Quantum Anomalies in the Running Vacuum Universe and Matter-Antimatter Asymmetry

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King's College London,

Physics Dept., London UK

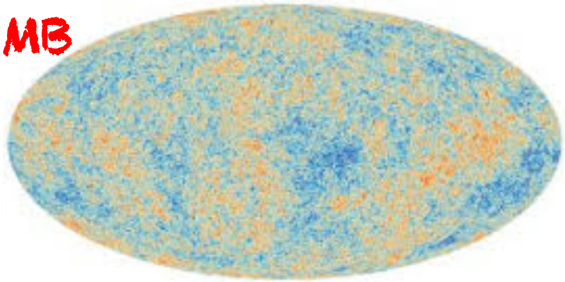
CA18108 - Quantum gravity phenomenology in the multi-messenger approach

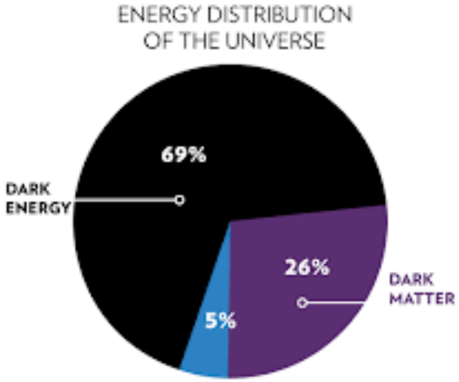
Workshop on Connecting Insights in Fundamental Physics: Standard Model and Beyond

August 31 - September 11, 2019

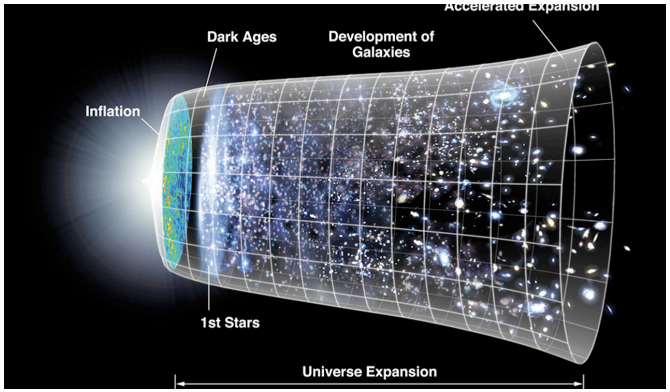
Important (last ~ 20 yrs) Discoveries in Cosmology/Astronomy

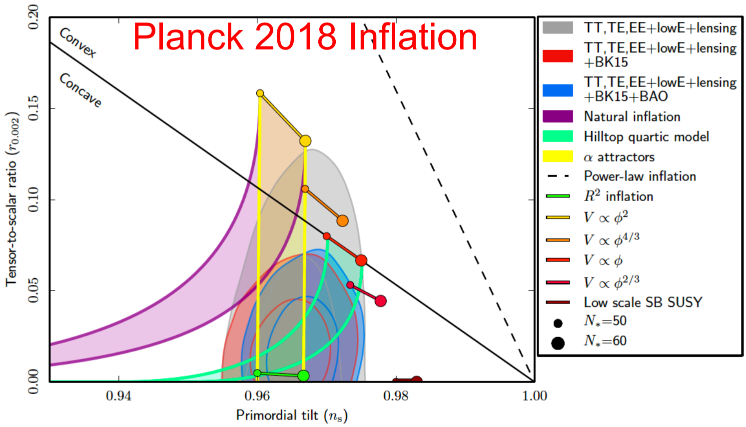
CMB





Helped towards better understanding of evolution of Universe, showed current acceleration ← cosmol. constant (?) dominance




Planck 2018 Inflation

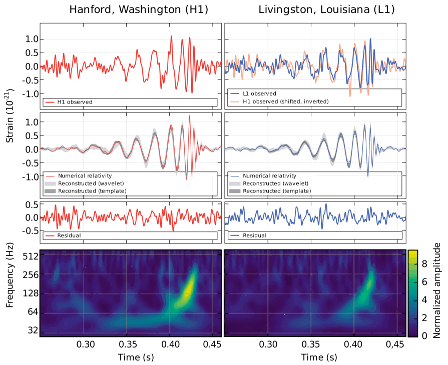
Cosmic time →

Inflation (de Sitter) → radiation-dominance → matter dominance → de Sitter (?) again

Gravitational Waves from Black Hole mergers

"Heard" for the first time by LIGO Inteferometer

Open new era in Astronomy



Important (last ~ 20 yrs) Discoveries in Cosmology/Astronomy

ENERGY DISTRIBUTION
OF THE UNIVERSE

What still we do not know/did not observe:

Nature of Dark Energy

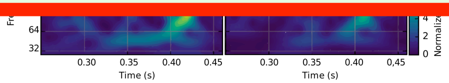
Nature of Dark matter

Primordial Gravitational Waves

(through detection of B-mode polarisation
in CMB from very early Universe)

Microscopic models of Inflation

(Is it due to fundamental inflatons or dynamical
e.g. Starobinsky type? ....)



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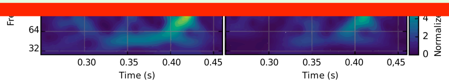
Microscopic understanding of **Matter/Antimatter asymmetry** in the Univerrse?



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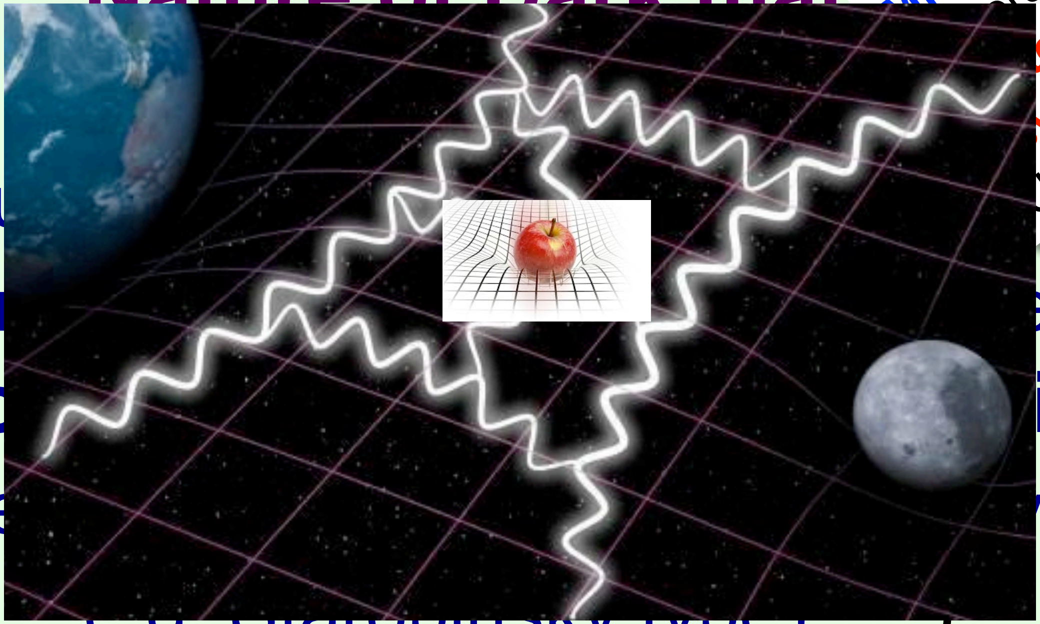

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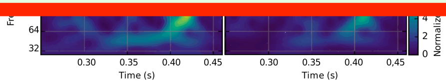
What is (if ...) the role of Quantum Gravity?



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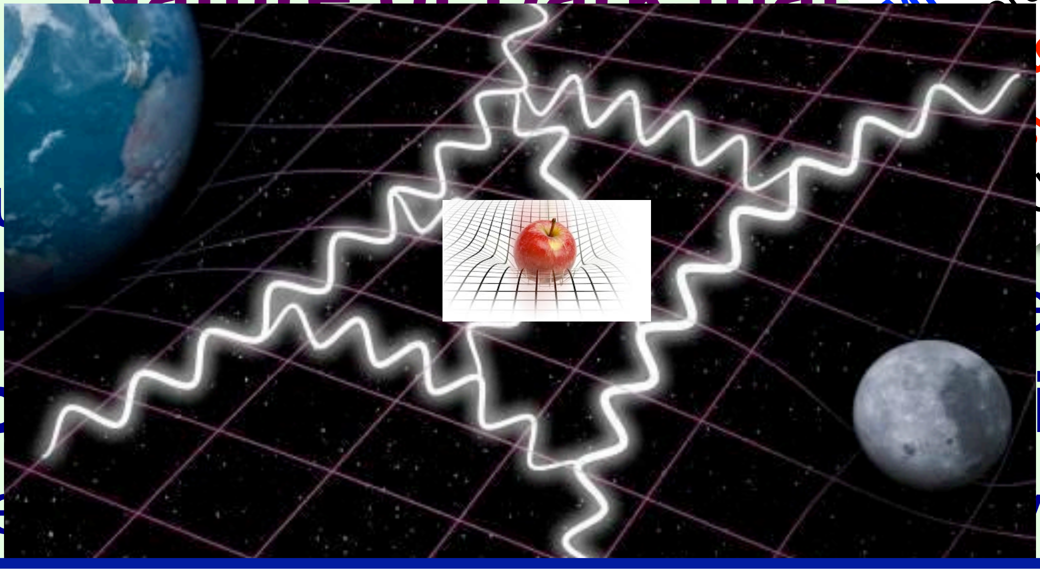
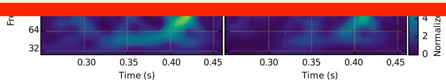


Effective field theories of Universe evolution as a first step?
Embed them in string theory (microscopic quantum gravity model)?

What is (if ...) the role of Quantum Gravity?



OUTLINE

I. **Running Vacuum Model of the Universe: general features**

II. **Quantum Anomalies: In (string-inspired) field theory + Torsion → appearance of (gravitational) axions**

III. **Primordial Gravitational Waves → Gravitational Anomalies-induced Inflation and Running Vacuum Model: Undiluted Lorentz- and CPT-violating gravitational axion backgrounds at end of inflation in string-inspired models**

IV. **Matter-Antimatter Asymmetry from CPT Violation in early Universe:** Lorentz Violating Background (flux) fields in string-inspired models & Baryogenesis through Leptogenesis in models with heavy RHN + gravitational axions

V. Conclusions-Outlook

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+ Basilakos, Sola (2019)

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+ Ellis,
de Cesare,
Sarkar,
Bossingham

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+ Basilakos
Sola (2019)

Source of
Dark Matter ?

V. Conclusions-Outlook

Part I

Running Vacuum Model of the Universe:

general features

RUNNING VACUUM MODEL (RVM) for the UNIVERSE

Shapiro + Sola
Sola + ...(2000)

Vacuum energy assumed de Sitter like but with time-dependent Cosmological parameter Λ(t) :

$$\rho^{\Lambda}_{\rm RVM}(t) = \Lambda(t)/\kappa^2 \qquad \kappa = \sqrt{8\pi G} = M_{\rm Pl}^{-1}$$

$$p(t)_{\rm RVM} = -\rho^{\Lambda(t)}_{\rm RVM}(t)$$

Renormalization-Group-like equation for the evolution of **vacuum energy density**
Hubble parameter H(τ) ←→ RG scale *μ*

$$\frac{d\,\rho_{\Lambda}^{\rm RVM}(t)}{d\,\ln H^2} = \frac{1}{(4\pi)^2}\sum_{i=F,B}\left[a_i M_i^2 H^2 + b_i H^4 + \mathcal{O}(\frac{H^6}{M_i^2})\right]$$

general covariance →
even powers of *H* ⚠

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Also in non-critical strings
Ellis NEM Nanopoulos ('98)

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Relevant for Cosmological observation/phenomenology up to and including H^4

$$\boxed{\rho^{\Lambda}_{\rm RVM}(H) = \frac{\Lambda(H)}{\kappa^2} = \frac{3}{\kappa^2}\Big(c_0 + \nu H^2 + \alpha\frac{H^4}{H_I^2}\Big)}$$

$$\nu = \mathcal{O}(10^{-3})$$

$$\mathcal{O}(10^{-4}) \lesssim \alpha \lesssim \mathcal{O}(1)$$

$$\frac{3}{\kappa^2}\,c_0 \,\simeq\, 10^{-122}\,M_{\rm Pl}^4$$

Total energy: $\rho^{\rm total} = \rho^{\Lambda}_{\rm RVM} + \rho^{\rm dust} + \rho^{\rm radiation}$

Cosmological Evolution of RVM

Basilakos, Lima, Sola + Gomez Valent + ... (2013 - 2018)

$$\dot{H} + \frac{3}{2}(1+\omega)H^2\left(1 - \nu - \frac{c_0}{H^2} - \alpha\frac{H^2}{H_I^2}\right) = 0$$

$\omega = \rho_m / p_m$ *m* = matter, radiation

$$\dot{\rho}_m + 3(1+\omega)H\rho_m = -\dot{\rho}^{\Lambda}_{\rm RVM}$$

Solution

$$H(a) = \left(\frac{1-\nu}{\alpha}\right)^{1/2} \frac{H_I}{\sqrt{D\, a^{3(1-\nu)(1+\omega_m)} + 1}}$$

$D > 0$

Early de Sitter (unstable)

$$D a^{4(1-\nu)} \ll 1 \quad \Longrightarrow \quad H^2 = (1-\nu)H_I^2/\alpha$$

Radiation

$$D a^{4(1-\nu)} \gg 1 \quad \Longrightarrow \quad H^2 \sim a^{3(1-\nu)(1+\omega_m)} \sim a^{-4}$$

$$\omega = 1/3$$

Late dark-Energy dominated era

$$H^2(a) = H_0^2\left[\tilde{\Omega}_{m0}\, a^{-3(1-\nu)} + \tilde{\Omega}_{\Lambda 0}\right]$$

c_0 dominant

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Dominant in early Universe → drives inflation
→ no need for external inflatons

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Relevant for Cosmological observation/phenomenology up to and including *H*⁴

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IN THIS TALK: present microscopic (string-inspired) model for RVM Universe….

