Aspects of non-perturbative unitarity in Quantum Field Theory

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Alexander von Humboldt
Stiftung / Foundation

Motivation

- **Einstein-Hilbert gravity**: unitary, but *perturbatively non-renormalizable*

- **Quadratic gravity** is a *renormalizable* theory, but has one *massive spin-2 ghost*

$$S = -\int d^4x \sqrt{-g}\, \{\gamma R + \alpha R_{\mu\nu} R^{\mu\nu} - \beta R^2\}$$

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→ Perturbative approaches could fail in the description of the UV behavior of a theory

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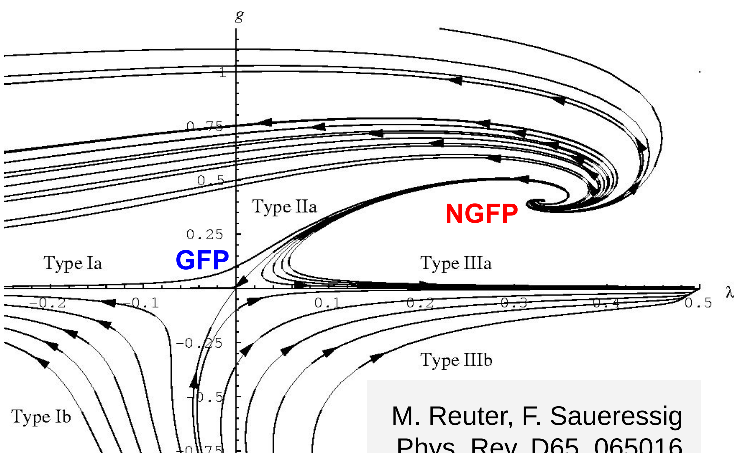

M. Reuter, F. Saueressig
Phys. Rev. D65, 065016

Functional (non-perturbative) RG

→ Fixed points of the RG flow:

- **GFP** → saddle point
- **NGFP** → UV-attractor

→ Extended truncations:

- **3 relevant directions**

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Non-perturbative unitarity: Although quadratic gravity has a ghost, *quantum effects* could make the ghost unstable, thus restoring unitarity

Salam, Strathdee (1978)
E. S. Fradkin, A. A. Tseytlin (1981)

Unitarity

- **Unitarity condition**

$$S^\dagger S = \mathbb{I} \qquad\qquad S = \mathbb{I} + iT$$

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- **Optical theorem**

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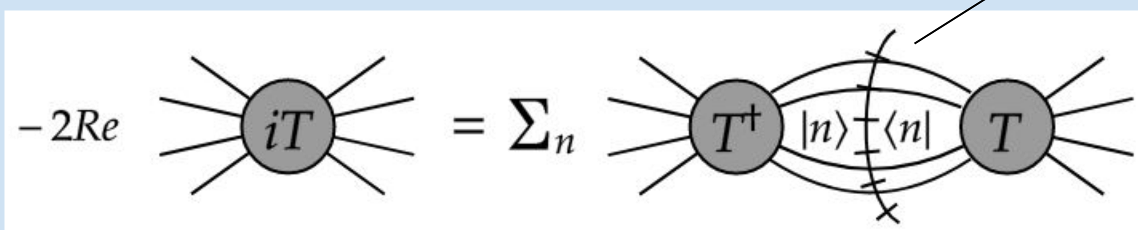
Cutting rules
T'Hooft, Veltman (1973)



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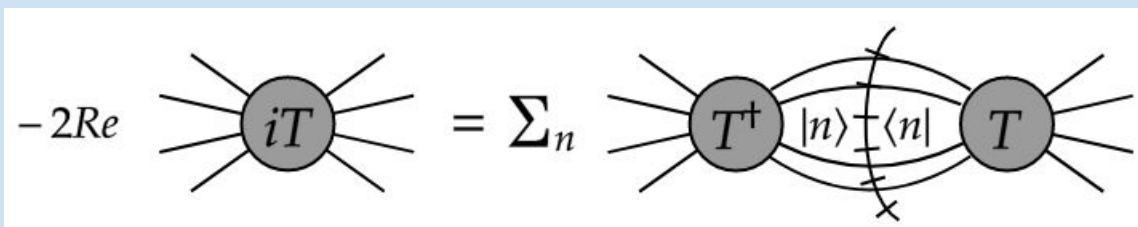

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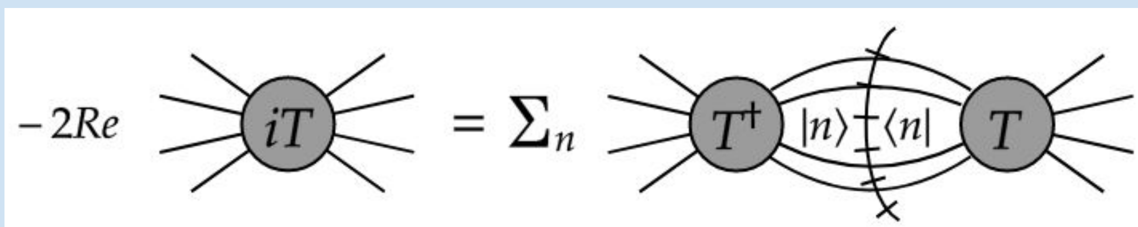

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If the space of asymptotic states contains **ghosts**

$$\langle m|n\rangle = (-1)^{\alpha_n}\delta_{mn} \qquad \text{“}\,\mathbb{I}\,\text{”} = \sum_n (-1)^{\alpha_n}|n\rangle\langle n|$$

