Higgs potential with S_3 symmetry

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Workshop on Connecting Insights in Fundamental Physics: Standard Model and Beyond

Outline

The Standard Model

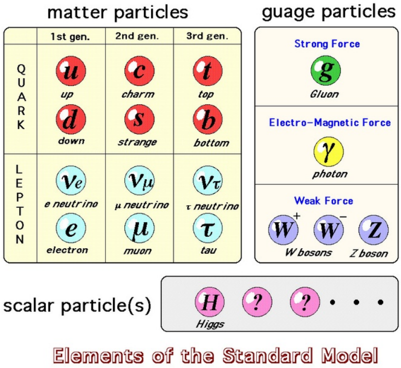


Figure 1: Elementary particles

Motivation

- Studies have been started in the 70's, hoping to find global symmetry that explains the mass and mixing patterns.

  Pakvasa et al (1978); Derman and Tsao (1979);Yahalom (1984); Wyler (1979); A. Mondragón et al (1999); Kubo et al (2004); etc

- S_3 is the smallest flavour symmetry suggested by data.
- Previous works in the quarks and neutrinos sector.

  Kubo et al (2004); A. Mondragón et al (2007); F. Gonzalez (2012); etc

- Extending the concept of flavour to the Higgs sector by adding two more EW doublets.
- Without the symmetry $\rightarrow$ 54 real parameters in the potential.
- Low energy model
- Testable model

The potential of three-Higgs-doublets under the symmetry S_3

- The lagrangian of the Higgs sector under the symmetry S_3 is given as:

$$\mathcal{L}_\phi = (D_\mu H_s)^2 + (D_\mu H_1)^2 + (D_\mu H_2)^2 - V(H_1, H_2, H_s). \tag{1}$$

- The potential $V(H_1, H_2.H_s)$ more general for the three higgs doublet model invariant under $SU(3)_c \times SU(2)_L \times U(1)_Y \times S_3$ is the following:

$$\begin{aligned} V &= \mu_1^2 \left(H_1^\dagger H_1 + H_2^\dagger H_2\right) + \mu_0^2 \left(H_s^\dagger H_s\right) + \frac{a}{2}\left(H_s^\dagger H_s\right)^2 + b\left(H_s^\dagger H_s\right)\left(H_1^\dagger H_1 + H_2^\dagger H_2\right) \\ &+ \frac{c}{2}\left(H_1^\dagger H_1 + H_2^\dagger H_2\right)^2 + \frac{d}{2}\left(H_1^\dagger H_2 - H_2^\dagger H_1\right)^2 + ef_{ijk}\left(\left(H_s^\dagger H_i\right)\left(H_j^\dagger H_k\right) + h.c.\right) \\ &+ f\left\{\left(H_s^\dagger H_1\right)\left(H_1^\dagger H_s\right) + \left(H_s^\dagger H_2\right)\left(H_2^\dagger H_s\right)\right\} + \frac{g}{2}\left\{\left(H_1^\dagger H_1 - H_2^\dagger H_2\right)^2 + \left(H_1^\dagger H_2 + H_2^\dagger H_1\right)^2\right\} \\ &+ \frac{h}{2}\left\{\left(H_s^\dagger H_1\right)\left(H_s^\dagger H_1\right) + \left(H_s^\dagger H_2\right)\left(H_s^\dagger H_2\right) + \left(H_1^\dagger H_s\right)\left(H_1^\dagger H_s\right) + \left(H_2^\dagger H_s\right)\left(H_2^\dagger H_s\right)\right\}. \end{aligned} \tag{2}$$

- The three Higgs doubles of $SU(2)$: H_1, H_2 and H_s can be writing in the following way:

$$H_1 = \begin{pmatrix} \phi_1 + i\phi_4 \\ \phi_7 + i\phi_{10} \end{pmatrix} \quad , \quad H_2 = \begin{pmatrix} \phi_2 + i\phi_5 \\ \phi_8 + i\phi_{11} \end{pmatrix}, \qquad H_s = \begin{pmatrix} \phi_3 + i\phi_6 \\ \phi_9 + i\phi_{12} \end{pmatrix} \quad . \tag{3}$$

Kubo et al (2004); Felix-Beltrán, Rodríguez-Jáuregui, M.M (2009), Das and Dey (2014), Barradas et al (2014), Costa, Ogreid, Osland and Rebelo (2016), etc