Basilakos, NEM, Sola
arXiv:1907.04890
& IJMD28 (2019) 1944002

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H^4 – terms in $\rho^{\Lambda}_{\rm RVM}$ can be generated through **CP –Violating Gravitational-Anomaly condensates** induced by **primordial Gravitational Waves** in **string-inspired** models involving **gravitational** (Kalb-Ramond) **axions**

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Graceful Exit from Inflation with **undiluted** **Lorentz and CPT Violating gravitational axion backgrounds** → **Leptogenesis** through decays of heavy **Right-Handed sterile neutriinos**

Basilakos, NEM, Sola arXiv:1907.04890 & IJMD28 (2019) 1944002

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Gravitational axions can source DM

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Part II

Quantum Anomalies

in (string-inspired) field theory

Anomalies in Quantum Field Theory:

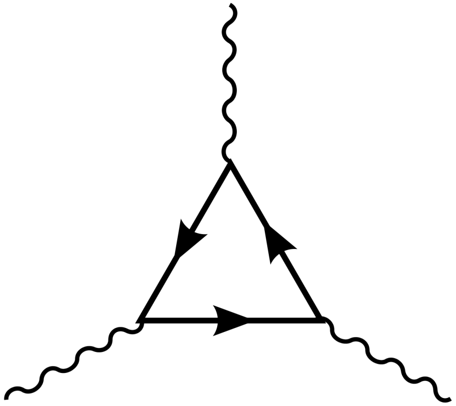
Classical Symmetry → Conserved Current

Quantum Theory: Failure of current conservation in ANY REGULARIZATION of the quantum theory

or equivalently:

Path-Integral measure NOT INVARIANT under symmetry transformation

OF INTEREST HERE: **GAUGE & GRAVITATIONAL CHIRAL ANOMALIES**



CHIRAL FERMIONIC LOOP in graphs with **1+D/2 external legs** (gauge fields or gravitons) in D- space-time dimensions **D=4 → triangular graphs**

Alvarez-Gaume, Witten

Mixed Anomalies (Gravitational + Gauge)

$$\nabla_\mu J^{5\mu} = \left(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\right)$$

gravitational
covariant derivative

$$J^{5\mu} = \sum_j \overline{\psi}_j\,\gamma^\mu\,\gamma^5\,\psi_j$$

Axial Current

$$\widetilde{R}_{\mu\nu\rho\sigma} = \frac{1}{2}\varepsilon_{\mu\nu\lambda\pi}R^{\lambda\pi}{}_{\rho\sigma}, \qquad \widetilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\,F^{\rho\sigma}$$

$$\varepsilon_{\mu\nu\rho\sigma} = \sqrt{-g}\,\epsilon_{\mu\nu\rho\sigma}, \qquad \varepsilon^{\mu\nu\rho\sigma} = \frac{\mathrm{sgn}(g)}{\sqrt{-g}}\,\epsilon^{\mu\nu\rho\sigma}$$

Anomaly terms are total derivatives:

$$\sqrt{-g}\left(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\right) = \sqrt{-g}\,\mathcal{K}^{\mu}_{\mathrm{mixed}}(\omega)_{;\mu} = \partial_\mu\left(\sqrt{-g}\,\mathcal{K}^{\mu}_{\mathrm{mixed}}(\omega)\right)$$

$$= 2\,\partial_\mu\left[\epsilon^{\mu\nu\alpha\beta}\,\omega^{ab}_{\nu}\left(\partial_\alpha\,\omega_{\beta ab} + \frac{2}{3}\,\omega_{\alpha a}{}^{c}\,\omega_{\beta cb}\right) - 2\epsilon^{\mu\nu\alpha\beta}\left(A^i_\nu\,\partial_\alpha A^i_\beta + \frac{2}{3}\,f^{ijk}\,A^i_\nu\,A^j_\alpha\,A^k_\beta\right)\right]$$

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Anomaly terms are total derivatives – can couple to axion-like fields *b*

$$S_B^{\rm eff} = \int d^4x \sqrt{-g}\Big[\, b(x)\, \Big(R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\, \widetilde{F}^{\mu\nu}\Big) + \ldots\Big],$$

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Contributions to Stress tensor **YES** **NO**

Gravitational Anomalies & Diffeomorphism Invariance

$$\int d^4x \sqrt{-g}\, b(x) \left(R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\, \widetilde{F}^{\mu\nu} \right)$$

Spoils conservation of stress tensor (**diffeomporphism invariance affected in quantum theory**)

Topological does NOT contribute to stress tensor

$$\delta\Big[\int d^4x \sqrt{-g}\, b\, R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \Big] = 4 \int d^4x \sqrt{-g}\, \mathcal{C}^{\mu\nu}\, \delta g_{\mu\nu} = -4 \int d^4x \sqrt{-g}\, \mathcal{C}_{\mu\nu}\, \delta g^{\mu\nu}$$

Cotton tensor

$$\mathcal{C}^{\mu\nu} = -\frac{1}{2}\Big[v_\sigma \left(\varepsilon^{\sigma\mu\alpha\beta} R^{\nu}{}_{\beta;\alpha} + \varepsilon^{\sigma\nu\alpha\beta} R^{\mu}{}_{\beta;\alpha} \right) + v_{\sigma\tau} \left(\widetilde{R}^{\tau\mu\sigma\nu} + \widetilde{R}^{\tau\nu\sigma\mu} \right) \Big] = -\frac{1}{2}\Big[\left(v_\sigma\, \widetilde{R}^{\lambda\mu\sigma\nu} \right)_{;\lambda} + (\mu \leftrightarrow \nu) \Big]$$

$$v_\sigma \equiv \partial_\sigma b = b_{;\sigma}, \;\; v_{\sigma\tau} \equiv v_{\tau;\sigma} = b_{;\tau;\sigma}$$

Traceless $$g_{\mu\nu}\, \mathcal{C}^{\mu\nu} = 0$$

Jackiw, Pi (2003)

Gravitational Anomalies & Diffeomorphism Invariance

Einstein's equation

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - \mathcal{C}^{\mu\nu} = \kappa^2 T^{\mu\nu}_{\rm matter}$$

$$\boxed{\mathcal{C}^{\mu\nu}{}_{;\mu} = - \frac{1}{8} v^{\nu} R^{\alpha\beta\gamma\delta} \widetilde{R}_{\alpha\beta\gamma\delta}}$$

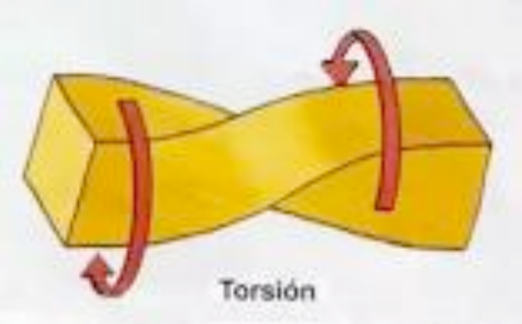
$$v_\sigma \equiv \partial_\sigma b$$

$$\boxed{\kappa^2 T^{\mu\nu}_{\rm matter\,;\mu} = - \mathcal{C}^{\mu\nu}{}_{;\mu} \neq 0}$$

Diffeomoprphism invariance breaking by gravitational anomalies

Torsion & Mixed Anomalies

Geometries with Torsion: **metric** and **spin-connection** **independent,**



Torsion 2-form $$\mathbf{T}^a = \mathbf{d}\ \mathbf{e}^a + \overline{\omega}^a \wedge \mathbf{e}^b$$

metric $$g_{\mu\nu} = e^a_\mu\, \eta_{ab}\, e^b_\nu$$

spin connection $$\overline{\omega}_{ab\mu} = \omega_{ab\mu} + K_{ab\mu}$$

Torsion-free spin connection

Contorsion

$$K_{abc} = \frac{1}{2}\Big(\mathrm{T}_{cab} - \mathrm{T}_{abc} - \mathrm{T}_{bcd}\Big)$$

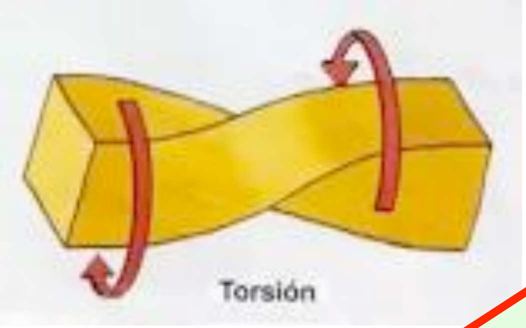
Christoffel symbol has **antisymmetric component**

$$\Gamma^\rho_{\mu\nu} \neq \Gamma^\rho_{\nu\mu}$$

Torsion & Mixed Anomali

Geometries with Torsion: **metric** and **spin-con** **t,**

Quantum Torsion is equivalent to **Axion-like** fields coupled to **mixed anomaly terms** (gauge and gravitational)



Torsion 2-form $\wedge \mathbf{e}^b$

$_\nu$

sp

$\omega_{ab\mu} + K_{ab\mu}$

$bc = \frac{1}{2}\Big(\mathrm{T}$

antisymmetric compor

$$\Gamma^\rho_{\mu\nu} \neq \Gamma^\rho_{\nu\mu}$$

Fermionic Field Theories with H-Torsion
EFFECTIVE ACTION AFTER INTEGRATING OUT QUANTUM TORSION FLUCTUATIONS

Fermions: $S_\psi \ni -\frac{3}{4}\int d^4\sqrt{-g}\,S_\mu\bar{\psi}\gamma^\mu\gamma^5\psi = -\frac{3}{4}\int S\wedge {}^\star J^5$

+ standard Dirac terms without torsion

$$\mathbf{S} = {}^\star\mathbf{T}$$

$$S_d = \frac{1}{3!}\epsilon^{abc}{}_d T_{abc}$$

$$T_{abc} \to H_{cab} = \epsilon_{cabd}\,\partial^d b$$

Bianchi identity

$$d\,{}^\star\mathbf{S} = 0$$

classical

conserved ``torsion ‘’ charge

$$Q = \int {}^\star\mathbf{S}$$

Postulate conservation at quantum level by adding counterterms

Implement $d\,{}^\star\mathbf{S} = 0$ via $\delta(d\,{}^\star\mathbf{S})$ constraint
→ lagrange multiplier in Path integral → b-field

Torsion & Fermions – QED WITH TORSION AS A STUDY CASE

Duncan, Kaloper, Olive (1992)

$$S^{\rm QED}_{\rm torsion} = - \int d^4x \, \sqrt{-g} \, \frac{1}{4} F_{\mu\nu} \, F^{\mu\nu} + S_\psi$$

$$S_\psi = \frac{i}{2} \int \left(\bar{\psi} \gamma^\mu \bar{\mathscr{D}}_\mu \psi - \left(\bar{\mathscr{D}}_\mu \bar{\psi} \right) \gamma^\mu \psi \right) \sqrt{-g} \;\, \mathrm{d}^4 x$$

$$\bar{\mathscr{D}}_\mu = \overline{\mathrm{D}}_\mu - ieA_\mu \qquad \overline{\mathrm{D}}_\mu = \partial_\mu + \frac{i}{4} [\gamma^a, \gamma^b] \, \overline{\omega}_{ab\mu}$$

$$S_\psi \ni -\frac{3}{4} \int d^4 \sqrt{-g} \, S_\mu \bar{\psi} \gamma^\mu \gamma^5 \psi = -\frac{3}{4} \int S \wedge {}^\star J^5$$

+ standard Dirac terms without torsion

$$\mathbf{S} = {}^\star\mathbf{T} \quad \boldsymbol{T} = (1/3!) T_{abc} \boldsymbol{e}^a \wedge \boldsymbol{e}^b \wedge \boldsymbol{e}^c \qquad S_d = \tfrac{1}{3!} \epsilon^{abc}{}_d T_{abc}$$

STRESS TENSOR

$$T^{A}_{\mu\nu} = F_{\mu\lambda}F_{\nu}{}^{\lambda} - \tfrac{1}{4}g_{\mu\nu}F_{\lambda\rho}F^{\lambda\rho},$$

$$T^{\psi}_{\mu\nu} = -\frac{i}{2}\Big(\bar{\psi}\gamma_{(\mu}\mathscr{D}_{\nu)}\psi - \big(\mathscr{D}_{(\mu}\bar{\psi}\big)\gamma_{\nu)}\psi\Big) + \tfrac{3}{4}S_{(\mu}\bar{\psi}\gamma_{\nu)}\gamma^{5}\psi.$$

$$T^{S}_{\mu\nu} = -\frac{3}{2\kappa^{2}}\Big(S_{\mu}S_{\nu} - \tfrac{1}{2}g_{\mu\nu}S_{\lambda}S^{\lambda}\Big)$$

Torsion is NON-PROPAGATING FIELD IN QED → Classical Eqs of motion

$$\boldsymbol{S} = \tfrac{1}{2}\kappa^{2}\boldsymbol{j}^{5} \quad \Rightarrow \quad \mathrm{d}\,{}^{\star}\mathrm{S} = 0 \qquad Q = \int {}^{\star}\mathrm{S}$$

classical

conserved
``torsion " charge

Postulate conservation at quantum level by adding counterterms

Implement $\mathrm{d}\,{}^{\star}\mathbf{S} = 0$ via $\delta(d\,{}^{\star}\mathbf{S})$ constraint

→ Lagrange multiplier in Path integral

→ pseudoscalar b(x)- field

$$\int DS\, Db \exp\Big[i\int \frac{3}{4\kappa^2}\mathbf{S}\wedge{}^{\star}\mathbf{S} - \frac{3}{4}\mathbf{S}\wedge{}^{\star}\mathbf{J}^5 + \Big(\frac{3}{2\kappa^2}\Big)^{1/2} b\, d^{\star}\mathbf{S}\Big]$$

$$= \int Db \exp\Big[-i\int \frac{1}{2}\mathrm{d}b\wedge{}^{\star}\mathrm{d}b + \frac{1}{f_b}\mathrm{d}b\wedge{}^{\star}\mathbf{J}^5 + \frac{1}{2f_b^2}\mathbf{J}^5\wedge\mathbf{J}^5\Big]$$

multiplier field $\Phi(x) \equiv (3/\kappa^2)^{1/2} b(x)$.

$$f_b = (3\kappa^2/8)^{-1/2} = \frac{M_P}{\sqrt{3\pi}}$$

Implement $\mathrm{d}\,{}^{\star}\mathbf{S} = 0$ via $\delta(d\,{}^{\star}\mathbf{S})$ constraint

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$$\int DS\,Db\,\exp\left[i\int \frac{3}{4\kappa^2}\mathbf{S}\wedge{}^{\star}\mathbf{S} - \frac{3}{4}\mathbf{S}\wedge{}^{\star}\mathbf{J}^5 + \left(\frac{3}{2\kappa^2}\right)^{1/2} b\, d{}^{\star}\mathbf{S}\right]$$

$$= \int Db\,\exp\left[-i\int \frac{1}{2}\mathrm{d}b\wedge{}^{\star}\mathrm{d}b + \frac{1}{f_b}\mathrm{d}b\wedge{}^{\star}\mathbf{J}^5 + \frac{1}{2f_b^2}\mathbf{J}^5\wedge\mathbf{J}^5\right]$$

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$$= \int Db \exp\left[-i \int \frac{1}{2}\mathrm{d}b\wedge {}^{*}\mathrm{d}b + \boxed{\frac{1}{f_b}\mathrm{d}b\wedge {}^{*}\mathbf{J}^5} + \frac{1}{2f_b^2}\mathbf{J}^5\wedge \mathbf{J}^5\right]$$

partial integrate

Implement $\mathrm{d}\,{}^\star\mathbf{S} = 0$ via $\delta(d\,{}^\star\mathbf{S})$ constraint

→ lagrange multiplier in Path integral

→ pseudoscalar b(x)- field

$$\int DS\,Db\,\exp\Big[i\int \frac{3}{4\kappa^2}\mathbf{S}\wedge{}^\star\mathbf{S} - \frac{3}{4}\mathbf{S}\wedge{}^\star\mathbf{J}^5 + \Big(\frac{3}{2\kappa^2}\Big)^{1/2} b\,d^\star\mathbf{S}\Big]$$

$$= \int Db\,\exp\Big[-i\int \frac{1}{2}\mathrm{d}b\wedge{}^\star\mathrm{d}b + \boxed{\frac{1}{f_b}\mathrm{d}b\wedge{}^\star\mathbf{J}^5} + \frac{1}{2f_b^2}\mathbf{J}^5\wedge\mathbf{J}^5\Big]$$

partial integrate

Use chiral anomaly equation (one-loop) in curved space-time:

$$\begin{aligned}\nabla_\mu J^{5\mu} &= \frac{e^2}{8\pi^2}F^{\mu\nu}\widetilde{F}_{\mu\nu} - \frac{1}{192\pi^2}R^{\mu\nu\rho\sigma}\widetilde{R}_{\mu\nu\rho\sigma}\\ &\equiv G(\mathbf{A},\omega)\,.\end{aligned}$$

