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Stability of electroweak vacuum in a scale invariant extension of the SM

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arXiv:1402.3826 (JHEP), arXiv:1505.05505, arXiv:1605.06713 (PRD),
arXiv:1411.6435 (PRD), arXiv:1606.07808 (JHEP), arXiv:1508.03297 (JHEP)
arXiv:1608.05719 (JCAP), arXiv:1711.08473 [astro-ph.CO] (JHEP)

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Outline:

- SM effective potential, tunneling and lifetime
- SM + dilaton
- Domain walls and gravitational waves
- Summary

SM Effective potential

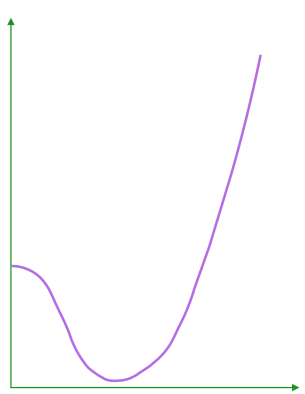
Standard Model Effective potential

$$V_{SM}(\mu) = -\frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \sum_i \frac{n_i}{64\pi^2} M_i^4 \left[\ln\left(\frac{M_i^2}{\mu^2}\right) - C_i\right]$$

For large field values $m^2 << \phi^2$ and $\mu = \phi$ the potential is very well approximated by

$$V_{SM}(\phi) \approx \phi^4 \left\{ \frac{\lambda}{4} + \frac{1}{64\pi^2}\left[6\left(\frac{g_2^2}{4}\right)^2\left(\ln\left(\frac{g_2^2}{4}\right) - \frac{5}{6}\right) + 3\left(\frac{g_1^2+g_2^2}{4}\right)^2\left(\ln\left(\frac{g_1^2+g_2^2}{4}\right) - \frac{5}{6}\right) \right.\right.$$
$$\left.\left. -12\left(\frac{y_t^2}{2}\right)^2\left(\ln\left(\frac{y_t^2}{2}\right) - \frac{3}{2}\right) + \left(\frac{3\lambda}{2}\right)^2\left(\ln\left(\frac{3\lambda}{2}\right) - \frac{3}{2}\right) + 3\left(\frac{\lambda}{2}\right)^2\left(\ln\left(\frac{\lambda}{2}\right) - \frac{3}{2}\right) \right]\right\}$$

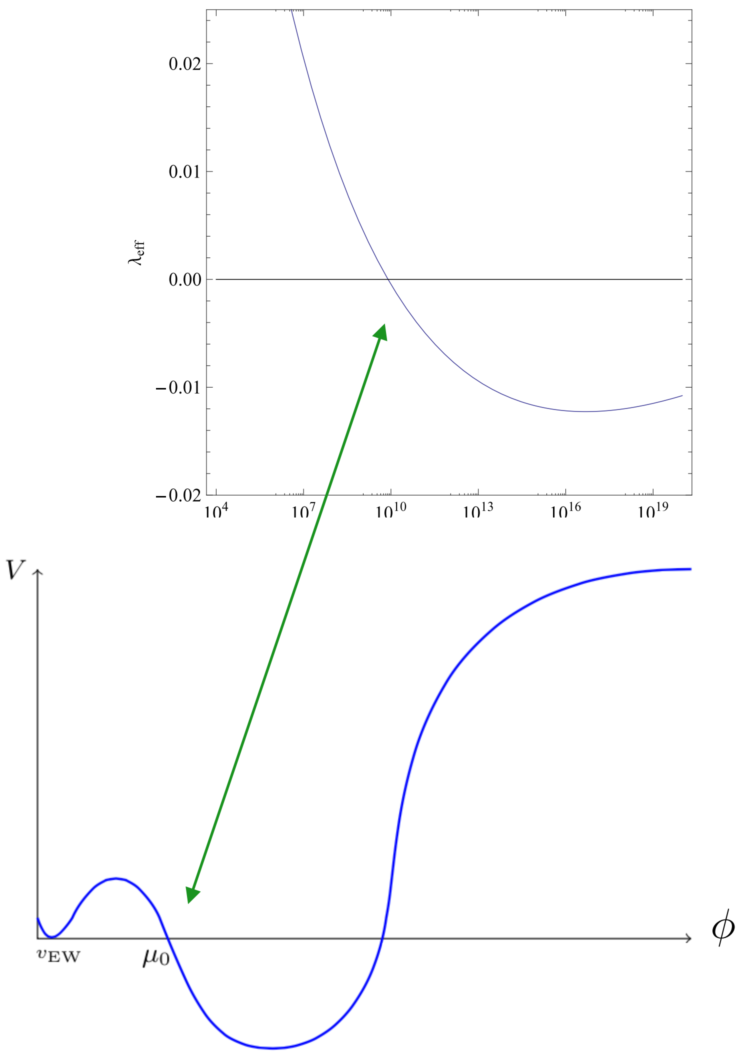
$$V_{SM}(\phi) \approx \frac{\lambda_{eff}(\phi)}{4}\phi^4$$

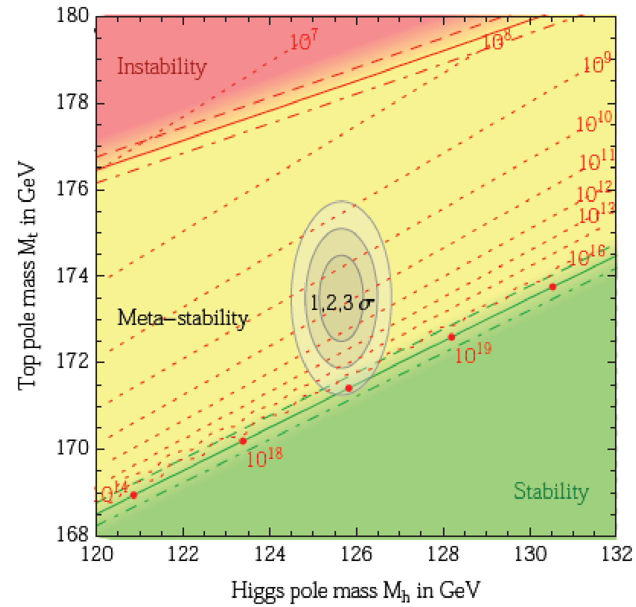


← classically quantum corrected ... →

SM Metastability

$$\lambda_{eff} < 0 \implies \text{Metastability}$$





D. Buttazzo, et al. [arXiv:1307.3536].
G. Degrassi, et al. [arXiv:1205.6497].

See lectures by G. Degrassi Corfu 2014

Tunneling

Standard semiclassical formalism

S. R. Coleman, Phys. Rev. D **15** (1977) 2929.
C. G. Callan, Jr. and S. R. Coleman, Phys. Rev. D **16** (1977) 1762.

$O(4)$ symmetric solution to euclidean equation of motion

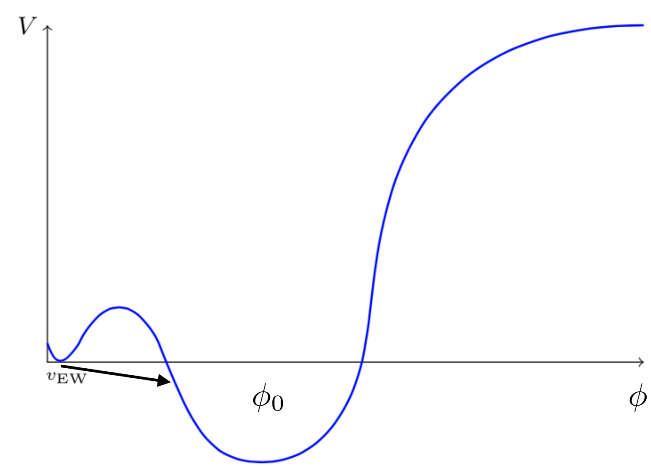
$$\ddot{\phi} + \frac{3}{s}\dot{\phi} = \frac{\partial V(\phi)}{\partial \phi},$$

$$s = \sqrt{\vec{x}^2 + x_4^2}.$$

with

- $\dot{\phi}(s = 0) = 0$ near the true vacuum
- $\phi(s = \infty) = \phi_{min}$ at the false vacuum

$= v_{EW}$



Tunneling

Action of the bounce solution

$$S_E = \int d^4x \left\{ \frac{1}{2} \sum_{\alpha=1}^{4} \left(\frac{\partial \phi(\mathbf{x})}{\partial x^\alpha} \right)^2 + V(\phi(\mathbf{x})) \right\}$$

$$= 2\pi^2 \int ds s^3 \left(\frac{1}{2} \dot{\phi}^2(s) + V(\phi(s)) \right),$$

allows us to calculate decay probability dp of a volume d^3x

$$dp = dt d^3x \frac{S_E^2}{4\pi^2} \left| \frac{det'[-\partial^2 + V''(\phi)]}{det[-\partial^2 + V''(\phi_0)]} \right|^{-1/2} e^{-S_E}.$$

Simplifying

- normalisation factor replaced with width of the barrier $\propto \phi_0$
- size of the universe is $T_U = 10^{10}$yr

we can calculate the lifetime of the false vacuum ($p(\tau) = 1$)

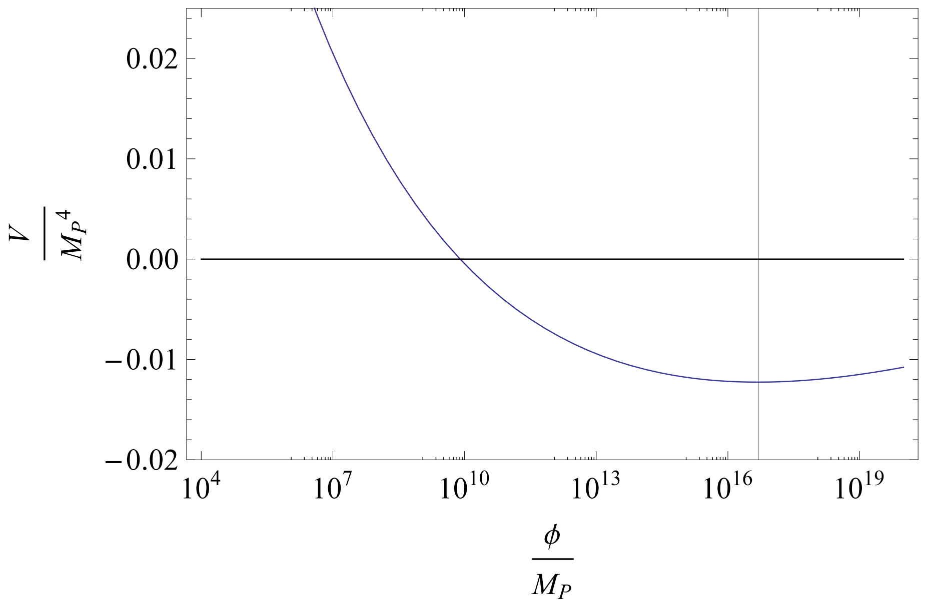
$$\frac{\tau}{T_U} = \frac{1}{\phi_0^4 T_U^4} e^{S_E}.$$

Standard Model

Approximating by a quartic potential:

$$\frac{\tau}{T_U} = \frac{1}{\phi^4(\lambda_{min})T_U^4} e^{\frac{8\pi^2}{3|\lambda_{min}|}} \approx 10^{540}.$$

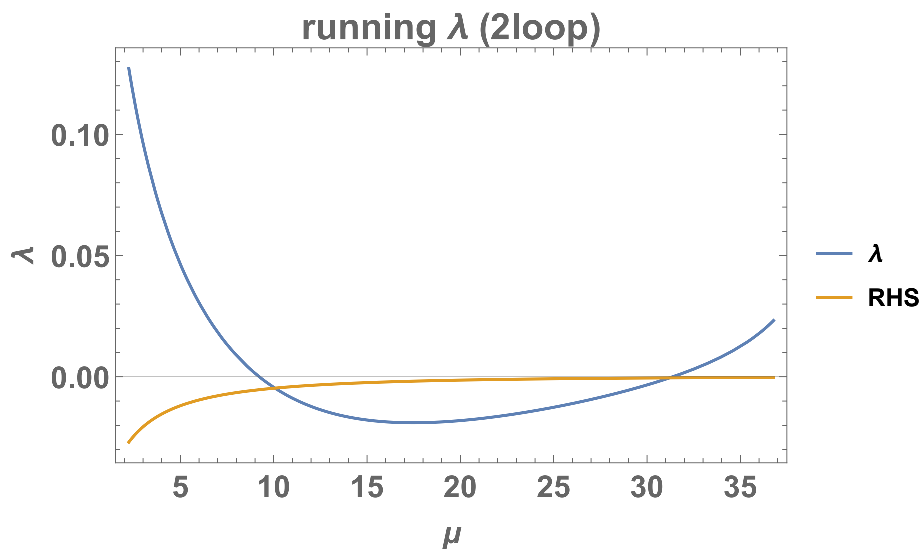
lifetime is minimal for ϕ that minimizes $\lambda_{eff}(\phi)$.