⇒ **Loss of physical unitarity**

Spectral representation and unitarity

- **Spectral representation**

$$\Delta(q^2) = i\int_0^\infty d\mu^2 \frac{\rho(\mu^2)}{q^2 - \mu^2 + i\varepsilon}$$

$$\rho(q^2) = \frac{1}{\pi}\mathrm{Im}\{i\Delta(q^2)\}$$

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$$\Delta(q^2) = i\left\{\underbrace{\sum_n \frac{R_n}{q^2-m_n^2}}_{\text{Real poles (stable particles)}} + \underbrace{\sum_n\left(\frac{\tilde{R}_n}{q^2-(\tilde{m}_n^2)} + \frac{\tilde{R}_n^*}{q^2-(\tilde{m}_n^2)^*}\right)}_{\text{Complex poles}} + \underbrace{\int_{m_{th}^2}^\infty \frac{\sigma(\mu^2)}{q^2-\mu^2}d\mu^2}_{\text{Unstable particles}}\right\}$$

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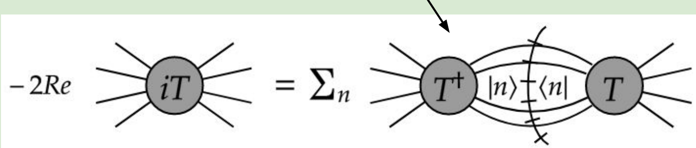

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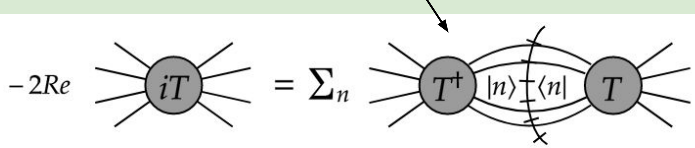

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$$\Delta(q^2) = \frac{i}{q^2-m_0^2}$$

Bare propagator

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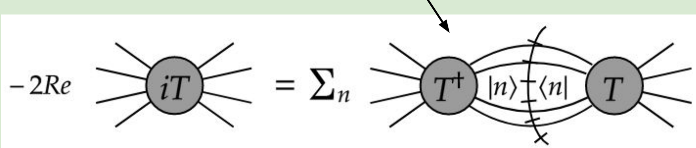

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$$\Delta(q^2) = \frac{i}{q^2-m_0^2} \longrightarrow \Delta(q^2) = \frac{i}{q^2-m_0^2-\underbrace{\Sigma(q^2)}_{\text{self-energy}}}$$

Bare propagator

Dressed propagator

Functional Renormalization Group

Solving the **quantum theory** is equivalent to solve the functional **renormalization group equation**

$$k\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\left\{\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} k\partial_k \mathcal{R}_k\right\}$$

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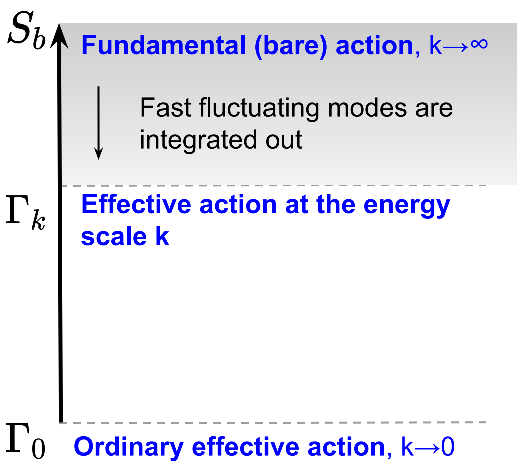

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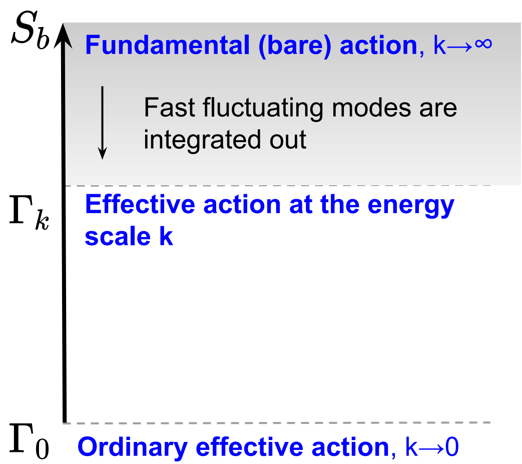

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S_b **Fundamental (bare) action**, $k \to \infty$

Fast fluctuating modes are integrated out

Γ_k **Effective action at the energy scale k** → All terms compatible with symmetry and field content of the theory are generated

Γ_0 **Ordinary effective action**, $k \to 0$ → *All quantum fluctuations are integrated out → non-locality*
Incorporates all quantum effects → fully-dressed quantities
This is the object to use to check **unitarity**!

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Problem: Need to work within truncation ⇒ higher-derivatives ⇒ Poles

Questions:

- What is the nature of these poles?
- Are these poles removed by quantum effects?
- Connection between poles in finite truncation and poles in the effective action?
- How do we understand, within truncation, if these poles are dangerous for unitarity?