Hence, effective action of torsion-full QED contains terms:

$$\int Db \, \exp\left[-i\int \frac{1}{2}\mathbf{d}b\wedge{}^{\star}\mathbf{d}b-\frac{1}{f_b}bG(\mathbf{A},\omega)+\frac{1}{2f_b^2}\mathbf{J}^5\wedge\mathbf{J}^5\right] .$$

$$\begin{aligned}\nabla_\mu J^{5\mu} &= \frac{e^2}{8\pi^2}F^{\mu\nu}\widetilde{F}_{\mu\nu}-\frac{1}{192\pi^2}R^{\mu\nu\rho\sigma}\widetilde{R}_{\mu\nu\rho\sigma}\\ &\equiv G(\mathbf{A},\omega) .\end{aligned}$$

Hence, effective action of torsion-full QED contains terms:

$$\int Db \, \exp\left[-i\int \frac{1}{2}\mathrm{d}b\wedge{}^{\star}\mathrm{d}b - \frac{1}{f_b}bG(\mathbf{A},\omega) + \frac{1}{2f_b^2}\mathbf{J}^5\wedge\mathbf{J}^5\right] .$$

$$\boxed{bR\tilde{R} - bF\tilde{F}}$$

Equivalent to: b → dynamical (massless axion-like) field with anomalous CP-Violating interactions

$$f_b = (3\kappa^2/8)^{-1/2} = \frac{M_P}{\sqrt{3\pi}}$$

$$\begin{aligned}\nabla_\mu J^{5\mu} &= \frac{e^2}{8\pi^2}F^{\mu\nu}\widetilde{F}_{\mu\nu} - \frac{1}{192\pi^2}R^{\mu\nu\rho\sigma}\widetilde{R}_{\mu\nu\rho\sigma} \\ &\equiv G(\mathbf{A},\omega) .\end{aligned}$$

A non-trivial example of Torsion: String Theories with Antisymmetric Tensor Backgrounds

Gross and Sloan, Metsaev and Tseytlin
Campbell, Duncan, Kaloper and Olive

NEM & Sarben Sarkar, arXiv:1211.0968 (EPJC)
John Ellis, NEM & Sarkar, arXiv:1304.5433 (PLB)
De Cesare, NEM & Sarkar arXiv:1412.7077 (EPJC)
Bossingham, NEM & Sarkar, arXiv:1712.03312 (EPJC)
Bossingham, NEM & Sarkar, arXiv:1810.13384 (EPJC)

A non-trivial example of Torsion: String Theories with Antisymmetric Tensor Backgrounds

Massless Gravitational multiplet of (closed) strings: spin 0 scalar (dilaton)
spin 2 traceless symmetric rank 2 tensor (graviton)
spin 1 antisymmetric rank 2 tensor

A non-trivial example of Torsion: String Theories with Antisymmetric Tensor Backgrounds

Massless Gravitational multiplet of (closed) strings: **spin 0 scalar (dilaton)**

spin 2 traceless symmetric rank 2 tensor (graviton)

spin 1 antisymmetric rank 2 tensor

KALB-RAMOND FIELD $B_{\mu\nu} = -B_{\nu\mu}$

Effective field theories (low energy scale E << M_s) `` **gauge**" invariant

$$B_{\mu\nu} \to B_{\mu\nu} + \partial_{[\mu}\theta(x)_{\nu]}$$

Depend only on field strength : $H_{\mu\nu\rho} = \partial_{[\mu}B_{\nu\rho]}$

Bianchi identity : $$\boxed{\partial_{[\sigma}H_{\mu\nu\rho]} = 0 \to d\star\mathbf{H} = 0}$$

ROLE OF H-FIELD AS TORSION

EFFECTIVE GRAVITATIONAL ACTION IN STRING LOW-ENERGY LIMIT

4-DIM PART

$$S^{(4)} = \int d^4x\sqrt{-g}\Big(\frac{1}{2\kappa^2}R - \frac{1}{6}H_{\mu\nu\rho}\,H^{\mu\nu\rho}\Big)$$

$$= \int d^4x\sqrt{-g}\Big(\frac{1}{2\kappa^2}\overline{R}\ \Big)$$

$\kappa^2 = 8\pi\, G$

$$\overline{R}(\overline{\Gamma})$$

generalised curvature

$$\overline{\Gamma}^{\mu}_{\nu\rho} = \Gamma^{\mu}_{\nu\rho} + \frac{\kappa}{\sqrt{3}}H^{\mu}_{\nu\rho} \neq \overline{\Gamma}^{\mu}_{\rho\nu}$$

Contorsion

ROLE OF H-FIELD AS TORSION – AXION FIELD

EFFECTIVE GRAVITATIONAL ACTION IN STRING LOW-ENERGY LIMIT

$$\sim \frac{1}{2}\partial^\mu b\, \partial_\mu b$$

4-DIM PART

$$S^{(4)} = \int d^4x\sqrt{-g}\Big(\frac{1}{2\kappa^2}R - \frac{1}{6}H_{\mu\nu\rho}\, H^{\mu\nu\rho}\Big)$$

$$= \int d^4x\sqrt{-g}\Big(\frac{1}{2\kappa^2}\overline{R}\ \Big)$$

$\kappa^2 = 8\pi\, G$

$$\overline{R}(\overline{\Gamma}) \qquad \overline{\Gamma}^\mu_{\nu\rho} = \Gamma^\mu_{\nu\rho} + \frac{\kappa}{\sqrt{3}}H^\mu_{\nu\rho} \neq \overline{\Gamma}^\mu_{\rho\nu}$$

IN 4-DIM DEFINE DUAL OF H AS :

$$-3\sqrt{2}\partial_\sigma b = \sqrt{-g}\,\epsilon_{\mu\nu\rho\sigma}H^{\mu\nu\rho}$$

b(x) **= Pseudoscalar**
(Kalb-Ramond (KR) axion)

Fermions and (generic) Torsion

Dirac Lagrangian (for concreteness, it can be extended to Majorana neutrinos)

$$\mathcal{L} = \sqrt{-g}\,\left(i\,\bar{\psi}\,\gamma^a D_a \psi - m\,\bar{\psi}\psi\right)$$

$$\gamma^a\gamma^b\gamma^c = \eta^{ab}\,\gamma^c + \eta^{bc}\,\gamma^a - \eta^{ac}\,\gamma^b - i\,\epsilon^{dabc}\,\gamma_d\,\gamma^5$$

$$D_a = \left(\partial_a - \frac{i}{4}\omega_{bca}\sigma^{bc}\right),$$

Gravitational covariant derivative including spin connection

$$\sigma^{ab} = \tfrac{i}{2}\left[\gamma^a, \gamma^b\right]$$

$$g_{\mu\nu} = e^a_\mu\,\eta_{ab}\,e^b_\nu$$

$$\omega_{bca} = e_{b\lambda}\left(\partial_a e^\lambda_c + \Gamma^\lambda_{\gamma\mu} e^\gamma_c e^\mu_a\right).$$

$$\mathcal{L} = \mathcal{L}_f + \mathcal{L}_I = \sqrt{-g}\bar{\psi}\left[(i\gamma^a\partial_a - m) + \gamma^a\gamma^5 B_a\right]\psi,$$

$$B^d = \epsilon^{abcd} e_{b\lambda}\left(\partial_a e^\lambda_c + \Gamma^\lambda_{\alpha\mu} e^\alpha_c e^\mu_a\right)$$

If **torsion** then $\Gamma_{\mu\nu} \neq \Gamma_{\nu\mu}$ ***antisymmetric*** part is the contorsion tensor, contributes

FERMIONS COUPLE TO H –TORSION VIA GRAVITATIONAL COVARIANT DERIVATIVE

$$S_\psi = \frac{i}{2}\int d^4x\sqrt{-g}\Big(\overline{\psi}\gamma^\mu\overline{\mathcal{D}}_\mu\psi - (\overline{\mathcal{D}}_\mu\overline{\psi})\gamma^\mu\psi\Big)$$

TORSIONFUL CONNECTION, FIRST-ORDER FORMALISM

$$\overline{\mathcal{D}}_a = \partial_a - \frac{i}{4}\overline{\omega}_{bca}\sigma^{bc}$$

$$\overline{\omega}_{ab\mu} = \omega_{ab\mu} + K_{ab\mu}$$

contorsion

$$K_{abc} = \frac{1}{2}\Big(T_{cab} - T_{abc} - T_{bca}\Big)$$

FERMIONS COUPLE TO H –TORSION VIA GRAVITATIONAL COVARIANT DERIVATIVE

$$S_\psi = \frac{i}{2}\int d^4x\sqrt{-g}\Big(\overline{\psi}\gamma^\mu\overline{\mathcal{D}}_\mu\psi - \boxed{(\overline{\mathcal{D}}_\mu\overline{\psi})}\gamma^\mu\psi\Big)$$

TORSIONFUL CONNECTION, FIRST-ORDER FORMALISM

$$\boxed{\overline{\mathcal{D}}_a = \partial_a - \frac{i}{4}\overline{\omega}_{bca}\sigma^{bc}}$$

$$\overline{\omega}_{ab\mu} = \omega_{ab\mu} + \boxed{K_{ab\mu}}$$

contorsion

$$\sigma^{ab} = \tfrac{i}{2}\left[\gamma^a, \gamma^b\right]$$

$$K_{abc} = \frac{1}{2}\Big(T_{cab} - T_{abc} - T_{bca}\Big)$$

$$\boxed{\begin{aligned}&\gamma^a\gamma^b\gamma^c = \\ &\eta^{ab}\,\gamma^c + \eta^{bc}\,\gamma^a - \eta^{ac}\,\gamma^b - i\,\epsilon^{dabc}\,\gamma_d\,\gamma^5\end{aligned}}$$

FERMIONS COUPLE TO H –TORSION VIA GRAVITATIONAL COVARIANT DERIVATIVE

$$S_\psi = \frac{i}{2}\int d^4x\sqrt{-g}\Big(\overline{\psi}\gamma^\mu\overline{\mathcal{D}}_\mu\psi - \boxed{(\overline{\mathcal{D}}_\mu\overline{\psi})}\gamma^\mu\psi\Big)$$

TORSIONFUL CONNECTION, FIRST-ORDER FORMALISM

$$\boxed{\overline{\mathcal{D}}_a = \partial_a - \frac{i}{4}\overline{\omega}_{bca}\sigma^{bc}}$$

$$\overline{\omega}_{ab\mu} = \omega_{ab\mu} + \boxed{K_{ab\mu}}$$

contorsion

$$K_{abc} = \frac{1}{2}\Big(T_{cab} - T_{abc} - T_{bca}\Big)$$

$$\boxed{H_{cab}}$$

Non-trivial contributions to ***B*^μ**

$$B^d = \epsilon^{abcd}e_{b\lambda}\left(\partial_a e^\lambda_c + \Gamma^\lambda_{\alpha\mu}e^\alpha_c e^\mu_a\right)$$

$$\overline{\Gamma}^\mu_{\nu\rho} = \Gamma^\mu_{\nu\rho} + \frac{\kappa}{\sqrt{3}}H^\mu_{\nu\rho} \neq \overline{\Gamma}^\mu_{\rho\nu}$$

FERMIONS COUPLE TO H –TORSION VIA GRAVITATIONAL COVARIANT DERIVATIVE

$$S_\psi = \frac{i}{2}\int d^4x\sqrt{-g}\Big(\overline{\psi}\gamma^\mu\overline{\mathcal{D}}_\mu\psi - (\overline{\mathcal{D}}_\mu\overline{\psi})\gamma^\mu\psi\Big)$$

TORSIONFUL CONNECTION, FIRST-ORDER FORMALISM

$$\boxed{S_\psi \ni \int d^4x B_a\,\overline{\psi}\gamma^a\gamma^5\psi}$$

$$\overline{\omega}_{ab\mu} = \omega_{ab\mu} + K_{ab\mu}$$

contorsion

$$K_{abc} = \frac{1}{2}\Big(T_{cab} - T_{abc} - T_{bca}\Big)$$

$$B^d \sim \epsilon^{abcd}H_{bca}$$

$$H_{cab}$$

Non-trivial contributions to ***B***$^\mu$

$$B^d = \epsilon^{abcd}e_{b\lambda}\left(\partial_a e^\lambda_c + \Gamma^\lambda_{\alpha\mu}e^\alpha_c e^\mu_a\right)$$

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FERMIONS COUPLE TO H –TORSION VIA GRAVITATIONAL COVARIANT DERIVATIVE

$$S_\psi = \frac{i}{2}\int d^4x\sqrt{-g}\Big(\overline{\psi}\gamma^\mu\overline{\mathcal{D}}_\mu\psi - (\overline{\mathcal{D}}_\mu\overline{\psi})\gamma^\mu\psi\Big)$$

TORSIONFUL CONNECTION

$$\boxed{S_\psi \ni \int d^4x B_a\, \overline{\psi}\gamma^a\gamma^5\psi}$$

$$B^d \sim \epsilon^{abcd}H_{bca} \longrightarrow \boxed{-3\sqrt{2}\partial_\sigma b \;=\; \sqrt{-g}\,\epsilon_{\mu\nu\rho\sigma}H^{\mu\nu\rho}}$$

b(x) = KR (gravitational) axion

Non-trivial contributions to $\boldsymbol{B^\mu}$

$$B^d = \epsilon^{abcd}e_{b\lambda}\left(\partial_a e^\lambda_c + \Gamma^\lambda_{\alpha\mu}e^\alpha_c e^\mu_a\right)$$

$$\overline{\Gamma}^\mu_{\nu\rho} \;=\; \Gamma^\mu_{\nu\rho} + \frac{\kappa}{\sqrt{3}}H^\mu_{\nu\rho} \neq \overline{\Gamma}^\mu_{\rho\nu}$$

FERMIONS COUPLE TO H –TORSION VIA GRAVITATIONAL COVARIANT DERIVATIVE

$$S_\psi = \frac{i}{2}\int d^4x\sqrt{-g}\Big(\overline{\psi}\gamma^\mu\overline{\mathcal{D}}_\mu\psi - (\overline{\mathcal{D}}_\mu\overline{\psi})\gamma^\mu\psi\Big)$$

TORSIONFUL CONNECTION → **AXION-LIKE CP-VIOLATING INTERACTION**

$$\boxed{S_\psi \ni \int d^4x B_a\,\overline{\psi}\gamma^a\gamma^5\psi} \longrightarrow \boxed{-\int d^4x\sqrt{-g}\,\partial_\alpha b\,\big(\overline{\psi}\,\gamma^\alpha\,\gamma^5\,\psi\big)}$$

Universal (gravitational) Coup[ling

$$B^d \sim \epsilon^{abcd}H_{bca} \longrightarrow \boxed{-3\sqrt{2}\partial_\sigma b \;=\; \sqrt{-g}\,\epsilon_{\mu\nu\rho\sigma}H^{\mu\nu\rho}}$$

***b(x)* = KR (gravitational) axion**

Non-trivial contributions to ***B^μ***

$$B^d = \epsilon^{abcd}e_{b\lambda}\left(\partial_a e^\lambda_c + \Gamma^\lambda_{\alpha\mu}e^\alpha_c e^\mu_a\right)$$

$$\overline{\Gamma}^\mu_{\nu\rho} \;=\; \Gamma^\mu_{\nu\rho} + \frac{\kappa}{\sqrt{3}}H^\mu_{\nu\rho} \neq \overline{\Gamma}^\mu_{\rho\nu}$$

Antoniadis, Bachas,
Ellis, Nanopoulos

NEM + Sarkar
de Cesare
Bossingham

In string-theory (inspired) Cosmologies with such KR-b axions there are solutions with

$$\boxed{b(t) = \text{constant x } t\ , \quad t = \text{FLRW cosmic time}}$$

→ Spontaneously Broken Lorentz (& CPT) Symmetry (SBL)

→ massless KR axion = Goldstone Boson of SBL

When **db/dt = constant** → Torsion is constant

Covariant Torsion tensor

$$\overline{\Gamma}^{\lambda}_{\ \mu\nu} = \Gamma^{\lambda}_{\ \mu\nu} + e^{-2\Phi} H^{\lambda}_{\mu\nu} \equiv \Gamma^{\lambda}_{\ \mu\nu} + T^{\lambda}_{\ \mu\nu}$$

$$T_{ijk} \sim \epsilon_{ijk}\, \dot{b}$$

Constant

$$\boxed{S_\psi \ni \int d^4x B_a \overline{\psi}\gamma^a\gamma^5\psi}$$

constant $\boldsymbol{B^0} \propto \dot{b}$

When **db/dt = constant** → Torsion is constant

Covariant Torsion tensor

$$\overline{\Gamma}^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} + e^{-2\Phi} H^{\lambda}_{\mu\nu} \equiv \Gamma^{\lambda}_{\mu\nu} + T^{\lambda}_{\mu\nu}$$

$$T_{ijk} \sim \epsilon_{ijk}\, \dot{b}$$

Constant

$$\boxed{S_\psi \ni \int d^4x B_a \overline{\psi} \gamma^a \gamma^5 \psi}$$

constant $B^0 \propto \dot{b}$

$$\mathcal{L} = \tfrac{1}{2} i \bar{\psi}\, \gamma^\nu\, \bar{\partial}_\nu \psi - \bar{\psi} M \psi, \qquad M \equiv m + b_\mu \gamma^5 \gamma^\mu$$

LV & CPTV

Standard Model Extension type with CPT and Lorentz Violating background $b^0 = B^0$

Kostelecky et al.