New extrema created by quantum corrections (Coleman-Weinberg mechanism)

condition for cancellation of corrections to the derivative of SM

$$\lambda = \frac{\hbar}{256\pi^2}\left[g_1^4 + 2g_1^2g_2^2 + 3g_2^4 - 48h_t^4 - 3(g_1^2+g_2^2)^2\log\frac{g_1^2+g_2^2}{4} - 6g_2^4\log\frac{g_2^2}{4} + 48y_t^4\log\frac{y_t^2}{2}\right]$$



Hence sensitivity to New Physics

Effective potential with nonrenormalisable interactions

We add new nonrenormalisable couplings
(similar to V. Branchina and E. Messina, [arXiv:1307.5193].)

$$V \approx \frac{\lambda_{eff}(\phi)}{4}\phi^4 + \frac{\lambda_6}{6!}\frac{\phi^6}{M_p^2} + \frac{\lambda_8}{8!}\frac{\phi^8}{M_p^4}.$$

New Physics at Planck scale

That modify the potential around the Planck scale:

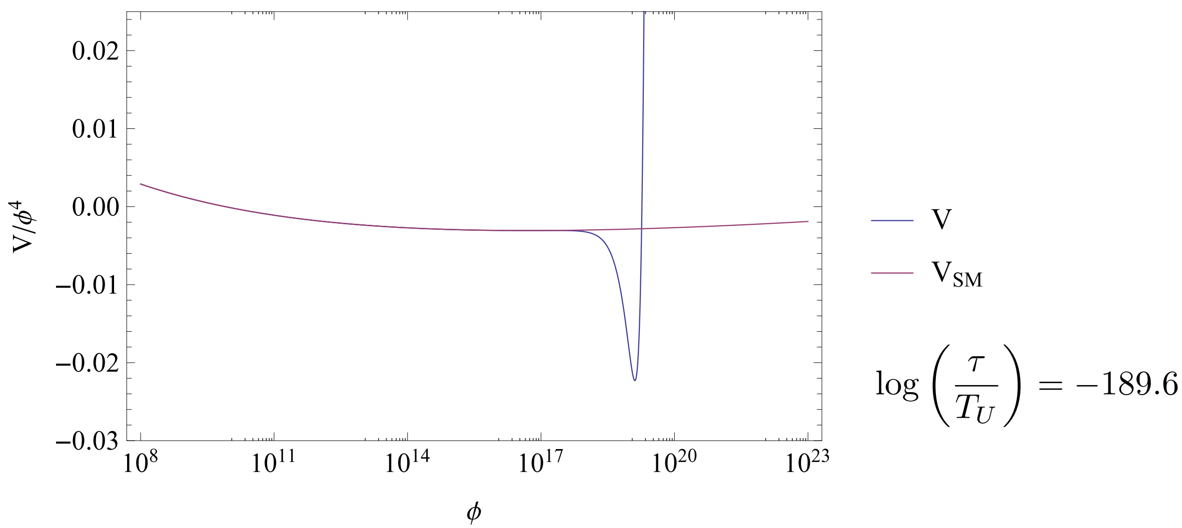


Figure: effective potential with $\lambda_6 = -1$ and $\lambda_8 = 1$.

Magnitude of the suppression scale

Approximate lifetime:

$$\frac{\tau}{T_U} = \frac{1}{\mu^4(\lambda_{min})\, T_U^4} e^{\frac{8\pi^2}{3|\lambda_{min}|}}.$$

Positive λ_6 and $\lambda_8 \rightarrow$ stabilizing the potential

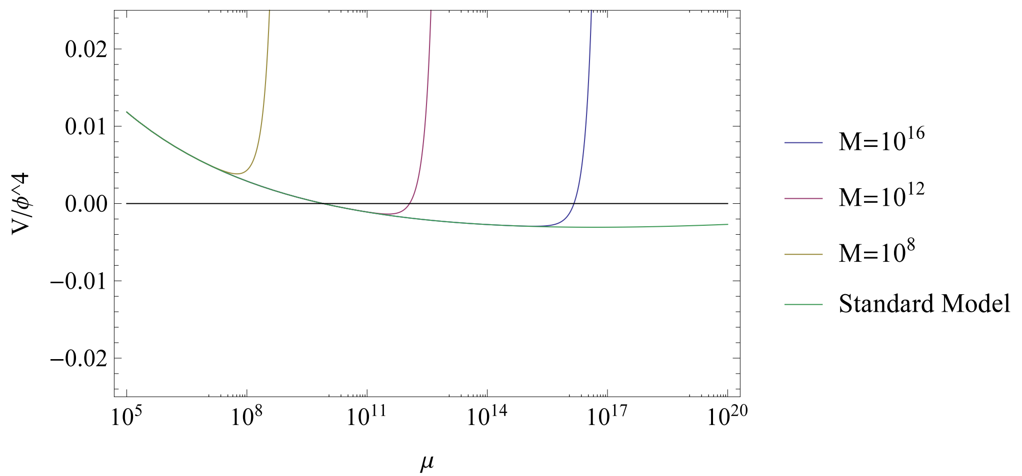


Figure: Scale dependence of $\frac{\lambda_{eff}}{4} = \frac{V}{\phi^4}$ with $\lambda_6 = \lambda_8 = 1$ for different values of suppression scale M. The lifetimes corresponding to suppression scales $M = 10^8, 10^{12}, 10^{16}$ are, respectively, $\log_{10}(\frac{\tau}{T_U}) = \infty, 1302, 581$ while for the Standard Model $\log_{10}(\frac{\tau}{T_U}) = 540$.

Magnitude of the suppression scale

Positive λ_8 and negative λ_6 $\rightarrow$ New Minimum

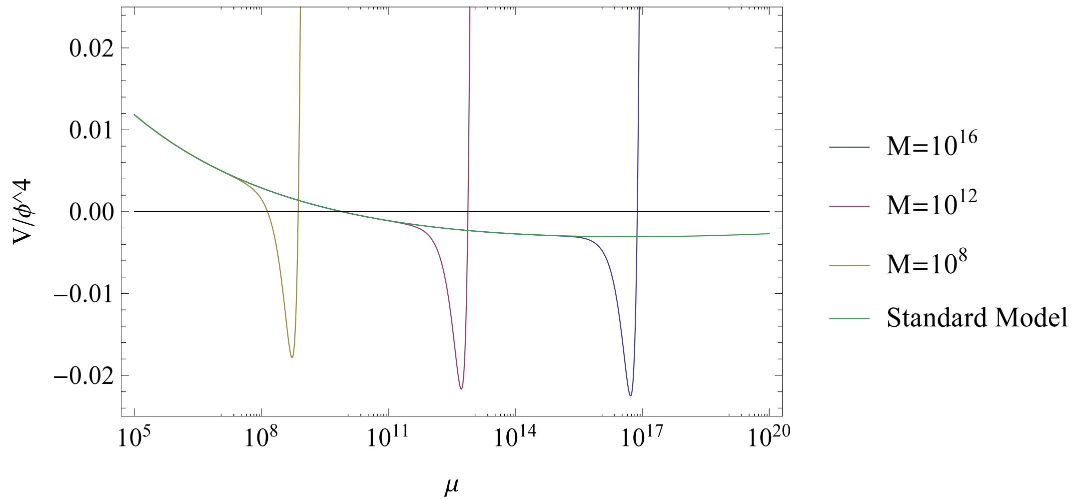


Figure: Scale dependence of $\frac{\lambda_{eff}}{4} = \frac{V}{\phi^4}$ with $\lambda_6 = -1$ and $\lambda_8 = 1$ for different values of suppression scale M. The lifetimes corresponding to suppression scales $M = 10^8, 10^{12}, 10^{16}$, are, respectively, $\log_{10}(\frac{\tau}{T_U}) = -45, -90, -110$ while for the Standard Model $\log_{10}(\frac{\tau}{T_U}) = 540$.

SM + dilaton

Quantum Scale Invariant SM

Additional singlet scalar

$$\mathcal{L} = \frac{1}{2}(\partial h)^2 + \frac{1}{2}(\partial S)^2 - \frac{m^2}{2}h^2 - \frac{M^2}{2}S^2 + \frac{\lambda_{hS}}{4}h^2 S^2 + \frac{\lambda}{4}h^4 + + \frac{\lambda_S}{4}S^4$$

$$\lambda' = \frac{1}{4\pi^2}\left(24\lambda^2 - 6y_t^4 + 12\lambda y_t^2 + \lambda_{hS}^2 + \frac{9}{8}g_2^4 + \frac{3}{8}g_1^4 + \frac{3}{4}g_2^2 g_1^2 - 9\lambda g_2^2 - 3\lambda g_1^2\right)$$

$$\lambda'_S = \frac{1}{4\pi^2}\left(10 * \lambda_S^2 + \lambda_{hX}^2\right)$$

$$\lambda'_{hS} = \frac{\lambda_{hS}}{4\pi^2}\left((2\lambda_{hS} + 6\lambda + 4\lambda_S + 3y_t^2 - \frac{9}{4}g_2^2 - \frac{3}{4}g_1^2\right)$$

$$- \mathcal{M}^2 = \begin{pmatrix} -m^2 + 3\lambda v_0^2 & \lambda_{hS} v_0 v_S \\ \lambda_{hS} v_0 v_S & -M^2 + 3\lambda_S v_S \end{pmatrix}$$

$$v_S^2 = v_0^2 / \tan^2 \beta \qquad m_h = 125.09\text{GeV}$$

Additional singlet scalar

$$\mathcal{L} = \frac{1}{2}(\partial h)^2 + \frac{1}{2}(\partial S)^2 - \frac{m^2}{2}h^2 - \frac{M^2}{2}S^2 + \frac{\lambda_{hS}}{4}h^2S^2 + \frac{\lambda}{4}h^4 + + \frac{\lambda_S}{4}S^4$$

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$$v_S^2 = v_0^2/\tan^2\beta \qquad m_h = 125.09\text{GeV}$$

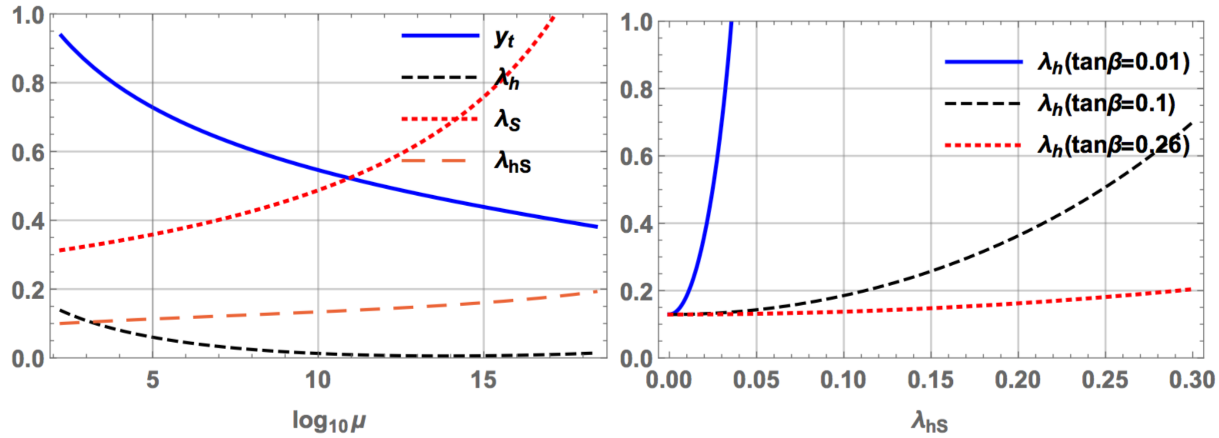


FIG. 1: Left panel shows an example of running of λs in our model. Right panel shows Higgs λ as a function of λ_{hS}.

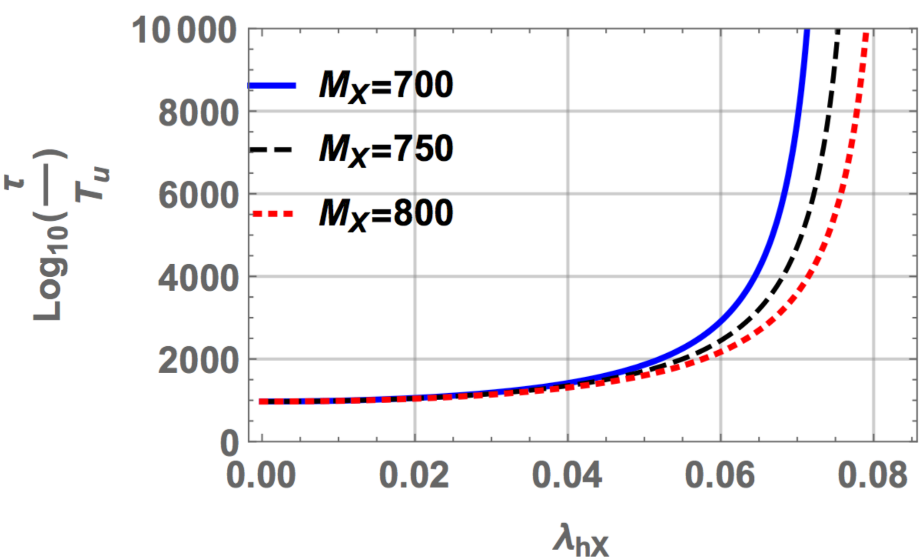


FIG. 2: lifetime of the vacuum as a function of λ_{hS}.

