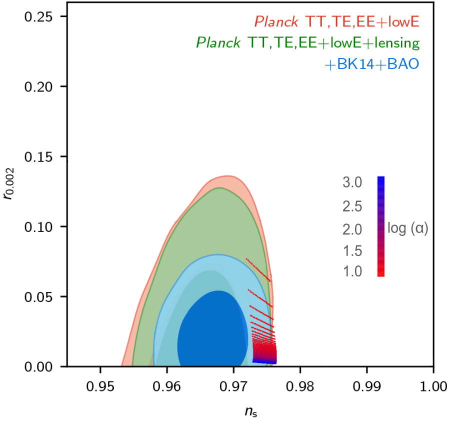


# Example: Linear Inflation Kannike, Racioppi, Raidal: 1509.05423

$$\mathcal{S} = \int d^4x\sqrt{-g}\left\{\frac{1}{2}\left(\xi\phi^2 + \alpha\chi^2\right)R - \frac{1}{2}(\nabla\phi)^2 - \frac{\lambda(\phi)}{4}\phi^4 - \frac{\alpha}{4}\chi^4 + \Lambda^4\right\}$$

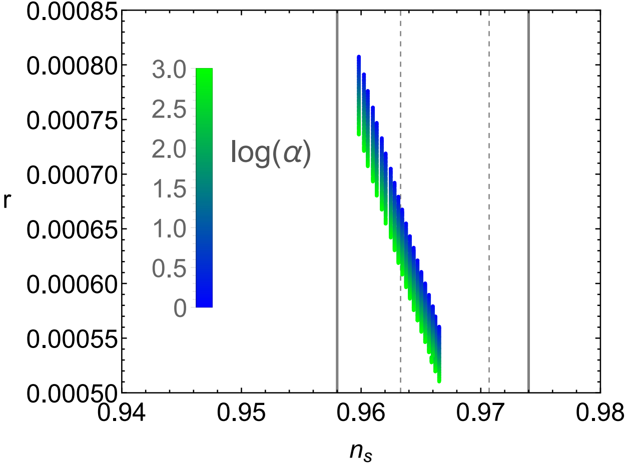
$$V_1(\phi) = \frac{1}{4}\lambda(\phi)\phi^4 + \Lambda^4 = \Lambda^4\left(1 + \frac{\xi^2\phi^4}{M_P^4}\left(2\ln(\xi\phi^2/M_P^2) - 1\right)\right), \quad \langle\phi\rangle^2 = M_\mathrm{P}^2/\xi$$



# Example: Non-minimal Higgs Inflation

$$\mathcal{L}/\sqrt{-g} = \frac{1}{2}\left(M_0^2 + \xi h^2\right) R + \frac{\alpha}{4} R^2 - \frac{1}{2} g^{\mu\nu}\partial_\mu h \partial_\nu h - V(h)\,, \quad V(h) = \frac{\lambda}{4}\left(h^2 - v^2\right)^2$$



# Summary and Conclusions

- $V(\phi) + R^2$ in **metric** leads to two-field inflation
- Exception: Coleman-Weinberg potential using Gildener-Weinberg approach
- $V(\phi) + R^2$ in **Palatini** leads to single-field inflation
- The effective potential is asymptotically flat and has a lower value
- The value of $r$ becomes smaller
- The values of $A_s$ and $n_s$ are unaffected (Enckell et al.: 1810.05536)

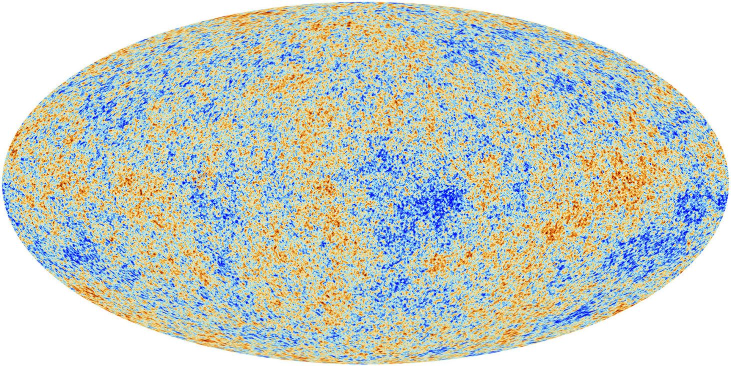
# Future directions

- Higher-order corrections to observables (AK, A. Lykkas, K. Tamvakis)
- Coleman-Weinberg $+R^2$: metric vs. Palatini (I. Gialamas, AK, A. Racioppi)

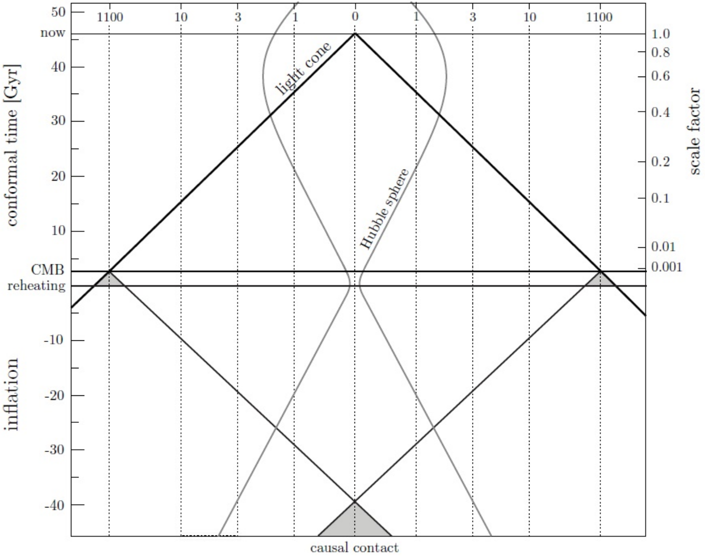
# Thank you!

☺

# Cosmic Microwave Background Radiation



# Horizon Problem



## Flatness Problem

The Friedmann equation (with zero cosmological constant),

$$H^2 = \frac{1}{3}\rho - \frac{\mathcal{K}}{a^2}\,, \qquad \Rightarrow \quad 1 - \Omega(t) = \frac{-\mathcal{K}}{(aH)^2}$$

For some earlier time $t_i$ we can write

$$1 - \Omega(t_i) = [1 - \Omega(t_0)] \left( \frac{\dot{a}(t_0)}{\dot{a}(t_i)} \right)^2 .$$

If we go back to the Planck time, $t_{\rm Pl} \sim 5 \times 10^{-44}{\rm s}$, we find

$$1 - \Omega(t_{\rm Pl}) < 10^{-64} .$$

During inflation, the Hubble parameter $H_I$ is almost constant and the scale factor grows as

$$a(t) \simeq a_{\rm end} \exp\left[H_I \left(t - t_{\rm end}\right)\right] = \exp\left[-N(t)\right] , \quad 1 - \Omega \propto e^{-2N} \to 0$$