Antoniadis, Bachas,
Ellis, Nanopoulos

NEM + Sarkar
de Cesare
Bossingham

In string-theory (inspired) Cosmologies with such KR-b axions there are solutions with

b(t) = constant x t , t = FLRW cosmic time

→ Spontaneously Broken Lorentz (& nCPT) Symmetry (SBL)

→ massless KR axion = Goldstone Boson of SBL

Gravitational Anomalies-induced condensates
due to primordial gravitational waves lead to
such Backgrounds for the b –field

Basilakos, NEM, Sola
(2019)

Effective Actions & Anomaly Cancellation – Addition of Counterterms

Green, Schwarz

$$S_B = \int d^4x\sqrt{-g}\Big(\frac{1}{2\kappa^2}[-R+2\,\partial_\mu\Phi\,\partial^\mu\Phi] - \frac{1}{6\kappa^2}\,e^{-4\Phi}\,H_{\lambda\mu\nu}H^{\lambda\mu\nu}+\dots\Big)$$

String Anomaly Cancellation requires modification in definition of $H_{\mu\nu\rho}$

$$H_{\mu\nu\rho} = \partial_{[\mu}\,B_{\nu\rho]} \qquad \Longrightarrow \qquad \boxed{\mathcal{H} = \mathrm{dB} + \frac{\alpha'}{8\,\kappa}\Big(\Omega_{3\mathrm{L}} - \Omega_{3\mathrm{Y}}\Big)}$$

$$\Omega_{3\mathrm{L}} = \omega^a{}_c \wedge \mathrm{d}\omega^c{}_a + \frac{2}{3}\omega^a{}_c \wedge \omega^c{}_d \wedge \omega^d{}_a, \qquad \Omega_{3\mathrm{Y}} = \mathbf{A} \wedge \mathrm{d}\mathbf{A} + \mathbf{A} \wedge \mathbf{A} \wedge \mathbf{A},$$

Modified Bianchi Constraint

$$\boxed{\varepsilon_{abc}{}^{\mu}\,\nabla_\mu\, H^{abc} = \frac{\alpha'}{32}\,\sqrt{-g}\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) \equiv \sqrt{-g}\,\mathcal{G}(\omega,\mathbf{A})}$$

Implement in path-integral as a field theory δ(…) via Lagrange multiplier b(x) pseudoscalar (axion-like) field (Kalb-Ramond (KR) Axion) becomes dynamical after H-torsion integration

$$\Pi_x\, \delta\Big(\varepsilon^{\mu\nu\rho\sigma}\, \mathcal{H}_{\nu\rho\sigma}(x)_{;\mu} - \mathcal{G}(\omega, \mathbf{A})\Big) = \exp\Big[i \int d^4x \sqrt{-g}\, \frac{1}{\sqrt{3}}\, b(x) \Big(\mathcal{H}_{\nu\rho\sigma}(x)_{;\mu} - \mathcal{G}(\omega,\mathbf{A})\Big)\Big]$$

$$= \exp\Big[-i \int d^4x\sqrt{-g}\Big(\partial^\mu b(x)\,\frac{1}{\sqrt{3}}\,\epsilon_{\mu\nu\rho\sigma}\,\mathcal{H}^{\nu\rho\sigma} + \frac{b(x)}{\sqrt{3}}\,\mathcal{G}(\omega,\mathbf{A})\Big)\Big]$$

$$\mathcal{Z} = \int DH\, Db\, \exp(-H \wedge *H + c_1 b(dH - \mathcal{G}) + \dots)$$

Effective action after H-torsion (exact) path-integration

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

KR-axion anomalous
CP-Violating interaction

Inclusion of Fermions

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

$$+\, S_{Dirac}^{Free} + \int d^4x\sqrt{-g}\,\Big(\mathcal{F}_\mu + \frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\Big)\,J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\,J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

KR-axion anomalous CP-Violating interaction

$$J^{5\,\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$

Axial Current

We shall use this later on to construct our Cosmological model leading to **CPT-Violation-induced Matter-Antimatter Asymmetry** in the (string) Universe

The Model

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[- \frac{1}{2\kappa^2}\, R + \frac{1}{2}\,\partial_\mu b\, \partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\, b(x)\, \Big(R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\, \widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

$$+ \, S^{Free}_{Dirac} + \int d^4x\sqrt{-g}\, \Big(\mathcal{F}_\mu + \frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\Big)\, J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\, J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

$$J^{5\,\mu} = \bar{\psi}_j\, \gamma^\mu\, \gamma^5 \psi_j$$

All fermion species

Part III

Gravitational Anomalies: SLSB & induced Inflation

The Model in Early Universe:
only gravitational d.o.f. (*b*, $g_{\mu\nu}$)

$$S^{\rm eff}_B = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\, R + \frac{1}{2}\, \partial_\mu b\, \partial^\mu b + \sqrt{\frac{2}{3}}\, \frac{\alpha^\prime}{96\, \kappa}\, b(x)\, R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} + \dots \Big]$$

$$= \int d^4x\, \sqrt{-g}\Big[-\frac{1}{2\kappa^2}\, R + \frac{1}{2}\, \partial_\mu b\, \partial^\mu b - \sqrt{\frac{2}{3}}\, \frac{\alpha^\prime}{96\, \kappa}\, \partial_\mu b(x)\, \mathcal{K}^\mu + \dots \Big]\,,$$

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Gravitational
Chern-Simons (gCS)

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

$$= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu + \dots\Big]\,,$$

Primordial Gravitational Waves →
Condensate < ...> of Gravitational Anomalies

$$g\mathcal{CS} = \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\int d^4x\,\sqrt{-g}\,\Big(\langle\, b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\,\rangle + \,:b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}:\Big)$$

quantum ordered

The Model in Early Universe:
only gravitational d.o.f. (b, $g_{\mu\nu}$)

Gravitational
Chern-Simons (gCS)

$$S_B^{\rm eff} = \int d^4x \sqrt{-g}\Big[-\frac{1}{2\kappa^2}\, R + \frac{1}{2}\, \partial_\mu b\, \partial^\mu b + \sqrt{\frac{2}{3}}\, \frac{\alpha'}{96\,\kappa}\, b(x)\, R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} + \dots \Big]$$

$$= \int d^4x\, \sqrt{-g}\Big[-\frac{1}{2\kappa^2}\, R + \frac{1}{2}\, \partial_\mu b\, \partial^\mu b - \sqrt{\frac{2}{3}}\, \frac{\alpha'}{96\,\kappa}\, \partial_\mu b(x)\, \mathcal{K}^\mu + \dots \Big]\,,$$

$$+ \sqrt{\frac{2}{3}}\, \frac{\alpha'}{96\,\kappa}\, \int d^4x \sqrt{-g}\, \langle b(x)\, R_{\mu\mu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \rangle$$

Condensate < ...> of
Gravitational Anomalies

Cosmological-
Constant-like

$$gCS = \sqrt{\frac{2}{3}}\, \frac{\alpha'}{96\,\kappa}\, \int d^4x \sqrt{-g}\, \Big(\langle\, b(x)\, R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma}\, \rangle + \;: b(x)\, R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} :\Big)$$

quantum ordered

Equations of Motion

Axion b(x)-Field

$$\partial_\alpha\Big[\sqrt{-g}\Big(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\mathcal{K}^\alpha\Big)\Big] = 0$$

Metric tensor (Einstein eqs)

$$\boxed{R^{\mu\nu} - \frac{1}{2}\,g^{\mu\nu}\,R - \sqrt{\frac{2}{3}}\,\frac{\alpha'\,\kappa}{12}\,\mathcal{C}^{\mu\nu} = \kappa^2\,T_b^{\mu\nu} + \Lambda\,g^{\mu\nu}}$$

$$\Lambda = +\sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\langle b(x)\,R_{\mu\mu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\rangle$$

Diffeo-invariance breaking

$$\kappa^2\,\widetilde{T}^{\mu\nu}_{b+g\mathcal{CS}} \equiv \sqrt{\frac{2}{3}}\,\frac{\alpha'\,\kappa}{12}\mathcal{C}^{\mu\nu} + \kappa^2 T_b^{\mu\nu} \quad \Rightarrow \quad \widetilde{T}^{\mu\nu}_{b+g\mathcal{CS}\,;\mu} = 0$$

Equations of Motion

Axion b(x)-Field

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha\bar{b} - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\mathcal{K}^\alpha\right)\right] = 0$$

If anomaly condensate → constant b
(LV solution) ⚠

Metric tensor (Einstein eqs)

$$\boxed{R^{\mu\nu} - \frac{1}{2}\,g^{\mu\nu}\,R - \sqrt{\frac{2}{3}}\,\frac{\alpha'\,\kappa}{12}\,\mathcal{C}^{\mu\nu} = \kappa^2\,T_b^{\mu\nu} + \Lambda\,g^{\mu\nu}}$$

$$\Lambda = +\sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\langle b(x)\,R_{\mu\mu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\rangle$$

Diffeo-invariance breaking

$$\kappa^2\,\widetilde{T}^{\mu\nu}_{b+g\mathcal{CS}} \equiv \sqrt{\frac{2}{3}}\,\frac{\alpha'\,\kappa}{12}\mathcal{C}^{\mu\nu} + \kappa^2 T^{\mu\nu}_{b} \quad \Rightarrow \quad \widetilde{T}^{\mu\nu}_{b+g\mathcal{CS}\,;\mu} = 0$$

Because:

Effective action contains **CP violating axion-like coupling**

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

Effective action contains **CP violating axion-like coupling**

$$\sqrt{-g}\,\mathcal{K}^{\mu}(\omega)_{;\mu}$$

$$S^{\rm eff}_{B} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\, R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha^\prime}{96\,\kappa}\, b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

0 during inflation

Effective action contains **CP violating axion-like coupling**

$$\partial_\mu\left(\sqrt{-g}\,\mathcal{K}^\mu(\omega)\right)$$

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

0 during inflation

$$\begin{aligned} ds^2 = dt^2 - a^2(t)\Big[&(1-h_+(t,z))\,dx^2 \\ &+ (1+h_+(t,z))\,dy^2 \\ &+ 2h_\times(t,z)\,dx\,dy + dz^2\Big] \end{aligned}$$

Average over inflationary space time in the presence of **primordial Gravitational waves**

b(x)-b(t)

Alexander, Peskin, Sheikh -Jabbari

$$\boxed{\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle = \frac{16}{a^4}\kappa^2\int\frac{d^3k}{(2\pi)^3}\frac{H^2}{2k^3}k^4\Theta + O(\Theta^3)}$$

$$\Theta = \sqrt{\frac{2}{3}}\frac{\kappa^3}{12}H\dot{b} \ll 1$$

$\kappa = M_{\rm Pl}^{-1}$,

$\dot{b} \equiv db/dt$

$a(t) \sim e^{Ht}$

H ≈ const. (inflation)

Effective action contains **CP violating axion-like coupling**

$$\partial_\mu\left(\sqrt{-g}\,\mathcal{K}^\mu(\omega)\right)$$

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \ldots\Big]$$

$$ds^2 = dt^2 - a^2(t)\Big[(1 - h_+(t,z))\,dx^2 + (1 + h_+(t,z))\,dy^2 + 2h_\times(t,z)\,dx\,dy + dz^2\Big]$$

Average over inflationary space time in the presence of **primordial Gravitational waves**

0 during inflation

b(x)-b(t)

Alexander, Peskin, Sheikh -Jabbari

$$\frac{d}{dt}\left(\sqrt{-g}\,\mathcal{K}^0(t)\right) = \boxed{\langle R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\rangle = \frac{16}{a^4}\,\kappa^2\int\frac{d^3k}{(2\pi)^3}\,\frac{H^2}{2k^3}\,k^4\,\Theta + O(\Theta^3)}$$

Homogeneity & Isotropy

$$\Theta = \sqrt{\frac{2}{3}}\,\frac{\kappa^3}{12}\,H\dot{b} \ll 1$$

H* ≈ const. *(inflation)

$$\kappa = M_{\rm Pl}^{-1},$$

$$\dot{b} \equiv db/dt$$

$$a(t) \sim e^{Ht}$$

Solutions (backgrounds) to the Eqs of Motion

$$\boxed{\alpha' = M_s^{-2}}$$

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^0$$

Grav. Anomaly Equation

$$\frac{d}{dt}\left(\sqrt{-g}\,\mathcal{K}^0(t)\right) = \boxed{\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle = \frac{16}{a^4}\kappa^2\int\frac{d^3k}{(2\pi)^3}\frac{H^2}{2k^3}k^4\Theta + O(\Theta^3)}$$

$$\Theta = \sqrt{\frac{2}{3}}\frac{\kappa^3}{12}H\dot{b} \propto \mathcal{K}^0$$

time evolution of Anomaly

μ = UV k-momentum Cut-off

$$\boxed{\mathcal{K}^0(t) \simeq \mathcal{K}^0_{\rm begin}(0)\exp\left[-3Ht\left(1 - 0.73\times10^{-4}\left(\frac{H}{M_{\rm Pl}}\right)^2\left(\frac{\mu}{M_{\rm Pl}}\right)^4\right)\right]}$$

$$\boxed{\frac{\mu}{M_s} \simeq 15\left(\frac{M_{\rm Pl}}{H}\right)^{1/2}} \Rightarrow \mathcal{K}^0 = \text{const.}$$

Spontaneous LV solution

Planck Data $H/M_{\rm Pl} < 10^{-4}$ ⇒ **to ensure constant anomaly $\mu / M_s = O(10^3)$**