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Take the **one-loop effective action** as a **toy model for the full effective action**

$$\Gamma_{QED} = -\frac{1}{4}\int d^4x \, \{F_{\mu\nu} P(\Box) F^{\mu\nu}\} \qquad P(q^2) = 1 - \frac{\alpha}{3\pi}\log\left(\frac{m_{th}^2 - q^2}{m_{th}^2}\right) - \frac{q^2}{M^2} \qquad m_{th} = 2m_f$$

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In this case the propagator has one massless pole and one massive ghost pole

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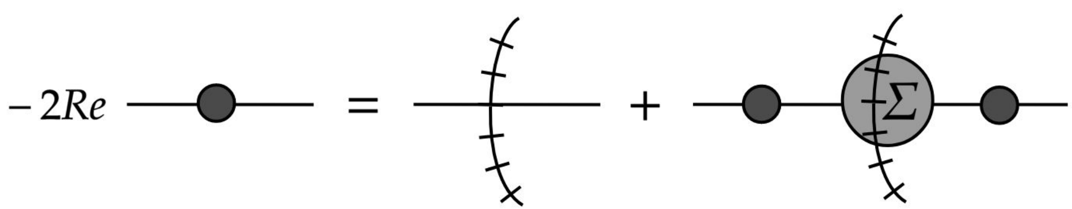
Absorptive part of the propagator



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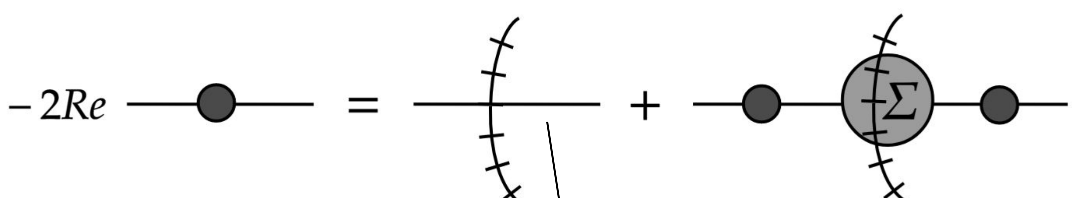

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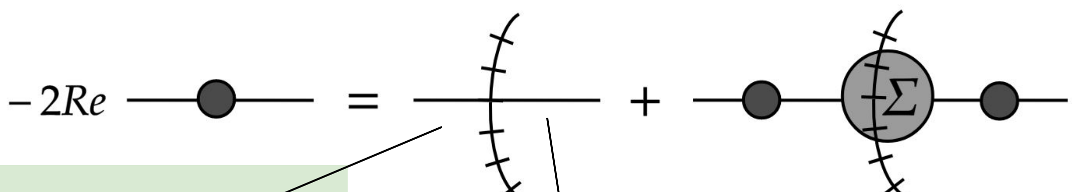
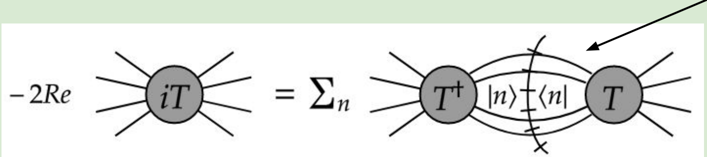




$\rho \sim \delta(q^2)$

What happens within truncations?

$$\Gamma_k = -\frac{1}{4}\int d^4x\{F_{\mu\nu}P_kF^{\mu\nu}\} \qquad P_k(z) = \sum_{n=1}^{\infty} a_n(k)\, z^n \qquad z = q^2/m_{th}^2$$

$$k = 0 \qquad \boxed{P(z) = 1 - z + \frac{\alpha}{3\pi}\sum_{n=1}^{\infty}\frac{z^n}{n} \equiv 1 - z - \frac{\alpha}{3\pi}\log(1-z)}$$

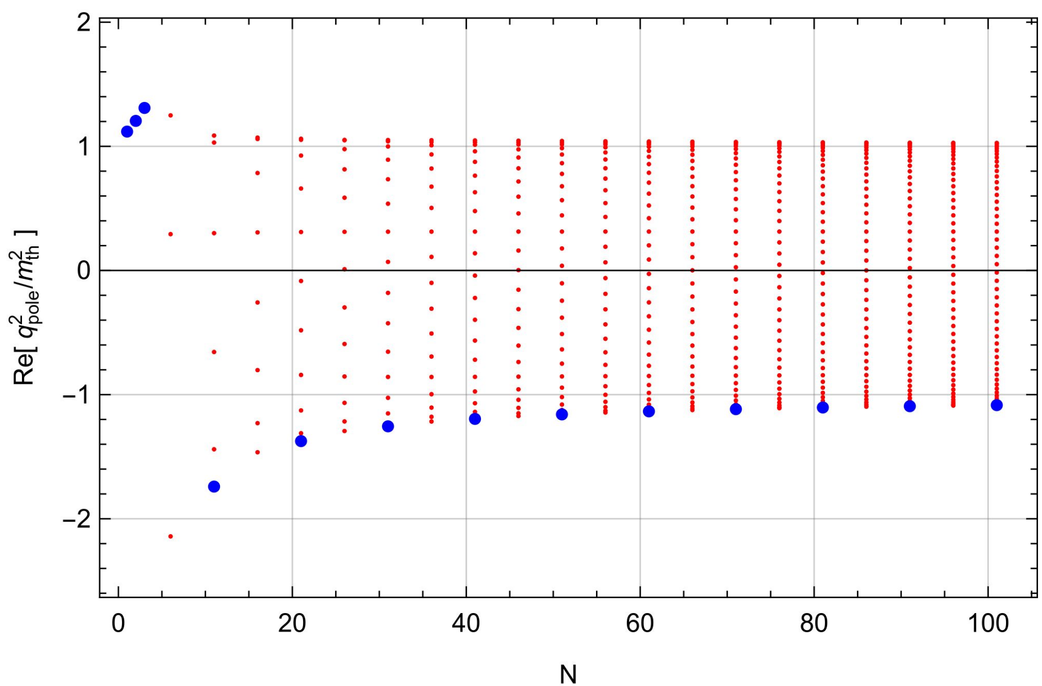
If all terms are included

Truncation of the action N (derivative expansion of the action)

$$P^N(z) = 1 - z + \frac{\alpha}{3\pi}\sum_{n=1}^{N}\frac{z^n}{n}$$

What happens within truncations?