The potential of three-Higgs-doublets under the symmetry S_3

- We introduce the variables:

$$
\begin{aligned}
x_1 &= H_1^\dagger H_1, & x_4 &= Re(H_1^\dagger H_2), & x_7 &= Im(H_1^\dagger H_2),\\
x_2 &= H_2^\dagger H_2, & x_5 &= Re(H_1^\dagger H_s), & x_8 &= Im(H_1^\dagger H_s),\\
x_3 &= H_s^\dagger H_s, & x_6 &= Re(H_2^\dagger H_s), & x_9 &= Im(H_2^\dagger H_s).
\end{aligned} \tag{4}
$$

- The potential can be rewritten as:

$$
\begin{aligned}
V \;=\; & \mu_1^2(x_1+x_2)+\mu_0^2 x_3 + a x_3^2 + b(x_1+x_2)x_3 + c(x_1+x_2)^2\\
& -4d x_7^2 + 2e\left[(x_1-x_2)x_6 + 2x_4x_5\right] + f(x_5^2+x_6^2+x_8^2+x_9^2)\\
& +g\left[(x_1-x_2)^2+4x_4^2\right] + 2h(x_5^2+x_6^2-x_8^2-x_9^2).
\end{aligned} \tag{5}
$$

- We worked in the normal minimum, where the configurations of the fields are:

$$
\phi_7 = v_1, \phi_8 = v_2, \phi_9 = v_3, \phi_i = 0, \;\; i \neq 7,8,9, \tag{6}
$$

they must satisfy the condition $v = \sqrt{v_1^2+v_2^2+v_3^2} = 246\ GeV$.

- The condition of the minimum fixes :

$$
v_1^2 = 3v_2^2, \quad \wedge \quad e = 0. \tag{7}
$$

Felix-Beltrán, Rodríguez-Jáuregui, M.M; Costa et al

- The minimum of potential can be parameterized in spherical coordinates, two angles and v.

$$v_1 = v\cos\varphi\sin\theta, \quad v_2 = v\sin\varphi\sin\theta \quad v_3 = v\cos\theta. \tag{8}$$

$\tan\varphi = \pm\frac{1}{\sqrt{3}}$, which means $\varphi = \pi/6$, therefore:

$$\tan\varphi = 1/\sqrt{3} \quad \Rightarrow \quad \sin\varphi = \frac{1}{2} \quad \& \quad \cos\varphi = \frac{\sqrt{3}}{2} \tag{9}$$

$$\tan\theta = \frac{2v_2}{v_3} \quad \Rightarrow \quad \sin\theta = \frac{2v_2}{v} \quad \& \quad \cos\theta = \frac{v_3}{v} \tag{10}$$

Mass Matrix

- The properties Higgs and Goldstone bosons have been found after diagonalize the 12×12 matrix $(\mathcal{M}_H^2)_{ij} = \frac{1}{2}\left.\frac{\partial^2 V}{\partial\phi_i\partial\phi_j}\right|_{\text{min}}$. We took the next convention:

$$[\mathcal{M}^2_{diag}]_i = R_i^T \mathcal{M}_i^2 R_i \quad i = s, a, c. \tag{11}$$

- The rotation matrix is the product of two rotations, i.e., $R_i = A * B_i$. The rotation matrix R which diagonalize $\mathcal{M}_A^2$ and $\mathcal{M}_C^2$ is:

$$R_{a,c} = \begin{pmatrix} \sin\theta\cos\varphi & -\sin\varphi & -\cos\theta\cos\varphi \\ \sin\theta\sin\varphi & \cos\varphi & -\cos\theta\sin\varphi \\ \cos\theta & 0 & \sin\theta \end{pmatrix}. \tag{12}$$

Higgs masses

The rotation matrix R which diagonalize $\mathcal{M}_S^2$ is:

$$R_s = \begin{pmatrix} \sin\alpha\cos\varphi & -\sin\varphi & -\cos\alpha\cos\varphi \\ \sin\alpha\sin\varphi & \cos\varphi & -\cos\alpha\sin\varphi \\ \cos\alpha & 0 & \sin\alpha \end{pmatrix}. \qquad (13)$$

we define the angle $\tan(2\alpha) = \frac{M_a^2 - M_c^2}{M_b^2}$.