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\mathcal{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \boxed{\dot{\bar{b}} = \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\mathcal{K}^0 \sim \text{constant}}$$

$$\frac{d}{dt}\left(\sqrt{-g}\,\mathcal{K}^0(t)\right) = \boxed{\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle = \frac{16}{a^4}\,\kappa^2 \int \frac{d^3k}{(2\pi)^3}\,\frac{H^2}{2k^3}\,k^4\,\Theta + O(\Theta^3)}$$

$$\Theta = \sqrt{\frac{2}{3}}\,\frac{\kappa^3}{12}\,H\dot{b} \propto \mathcal{K}^0$$

time evolution of Anomaly

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$$\boxed{\mathcal{K}^0(t) \simeq \mathcal{K}^0_{\rm begin}(0)\,\exp\left[-3Ht\left(1 - 0.73\times 10^{-4}\left(\frac{H}{M_{\rm Pl}}\right)^2\left(\frac{\mu}{M_{\rm Pl}}\right)^4\right)\right]}$$

$$\boxed{\frac{\mu}{M_s} \simeq 15\left(\frac{M_{\rm Pl}}{H}\right)^{1/2}}$$

$$\boxed{\mathcal{K}^0 = \text{const.}}$$

Spontaneous LV solution

Planck Data $H/M_{\rm Pl} < 10^{-4}$

to ensure constant anomaly
$\mu / M_s = O(10^3)$

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\Big[\sqrt{-g}\Big(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\mathcal{K}^\alpha(t)\Big)\Big] = 0 \quad \Rightarrow \quad \boxed{\dot{\bar{b}} = \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\mathcal{K}^0 \sim \text{constant}}$$

Using **slow-roll assumption** for both fields, *b* and the inflaton φ

$$\varepsilon = \frac{1}{2}\,\frac{1}{(HM_{\rm Pl})^2}\,\dot{\varphi}^2 \sim \frac{1}{2}\,\frac{1}{(HM_{\rm Pl})^2}\,\dot{\bar{b}}^2 \sim 10^{-2}$$

Planck Data

@ end of Inflationary era

$$\boxed{\dot{\bar{b}} \sim \sqrt{2\,\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H}$$

$$H = H_{\rm infl} \simeq \text{const.}$$

Shapiro-Sola

Running-vacuum type contribution of GA to energy density

$$\boxed{\rho^{\varphi+b} \simeq 3M_{\rm Pl}^4\Big[3.33\times 10^{-3}\,\Big(\frac{H_{\rm infl}}{M_{\rm Pl}}\Big)^2 + \frac{\mathcal{U}(\varphi)}{3M_{\rm Pl}^4}\Big]}$$

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\Big[\sqrt{-g}\Big(\partial^\alpha\bar{b}-\sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\mathcal{K}^\alpha(t)\Big)\Big]=0 \quad \Rightarrow \quad \boxed{\dot{\bar{b}}=\sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\mathcal{K}^0\sim\text{constant}}$$

Using **slow-rol** … ton φ

$$\varepsilon \quad \ldots \quad 2\,(HM_{\rm Pl})^2\,\varphi \quad \ldots \quad 2\,(HM_{\rm Pl})^2\,b \quad \ldots$$

Undiluted KR axion background at the end of Inflation

Planck Data

@ end of Inflationary era

$$\boxed{\dot{\bar{b}}\sim\sqrt{2\,\varepsilon}\,M_{\rm Pl}\,H\sim 0.14\,M_{\rm Pl}\,H}$$

$$H = H_{\rm infl} \simeq \text{const.}$$

Shapiro-Sola

Running-vacuum type contribution of GA to energy density

$$\rho^{\varphi+b}\simeq 3M_{\rm Pl}^4\left[3.33\times10^{-3}\left(\frac{H_{\rm infl}}{M_{\rm Pl}}\right)^2+\frac{\mathcal{U}(\varphi)}{3M_{\rm Pl}^4}\right].$$

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\mathcal{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \boxed{\dot{\bar{b}} = \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\mathcal{K}^0 \sim \text{constant}}$$

Using **slow-roll assumption** for both fields, *b* and the inflaton *φ*

$$\varepsilon = \frac{1}{2}\,\frac{1}{(HM_{\rm Pl})^2}\,\dot{\varphi}^2 \sim \frac{1}{2}\,\frac{1}{(HM_{\rm Pl})^2}\,\dot{\bar{b}}^2 \sim 10^{-2}$$

Planck Data

@ end of Inflationary era

$$\boxed{\dot{\bar{b}} \sim \sqrt{2\,\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H}$$

Can be generated by gravitnl. anomaly condensates ~ H^4

$$H = H_{\rm infl} \simeq {\rm const.}$$

Shapiro-Sola

Running-vacuum type contribution of GA to energy density

$$\rho^{\varphi+b} \simeq 3M_{\rm Pl}^4\left[3.33\times10^{-3}\left(\frac{H_{\rm infl}}{M_{\rm Pl}}\right)^2 + \frac{\mathcal{U}(\varphi)}{3M_{\rm Pl}^4}\right].$$

Gravitational Anomaly Condensates → Dynamical Inflation

Basilakos, NEM, Sola

Cotton tensor properties

$$C^{\mu 0}{}_{;\mu} = \frac{d}{dt}\mathcal{C}^{00} + 4H\,\mathcal{C}^{00} \simeq -\frac{1}{8}\,\dot{\overline{b}}\,\langle R^{\alpha\beta\gamma\delta}\,\widetilde{R}_{\alpha\beta\gamma\delta}\rangle \simeq -\frac{1}{8}\sqrt{\frac{2}{3}}\,\frac{\alpha'\,\kappa}{12}\,H\,\frac{1}{\pi^2}\Big(\frac{H}{M_{\rm Pl}}\Big)^2\,\mu^4\,\dot{\overline{b}}^2\,;$$

Gravitational Anomaly Condensates → Dynamical Inflation

Basilakos, NEM, Sola

Cotton tensor properties

$$\mathcal{C}^{\mu 0}{}_{;\mu} = \frac{d}{dt}\mathcal{C}^{00} + 4H\,\mathcal{C}^{00} \simeq -\frac{1}{8}\dot{\bar{b}}\,\langle R^{\alpha\beta\gamma\delta}\,\widetilde{R}_{\alpha\beta\gamma\delta}\rangle \simeq -\frac{1}{8}\sqrt{\frac{2}{3}}\,\frac{\alpha'\,\kappa}{12}\,H\,\frac{1}{\pi^2}\Big(\frac{H}{M_{\rm Pl}}\Big)^2\,\mu^4\,\dot{\bar{b}}^2$$

Recall

$$\boxed{\mathcal{C}^{\mu\nu}{}_{;\mu} = -\frac{1}{8}\,v^\nu\,R^{\alpha\beta\gamma\delta}\,\widetilde{R}_{\alpha\beta\gamma\delta}}$$

$$v_\sigma \equiv \partial_\sigma b$$

Gravitational Anomaly Condensates → Dynamical Inflation

Basilakos, NEM, Sola

Cotton tensor properties

$$C^{\mu 0}{}_{;\mu} = \cancel{\frac{d}{dt}\mathcal{C}^{00}} + 4H\,\mathcal{C}^{00} \simeq -\frac{1}{8}\,\dot{\overline{b}}\,\langle R^{\alpha\beta\gamma\delta}\,\widetilde{R}_{\alpha\beta\gamma\delta}\rangle \simeq -\frac{1}{8}\sqrt{\frac{2}{3}}\,\frac{\alpha'\,\kappa}{12}\,H\,\frac{1}{\pi^2}\Big(\frac{H}{M_{\rm Pl}}\Big)^2\,\mu^4\,\dot{\overline{b}}^2$$

0

Approx. constant

$$\Big\Downarrow \qquad \dot{\overline{b}} \sim \sqrt{2\,\varepsilon}\,M_{\rm Pl}\,H$$

$$\mathcal{C}^{00} \simeq -\epsilon\sqrt{\frac{2}{3}}\,\frac{\alpha'\,\kappa}{192}\,\frac{1}{\pi^2}\,\mu^4\,H^4$$

Gravitational Anomaly Condensates → Dynamical Inflation

Basilakos, NEM, Sola

Cotton tensor properties

$$C^{\mu 0}{}_{;\mu} = \cancelto{0}{\frac{d}{dt}\mathcal{C}^{00}} + 4H\,\mathcal{C}^{00} \simeq -\frac{1}{8}\dot{\bar{b}}\,\langle R^{\alpha\beta\gamma\delta}\,\widetilde{R}_{\alpha\beta\gamma\delta}\rangle \simeq -\frac{1}{8}\sqrt{\frac{2}{3}}\,\frac{\alpha'\,\kappa}{12}\,H\,\frac{1}{\pi^2}\Big(\frac{H}{M_{\rm Pl}}\Big)^2\mu^4\,\dot{\bar{b}}^2$$

Approx. constant

$$\dot{\bar{b}} \sim \sqrt{2\,\varepsilon}\,M_{\rm Pl}\,H$$

$$\mathcal{C}^{00} \simeq -\epsilon\sqrt{\frac{2}{3}}\,\frac{\alpha'\,\kappa}{192}\,\frac{1}{\pi^2}\,\mu^4\;H^4 \quad < 0$$

(cf. Gauss-Bonnet-dilaton coupling case)

Gravitational Anomaly Condensates → Dynamical Inflation

Basilakos, NEM, Sola

Cotton tensor properties

$$C^{\mu 0}{}_{;\mu} = \frac{d}{dt}\mathcal{C}^{00} + 4H\,\mathcal{C}^{00} \simeq -\frac{1}{8}\,\dot{b}\,\langle R^{\alpha\beta\gamma\delta}\,\widetilde{R}_{\alpha\beta\gamma\delta}\rangle \simeq -\frac{1}{8}\sqrt{\frac{2}{3}}\,\frac{\alpha'\,\kappa}{12}\,H\,\frac{1}{\pi^2}\left(\frac{H}{M_{\rm Pl}}\right)^2\mu^4\,\dot{b}^2$$

0

Approx. constant

$$\dot{b} \sim \sqrt{2\,\varepsilon}\,M_{\rm Pl}\,H$$

$$\mathcal{C}^{00} \simeq -\epsilon\sqrt{\frac{2}{3}}\,\frac{\alpha'\,\kappa}{192}\,\frac{1}{\pi^2}\,\mu^4\;H^4$$

$$\kappa^2\,\widetilde{T}^{\mu\nu}_{b+g\mathcal{CS}} \equiv \sqrt{\frac{2}{3}}\,\frac{\alpha'\,\kappa}{12}\mathcal{C}^{\mu\nu} + \kappa^2 T^{\mu\nu}_{b}$$

$$\rho^{g\mathcal{CS}} = +\sqrt{\frac{2}{3}}\,\frac{\alpha'}{12\,\kappa}\,\mathcal{C}^{00} \simeq -2.932\times 10^{-5}\,\epsilon\left(\frac{\mu}{M_s}\right)^4 H^4$$

$$\frac{\mu}{M_s} \simeq 15\left(\frac{M_{\rm Pl}}{H}\right)^{1/2}$$

$$\rho^{g\mathcal{CS}} \simeq -1.484\,\epsilon\,M_{\rm Pl}^2\,H^2$$

Equations of Motion

Metric tensor (Einstein eqs)

$$\boxed{R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}\,R - \sqrt{\frac{2}{3}}\,\frac{\alpha'\,\kappa}{12}\,\mathcal{C}^{\mu\nu} = \kappa^2\,T_b^{\mu\nu} + \Lambda\,g^{\mu\nu}}$$

$$\Lambda = +\sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\langle b(x)\,R_{\mu\mu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\rangle$$

$$\kappa^2\,\widetilde{T}^{\mu\nu}_{b+g\mathcal{CS}} \equiv \sqrt{\frac{2}{3}}\,\frac{\alpha'\,\kappa}{12}\mathcal{C}^{\mu\nu} + \kappa^2 T_b^{\mu\nu} \quad \Rightarrow \quad \widetilde{T}^{\mu\nu}_{b+g\mathcal{CS}\,;\mu} = 0$$

$$\Downarrow$$

$$\frac{d}{dt}(\rho^b + \rho^{g\mathcal{CS}}) + 3H\Big((1 + \underset{w_b = 1}{w_b})\,\rho^b + \frac{4}{3}\rho^{g\mathcal{CS}}\Big) \simeq 0 \quad \Rightarrow \quad \boxed{\rho^b \simeq -\frac{2}{3}\rho^{g\mathcal{CS}}}$$

Equations of Motion

Metric tensor (Einstein eqs)

$$\boxed{R^{\mu\nu} - \frac{1}{2}\, g^{\mu\nu}\, R - \sqrt{\frac{2}{3}}\, \frac{\alpha'\, \kappa}{12}\, \mathcal{C}^{\mu\nu} = \kappa^2\, T_b^{\mu\nu} + \Lambda\, g^{\mu\nu}}$$

$$\Lambda = +\sqrt{\frac{2}{3}}\, \frac{\alpha'}{96\,\kappa}\, \langle b(x)\, R_{\mu\mu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \rangle$$

$$\kappa^2\, \widetilde{T}^{\mu\nu}_{b+g\mathcal{CS}} \equiv \sqrt{\frac{2}{3}}\, \frac{\alpha'\, \kappa}{12} \mathcal{C}^{\mu\nu} + \kappa^2 T_b^{\mu\nu} \quad \Rightarrow \quad \widetilde{T}^{\mu\nu}_{b+g\mathcal{CS}\,;\mu} = 0$$

$$\Big\Downarrow$$

$$\frac{d}{dt}(\rho^b + \rho^{g\mathcal{CS}}) + 3H\Big((1 + \underset{w_b\,=\,1}{w_b})\, \rho^b + \frac{4}{3}\rho^{g\mathcal{CS}}\Big) \simeq 0 \quad \Rightarrow \quad \boxed{\rho^b \simeq -\frac{2}{3}\rho^{g\mathcal{CS}}}$$

$$\rho^{g\mathcal{CS}} \simeq -1.484\, \epsilon\, M_{\rm Pl}^2\, H^2$$

$$\boxed{\rho^b \simeq 0.9895\, \epsilon\, M_{\rm Pl}^2\, H^2}$$

Role of Anomaly Condensate

$$\Lambda \equiv \langle b(x)\, R_{\mu\mu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \rangle \simeq 5.86 \times 10^{7}\, \epsilon \mathcal{N}\, H^4 > 0$$

Positive Cosmological Constant-like

Positive total energy density since Λ-term dominates

$$\rho_{\rm total} = \rho_b + \rho_{gCS} + \rho_\Lambda \simeq 3M_{\rm Pl}^4 \Big[-1.7 \times 10^{-3} \Big(\frac{H}{M_{\rm Pl}}\Big)^2 + \Big(1.17 - 1.37\Big) \times 10^7 \, (\frac{H}{M_{\rm Pl}})^4 \Big] > 0$$

Basilakos,NEM, Sola ....