Quantum scale symmetric effective lagrangian

See also papers by G.G. Ross and D. Ghilencea

No scale anomaly in

„dynamical" regulator

$$\mathcal{L}^{(0)}(\phi,\sigma) = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\sigma)^2 - \mu^{2\varepsilon}(\sigma)\left[V(\phi,\sigma) + \sum_{n=0}\lambda_n \frac{\phi^{4+2n}}{\sigma^{2n}}\right]$$

renormalizable, classicaly scale-invariant

$\mathbb{Z}^2 \times \mathbb{Z}^2$
$\phi \to -\phi$
$\sigma \to -\sigma$

go to *broken phase*

eg. $\mu = e^t\,\sigma^{\frac{1}{1-\epsilon}}$

$$\mathcal{L}^{(0)}(\phi_0+\phi',\sigma_0+\phi')$$

compute loop corrections (in momentum expansion) & RGE functions *β, γ*

$$\mathcal{L}_{\text{eff}}(\phi,\sigma) = -V_{\text{eff}}(\phi,\sigma) + \dots$$

- Homogenous function (no mass-parameters, only vev's)
- $\mathbb{Z}^2 \times \mathbb{Z}^2$ sym.
- Satisfies Callan-Symanzik eq.

Quantum scale symmetric effective lagrangian

RG-improvement:

$$\mu = e^{t}\mu_0 \,, \qquad \lambda(t)\,\phi^4 + \frac{\lambda^2(t)\phi^4}{64\pi^2}\log\left(\frac{\phi}{e^t\sigma}\right)^2 + \dots$$

Choose

$$t = t(\phi,\sigma) \sim \log\frac{\phi}{\sigma}$$

to avoid large logs.

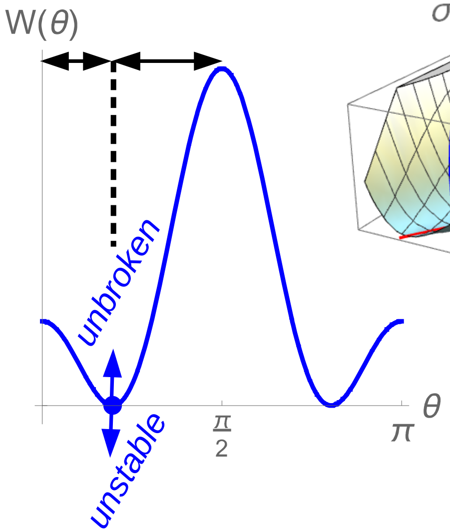
Spontanous scale-symmetry breaking:

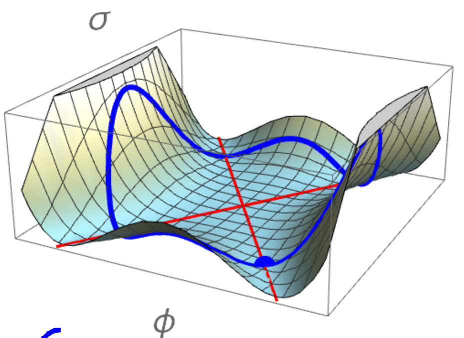
$$\begin{pmatrix}\phi\\ \sigma\end{pmatrix} = M\begin{pmatrix}\sin\theta\\ \cos\theta\end{pmatrix}\,, \quad V_{\rm eff} = M^4\,W(\theta)\,,$$

flat direction in $V_{\rm eff}$ $\Rightarrow$

$$\exists_{\theta=\theta_0}\, W(\theta_0) = W'(\theta_0) = 0$$

renormalization condition,
similar to choosing C.C.





- **Hierarchy** of scales via **aligning** the flat direction $\perp\,\phi$ → $\theta_0 \approx \dfrac{\phi_0}{\sigma_0} \ll 1$
- New perspective on **naturalness**: is this alignement stable wrt. embedding in a UV completion?

Standard Model

$$\mathcal{L}_{SM'+\sigma} = \mathcal{L}_{scalar} + \mathcal{L}_{top} + \mathcal{L}_{gauge} + \mathcal{L}_{\text{gauge-fixing}} + \mathcal{L}_{other}$$

$$\mu = \mu(t,\sigma) = \mathrm{e}^{t}\sigma^{\frac{1}{1-\varepsilon}}$$

$$\mathcal{L}_{scalar} = (D_\mu H)^\dagger (D_\mu H) + \frac{1}{2}(\partial\sigma)^2 - V(H^\dagger H, \sigma^2)\mu^{2\varepsilon}$$

$$V = \sum_{n=0}^{\infty} \lambda_{2n} \frac{(H^\dagger H)^n}{\sigma^{2n-4}}\,, \quad H = \begin{pmatrix} \chi^1 + i\chi^2 \\ \varphi + i\chi^3 \end{pmatrix}$$

$$D_\mu H = \left(\partial_\mu - i\frac{\sigma_a}{2}W_\mu^a + iYB_\mu\right) H$$

$$\mathcal{L}_{top} = \overline{Q}_L i\gamma^\mu D_\mu Q_L + \bar{t}_R i\gamma^\mu D_\mu t_R$$

$$+ \left(-y\mu^\varepsilon \overline{Q}_L \widetilde{H} t_R + h.c.\right)\left(1 + \sum_{n=1}^{\infty} \tau_{2n} \frac{(H^\dagger H)^n}{\sigma^{2n}}\right)$$

Standard Model cd

$$\mathcal{L}_{gauge} = -\frac{1}{4g^2}\mu^{-2\varepsilon}\left(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu + \varepsilon^{abc} W^b_\mu W^c_\nu\right)\left(1 + \sum_{n=1}^{\infty} \eta_{2n}\frac{(H^\dagger H)^n}{\sigma^{2n}}\right)$$
$$-\frac{1}{4g'^2}\mu^{-2\varepsilon}\left(\partial_\mu B_\nu - \partial_\nu B_\mu\right)^2\left(1 + \sum_{n=1}^{\infty} \theta_{2n}\frac{(H^\dagger H)^n}{\sigma^{2n}}\right) \tag{6.19}$$

$$\mathcal{L}_{gauge-fixing} = \lim_{\substack{\xi, v\to 0\\ \xi\sim v}}\left[-\frac{1}{\xi}F^+F^- - \frac{1}{2\xi}(F^3)^2 - \frac{1}{2\xi}(F^B)^2\right]\left(1 + \sum_{n=1}^{\infty} \zeta_{2n}\frac{(H^\dagger H)^n}{\sigma^{2n}}\right) \tag{6.20}$$

$$F^\pm \equiv \frac{\mu^{-\varepsilon}}{g}\partial^\mu W^\pm_\mu \mp i\frac{gv\mu^\varepsilon}{2}\chi^\pm\ , \quad F^3 \equiv \frac{\mu^{-\varepsilon}}{g}\partial^\mu W^3_\mu - \frac{gv\mu^\varepsilon}{2}\chi^3$$
$$F^B = \frac{\mu^{-\varepsilon}}{g'}\partial^\mu B_\mu - \frac{g' v\mu^\varepsilon}{2}\chi^3 \tag{6.21}$$

$$W^\pm = \frac{1}{\sqrt{2}}\left(W^1_\mu - W^2_\mu\right)\ , \quad \chi^\pm = \frac{1}{\sqrt{2}}\left(\chi^1 \mp i\chi^2\right) \tag{6.22}$$

Standard Model cd

$$\text{expanding } (\langle\sigma\rangle + \sigma')^{f(\varepsilon)} = \langle\sigma\rangle^{f(\varepsilon)} \left(1 + \tfrac{\sigma'}{\langle\sigma\rangle}\right)^{f(\varepsilon)} \text{ for small } \tfrac{\sigma'}{\langle\sigma\rangle}$$

$$\sim \frac{1}{g^2}\frac{\sigma'}{\langle\sigma\rangle}\left(\partial_\mu W^a_\nu\right)^2 \ , \quad \sim \frac{\sigma'}{\langle\sigma\rangle}\bar{t}_L t_R \varphi$$

$$\sim \frac{(D_\mu H)^\dagger (D_\mu H)\left(H^\dagger H\right)}{\sigma^2} , \quad \sim \frac{(\partial_\mu B_\nu - \partial_\nu B_\mu)^4}{\sigma^4}\mu^{-6\varepsilon} , \quad \sim \frac{\left(\overline{Q}_L \widetilde{H} t_R\right)^2}{\sigma^4}$$

$$\text{but not} \sim \frac{\sigma^6}{H^\dagger H}$$

Chiral symmetry

$$\mu = e^t \sigma$$

$$\frac{dg}{dt} = -\frac{b_0 g^3}{2}$$

$$\Lambda = < \sigma > e^{-\frac{1}{b_0 g^2(t)}}$$

$$\delta L = -\sigma e^{-\frac{1}{b_0 g^2}} \bar{q} q$$

Spontaneous breaking of scale invariance

Standard Model cd

$$\tan\theta_0 = \frac{\langle\varphi\rangle}{\langle\sigma\rangle} \lesssim 10^{-7}$$

$$h = \varphi\cos\theta_0 - \sigma\sin\theta_0 \approx \varphi$$

$$\rho = \sigma\cos\theta_0 + \varphi\sin\theta_0 \approx \sigma$$

$$m_h^2 \approx \mathrm{tr}\, V''(\langle\varphi\rangle, \langle\sigma\rangle)\ , \quad m_\rho^2 = 0$$

Conjecture: a version of QSI SM can be coupled consistently to Weyl gravity, hence all scales, including Planck scale, would be proportional to the vev of sigma and a combination of dimensionless couplings

see

D. M. Ghilencea, H. M. Lee, Phys. Rev. D 99 (2019)
D. M. Ghilencea, JHEP 1903 (2019)
P. G. Ferreira, C. T. Hill, J. Noller and G. G. Ross, arXiv:1906.03415 [gr-qc]

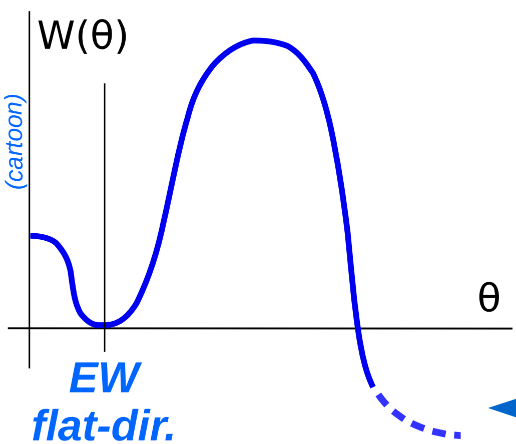
Quantum scale symmetric SM + σ

$$H = \begin{pmatrix} 0 \\ \frac{\phi}{\sqrt{2}} \end{pmatrix}$$

(electroweak vacuum → electroweak flat direction)

$$\mathcal{L}_{SM}\Big|_{\substack{m^2=0 \\ \mu=\mu(\sigma)}} + \frac{1}{2}(\partial\sigma)^2 - \lambda_m |H|^2\sigma^2 - \frac{\lambda_\sigma}{4}\sigma^4 + \sum_{n=0}\lambda_n \frac{|H|^{4+2n}}{\sigma^{2n}}$$

$$V_{\rm eff}^{\rm SM}(\phi,\sigma) \approx \frac{1}{4}\lambda_{\rm eff}\left(\log\frac{\phi}{\sigma}\right)\phi^4 = M^4 \underbrace{\lambda_{\rm eff}(\log\tan\theta)\frac{\tan^4\theta}{(1+\tan^2\theta)^2}}_{W(\theta)}$$



$V_{\rm eff}$ ***is:***

- ***unstable*** → Tunneling via **2-dim instanton** (Coleman's bounce), in the presence of nonrerm. terms.
- ***unbounded below*** → (Even stronger) motivation to stabilise the $V_{\rm eff}$ completely: $\lambda_{\rm eff} \overset{!}{>} 0$