$$P^N(z) = 1 - z + \frac{\alpha}{3\pi}\sum_{n=1}^{N}\frac{z^n}{n} \qquad z = q^2/m_{th}^2 \qquad \alpha = 1 \qquad \boxed{\Delta(q^2) \sim \frac{i}{q^2 P^N(q^2)}}$$



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$$q^2 \sim -m_{th}^2$$

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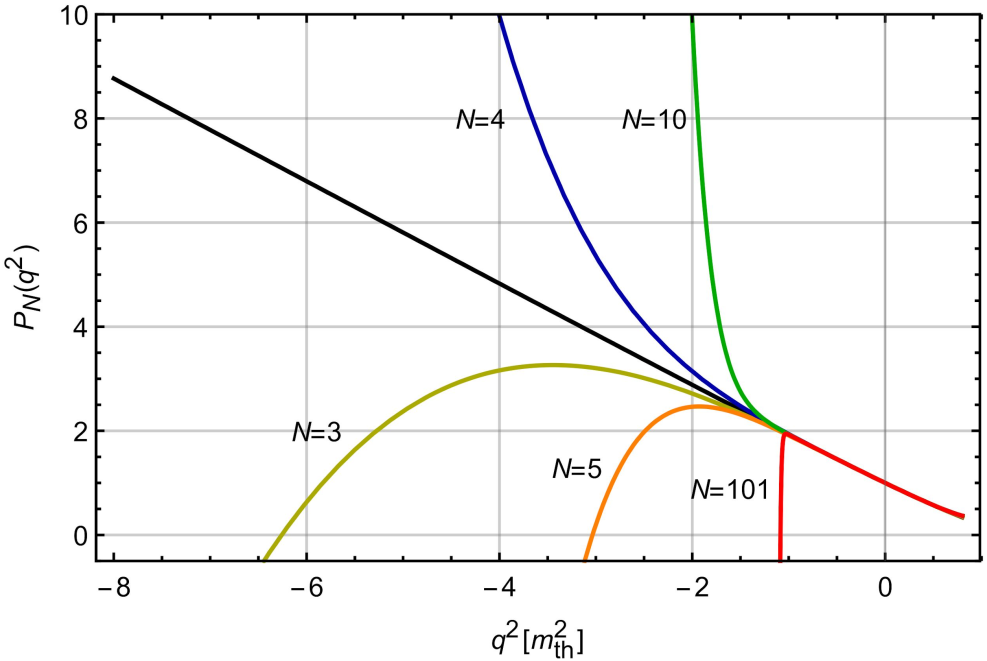

The apparent ghost pole is generated by the convergence properties of the function P(z)

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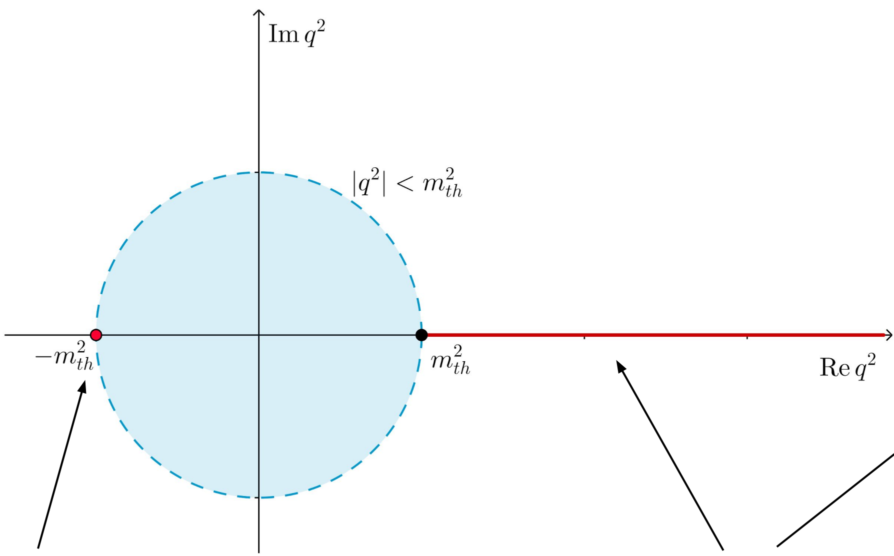
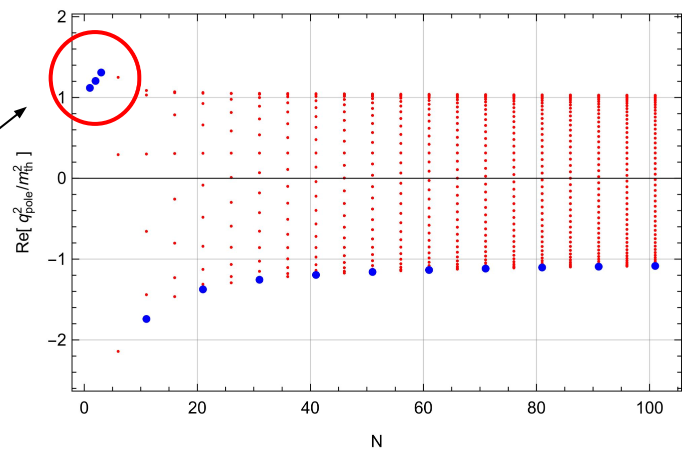