- The Higgs masses can be rewritten in the following way:

$$m^2_{h_0} = -9ev^2 \sin\theta\cos\theta, \qquad (14)$$

$$m^2_{H_1,H_2} = (M_a^2 + M_c^2) \pm \sqrt{(M_a^2 - M_c^2)^2 + (M_b^2)^2}, \qquad (15)$$

$$M_a^2 = \left[2(c+g)v^2\sin^2\theta + \frac{3}{2}ev^2\sin\theta\cos\theta\right],$$

$$M_b^2 = \left[3ev^2\sin^2\theta + 2(b+f+2h)v^2\sin\theta\cos\theta\right],$$

$$M_c^2 = 2av^2\cos^2\theta - \frac{ev^2\tan\theta\sin^2\theta}{2}.$$

$$m^2_{A_1} = -4(d+g)v^2\sin^2\theta - 5ev^2\sin\theta\cos\theta - 4hv^2\cos^2\theta, \qquad (16)$$

$$m^2_{A_2} = -v^2(e\tan\theta + 4h). \qquad (17)$$

$$m^2_{H_1^\pm} = -\left[5ev^2\sin\theta\cos\theta + (f+2h)v^2\cos^2\theta\right] - 4gv^2\sin^2\theta, \qquad (18)$$

$$m^2_{H_2^\pm} = -v^2\left[e\tan\theta + (f+2h)\right]. \qquad (19)$$

Das and Dey (2014); Barradas et al (2014)

Higgs Basis

- This is the basis in which one of the Higgs' doublets has the complete vacuum expectation value, ϕ_{vev}, and the other doubles are perpendicular to the first one, ψ_1, ψ_2.
- The Higgs basis is define in the following form:

$$\begin{pmatrix} \phi_{vev} \\ \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} \cos\varphi\sin\theta & \sin\varphi\sin\theta & \cos\theta \\ -\sin\varphi & \cos\varphi & 0 \\ -\cos\varphi\cos\theta & -\sin\varphi\cos\theta & \sin\theta \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix}. \quad (20)$$

- The doublets in the Higgs basis are given as:

$$\phi_{vev} = \begin{pmatrix} G^{\pm} \\ \frac{1}{\sqrt{2}}(v + \widetilde{h} + iG_0) \end{pmatrix}, \quad \psi_1 = \begin{pmatrix} H_1^{\pm} \\ \frac{1}{\sqrt{2}}(\widetilde{H_1} + iA_1) \end{pmatrix}, \quad \psi_2 = \begin{pmatrix} H_2^{\pm} \\ \frac{1}{\sqrt{2}}(\widetilde{H_2} + iA_2) \end{pmatrix}.$$

where

$$\begin{pmatrix} \widetilde{h} \\ \widetilde{H_1} \\ \widetilde{H_2} \end{pmatrix} = \begin{pmatrix} \cos(\alpha-\theta) & 0 & \sin(\alpha-\theta) \\ 0 & 1 & 0 \\ -\sin(\alpha-\theta) & 0 & \cos(\alpha-\theta) \end{pmatrix} \begin{pmatrix} H_1 \\ h_0 \\ H_2 \end{pmatrix}. \quad (21)$$

Das and Dey (2014)

- On the other hand the kinetic term in this basis is given as follow:

$$\mathcal{L}_{kin} = |D_\mu \phi|^2 + \sum_{a=2}^{N} |D_\mu \psi_a|^2. \qquad (22)$$

- A summary of the couplings h_0, H_1 and H_2, with two vector bosons and with vector bosons and charged scalars.

$\cos(\alpha-\theta)$	$\sin(\alpha-\theta)$
$g_{H_1 W^+ W^-}$	$g_{H_2 W^+ W^-}$
$g_{H_1 ZZ}$	$g_{H_2 ZZ}$
$g_{ZA_2 H_2}$	$g_{ZA_2 H_1}$
$g_{W^\pm H_2^\mp H_2}$	$g_{W^\pm H_2^\mp H_1}$
$g_{ZW^\pm H_2^\mp H_2}$	$g_{ZW^\pm H_2^\mp H_1}$
$g_{\gamma W^\pm H_2^\mp H_2}$	$g_{\gamma W^\pm H_2^\mp H_1}$

- h0 has no trilinear gauge couplings, only:

$$g_{ZA_1 h_0},\; g_{ZW^\pm H_1^\pm h_0},\; g_{W^\pm H_1^\pm h_0} \text{ and } g_{\gamma W^\pm H_1^\pm h_0}. \qquad (23)$$