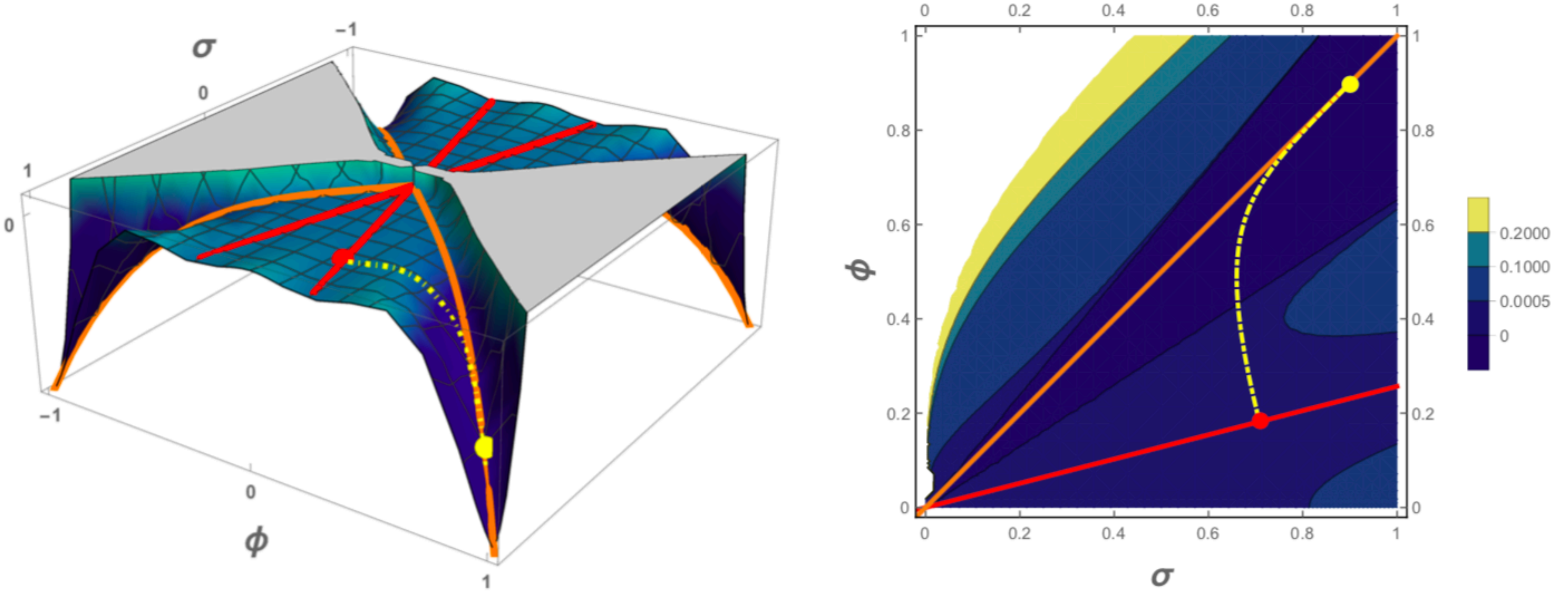


Figure 6.2: The scale-symmetric $V(\varphi, \sigma)$ function (6.6). It is $\mathbb{Z}^2$ symmetric under $\phi \to -\phi$ and $\sigma \to -\sigma$. The red lines mark the *flat directions*. The red point is connected with the yellow point by the bounce field configuration $(\varphi_B(s), \sigma_B(s))$ marked with yellow dashed line. The orange lines correspond to the global minima of $W(\tan\theta)$.

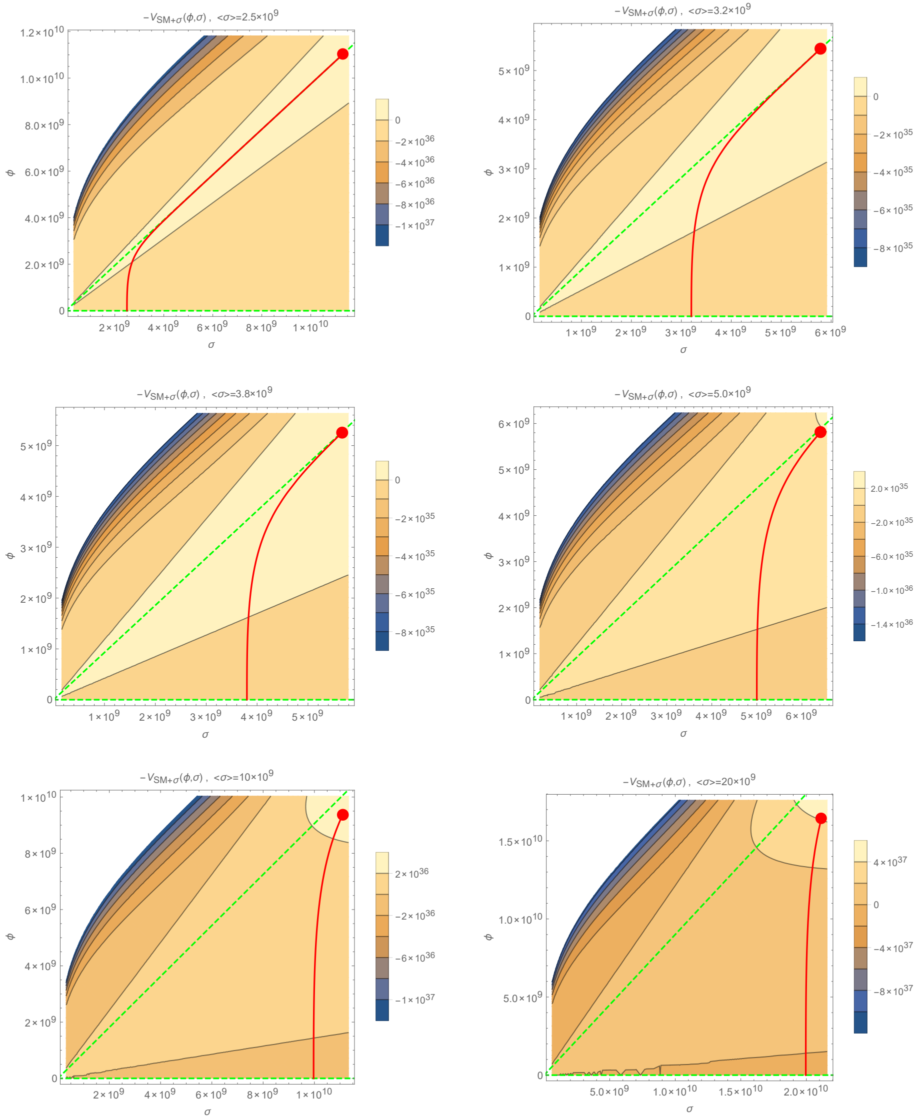


FIG. 2. Contour plots of the effective potentials $-V_{SM+\sigma}(\phi,\sigma)$ for various choices of $\langle\sigma\rangle$. Lower green dashed line marks the electroweak vacuum-direction, higher green dashed line marks the direction of greatest instability. Red continuous line is a plot of the bounce configuration (ϕ_B,σ_B). (Note that, mainly due to varying contribution of the nonrenormalizable interaction from one plot to another, the plots present differing potentials and it would be misleading to plot the bounce configurations in a single frame.)

$M_s[GeV]$	$\lambda_6(0)$	$S_B^{SM+\sigma}$	$S_B^{SM+\lambda_6}$
2.5×10^9	1.7×10^{-4}	1.18×10^6	4.38×10^6
3.2×10^9	2.2×10^{-4}	1.77×10^5	2.24×10^5
3.8×10^9	2.5×10^{-4}	9.13×10^4	1.02×10^5
5×10^9	3.0×10^{-4}	4.62×10^4	4.82×10^4
1×10^{10}	4.2×10^{-4}	1.83×10^4	1.85×10^4
2×10^{10}	5.3×10^{-4}	1.10×10^4	1.11×10^4

TABLE I. Action of tunneling bounce configurations found in the two described setups for six increasing values of $M_s = \langle\sigma\rangle = \Lambda$. This value has a different meaning in the two models. In $SM+\lambda_6$ it is simultaneously the suppression scale Λ in the ϕ^6 interaction and location of the global minimum (*true vacuum*), $M_s = \langle\phi\rangle_{tv}$. In $SM+\sigma$ it is the VEV of σ in the electroweak vacuum ray, $M_s = \langle\sigma\rangle$. The low scale physics described by the models is the same but the electroweak vacuum is less stable in $SM+\sigma$ for values of M_s only slightly above the instability scale.

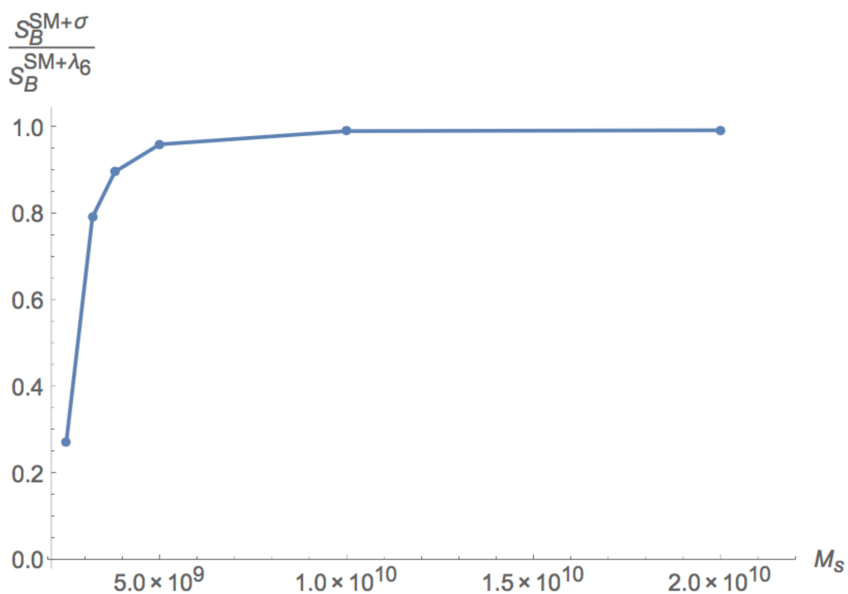


FIG. 3. Ratio of bounce's actions in the two discussed models. When potential in the unstable region becomes dominated by the new ϕ^6 term, there is a discrepancy between the two values of S_B. Compare with FIG. 1. and TABLE I. Note that the value M_S has slightly different meanings in the two setups.

Summary

SM + dilaton

1) You may use **a field as the scale *μ***
   in Dim-Reg to preserve scale symmetry at the quantum level.

2) The price to pay: infinitely many nonpolynomial ϕ/σ operators
   and corresponding couplings: **nonrenormalizability**.

3) Minimal subtraction scheme involves **evanescent interactions**.

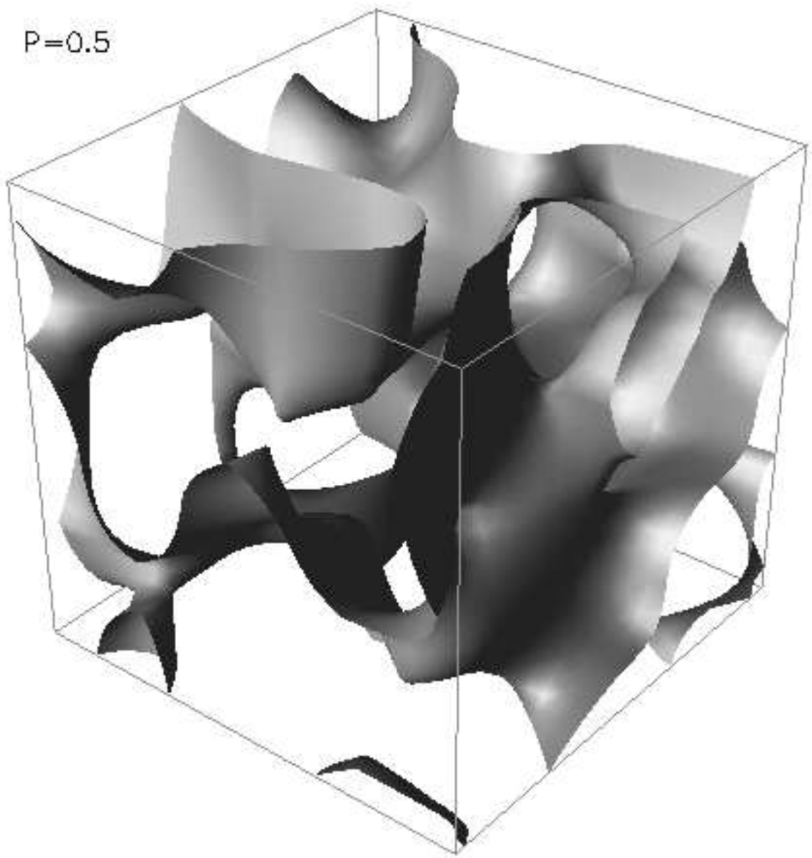
4) Presence of a **flat direction** $\leftarrow$ tuning.