The apparent ghost pole is generated by the convergence properties of the function P(z)



Fake ghost living in the principal branch of the Log (not appearing in the full theory)

Unstable ghost lives in the branch cut (cannot be seen in any perturbative expansion)

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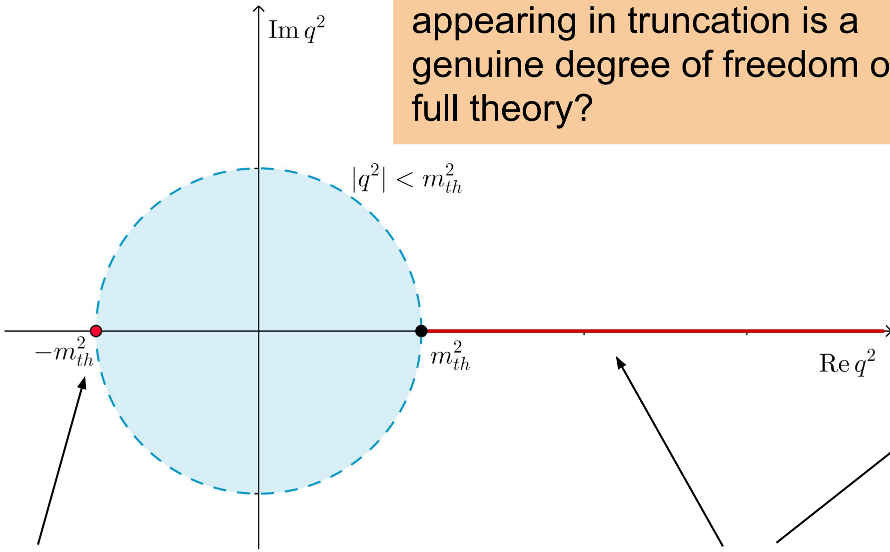
How to determine whether a pole appearing in truncation is a genuine degree of freedom of the full theory?



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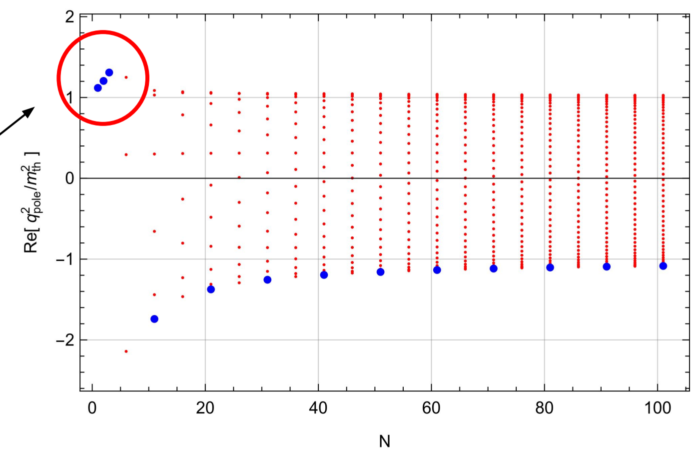