Higgs couplings

$$g_{H_1 W^\pm W^\mp} = \frac{4M_W^2 \cos(\alpha - \theta) g^{\mu\nu}}{v} \quad (24)$$

$$g_{H_2 W^\pm W^\mp} = \frac{4M_W^2 \sin(\alpha - \theta) g^{\mu\nu}}{v} \quad (25)$$

$$g_{h_0 W^\pm W^\mp} = 0 \quad (26)$$

$$g_{h_0 ZZ} = 0 \quad (27)$$

$$g_{H_1 H_1 W^\pm W^\mp} = \frac{2M_W^2 g^{\mu\nu}}{v^2} \quad (28)$$

$$g_{H_2 H_2 W^\pm W^\mp} = \frac{2M_W^2 g^{\mu\nu}}{v^2} \quad (29)$$

$$g_{h_0 h_0 W^\pm W^\mp} = \frac{2M_W^2 g^{\mu\nu}}{v^2} \quad (30)$$

$$g_{H_1^\pm H_1^\pm W^\pm W^\mp} = \frac{2M_W^2 g^{\mu\nu}}{v^2} \quad (31)$$

$$g_{H_2^\pm H_2^\pm W^\pm W^\mp} = \frac{2M_W^2 g^{\mu\nu}}{v^2} \quad (32)$$

$$g_{H_1 ZZ} = \frac{2M_Z^2 \cos(\alpha - \theta) g^{\mu\nu}}{v} \quad (33)$$

$$g_{H_2 ZZ} = \frac{2M_Z^2 \sin(\alpha - \theta) g^{\mu\nu}}{v} \quad (34)$$

$$g_{H_1 H_1 ZZ} = \frac{M_Z^2 g^{\mu\nu}}{v^2} \quad (35)$$

$$g_{H_2 H_2 ZZ} = \frac{M_Z^2 g^{\mu\nu}}{v^2} \quad (36)$$

$$g_{h_0 h_0 ZZ} = \frac{M_Z^2 g^{\mu\nu}}{v^2} \quad (37)$$

Higgs self-couplings

- The Higgs trilinear self-couplings are defining as:

$$\lambda_{ijk} = \frac{-i\partial^3 V}{\partial H_i \partial H_j \partial H_k}. \tag{38}$$

- Some of the trilinear self-couplings.

$$g_{h_0 h_0 h_0} = 0,$$

$$g_{H_1 H_1 H_1} = \frac{2}{v s_{2\theta}} \left(m_{h_0}^2 \left(\frac{s^3_{\alpha-\theta}}{9 c_\theta^2} \right) - m_{H_1}^2 \left(c_\alpha^2 s_{\alpha-\theta} - s_\alpha c_\theta \right) \right),$$

$$g_{A_1 A_1 A_1} = 0,$$

$$g_{h_0 h_0 H_1} = \frac{1}{v s_\theta} \left(m_{h_0}^2 \frac{s_{\alpha+\theta}}{c_\theta} + s_\alpha m_{H_1}^2 \right),$$

$$g_{H_2 H_2 H_2} = -\frac{2}{v s_{2\theta}} \left(m_{h_0}^2 \left(\frac{c^3_{\alpha-\theta}}{9 c_\theta^2} \right) + m_{H_2}^2 \left(c_\alpha^2 c_{\alpha-\theta} - s_\alpha s_\theta \right) \right),$$

$$g_{h_0 h_0 H_2} = -\frac{1}{v s_\theta} \left(m_{h_0}^2 \frac{c_{\alpha+\theta}}{c_\theta} + c_\alpha m_{H_2}^2 \right),$$

Per Osland et al (2008); John F. Gunion and Howard E. Haber (2003); Barradas-Guevara et al. (2014)

Higgs self-couplings

$$
\begin{aligned}
g_{H_2H_2H_1H_1} &= \frac{1}{8v^2 s_{2\theta}^2}\left(\frac{4m_{h_0}^2\, s_{2(\alpha-\theta)}}{3c_\theta^2}\left(2s_{2\alpha} + s_{2(\alpha-\theta)}\right) - 2m_{H_1}^2\, s_{2\alpha}(3c_{2\alpha}\, s_{2(\alpha-\theta)}\right. \\
&\left. -3s_{2\alpha} + s_{2\theta}) + 2m_{H_2}^2\, s_{2\alpha}(3c_{2\alpha}\, s_{2(\alpha-\theta)} + 3s_{2\alpha} + s_{2\theta})\right), \qquad (39)
\end{aligned}
$$