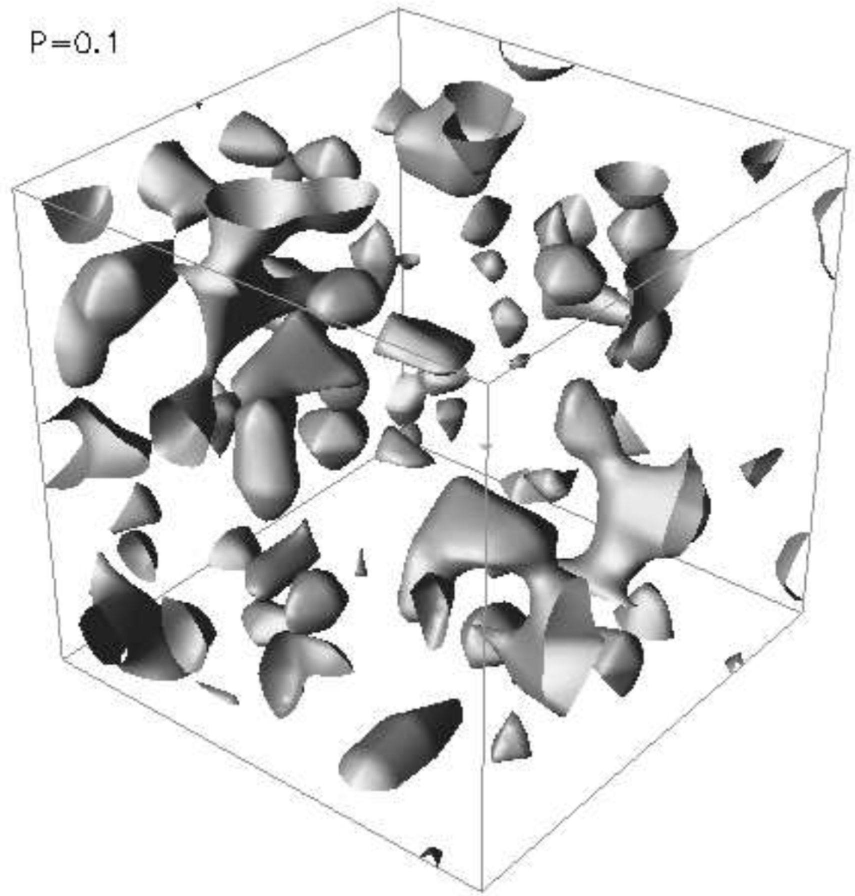
5) **Naturalness: aligning** the flat direction perpendicular to Higgs

6) **Instability = unboundedness below**

Domain walls and gravitational waves

Network of walls prefers the true vacuum!





About our simulation

- We modeled the Higgs field with a positive, real scalar ϕ.
- The evolution of ϕ is given by EOM:

$$\frac{\partial^2 \phi}{\partial \eta^2} + \frac{\alpha}{\eta}\left(\frac{d \ln a}{d \ln \eta}\right)\frac{\partial \phi}{\partial \eta} - \Delta\phi = -a^{\beta}\frac{\partial V}{\partial \phi},$$

  with a potential $V(\phi)$ equal to the RG improved potential of the SM Higgs $V_{SM}(|h|)$.
- The PRS algorithm[2] (with $\alpha = 3$, $\beta = 0$) was used.
- We used the optimization of a time step[3].
- Our simulations were run on a lattice of the size 512^3.

[2] William H. Press, Barbara S. Ryden, and David N. Spergel. "Dynamical Evolution of Domain Walls in an Expanding Universe". In: *Astrophys. J.* 347 (1989), pp. 590–604. DOI: 10.1086/168151.

[3] Z. Lalak, S. Lola, and P. Magnowski. "Dynamics of domain walls for split and runaway potentials". In: *Phys. Rev.* D78 (2008), p. 085020. DOI: 10.1103/PhysRevD.78.085020. arXiv: 0710.1233 [hep-ph].

Initial conditions

Following the general considerations[4] we assumed that the initial distribution of field strength is given by probability distribution:

$$P(h) = \frac{1}{\sqrt{2\pi\sigma_I}} e^{-\frac{(h-\theta)^2}{2\sigma_I^2}} \qquad \sigma_I \sim \frac{\sqrt{N} H_I}{2\pi}$$

We considered various combinations of values of σ and θ in order to cover the set of initial conditions which can be predicted by models of the early Universe.

Our simulations were initialized at different conformal times η_{start} ranging from 10^{-14} GeV^{-1} to 10^{-10} GeV^{-1}.

[4] Z. Lalak et al. "Large scale structure from biased nonequilibrium phase transitions: Percolation theory picture". In: *Nucl. Phys.* B434 (1995), pp. 675–696. DOI: 10.1016/0550-3213(94)00557-U. arXiv: hep-ph/9404218 [hep-ph].

Higgs domain walls after reheating

Higgs domain walls after reheating

- After reheating the early Universe was very hot and dense and it was better described that time by the thermal state with temperature T, than by the vacuum state.
- The dynamics of Higgs domain walls in the background of this thermal state could be different than in the vacuum state.
- The evolution of the domain walls in the cooling down Universe can be determined reliably only in lattice simulations.

Thermal effective potential

$$V_{\text{eff}}(h,T) = \frac{1}{2}m^2h^2 + \frac{1}{4}\lambda h^4 + \sum_{i=\varphi,\chi,W,Z,t} \frac{1}{2}n_i T \sum_{n=-\infty}^{\infty} \int \frac{d^3\vec{k}}{(2\pi)^3} \log\left[\vec{k}^2 + {\omega_n}^2 + {m_i}^2(\phi)\right]$$

$$\begin{aligned}
{m_h}^2(\phi) &= m^2 + 3\lambda h^2, \\
{m_{\chi_i}}^2(\phi) &= m^2 + \lambda h^2, \\
{m_W}^2(\phi) &= \frac{g^2}{4}hi^2, \\
{m_Z}^2(\phi) &= \frac{g^2 + g'^2}{4}h^2, \\
{m_t}^2(\phi) &= \frac{{y_t}^2}{2}h^2,
\end{aligned}$$

$$\begin{aligned}
V_{\text{SM}}(h,T) = \frac{1}{2}m^2h^2 + \frac{1}{4}\lambda h^4 &+ \sum_{i=\varphi,\chi,W,Z,t} n_i \frac{{m_i}^4(h)}{64\pi^2}\left[\log\frac{{m_i}^2(h)}{\mu^2} - C_i\right] \\
&+ \sum_{i=\varphi,\chi,W,Z} \frac{n_i T^4}{2\pi^2} J_b\left(\frac{{m_i}^2(h)}{T^2}\right) + \frac{n_t T^4}{2\pi^2} J_f\left(\frac{{m_t}^2(h)}{T^2}\right) \\
&+ \sum_{i=\varphi,\chi,W,Z,\gamma} \frac{\bar{n}_i T}{12\pi}\left[{m_i}^3(h) - \left({m_i}^2(h) + \Pi_i(T)\right)^{\frac{3}{2}}\right],
\end{aligned}$$

$$J_{b/f}(x) := \int_0^\infty dk k^2 \log\left[1 \mp e^{-\sqrt{k^2+x}}\right]$$

Removal of IR divergencies - resumption of daisy diagrams

- shift by Debye masses

$$\Pi_{\varphi,\chi_i}(T) = \frac{T^2}{4{h_{EW}}^2}({m_\varphi}^2(h) + 2{m_W}^2(h) + {m_Z}^2(h) + 2{m_t}^2(h)),$$

$$\Pi_W(T) = \frac{22}{3}\frac{{m_W}^2(h)}{{h_{EW}}^2}T^2,$$

$${m_{Z/\gamma}}^2 + \Pi_{Z/\gamma}(T)$$

$$\begin{pmatrix} \frac{1}{4}g^2h^2 + \frac{11}{6}g^2T^2 & -\frac{1}{4}{g'}^2g^2h^2 \\ -\frac{1}{4}{g'}^2g^2h^2 & \frac{1}{4}{g'}^2h^2 + \frac{11}{6}g^2T^2 \end{pmatrix}$$

Position of the local maximum

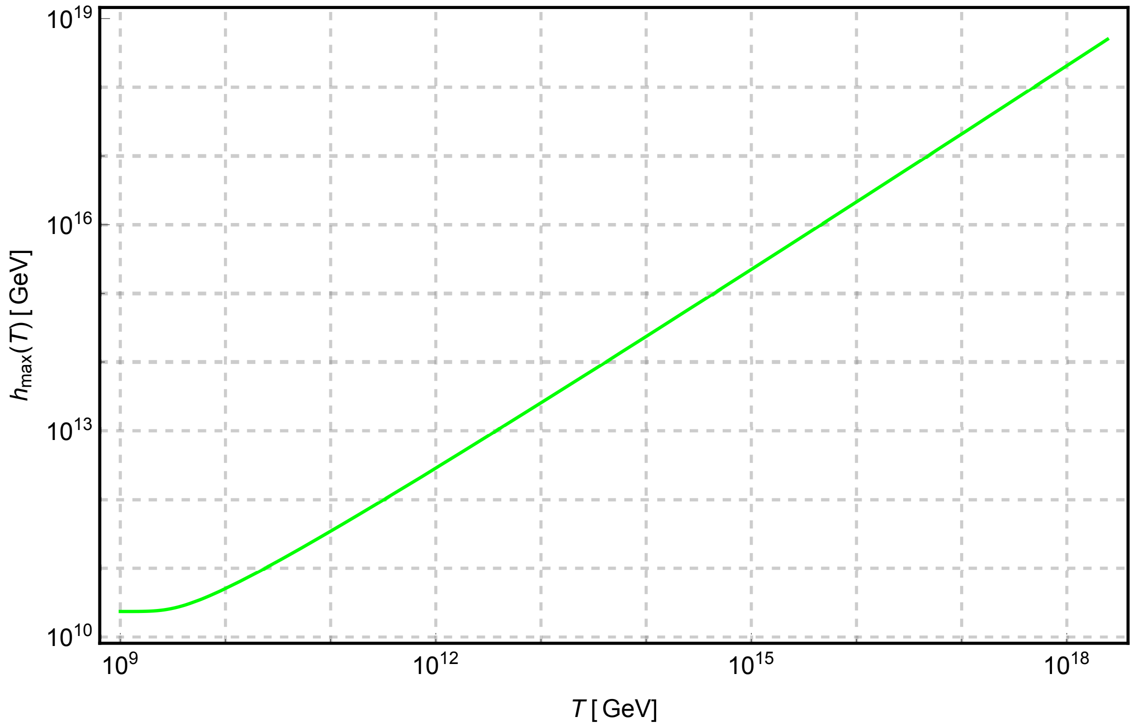


Figure : **The position h_{max} of the local maximum separating two minima of the RG improved effective potential as a function of the temperature of thermal bath T.**

Width of Higgs domain walls

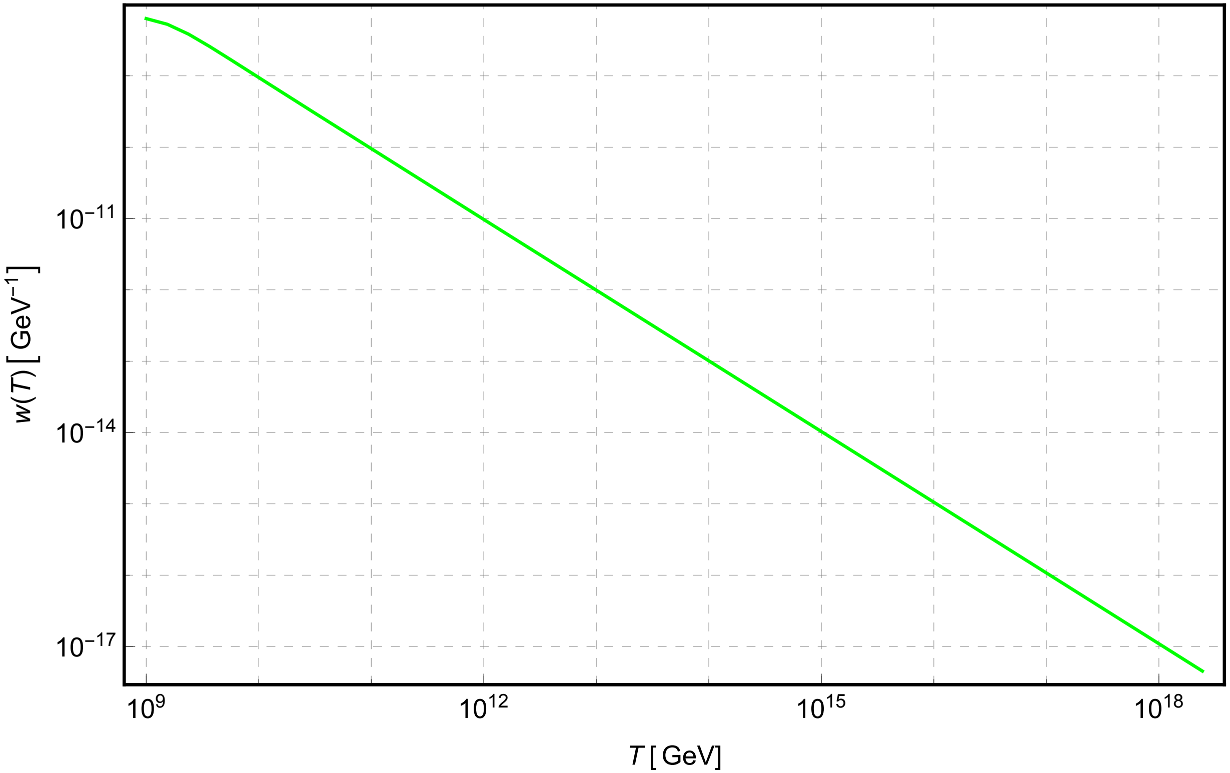


Figure : The width of domain walls w as a function of the temperature T.