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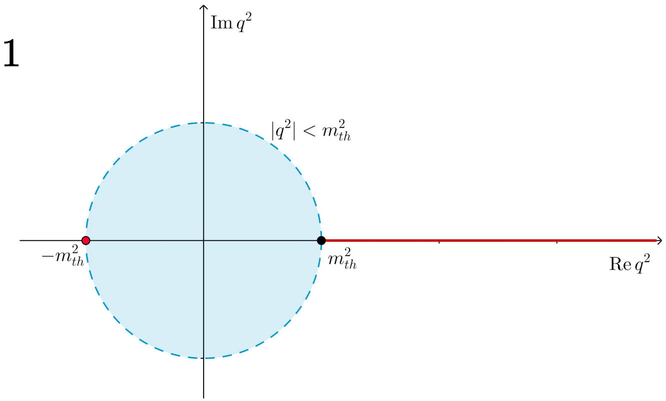
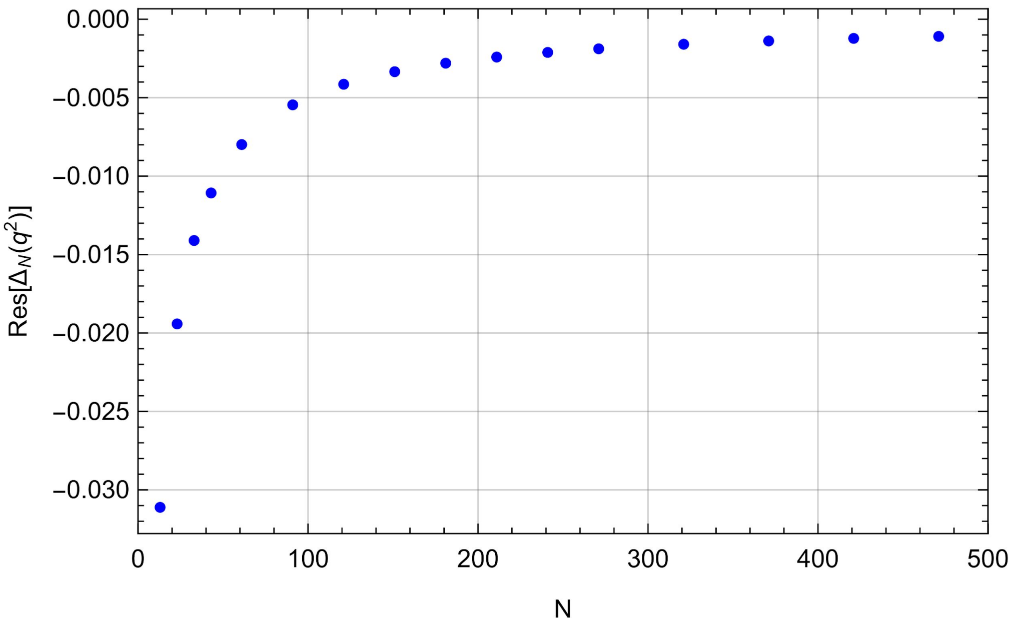
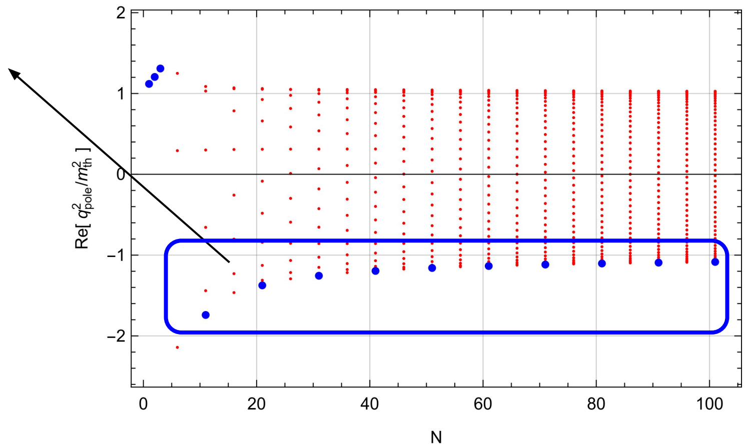


The answer lies in the **residue**!





What happens if the full theory has a stable ghost?

$$P(q^2) = 1 + \frac{\alpha}{3\pi}\log\left(\frac{m_{th}^2 - q^2}{m_{th}^2}\right) - \frac{q^2}{M^2} \longrightarrow$$

Flipping the sign of the log generates a **stable ghost**, living in the principal branch of the Log

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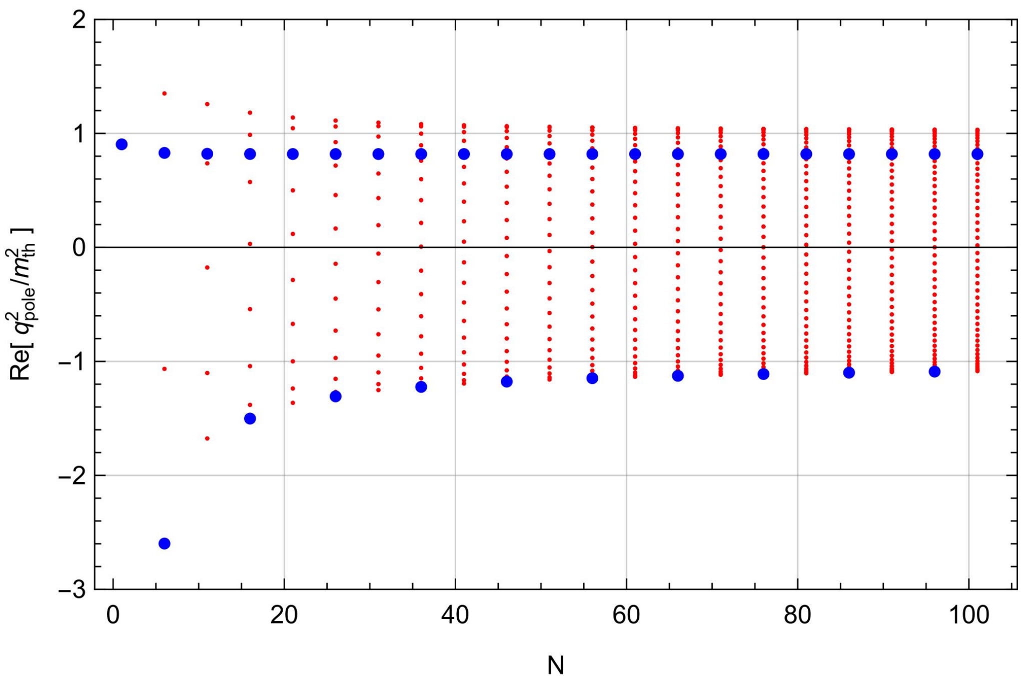

● Real poles

● Complex poles

Two persistent ghost poles!

What happens if the full theory has a stable ghost?