RVM-like terms drive inflation contain scalar d.o.f. ``vacuumon''

$$\rho_{\rm tot} \equiv \rho_\phi = \dot{\phi}^2/2 + V(\phi)$$

$$p_{\rm tot} \equiv p_\phi = \dot{\phi}^2/2 - V(\phi)$$

$$\dot{\phi}^2 = -\frac{2}{\kappa^2}\dot{H}$$

$$V = \frac{3H^2}{\kappa^2}\left(1 + \frac{\dot{H}}{3H^2}\right) = \frac{3H^2}{\kappa^2}\left(1 + \frac{a}{6H^2}\,\frac{dH^2}{da}\right)$$

$$U(\phi) = \frac{H_I^2}{\alpha\kappa^2}\,\frac{2 + \cosh^2(\kappa\phi)}{\cosh^4(\kappa\phi)}$$

$$\epsilon = -\frac{\dot{H}}{H} = \frac{1}{2}\frac{1}{(HM_{\rm Pl})^2}\,\dot{\phi}^2 \simeq 10^{-2} \quad \Longrightarrow \quad \epsilon = \frac{1}{2}\frac{1}{(HM_{\rm Pl})^2}\,\dot{\phi}^2 \sim \frac{1}{2}\frac{1}{(HM_{\rm Pl})^2}\,\dot{b}^2 \sim 10^{-2}$$

Post Inflationary Era

Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation

Basilakos, NEM,Sola (2019)

Required by consistency of quantum theory of matter and radiation (**diffeomorphism invariance**)

Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation

$$\boxed{\partial_\mu\left(\sqrt{-g}\,\mathcal{K}^\mu(\omega)\right)}$$

Recall:

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} \qquad\qquad \Big) + \dots\Big]$$

$$+\,S_{Dirac}^{Free} + \int d^4x\sqrt{-g}\,\Big(\mathcal{F}_\mu + \frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\Big)\,J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\,J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

$$J^{5\,\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$ **Axial Current**

KR-axion anomalous
CP-Violating interaction

Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation

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$$+ S_{Dirac}^{Free} + \int d^4x\sqrt{-g}\left(\mathcal{F}_\mu + \frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\right)J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\,J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

$$J^{5\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$

Axial Current

KR-axion anomalous
CP-Violating interaction

$$\nabla_\mu J^{5\mu} = \frac{e^2}{8\pi^2}F^{\mu\nu}\widetilde{F}_{\mu\nu} - \frac{1}{192\pi^2}R^{\mu\nu\rho\sigma}\widetilde{R}_{\mu\nu\rho\sigma} \equiv G(\mathbf{A},\omega)$$

includes possible
chiral U(1) or QCD-type
anomalies

Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation

$$\boxed{\partial_\mu\left(\sqrt{-g}\,\mathcal{K}^\mu(\omega)\right)}$$

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KR-axion anomalous CP-Violating interaction

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includes possible chiral U(1) or QCD-type anomalies

⚠

Gauge terms do **not** contribute to stress tensor
→ do **not** affect diffeomorphism invariance

Cancellation of Gravitational Anomalies in Radiation Era
by:
Chiral Fermionic Matter generation @ end of Inflation

Recall:

$$\partial_\mu\left(\sqrt{-g}\,\mathcal{K}^\mu(\omega)\right)$$

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa}\,b(x)\left(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\right) + \dots\Big]$$

$$+ S_{Dirac}^{Free} + \int d^4x\sqrt{-g}\left(\mathcal{F}_\mu + \frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\right)J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\,J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

Partial Integrate

$$J^{5\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$ **Axial Current**

$$\nabla_\mu J^{5\mu} = \frac{e^2}{8\pi^2}F^{\mu\nu}\widetilde{F}_{\mu\nu} - \frac{1}{192\pi^2}R^{\mu\nu\rho\sigma}\widetilde{R}_{\mu\nu\rho\sigma} \equiv G(\mathbf{A},\omega)$$

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Cancellation of Gravitational Anomalies in Radiation Era by:

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$$S^{\mathrm{eff}} = \int d^4x \sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\, \partial^\mu b + \kappa\, b(x)\, \nabla_\mu \Big(\sqrt{\frac{2}{3}}\, \frac{1}{96}\, \mathcal{K}^\mu - \sqrt{\frac{3}{8}}\, J^{5\mu} \Big) \Big] + \dots,$$

$$J^{5\mu} = \sum_j \bar{\psi}_j \gamma^\mu \gamma^5 \psi_j$$

Chiral current, including RHN

(Mixed) Anomaly equation

$$\nabla_\mu J^{5\mu} = \frac{e^2}{8\pi^2} F^{\mu\nu} \widetilde{F}_{\mu\nu} - \frac{1}{192\pi^2} R^{\mu\nu\rho\sigma} \widetilde{R}_{\mu\nu\rho\sigma} \equiv G(\mathbf{A}, \omega)$$

includes possible
chiral U(1) or QCD-type
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Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation

$$S^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \kappa\, b(x)\,\nabla_\mu\Big(\sqrt{\frac{2}{3}}\,\frac{1}{96}\,\mathscr{K}^\mu - \sqrt{\frac{3}{8}}\,J^{5\mu}\Big)\Big] + \dots,$$

$$J^{5\mu} = \textstyle\sum_j \bar{\Psi}_j\,\gamma^\mu\,\gamma^5\,\psi_j$$

Chiral current, including RHN

$$\partial_\mu\Big[\sqrt{-g}\Big(\sqrt{\frac{3}{8}}\frac{\alpha'}{\kappa}\,J^{5\mu} - \frac{\alpha'}{\kappa}\sqrt{\frac{2}{3}}\,\frac{1}{96}\,K^\mu\Big)\Big] = \sqrt{\frac{3}{8}}\,\frac{\alpha'}{\kappa}\,\Big(\frac{\alpha_{\rm EM}}{2\pi}\,\sqrt{-g}\,F^{\mu\nu}\,\widetilde{F}_{\mu\nu} + \frac{\alpha_s}{8\pi}\,\sqrt{-g}\,G^a_{\mu\nu}\,\widetilde{G}^{a\mu\nu}\Big)$$

chiral U(1) Gluon QCD

Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation

$$S^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \kappa\, b(x)\,\nabla_\mu\Big(\sqrt{\frac{2}{3}}\,\frac{1}{96}\,\mathscr{K}^\mu - \sqrt{\frac{3}{8}}\,J^{5\,\mu}\Big)\Big] + \dots,$$

$$J^{5\,\mu} = \sum_j \bar{\Psi}_j\,\gamma^\mu\,\gamma^5\,\psi_j;$$

Chiral current, including RHN

$$\boxed{\partial_\mu\Big[\sqrt{-g}\,\Big(\sqrt{\frac{3}{8}}\frac{\alpha'}{\kappa}\,J^{5\mu} - \frac{\alpha'}{\kappa}\sqrt{\frac{2}{3}}\,\frac{1}{96}\,\mathcal{K}^\mu\Big)\Big] = \sqrt{\frac{3}{8}}\,\frac{\alpha'}{\kappa}\,\Big(\frac{\alpha_{\rm EM}}{2\pi}\,\sqrt{-g}\,F^{\mu\nu}\,\widetilde{F}_{\mu\nu} + \frac{\alpha_s}{8\pi}\,\sqrt{-g}\,G^a_{\mu\nu}\,\widetilde{G}^{a\mu\nu}\Big)}$$

Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation

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$$\boxed{\partial_\mu\Big[\sqrt{-g}\,\Big(\sqrt{\frac{3}{8}}\frac{\alpha'}{\kappa}\,J^{5\mu} - \frac{\alpha'}{\kappa}\sqrt{\frac{2}{3}}\,\frac{1}{96}\,K^\mu\Big)\Big] = \sqrt{\frac{3}{8}}\,\frac{\alpha'}{\kappa}\,\Big(\frac{\alpha_{\rm EM}}{2\pi}\,\sqrt{-g}\,F^{\mu\nu}\,\widetilde{F}_{\mu\nu} + \frac{\alpha_s}{8\pi}\,\sqrt{-g}\,G^a_{\mu\nu}\,\widetilde{G}^{a\mu\nu}\Big)}$$

Eqs of Motion for b-field →

$$\partial_\mu\Big(\sqrt{-g}\,\partial^\mu b(x)\Big) = \text{“chiral U(1) anomalies''},$$

and/or QCD type

Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation

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$$J^{5\mu} = \sum_j \bar{\psi}_j\, \gamma^\mu\, \gamma^5\, \psi_j;$$

Chiral current, including RHN

$$\boxed{\partial_\mu\Big(\sqrt{-g}\Big[\sqrt{\frac{3}{8}}\,J^{5\mu} - \sqrt{\frac{2}{3}}\,\frac{1}{96}\,\mathcal{K}^\mu\Big]\Big) = \text{“chiral U(1) anomalies”}.}$$

Possibly also QCD type

Eqs of Motion for b-field → $\partial_\mu\Big(\sqrt{-g}\,\partial^\mu b(x)\Big) = \text{“chiral U(1) anomalies”},$

and/or QCD type

Scale factor $a(t) \sim T^{-1}$

⇓ $b=b(t)$,

$$\dot{b} = \frac{\rm const.}{a^3(t)} + \dots$$

Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation

$$S^{\rm eff} = \int d^4x \sqrt{-g}\Big[-\frac{1}{2\kappa^2} R + \frac{1}{2}\partial_\mu b\, \partial^\mu b + \kappa\, b(x)\, \nabla_\mu \Big(\sqrt{\frac{2}{3}}\, \frac{1}{96}\, \mathcal{K}^\mu - \sqrt{\frac{3}{8}}\, J^{5\mu}\Big)\Big] + \dots,$$

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$$\boxed{\partial_\mu \Big(\sqrt{-g}\Big[\sqrt{\frac{3}{8}}\, J^{5\mu} - \sqrt{\frac{2}{3}}\, \frac{1}{96}\, \mathcal{K}^\mu\Big]\Big) = \text{“chiral U(1) anomalies”}.}$$

Possibly also QCD type

Eqs of Motion for b-field → $$\partial_\mu\Big(\sqrt{-g}\, \partial^\mu b(x)\Big) = \text{“chiral U(1) anomalies”},$$

Scale factor $a(t) \sim T^{-1}$

Possibly also QCD

$$\boxed{\dot{b} \propto T^3 + \text{subleading } (\sim T^2) \text{ chiral U(1) anomaly terms}}$$

and/or QCD type

Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation

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$$\partial_\mu\Big(\sqrt{-g}\Big[\sqrt{\frac{3}{8}}\,J^{5\mu} - \sqrt{\frac{2}{3}}\,\frac{1}{96}\,\mathcal{K}^\mu\Big]\Big) = \text{“chiral U(1) anomalies''}.$$

Possibly also QCD type

Eqs of Motion for b-field → $\partial_\mu\Big(\sqrt{-g}\,\partial^\mu b(x)\Big) = \text{“chiral U(1) anomalies''},$

Scale factor *a(t)* ~ T^{-1}

Possibly also QCD

$$\boxed{\dot{\overline{b}} \propto T^3 + \text{subleading } (\sim T^2) \text{ chiral U(1) anomaly terms}}$$

Viewed as sufficiently slow moving to induce Leptogenesis

Bossingham, NEM, Sarkar (2018)

Part IV

Gravitational Anomalies

&

Lorentz- & CPT-Violating

Leptogenesis

CPT VIOLATING LRPTOGENESIS & GRAVITATIONAL ANOMALIES IN THE EARLY UNIVERSE

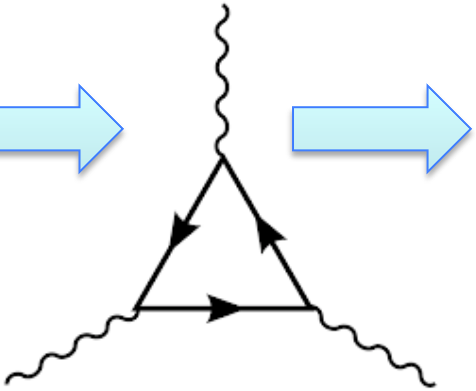
S Basilakos, NEM, J Sola (2019)

Microscopic Mechanism For LV & CPTV H-Torsion Background

Inflationary (de Sitter) Phase

Radiation Era

Primordial Gravitational Waves



Gravitational anomaly (GA)

Undiluted constant

H-torsion (B_0)

background

+ chiral matter generation

@ inflation exit

Cancellation of GA (consitency of QFT)

$B_0 \propto T^3$

(slowly varying)

Leptogenesis

(decays of RHN)

NEM, Sarkar Bossnigham

NEM, Sarkar,
+ de Cesare, Bossingham

Early Universe
T >> T_{EW}

$$\mathcal{L} = i\overline{N}\not{\partial}N - \frac{M}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not{B}\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Heavy RHN interact with axial constant background

with only temporal component $B_0 \neq 0$

NEM, Sarkar,
+ de Cesare, Bossingham

Early Universe
T >> T_{EW}

$$\mathcal{L} = i\overline{N}\partial\!\!\!/ N - \frac{M}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Heavy RHN interact with axial constant background

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Lorentz & CPT Violation

NEM, Sarkar,
+ de Cesare, Bossingham

Early Universe
$T >> T_{EW}$

$$\mathcal{L} = i\overline{N}\partial\!\!\!/ N - \frac{M}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Heavy RHN interact with axial constant background

with only temporal component $B_0 \neq 0$

STANDARD MODEL EXTENSION EFT

Kostelecky, Bluhm, Colladay, Lehnert, Potting, Russell *et al.*

Lorentz & CPT Violation

$$\mathcal{L} = \frac{1}{2}i\bar{\psi}\Gamma^\nu\bar{\partial}_\nu\psi - \bar{\psi}M\psi,$$

$$M \equiv m + a_\mu\gamma^\mu + b_\mu\gamma_5\gamma^\mu + \frac{1}{2}H^{\mu\nu}\sigma_{\mu\nu}$$

Spontaneous Violation of Lorentz Symmetry
(LV coefficients are v.e.v. of tensor-valued field quantities)
$B_0 \approx$ constant is H-torsion background in our model

CPT Violation

de Cesare, NEM, Sarkar
Eur.Phys.J. C75, 514 (2015)

Early Universe
T >> T_{EW}

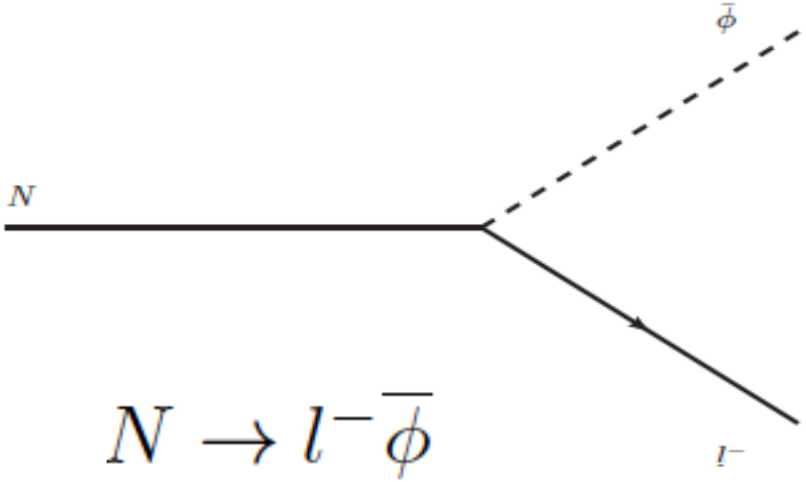
$$\mathcal{L} = i\overline{N}\not\partial N - \frac{M}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not B\gamma^5 N - \boxed{Y_k\overline{L}_k\tilde{\phi}N} + h.c.$$

Heavy RHN interact with axial constant background

with only temporal component $B_0 \neq 0$

Produce Lepton asymmetry

Lepton number & CP Violations
@ tree-level due to
Lorentz/CPTV Background



$$N \to l^+\phi \qquad\qquad N \to l^-\overline{\phi}$$

$$\Gamma_1 = \sum_k \frac{|Y_k|^2}{32\pi^2}\frac{M^2}{\Omega}\frac{\Omega + B_0}{\Omega - B_0} \neq \Gamma_2 = \sum_k \frac{|Y_k|^2}{32\pi^2}\frac{M^2}{\Omega}\frac{\Omega - B_0}{\Omega + B_0}$$

$B_0 \neq 0$

CPV & LV

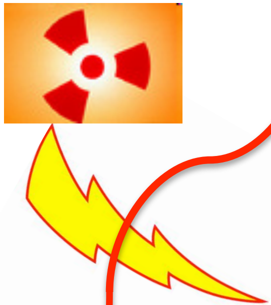
$$\Omega = \sqrt{B_0^2 + M^2}$$

$$\mathcal{L} = i\overline{N}\not{\partial}N - \frac{M}{2}(\overline{N^c}N\ {}^{m}\overline{N}N^c) - \overline{N}\not{B}\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Constant B_0 Background