Bounds on the standard deviation σ

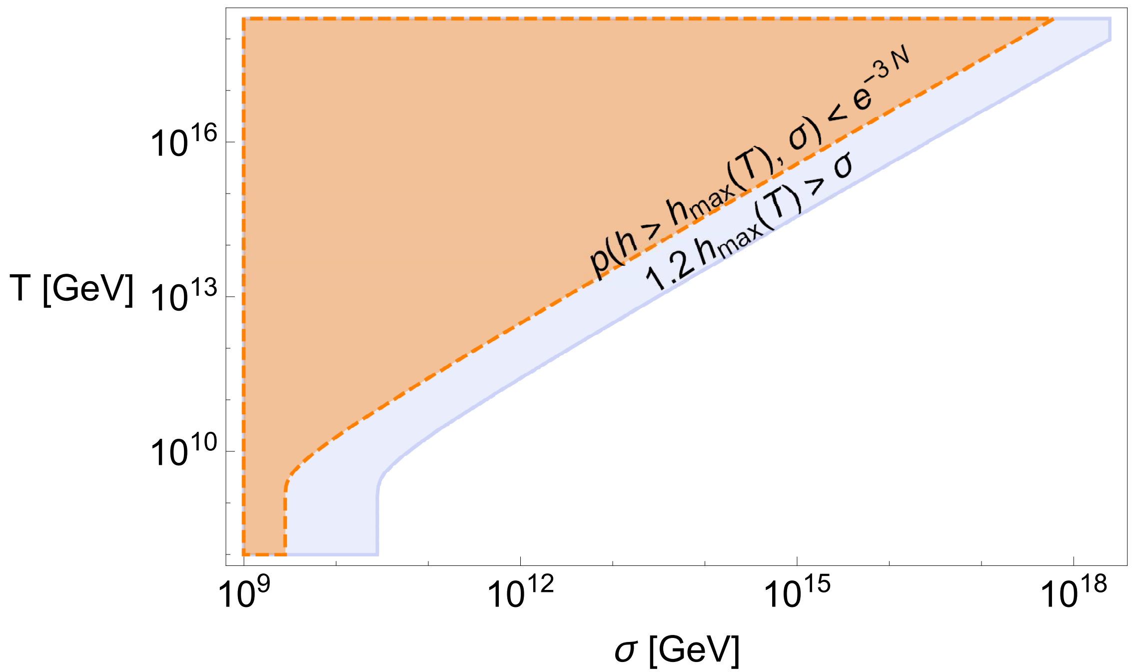


Figure : Maximal value of the standard deviation σ_I of initial distribution for given temperature T.

Bounds on the reheating temperature T_{RH}

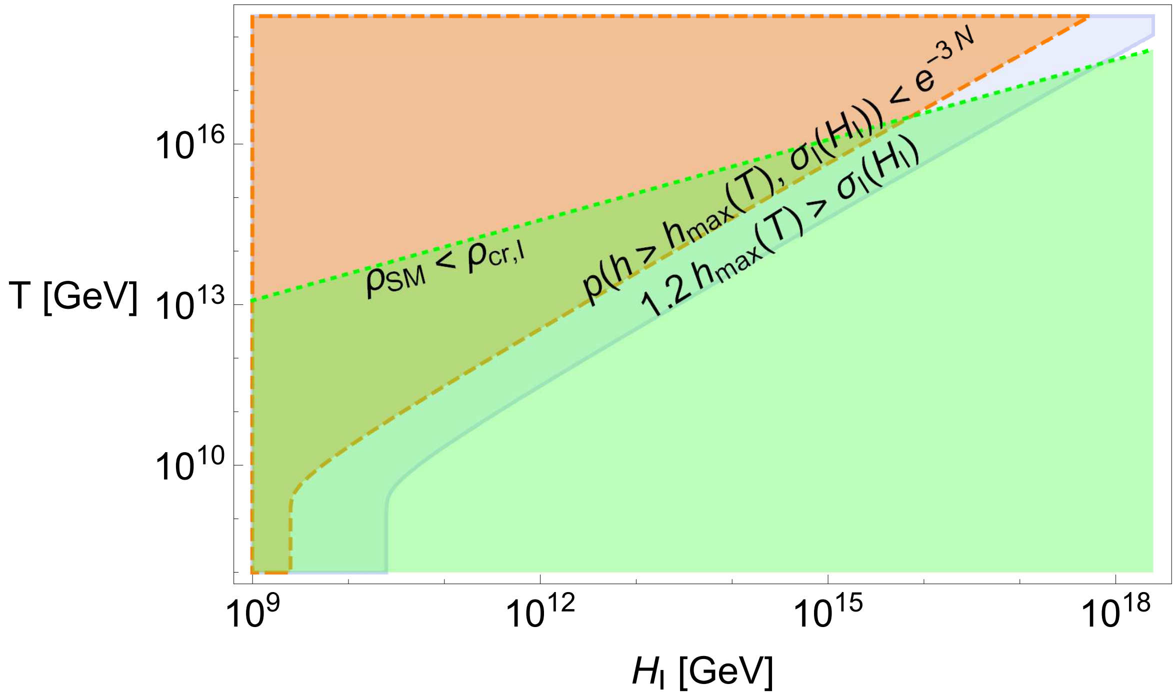


Figure : Maximal value of the reheating temperature from inflation with Hubble parameter value H_I.

Bounds from the evolution of Higgs domain walls

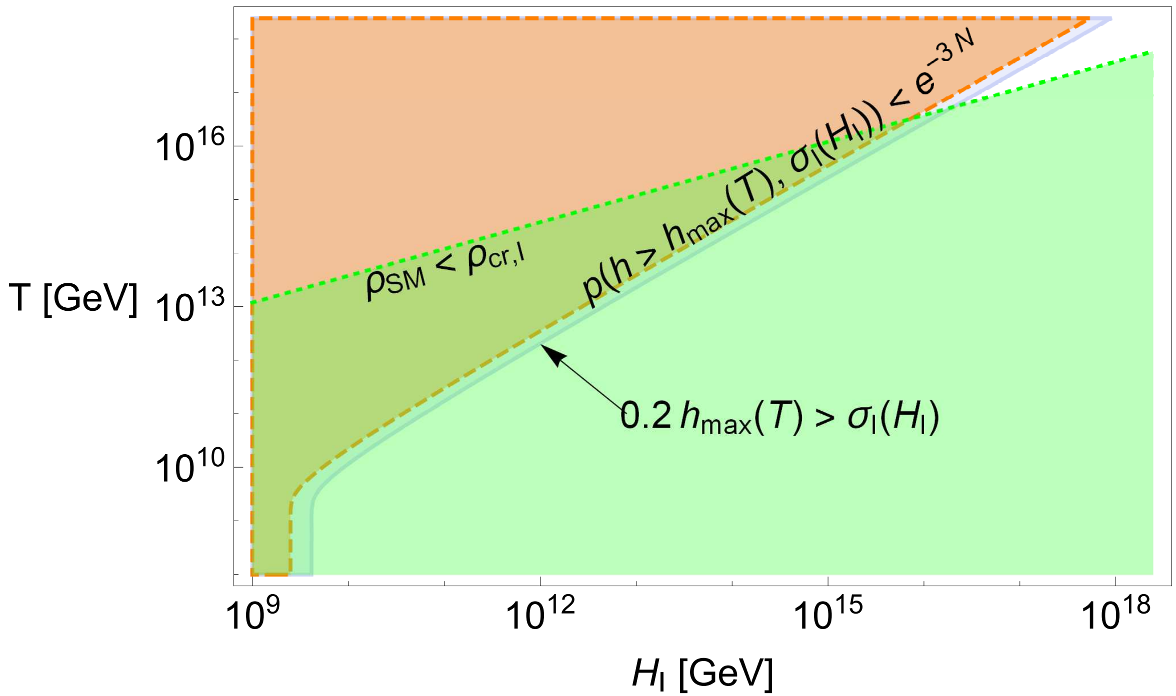


Figure : **Bounds on inflationary models from the evolution of Higgs domain walls.**

Summary - thermal bath

- SM thermal corrections to the effective potential enlarge the basin of attraction of the EWSB vacuum
- Higgs domain walls in the thermal bath are highly unstable

Gravitational waves from domain walls

Gravitational waves from domain walls

Energy density generated by one mode $\varrho_{gw}(\eta, k)$ can be expressed as:

$$\varrho_{gw}(\eta, k) = \frac{1}{16\pi^3 {M_{Pl}}^2 a(\eta)^4 V} \sum_{i,j} \left[\left| \int_{\eta_i}^{\eta_f} d\eta' \cos\left(|k| \left(\eta - \eta'\right)\right) a(\eta') \widehat{T^{TT}}_{ij}(\eta', k) \right|^2 \right.$$
$$\left. + \left| \int_{\eta_i}^{\eta_f} d\eta' \sin\left(|k| \left(\eta - \eta'\right)\right) a(\eta') \widehat{T^{TT}}_{ij}(\eta', k) \right|^2 \right],$$

after redshift

$$\frac{d\rho_{gw}}{d\log|k|}(\eta_0, k) = (1 + z_{EQ})^{-4} \frac{a(\eta_{dec})^4}{a(\eta_{EQ})^4} \frac{d\rho_{gw}}{d\log|k|}(\eta_{dec}, k),$$

$$f_0 = \frac{a(\eta_{dec})}{a(\eta_0)} \frac{k}{2\pi} = 5.07 \times 10^6 \left(\frac{10^{19} \frac{\mathrm{eV}}{\hbar}}{H_{dec}} \right)^{\frac{1}{2}} \left(\frac{k}{10^{10} \ \frac{\mathrm{GeV}}{\hbar c}} \right) \ \mathrm{Hz}.$$

Expectations:

N. Kitajima and F. Takahashi, *Gravitational waves from Higgs domain walls*, *Phys. Lett.* **B745** (2015) 112–117, [1502.03725].

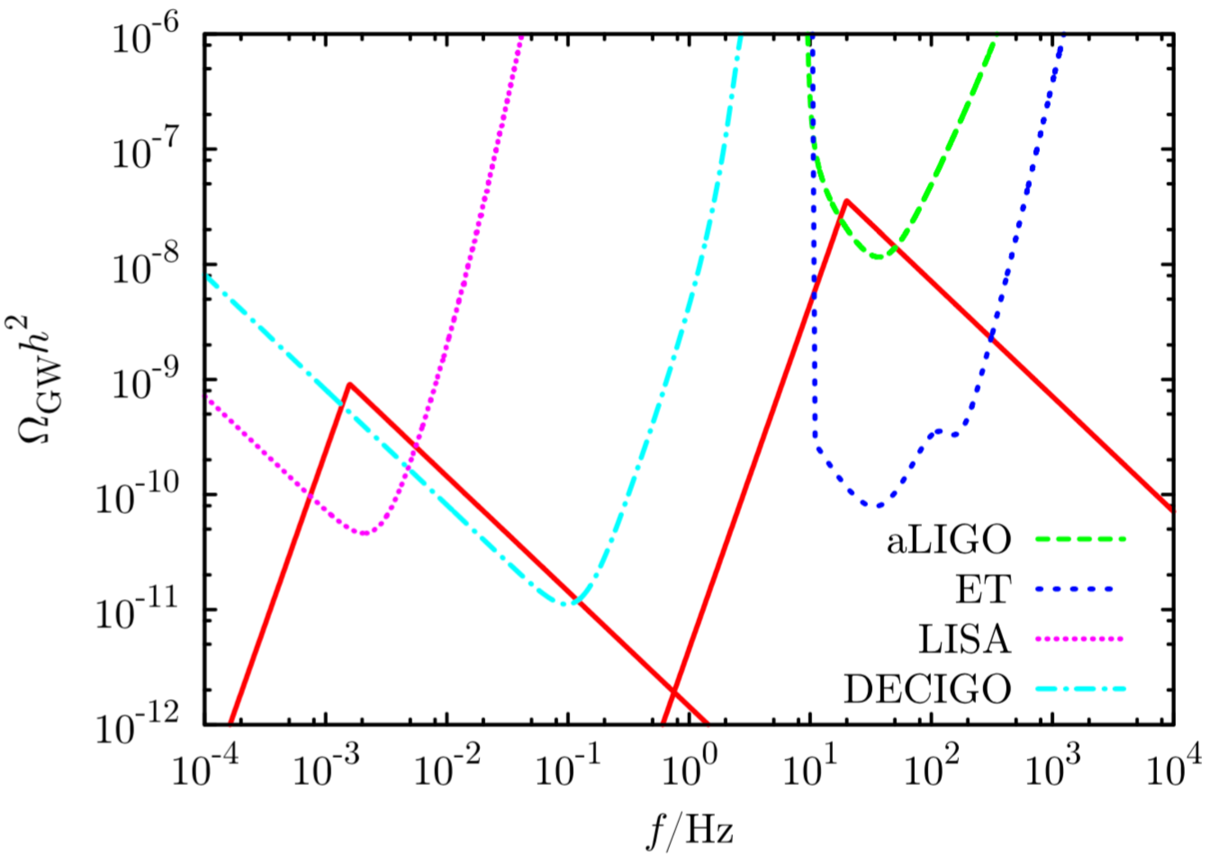


Fig. 3. The typical spectrum of the gravitational waves is shown by the solid (red) lines. We have taken $\varphi_f = 2 \times 10^9$ GeV and $(V_f/V_{\mathrm{max}})^{1/4} = 5 \times 10^{-5}$ for the left line and $\varphi_f = 2 \times 10^{12}$ GeV and $(V_f/V_{\mathrm{max}})^{1/4} = 10^{-3}$ for the right line.