$$
\begin{aligned}
g_{H_2H_2H_2H_2} &= \frac{1}{v^2 s_{2\theta}^2}\left(m_{h_0}^2\, c_{\alpha-\theta}^3\,\frac{(c_{\alpha-\theta} + 2c_{\alpha+\theta})}{9c_\theta^2}\right. \\
&\left. + m_{H_1}^2\,\frac{s_{2\alpha}^2\, c_{\alpha-\theta}^2}{4} + m_{H_2}^2\,(c_\alpha^2\, c_{\alpha-\theta} - s_\alpha\, s_\theta)^2\right), \qquad (40)
\end{aligned}
$$

- After EW symmetry breaking, S_3 breaks $\rightarrow$ residual Z_2 symmetry.

  Das and Dey (2014); Ivanov(2017)

- h_0, A_1, $H_1^\pm$ have Z_2 parity -1,
  H_1, H_2 parity +1,
  $H_2^\pm$, A_2 parity +1.
- h_0 decoupled from gauge bosons.
- There is an "alignment" limit, where H_2 is the SM Higgs boson $\rightarrow$ H_1 also decoupled from the gauge bosons.

Higgs one loop self energy

- For the scalars bosons the self energy in the 3HDM-S_3 are given as:

$$\Pi^S(s) = \begin{pmatrix} \Pi^{S,(\tilde{q})}_{h_0 h_0}(s) & \Pi^{S,(\tilde{q})}_{h_0 H_1}(s) & \Pi^{S,(\tilde{q})}_{h_0 H_2}(s) \\ \Pi^{S,(\tilde{q})}_{H_1 h_0}(s) & \Pi^{S,(\tilde{q})}_{H_1 H_1}(s) & \Pi^{S,(\tilde{q})}_{H_1 H_2}(s) \\ \Pi^{S,(\tilde{q})}_{H_2 h_0}(s) & \Pi^{S,(\tilde{q})}_{H_2 H_1}(s) & \Pi^{S,(\tilde{q})}_{H_2 H_2}(s) \end{pmatrix}; \qquad (41)$$

- The self energy for the H_2 Higgs is given by:

$$\begin{aligned} \Pi^{S,(\tilde{q})}_{H_2 H_2}(s) &= \sum_i \frac{g_{H_2 H_2 \phi_i^0 \phi_i^0}}{16\pi^2} A0(m^2_{\phi_i^0}) + \sum_i \frac{g^2_{H_2 \phi_i^0 \phi_i^0}}{8\pi^2} B0(s^2, m^2_{\phi_i^0}, m^2_{\phi_i^0}) \\ &+ \sum_j \frac{g^2_{H_2 H_j^\pm H_j^\mp}}{8\pi^2} B0(s^2, m^2_{H_j^\pm}, m^2_{H_j^\pm}) \qquad (42) \end{aligned}$$

with $\phi_i^0 = h_0, H_1, H_2, A_1, A_2$ and $j = 1, 2$.

- The one-loop corrected Higgs mass H_2 can be obtain as the pole of the propagator of the block 2×2 matrix given above in the addition of the tree level Higgs mass terms

$$det\left[\hat{\Delta}^{-1}_{H_1 H_2}(s^\star)\right] = det\left[(s^\star)\mathbf{1}_{2\times 2} - \mathcal{M}^2_{H_1 H_2}(s^\star)\right] = 0, \qquad (43)$$

Work in progress.

Proceeding

- We made a scan in the parameter space, we imposed the stability and unitarity conditions.

  Das and Dey (2014), Barradas et al (2014)
- We imposed the alignment limit, which means $(\alpha - \theta) = \frac{\pi}{2}$.
- We imposed a decoupling limit by hand, i.e., from the three scalars bosons that we have in the model, we took H_2 as the Higgs of the SM, which the mass is $125\ GeV$, while the others two Higgs must have larger masses to $500\ GeV$.