Early Universe
T > 10^5 GeV

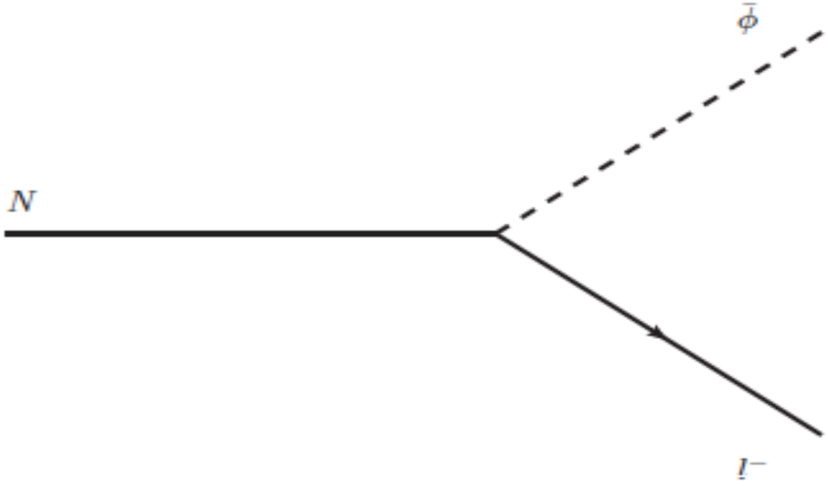
CPT Violation



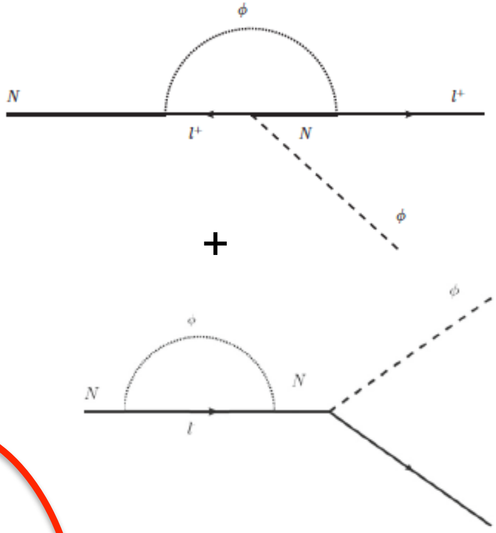
Lepton number & CP Violations @ tree-level
due to Lorentz/CPTV Background

$$N_I \rightarrow \overline{\phi}\,\ell\,,\ \phi\,\overline{\ell}$$

Produce Lepton asymmetry



Contrast with one-loop conventional CPV Leptogenesis (in absence of H-torsion)



Fukugita, Yanagida,

$$\mathcal{L} = i\overline{N}\not{\partial}N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not{B}\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Early Universe
T > 10^5 GeV

CPT Violating Leptogenesis

Lepton number & CP Violations @ tree-level due to Lorentz/CPTV Background

$$N_I \rightarrow \overline{\phi}\,\ell\,,\ \phi\,\overline{\ell}$$

$$\mathcal{L} = i\overline{N}\not{\partial}N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not{B}\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

CPT Violation

Early Universe
$T > 10^5$ GeV

One generation of *massive neutrinos N* *suffices* *for generating CPTV Leptogenesis;*

Lepton number & CP Violations @ tree-level due to Lorentz/CPTV Background

$$N_I \rightarrow \overline{\phi}\,\ell\,,\ \phi\,\overline{\ell}$$

$$\mathcal{L} = i\overline{N}\not{\partial}N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not{B}\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

CPT Violation

Early Universe
$T > 10^5$ GeV

***One generation* of**
massive neutrinos N
suffices for generating
CPTV Leptogenesis;
mass m free
to be fixed

Lepton number & CP Violations @ tree-level
due to Lorentz/CPTV Background

$$N_I \rightarrow \overline{\phi}\,\ell\,,\ \phi\,\overline{\ell}$$

$$\mathcal{L} = i\overline{N}\not{\partial}N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not{B}\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Constant $B^0 \neq 0$ background

Early Universe
$T > 10^5$ GeV

CPT Violation

Lepton number & CP Violations @ tree-level
due to Lorentz/CPTV Background

$$N_I \rightarrow \overline{\phi}\,\ell\,,\ \phi\,\overline{\ell}$$

Produce Lepton asymmetry

$$\mathcal{L} = i\overline{N}\partial\!\!\!/ N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Early Universe
T > 10^5 GeV

CPT Violation

Constant $B^0 \neq 0$ background

Lepton number & CP Violations @ tree-level due to Lorentz/CPTV Background

$$N_I \rightarrow \overline{\phi}\,\ell\,,\ \phi\,\overline{\ell}$$

$$\frac{\Delta L}{n_\gamma} \simeq 10^{-10},$$

* **Solving system of Boltzmann eqs**

Produce Lepton asymmetry

*

Decoupling Temperature T_D : **decay process out of equilibrium**
@ which Lepton asymmetry is evaluated

$$\Gamma \simeq H = 1,66\, T_D^2 \underset{\text{d.o.f.}}{\mathcal{N}^{1/2}} m_P^{-1}$$

assume standard cosmology

$$T_D \simeq 6.2 \cdot 10^{-2} \frac{|Y|}{\mathcal{N}^{1/4}} \sqrt{\frac{m_P(\Omega^2 + B_0^2)}{\Omega}}$$

for one generation of RH heavy neutrino

$$\Omega = \sqrt{B_0^2 + m_N^2} \ .$$

Estimate: Total Lepton number asymmetry

$$\boxed{\left(N \to \ell^- \phi^+, \nu\ \phi^0\right) - \left(N \to \ell^+ \phi^-, \overline{\nu}\ \phi^0\right)}$$

via solving the appropriate system of **Boltzmann equations:**

CPTV Thermal

$$\mathcal{L} = i\overline{N}\partial\!\!\!/ N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Early Universe
$T > 10^5$ GeV

CPT Violation

Constant $B^0 \neq 0$ background

Lepton number & CP Violations @ tree-level due to Lorentz/CPTV Background

$$N_I \rightarrow \overline{\phi}\,\ell\,,\;\; \phi\,\overline{\ell}$$

$$\frac{\Delta L}{n_\gamma} \simeq 10^{-10},$$

$$\frac{B_0}{m} \simeq 10^{-8}$$

Produce Lepton asymmetry

$$\frac{\Delta L^{TOT}}{s} \simeq \frac{g_N}{7\,e\,(2\pi)^{3/2}}\frac{B_0}{m_N} \simeq 0.007\,\frac{B_0}{m_N}$$

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$$Y_k \sim 10^{-5}$$

$$m \geq 100\text{TeV} \rightarrow$$

$$B^0 \sim 1\text{MeV}$$

$$T_D \simeq m \sim 100\ \text{TeV}$$

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Similar order of magnitude estimates if $B^0 \sim T^3$ during Leptogenesis era

Bossingham, NEM, Sarkar

$$\mathcal{L} = i\overline{N}\not{\partial}N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not{B}\gamma^5 N - Y_k \overline{L}_k \tilde{\phi} N + h.c.$$

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Produce Lepton asymmetry

Equilibrated electroweak
B+L violating sphaleron interactions

B-L conserved

Environmental Conditions Dependent

Observed Baryon Asymmetry In the Universe (BAU)

$$L = \frac{2}{M} l_L l_L \phi\phi + \text{H.c.}$$

where

$$l_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

Fukugita, Yanagida,

$$\mathcal{L} = i\overline{N}\not{\partial}N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not{B}\gamma^5 N - Y_k \overline{L}_k \tilde{\phi} N + h.c.$$

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Equilibrated electroweak
B+L violating sphaleron interactions

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Environmental
Conditions Dependent

Observed Baryon Asymmetry
In the Universe (BAU)

Estimate BAU by fixing CPTV background parameters
In some models this means fine tuning ….

B^0 : (string) model: cancellation of gravity anonaly @ exit from inflation implies

$$B^0 \sim \dot{\bar{b}} \sim 1/a^3(t) \sim T^3$$

i.e. scales @ leptogenesis era @ T ≈ T_d = 10^5 GeV, from B^0 = const = 1 MeV **to** :

$$B_0 = c_0 T^3 \qquad c_0 = 10^{-42}\,\mathrm{meV}^{-2}$$

(ii) **or B^0 small today but non zero**

$$\boxed{B_{0\,\mathrm{today}} = \mathcal{O}\left(10^{-44}\right)\ \mathrm{meV}}$$

Quite safe from stringent Experimental Bounds

$$|B^0| < 10^{-2}\,\mathrm{eV}$$
$$B_i \equiv b_i < 10^{-31}\,\mathrm{GeV}$$

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$$B_0 = c_0 \, T^3 \qquad c_0 = 10^{-42} \, \mathrm{meV}^{-2}$$

If chiral U(1) anomalies present

$$B^0 \sim T^2$$

(ii) **or B^0 small today but non zero**

$$\boxed{B_{0\,\mathrm{today}} = \mathcal{O}\left(10^{-44}\right) \, \mathrm{meV}}$$

$$B_0\Big|_{\mathrm{today}} \sim 2.435 \times 10^{-34} \, \mathrm{eV}$$

Quite safe from stringent Experimental Bounds

$$|B^0| \, < \, 10^{-2} \, \mathrm{eV}$$
$$B_i \equiv b_i \, < \, 10^{-31} \, \mathrm{GeV}$$

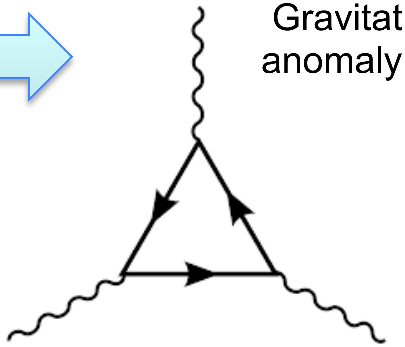
Summary of Cosmological Evolution

Cosmic Time

Basilakos, NEM, Sola

Inflationary (de Sitter) Phase

Primordial Gravitational Waves →



Gravitational anomaly (GA) →

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$$\dot{\overline{b}} \sim \sqrt{2\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H$$

chiral matter generation @ inflation exit

Cancellation of GA

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$$B_0 \propto T^3$$

Leoptogenesis induced by RHN (tree-level) decays

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B-L conserving sphelaron processes → **Baryongenesis**

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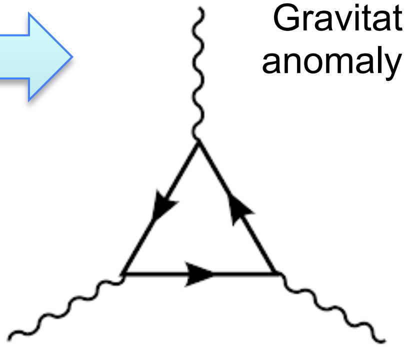

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Matter Era

Possible potential generation for b – mixing with axion Dark matter

Matter Era and KR axion as source of Dark Matter

$$
\begin{aligned}
S^{\rm eff} = & \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu\Big] \\
& + S_{Dirac}^{Free} + \int d^4x\sqrt{-g}\,\frac{\alpha'}{\kappa}\,\sqrt{\frac{3}{8}}\,\partial_\mu b\,J^{5\mu} - \frac{3\,\alpha'^2}{16\,\kappa^2}\,\int d^4x\sqrt{-g}\,J^5_\mu J^{5\mu} + \dots,
\end{aligned}
$$

$$
\partial_\mu\Big[\sqrt{-g}\,\Big(\sqrt{\frac{3}{8}}\frac{\alpha'}{\kappa}\,J^{5\mu} - \frac{\alpha'}{\kappa}\sqrt{\frac{2}{3}}\,\frac{1}{96}\,\mathcal{K}^\mu\Big)\Big] = \sqrt{\frac{3}{8}}\,\frac{\alpha'}{\kappa}\,\Big(\frac{\alpha_{\rm EM}}{2\pi}\,\sqrt{-g}\,F^{\mu\nu}\,\widetilde{F}_{\mu\nu} + \frac{\alpha_s}{8\pi}\,\sqrt{-g}\,G^a_{\mu\nu}\,\widetilde{G}^{a\mu\nu}\Big)
$$

Matter Era and KR axion as source of Dark Matter

$$S^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu\Big]$$

$$+\,S_{\tt Dirac}^{\tt Free} + \int d^4x\sqrt{-g}\,\boxed{\frac{\alpha'}{\kappa}\,\sqrt{\frac{3}{8}}\,\partial_\mu b\,J^{5\mu}} - \frac{3\alpha'^2}{16\,\kappa^2}\,\int d^4x\sqrt{-g}\,J^5_\mu J^{5\mu} + \dots,$$

$$\partial_\mu\Big[\sqrt{-g}\,\Big(\sqrt{\frac{3}{8}}\frac{\alpha'}{\kappa}\,J^{5\mu} - \frac{\alpha'}{\kappa}\sqrt{\frac{2}{3}}\,\frac{1}{96}\,\mathcal{K}^\mu\Big)\Big] = \sqrt{\frac{3}{8}}\,\frac{\alpha'}{\kappa}\,\Big(\frac{\alpha_{\rm EM}}{2\pi}\,\sqrt{-g}\,F^{\mu\nu}\,\widetilde{F}_{\mu\nu}\,\boxed{+\,\frac{\alpha_s}{8\pi}\,\sqrt{-g}\,G^a_{\mu\nu}\,\widetilde{G}^{a\mu\nu}}\Big)$$

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$$\Downarrow$$

$$\boxed{S^{\rm eff}_b = \int d^4x\sqrt{-g}\Big[\frac{1}{2}\,\partial_\mu b\,\partial^\mu b - \frac{\alpha'}{\kappa}\,\sqrt{\frac{3}{8}}\,\frac{\alpha_s}{8\pi}\,b(x)\,G^a_{\mu\nu}\,\widetilde{G}^{a\mu\nu}\Big]}$$

During QCD era

Matter Era and KR axion as source of Dark Matter

$$\boxed{S_b^{\rm eff} = \int d^4x\sqrt{-g}\Big[\frac{1}{2}\partial_\mu b\,\partial^\mu b - \frac{\alpha'}{\kappa}\,\sqrt{\frac{3}{8}}\,\frac{\alpha_s}{8\pi}\,\boxed{b(x)\,G^a_{\mu\nu}\,\widetilde{G}^{a\mu\nu}}\Big]}$$

During QCD era

QCD **Instantons** can generate an axion-shift-symmetry breaking potential

$$\boxed{V_b^{\rm QCD} \simeq \Lambda^4_{\rm QCD}\Big(1 - \cos\big(\frac{b}{f_b}\big)\Big)}, \quad f_b \equiv \sqrt{\frac{8}{3}}\,\frac{\kappa}{\alpha'} = \sqrt{\frac{8}{3}}\,\Big(\frac{M_s}{M_{\rm Pl}}\Big)^2 M_{\rm Pl}$$

$$\Lambda_{\rm QCD} \sim 218 \ {\rm MeV}$$

Matter Era and KR axion as source of Dark Matter

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Our linear KR background in inflationary era requires

$$M_{\rm Pl} \gtrsim M_s \gtrsim 10^{-3}\,M_{\rm Pl}$$

$$3.9 \times 10^{18}\;{\rm GeV} \gtrsim\; f_b\; \gtrsim 3.9 \times 10^{12}\;{\rm GeV},$$

Matter Era and KR axion as source of Dark Matter

$$S_b^{\rm eff} = \int d^4x\sqrt{-g}\Big[\frac{1}{2}\,\partial_\mu b\,\partial^\mu b - \frac{\alpha'}{\kappa}\,\sqrt{\frac{3}{8}}\,\frac{\alpha_s}{8\pi}\,b(x)\,G^a_{\mu\nu}\,\widetilde{G}^{a\mu\nu}\Big]$$

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$$10^9 \ {\rm GeV} < f_a < 10^{12} \ {\rm GeV}$$

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Phenomenology
OK !