Numerical simulations:

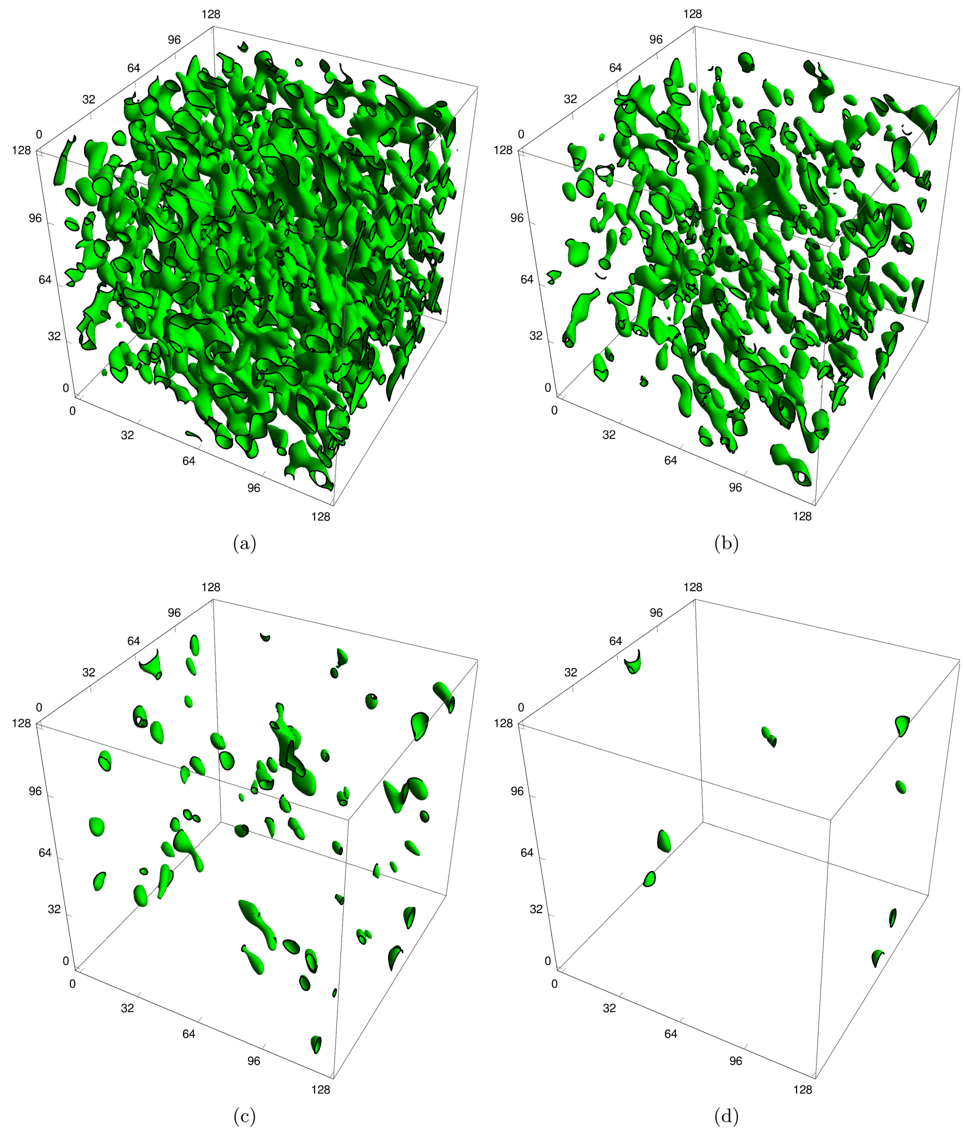


Figure 11: **Visualization of the isosurface of the field strength ϕ corresponding the value v_{max} at four different conformal times: $\eta = 10^{-9}$ GeV^{-1} (a) and $\eta = 1.2 \times 10^{-9}$ GeV^{-1} (b), $\eta = 1.3 \times 10^{-9}$ GeV^{-1} (c), $\eta = 1.4 \times 10^{-9}$ GeV^{-1}(d). Lengths are given in units of the lattice spacing i.e. 10^{-10} GeV^{-1}.**

Spectrum of GWs after emission

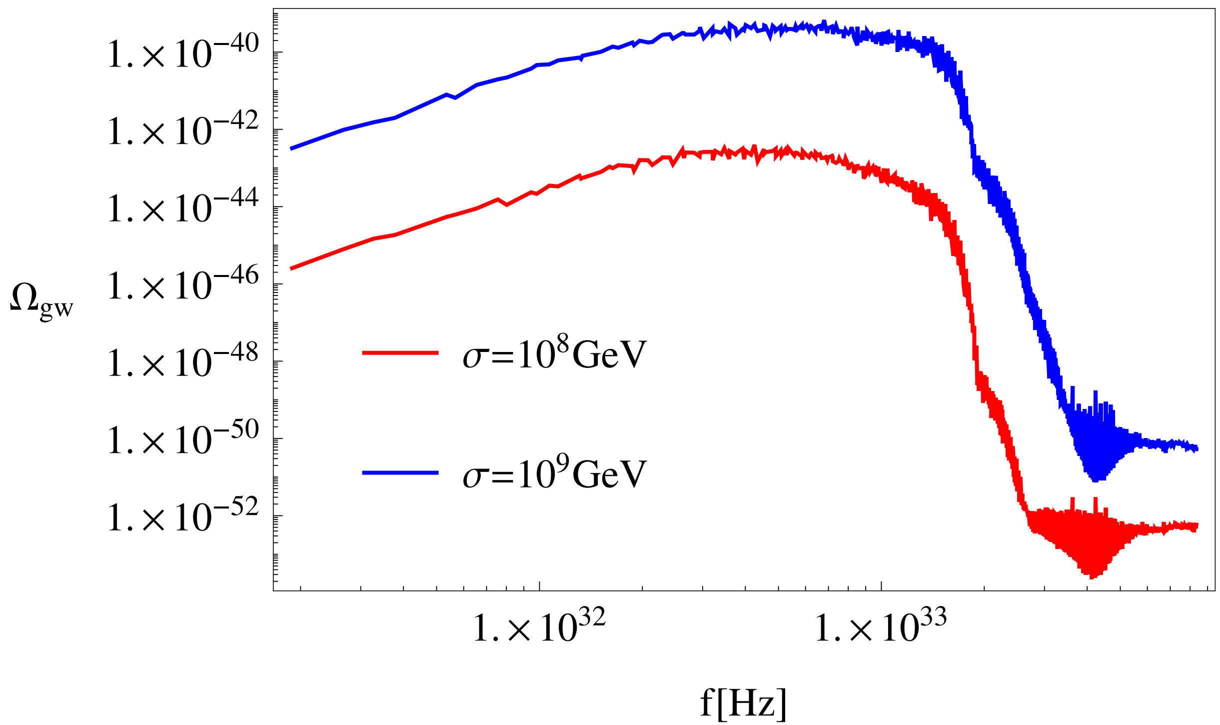


Figure: Spectrum of gravitational waves Ω_{gw} emitted from SM domain walls at the time of the decay.

Present spectrum of GWs

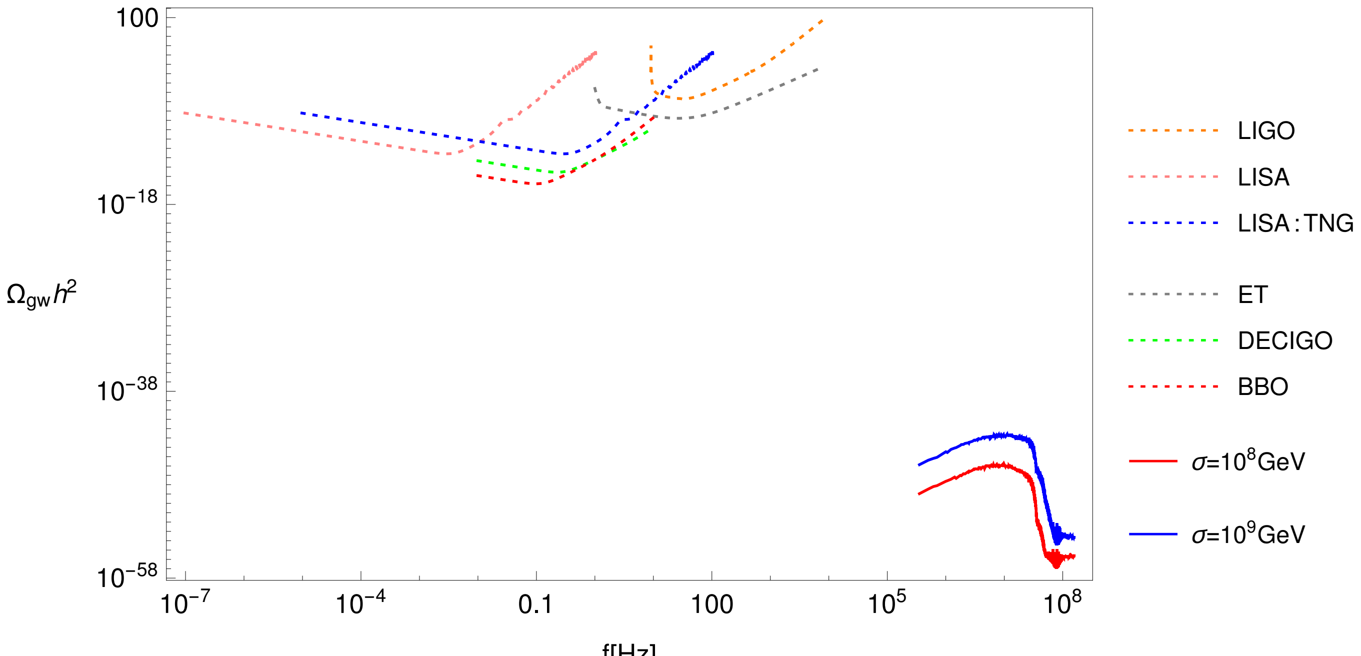
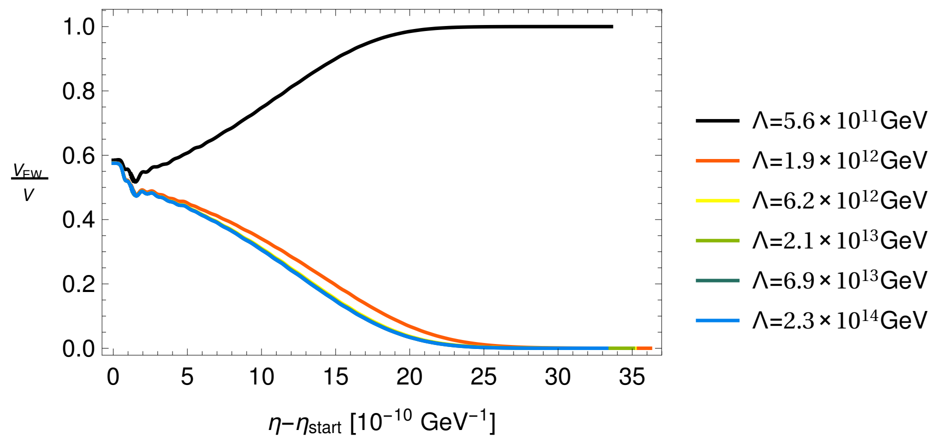
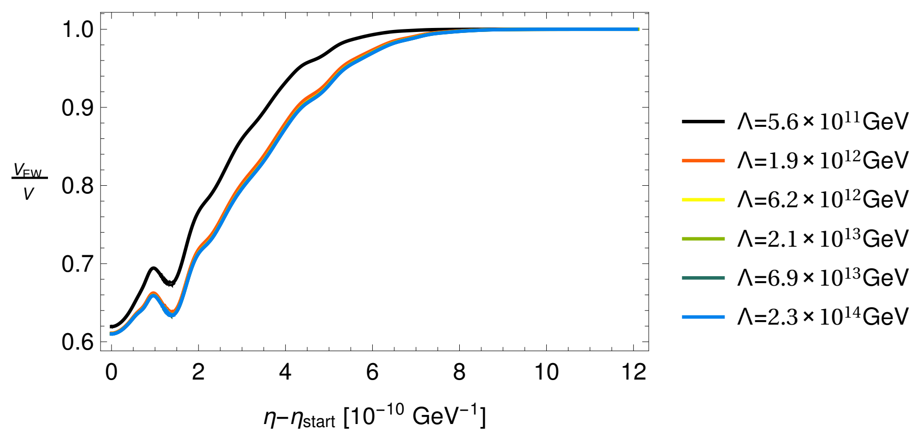


Figure: Predicted sensitivities (dashed) for future GWs detectors: aLIGO, ET, LISA, LISA:TNG, DECIGO and BBO compared with the spectrum of GWs (solid) calculated in lattice simulations for the initial values of $\sigma = 10^8,\ 10^9$ GeV and the standard cosmology.