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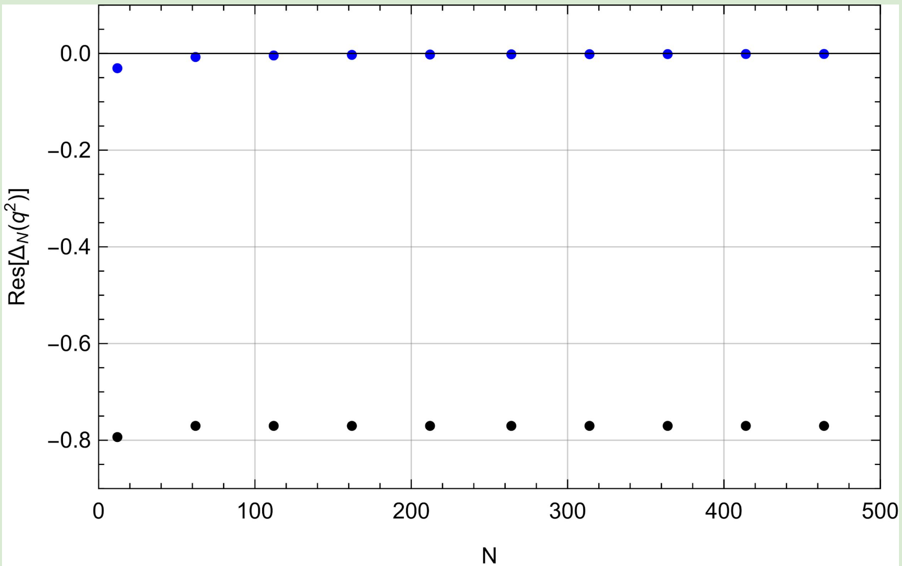
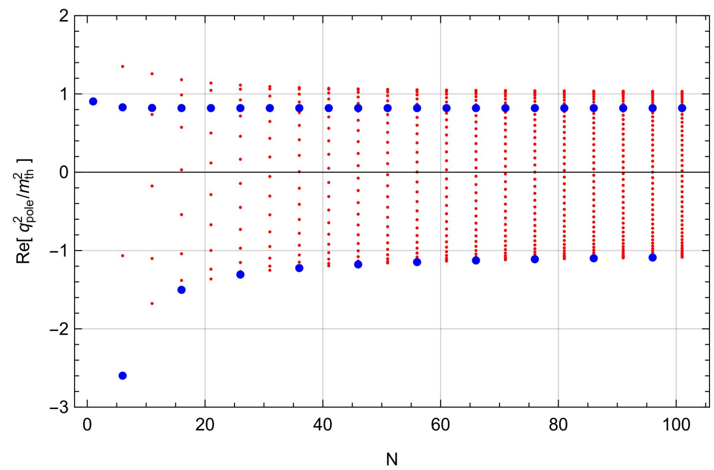