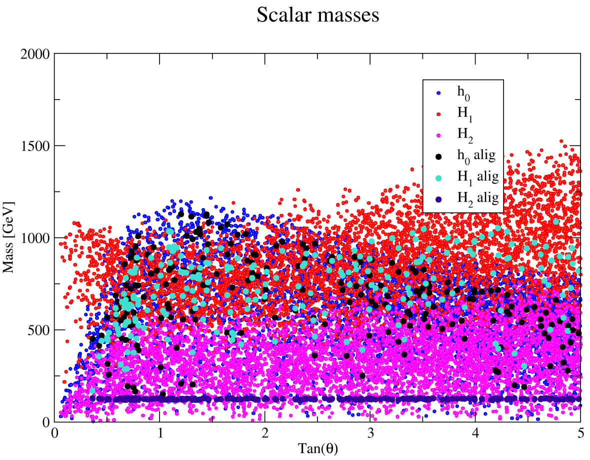
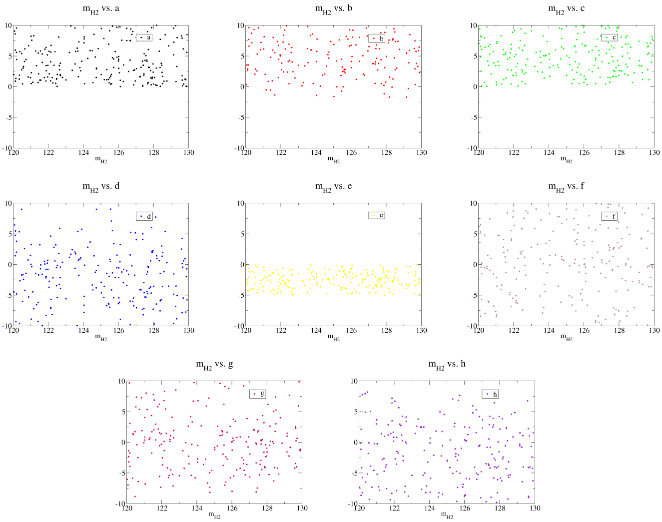


Figure 2: Higgs scalars masses h_0, H_1, H_2. Imposing unitarity and stability conditions.

Mass H_2 vs. parameters



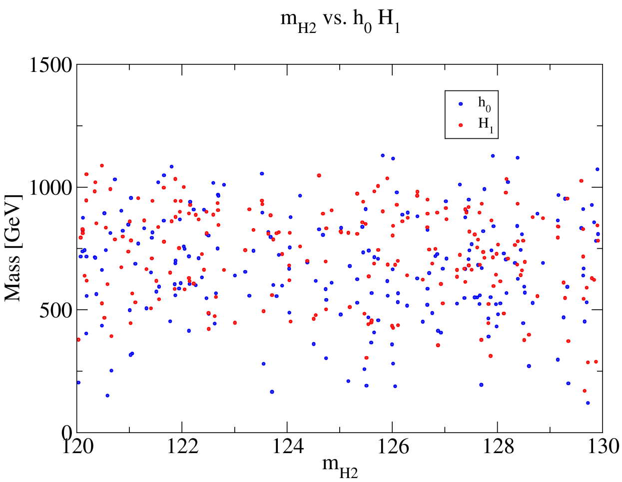


Figure 3: Comparison between m_{H_2} and the other scalar Higgs in the alignment limit.

Imposing H_2 as the SM Higgs

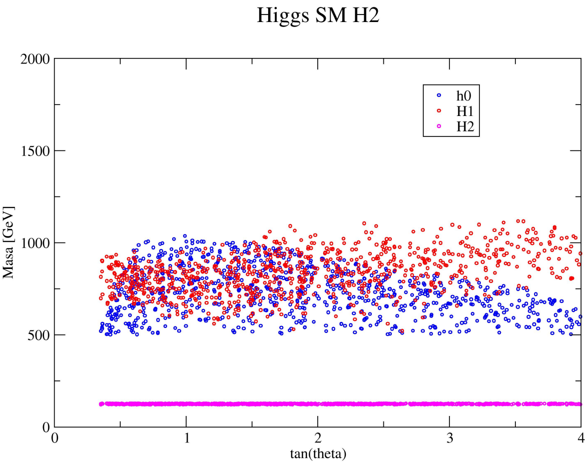


Figure 4: Higgs scalars masses h_0, H_1, H_2, $\tan\theta \in (0, 2)$

Summary

- We compute the Higgs self-couplings and the Higgs couplings with the vector bosons.
- We found values for the parameters of the model and the angle θ, that pass all Higgs bounds and satisfy the alignment limit.
- We are able to compute different process in order to make boundaries to the parameters of the model.
- The model has favorable results.

Thank you so much !