Matter Era and KR axion as source of Dark Matter

$$S_b^{\rm eff} = \int d^4x \sqrt{-g}\Big[\frac{1}{2}\partial_\mu b\, \partial^\mu b - \frac{\alpha'}{\kappa}\,\sqrt{\frac{3}{8}}\,\frac{\alpha_s}{8\pi}\, b(x)\, G^a_{\mu\nu}\, \widetilde{G}^{a\mu\nu}\Big]$$

During QCD era

QCD **Instantons** can generate an axion-shift-symmetry breaking potential

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$$\Lambda_{\rm QCD} \sim 218\ \ {\rm MeV}$$

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b-axion Mass

$$m_b = \sqrt{\frac{\partial^2 V_b^{\rm QCD}}{\partial b^2}\Big|_{b=0}} = \frac{\Lambda_{\rm QCD}^2}{f_b} = \sqrt{\frac{3}{8}}\Big(\frac{\Lambda_{\rm QCD}}{M_s}\Big)^2 M_{\rm Pl} = \sqrt{\frac{3}{8}}\Big(\frac{\Lambda_{\rm QCD}}{M_{\rm Pl}}\Big)^2\Big(\frac{M_{\rm Pl}}{M_s}\Big)^2 M_{\rm Pl}$$

Matter Era and KR axion as source of Dark Matter

$$S_b^{\rm eff} = \int d^4x\sqrt{-g}\Big[\frac{1}{2}\partial_\mu b\,\partial^\mu b - \frac{\alpha'}{\kappa}\,\sqrt{\frac{3}{8}}\,\frac{\alpha_s}{8\pi}\,b(x)\,G^a_{\mu\nu}\,\widetilde{G}^{a\mu\nu}\Big]$$

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$$\Lambda_{\rm QCD}\ \sim\ 218\ \ {\rm MeV}$$

Our linear KR background in inflationary era requires

$$1.17\times 10^{-5}\ {\rm eV} \gtrsim m_b \gtrsim 1.17\times 10^{-11}\ {\rm eV}$$

$$M_{\rm Pl} \gtrsim M_s \gtrsim 10^{-3}\,M_{\rm Pl}$$

b-axion Mass

$$m_b = \sqrt{\frac{\partial^2 V_b^{\rm QCD}}{\partial b^2}\Big|_{b=0}} = \frac{\Lambda^2_{\rm QCD}}{f_b} = \sqrt{\frac{3}{8}}\,\Big(\frac{\Lambda_{\rm QCD}}{M_s}\Big)^2\,M_{\rm Pl} = \sqrt{\frac{3}{8}}\,\Big(\frac{\Lambda_{\rm QCD}}{M_{\rm Pl}}\Big)^2\,\Big(\frac{M_{\rm Pl}}{M_s}\Big)^2\,M_{\rm Pl}$$

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$$S_b^{\rm eff} = \int d^4x \sqrt{-g}\Big[\frac{1}{2}\partial_\mu b\,\partial^\mu b - \frac{\alpha'}{\kappa}\,\sqrt{\frac{3}{8}}\,\frac{\alpha_s}{8\pi}\,b(x)\,G^a_{\mu\nu}\,\widetilde{G}^{a\mu\nu}\Big]$$

During QCD era

QCD **Instantons** can generate an axion-shift-symmetry breaking potential

$$m_a \sim 5.7\,\Big(\frac{10^{12}\ {\rm GeV}}{f_a}\Big)\ \times 10^{-6}\ {\rm eV}$$

Compatible with QCD-Axion Phenomenology

$$3.9 \times 10^{18}\ {\rm GeV} \gtrsim f_b \gtrsim 3.9 \times 10^{12}\ {\rm GeV}$$

$$1.17 \times 10^{-5}\ {\rm eV} \gtrsim m_b \gtrsim 1.17 \times 10^{-11}\ {\rm eV}$$

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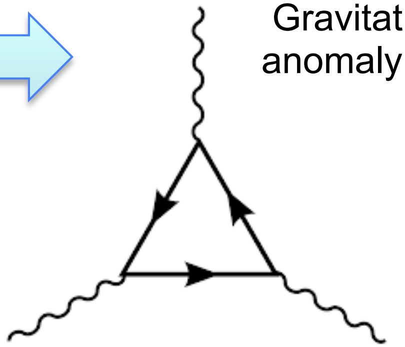
Inflationary (de Sitter) Phase

Primordial Gravitational Waves



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chiral matter generation @ inflation exit

Cancellation of GA

Radiation Era

$$B_0 \propto T^3$$

Leoptogenesis induced by RHN (tree-level) decays

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B-L conserving sphelaron processes → Baryongenesis

Matter Era

Possible potential generation for b – mixing with axion Dark matter

Modern de-Sitter Era

GA resurfacing

$$\dot{b}_{\rm today} \sim \sqrt{2\,\varepsilon'}\,M_{\rm Pl}\,H_0$$

$$\varepsilon' \sim \varepsilon = \mathscr{O}(10^{-2})$$ **Phenomenology**

Summary of Cosmological Evolution

Cosmic
Time

Basilakos, NEM, Sola

Inflationary (de Sitter) Phase

Primordial
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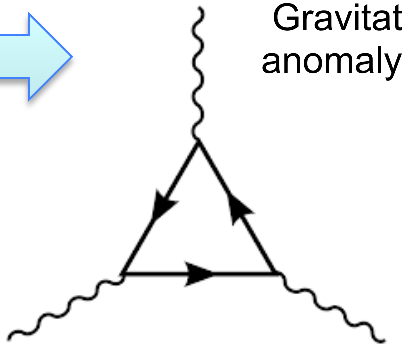

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**chiral matter
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$$B_0 \propto T^3$$

**Leoptogenesis induced by
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$$N_I \;\rightarrow\; \overline{\phi}\,\ell\,,\;\; \phi\,\overline{\ell}$$

B-L conserving sphelaron processes → **Baryongenesis**

**Consistent with current
bounds on LV & CPTV
$B_0 < 10^{-2}$ eV,
$B_i < 10^{-22}$ eV**

Matter Era

Possible potential generation for b – mixing with axion Dark matter

Modern de-Sitter Era

GA resurfacing

$$\dot{b}_{\rm today} \sim \sqrt{2\,\varepsilon'}\,M_{\rm Pl}\,H_0$$

$$\varepsilon' \sim \varepsilon = \mathcal{O}(10^{-2})$$

$H_0 \sim 10^{-42}$ GeV
$\approx 10^{-60}\,M_{\rm Pl} \approx$!0^{-33} eV

Phenomenology

V. CONCLUSIONS

- CPT Violation (CPTV) due to (strong) quantum fluctuations in space-time at early eras or LV early Universe Geometries (with background flux fields) is a possibility and may explain the observed matter-antimatter asymmetry in the cosmos, consistently with current-era bounds of CPTV
- One framework for early universe CPTV: Standard Model Extension (SME)
- A string-inspired model of the Early Universe entailing CPT and Lorentz Violation due to Kalb-Ramond-axion- modified background geometries – Consistent phenomenology in current era
- GRAVITATIONAL ANOMALIES induced by PRIMORDIAL GRAVITY WAVES during inflation → UNDILUTED Kalb-Ramon Axion Background @inflation exit
- crucial for :

  BARYOGENESIS THROUGH LEPTOGENESIS via Supermassive Right Handed Neutrinos during Radiation-dominace era

- OUTLOOK: UNDERSTANDING POTENTIAL ROLE OF KR AXIONS AS SOURCES OF (AXIONIC) DARK MATTER IN THE UNIVERSE

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  BARY GH LEPTOGENESIS via Supermassive Right Handed Neutrin adiation-dominace era
- OUTLOOK: UNDERSTANDING POTENTIAL ROLE OF KR AXIONS AS SOURCES OF (AXIONIC) DARK MATTER IN THE UNIVERSE

Thank you !

SPARES

Part IV

Matter-Antimatter Asymmetry in the Universe

STANDARD MODEL INCOMPATIBLE WITH BARYOGENESIS

- Matter-Antimatter asymmetry in the Universe → Violation of Baryon # (B), C & CP

- Tiny CP violation ($O(10^{-3})$) in Labs: e.g. $K^0 \overline{K}^0$

- But Universe consists only of matter

$$\boxed{\frac{n_B - \overline{n}_B}{n_B + \overline{n}_B} \sim \frac{n_B - \overline{n}_B}{s} = (8.4 - 8.9) \times 10^{-11}}$$

T > 1 GeV

Sakharov : Non-equilibrium physics of early Universe, **B, C, CP violation**

→ $n_B - \bar{n}_B$ but **not quantitatively in SM**, still a mystery

Assume CPT invariance

Within the Standard Model, lowest CP Violating structures

$$d_{CP} = \sin(\theta_{12})\sin(\theta_{23})\sin(\theta_{13})\sin\delta_{CP}$$
$$\cdot(m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)$$

Kobayashi-Maskawa CP Violating phase

Rubakov, Kuzmin, Shaposhnikov,
Gavela, Hernandez, Orloff,Pene

Shaposhnikov

$$D = \mathrm{Im}\ \mathrm{Tr}\left[\mathcal{M}_u^2\mathcal{M}_d^2\mathcal{M}_u\mathcal{M}_d\right]$$

$$\boxed{\delta_{KM}^{CP} \sim \frac{D}{T^{12}} \sim 10^{-20}} \quad << \quad \frac{n_B - \overline{n}_B}{n_B + \overline{n}_B} \sim \frac{n_B - \overline{n}_B}{s} = (8.4 - 8.9) \times 10^{-11}$$

$$T \simeq T_{\mathrm{sph}}$$

T > T_{sph} sphalerons @ equilibrium

$$T_{\mathsf{sph}}(m_H) \in [130, 190]\mathrm{GeV}$$

This CP Violation Cannot be the Source of Baryon Asymmetry in The Universe

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$$\cdot(m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 \quad m_u^2)(m \quad\quad ^2 - m_d^2)$$

MUST GO BEYOND THE STANDARD MODEL TO UNDERSTAND BARYON ASYMMETRY IN THE UNIVERSE

Violating phase

Rubakov, Kuzmin, Shaposhnikov,
Gavela, Hernandez, Orloff,Pene

Shaposhnikov

$$D \quad\quad _u\mathcal{M}_d]$$

$$\delta_{KM}^{CP} \sim \frac{D}{T^{12}} \quad\quad \frac{\quad _B}{\quad _B + \overline{n}_B} \sim \frac{n_B - \overline{n}_B}{s} = (8.4 - 8.9) \times 10^{-11}$$

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Role of Neutrinos?

- Several Ideas to go beyond the SM (e.g. GUT models, Supersymmetry, extra dimensional models etc.)
- Massive ν are **simplest** extension of SM
- Right-handed supermassive ν may provide extensions of SM with:

  extra CP Violation and thus Origin of Universe’s **matter-antimatter asymmetry** due to neutrino masses, **Dark Matter**

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SM Extension with N extra right-handed neutrinos (RHN)

$$L = L_{SM} + \bar{N}_I i \partial_\mu \gamma^\mu N_I - F_{\alpha I} \, \bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2} \, \bar{N}_I^c N_I + \mathrm{h.c.}$$

**Minkowski, Fukugita, Yanagida,
Mohapatra, Senjanovic, Lazarides, Shafi, Wetterich,
Sechter, Valle, Paschos, Hill, Luty,
Vergados, de Gouvea…, Liao, Nelson,
Buchmuller, Anisimov, di Bari…, Akhmedov, Rubakov,
Smirnov, Davidson, Giudice, Notari, Raidal, Riotto,
Strumia, Hernandez, Giunti...
Pilaftsis, Underwood (heavy RHN → leptogenesis),
Shaposhnikov, Asaka, Blanchet, Boyarski, Ruchayskiy
(vMSM, light RHN)…**

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$$m_\nu = -M^D \frac{1}{M_I} [M^D]^T .$$

explain light SM ν masses

seesaw

Minkowski,
Fukugita, Yanagida,
Mohapatra, Senjanovic,
Lazarides, Shafi,
Wetterich,
Sechter, Valle,

$$M_D = F_{\alpha I} v$$

$$v = \langle \phi \rangle \sim 175 \text{ GeV} \qquad M_D \ll M_I$$

$F_{\alpha 1} \approx 10^{-10}$ → $m_\nu^2 \approx 10^{-3}\ \mathrm{eV}^2$

CPT CONSERVING Thermal Leptogenesis

Early Universe
T >> T_{EW} =10^2 GeV

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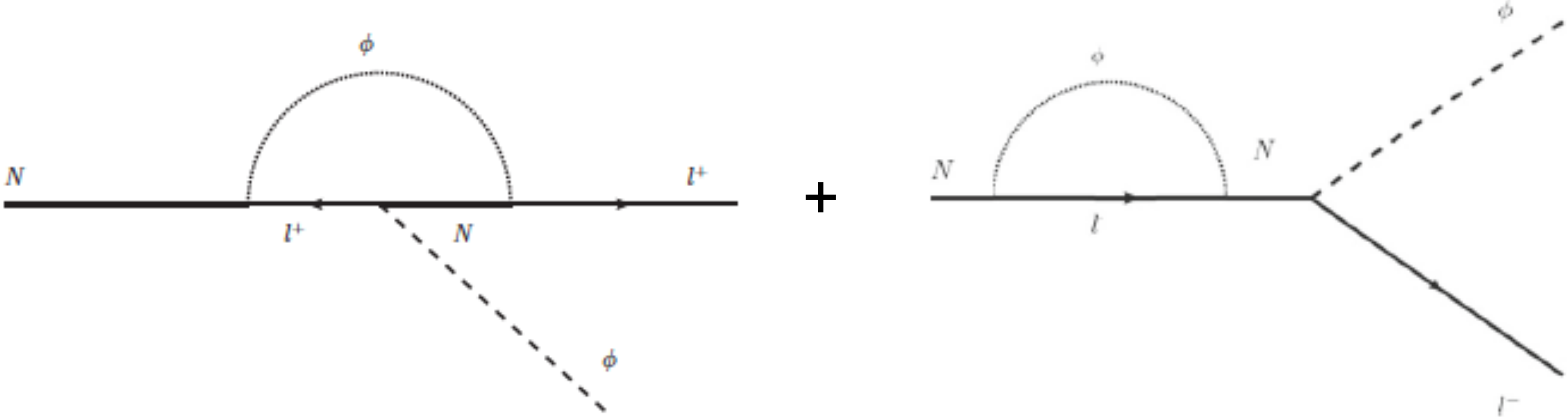
CPT Conserved

Lepton number & CP Violations @ LOOP LEVEL NECESSARILY,
WITH MORE THAN ONE FLAVOUR OF HEAVY RHN

$$N_I \to H\nu,\ \bar{H}\bar{\nu}$$

HEAVY M_I > M_W



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Embed this model into our String-Inspired Gravitationally Anomalous RVM framework

NEM + Sarkar, DeCesare, Bossingham,
+ Basilakos, Sola (2019)

CPT Theorem

Conditions for the Validity of CPT Theorem

$$P:\ \vec{x} \to -\vec{x}, \quad T:\ t \to -t(T), \quad C\psi(q_i) = \psi(-q_i)$$

CPT Invariance Theorem :
A quantum field theory lagrangian is invariant under CPT if it satisfies

(i) Flat space-times
(ii) Lorentz invariance
(iii) Locality
(iv) Unitarity

Schwinger, Pauli, Luders, Jost, Bell
revisited by:
Greenberg, Chaichian, Dolgov, Novikov, Fujikawa, Tureanu ...

(ii)-(iv) Independent reasons for violation

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