New Physics and domain walls

New physics $\quad \delta V_{SM}^{\Lambda}(h) = \frac{\lambda_6}{6\,!}\frac{h^6}{\Lambda_{NP}^2}$





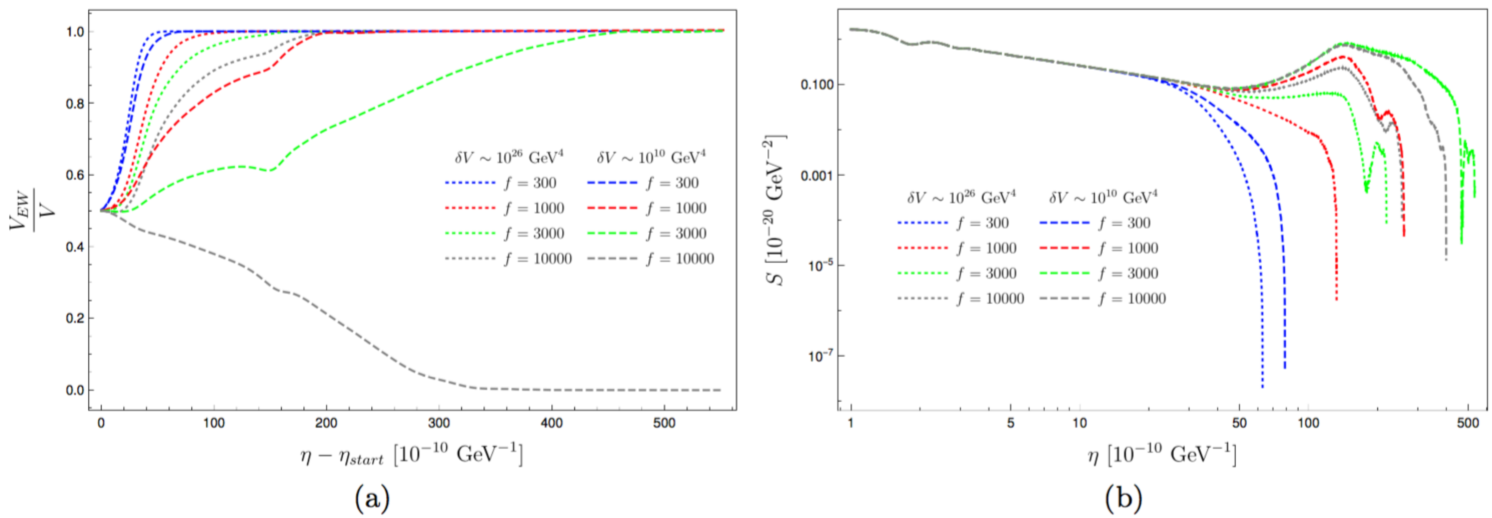


Figure 9: The time dependence of the fraction of lattice sites occupied by the field on the electroweak side of the barrier $\frac{V_{\rm EW}}{V}$ (a) and of the surface area of domain walls per lattice site (b) for two different precision levels of the fine-tuning of the degeneracy of the minima of the effective potential $\delta V = \mathcal{O}\left(10^{26}\ \mathrm{GeV}^4\right)$ (dotted) and $\delta V = \mathcal{O}\left(10^{10}\ \mathrm{GeV}^4\right)$ (dashed) and four different initialization probability distributions given by relation $\theta + \frac{\sigma}{f} = h_{max}$ for $f = 300$ (blue), $f = 1000$ (red), $f = 3000$ (green) and $f = 10000$ (gray).

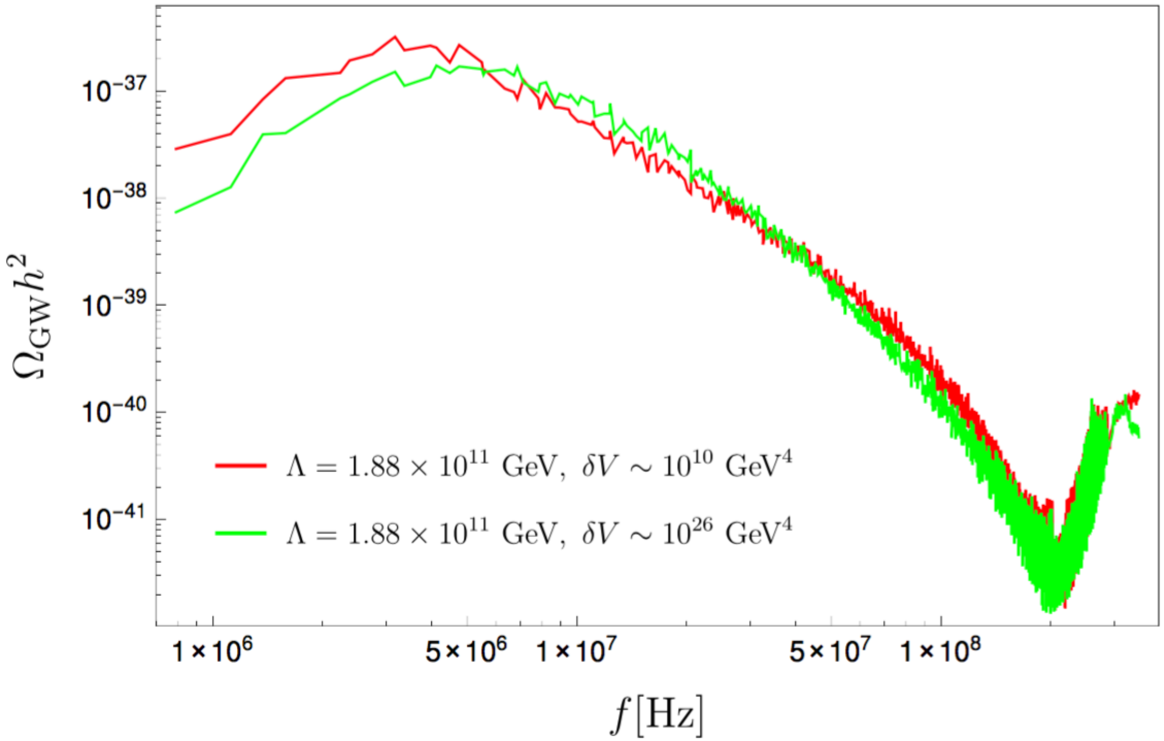
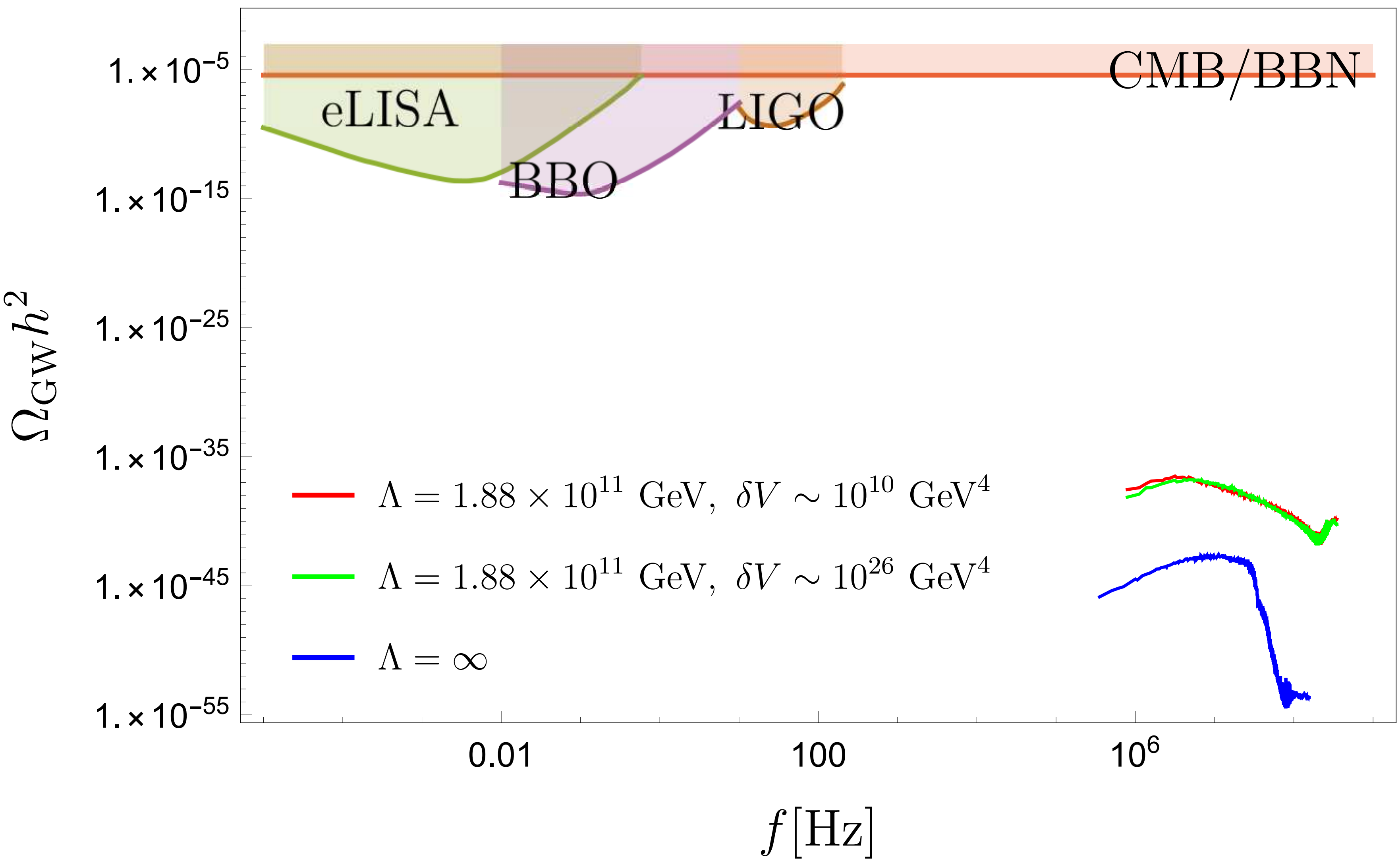


Figure 11: Present day spectrum of gravitational waves Ω_{gw} emitted from Higgs domain walls in the case of the Higgs potential with nearly degenerate minima with the difference in values of potential in minima of the order of $\delta V \sim 10^{10}$ GeV^4 (red) and $\delta V \sim 10^{26}$ GeV^4 (green).



CMB/BBN
eLISA
BBO
LIGO
$1.\times 10^{-5}$
$1.\times 10^{-15}$
$1.\times 10^{-25}$
$1.\times 10^{-35}$
$1.\times 10^{-45}$
$1.\times 10^{-55}$
$\Omega_{\rm GW}h^2$
$\Lambda = 1.88 \times 10^{11}$ GeV, $\delta V \sim 10^{10}$ GeV^4
$\Lambda = 1.88 \times 10^{11}$ GeV, $\delta V \sim 10^{26}$ GeV^4
$\Lambda = \infty$
0.01
100
10^6
$f\,[\mathrm{Hz}]$

Summary II

- SM vacuum can be stabilized by higher order operators if they appear at suffciently low energy scale $10^{10} - 10^{11}\,\mathrm{GeV}$

- SM vacuum lifetime can be dramatically shortened by higher order operators for any suppression scale

- Beyond the leading order one needs to define proper expansion of the action to demonstrate perturbatively the cancellation of gauge-dependent contributions to the lifetime of the EW vacuum. In the abelian Higgs model such a procedure can be carried out at the level of the renormalized effective action

- Peoperties of the electroweak vacuum - critical temperature and lifetime - can be modified by a fast expansion of the gravitational background

- Tunneling from Minkowski suppressed by gravity but tunnelling from dS enhanced by CDL bounces

- Decaying networks of domain walls produce a signal in the form of gravitational waves - too weak to be detected anytime soon - if a signal is detected then either fine-tuning or non-standard cosmology have occurred