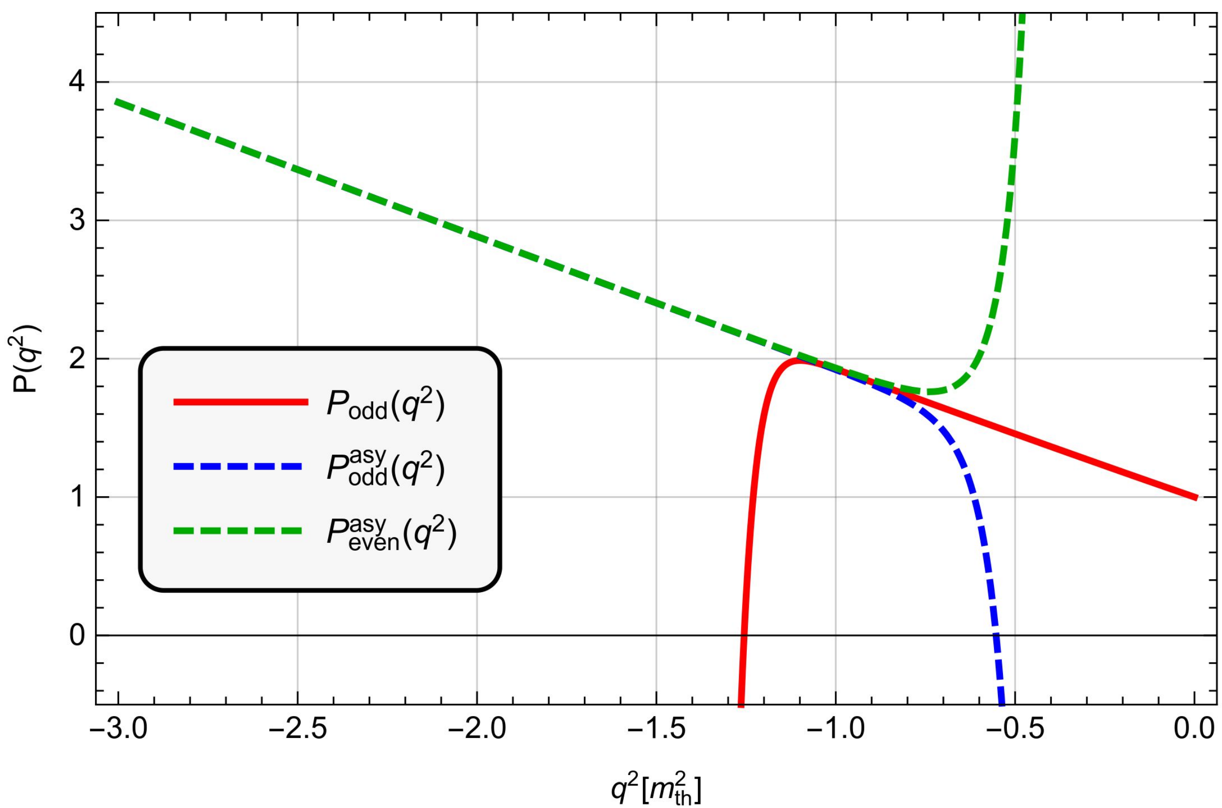


Ghost of the full theory
→ persistent **negative residue**

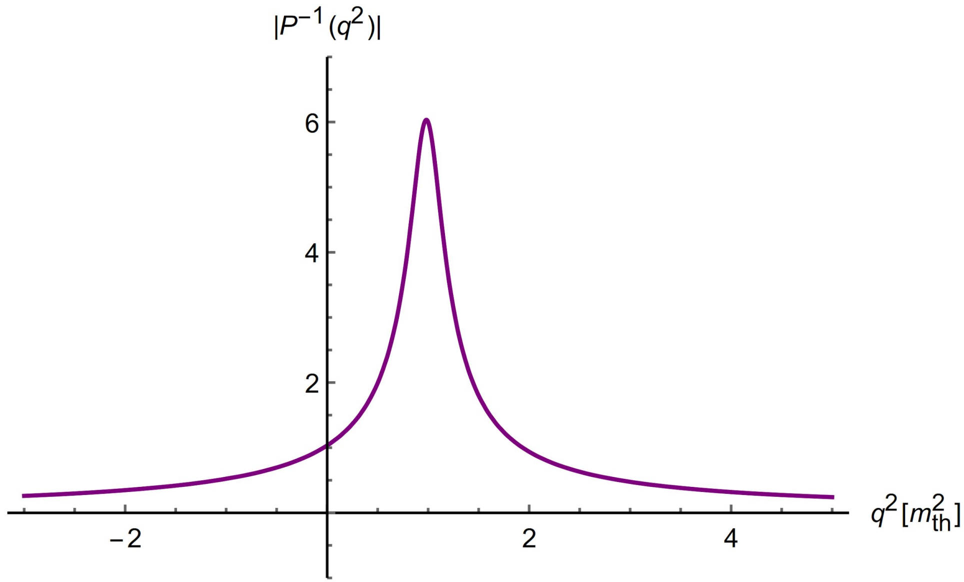
Fake ghost (generated by convergence properties of P(z))
→ **residue approaches zero**

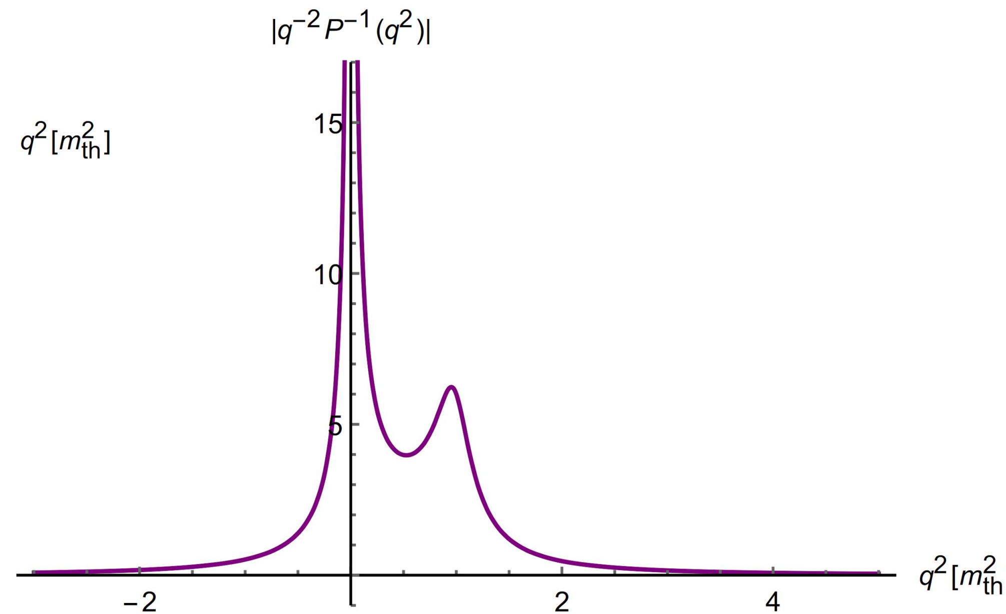
Summary

- We discussed **unitarity** from the point of view of the Functional Renormalization Group
- Including ***all quantum fluctuations*** is crucial for unitarity: it determines what states appear in the sum over states in the **optical theorem**
- **Truncations** / derivative expansion of the action ⇒ **fictitious poles**
- The fictitious pole is however a fake ghost: its **residue approaches zero** when a sufficiently large number of terms in the action are included.
- **Stable ghosts** in the full theory are instead characterized by a **persistent negative residue**.
- Most reliable instrument to check *unitarity in full glory*: fully-quantum effective action



4
3
2
1
0
P(q^2)
$P_{\rm odd}(q^2)$
$P_{\rm odd}^{\rm asy}(q^2)$
$P_{\rm even}^{\rm asy}(q^2)$
−3.0
−2.5
−2.0
−1.5
−1.0
−0.5
0.0
$q^2[m_{\rm th}^2]$


$|P^{-1}(q^2)|$
6
4
2
−2
2
4
$q^2\,[m_{\mathrm{th}}^2]$


$|q^{-2}P^{-1}(q^2)|$
15
10
5
−2
2
4
$q^2\,[m_{\mathrm{th}}^2]$