Axions and clockwork in heteroti M-theory

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Outline

- **Motivation**
- **Clockwork mechanism and General Linear Dilaton**
- **Possible solutions of the hierarchy problem**
  - **In minimal heterotic M-theory (Hořava-Witten model)**
  - **In heterotic M-theory with vector multiplets**
- **Axions**
  - **heterotic string theory**
  - **heterotic M-theory**
- **Summary**

Motivation

- **Understanding the origin of large hierarchies of scales (and small couplings) is a major challenge in theoretical physics**

- **Some of proposed mechanisms based on extra dimensions**
  - **large extra dimensions (LED)**
  - **warped extra dimensions (RS)**
  - **linear dilaton model (LD)**
- **They may be considered as various General Linear Dilaton (GLD) models**
  - **generalizations of continuous version of clockwork mechanism**

Motivation

- Understanding the origin of large hierarchies of scales (and small couplings) is a major challenge in theoretical physics
- Some of proposed mechanisms based on extra dimensions
  - large extra dimensions (LED)
  - warped extra dimensions (RS)
  - linear dilaton model (LD)
- They may be considered as various General Linear Dilaton (GLD) models
  - generalizations of continuous version of clockwork mechanism
- Which of such models may be derived from fundamental higher-dimensional theories like string- or M-theory?
- Which may accommodate interesting axions (QCD axion, DM axion etc.)?

Clockwork

Clockwork mechanism:

- device to obtain light degrees of freedom with (strongly) suppressed couplings within theory without small fundamental parameters

- generalization of aligned axion mechanism — Kim, Nilles, Peloso, 2004
- name suggested in — Kaplan, Rattazzi, 2015
- related to deconstruction
- generalization of discrete clockwork to continuous one proposed in — Giudice, McCullough, 2016
- problems of such generalization — Craig, Garcia, Sutherland, 2017
- description of General Continuous Clockwork (GCCW) — Choi, Im, Shin, 2017

Discrete clockwork

Discrete scalar clockwork action ($q > 1$)

$$\int d^4x \left[\sum_{i=0}^{N} \frac{1}{2}(\partial_\mu \phi_i)^2 + \sum_{i=0}^{N-1} \frac{1}{2} m^2 (\phi_{i+1} - q\phi_i)^2 \right]$$

Mass matrix

$$m^2 \begin{pmatrix} 1 & -q & 0 & \dots & 0 \\ -q & 1+q^2 & -q & \dots & 0 \\ 0 & -q & 1+q^2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 1+q^2 & -q \\ 0 & 0 & 0 & -q & q^2 \end{pmatrix}$$

has one massless eigenstate $\chi_0 = \mathcal{N} \sum_{i=0}^{N} \frac{\phi_n}{q^i}$

component at each successive site is q times smaller than at the previous site

for large N: coupling of χ_0 at 0-th and N-th sites are very different

Continuous clockwork

Sites in the field space $i = 0 \dots N$
may be interpreted as points in 5-th dimension $y = y_i$
$\phi_i(\vec{x}) \quad \longrightarrow \quad \Phi(\vec{x}, y_i)\Delta y^{1/2}$;
$\phi_{i+1}(\vec{x}) - \phi_i(\vec{x}) \quad \longrightarrow \quad \partial_y \Phi(\vec{x}, y_i)\Delta y^{3/2}$
etc.

Continuum limit ($\Delta y \equiv \pi R/N$, $N \to \infty$):

$$\int \mathrm{d}^5 x \, e^{2ky} \left[\frac{1}{2}(\partial_\mu \Phi)^2 + \frac{1}{2} e^{2py} (\partial_5 \Phi)^2 \right]$$

may be obtained from simple Lagrangian

$$\int \mathrm{d}^5 x \sqrt{-g} \, \frac{1}{2} \partial_\alpha \Phi \partial^\alpha \Phi$$

in the warped background

$$\mathrm{d}s^2 = e^{\frac{4}{3}ky} \left(e^{\frac{2}{3}py} \eta_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + e^{-\frac{4}{3}py} \mathrm{d}y^2 \right)$$

$\mu, \nu, \ldots = 0, 1, 2, 3 \qquad \alpha, \beta, \ldots = 0, 1, 2, 3, 5 \qquad x^5 \equiv y$

General Linear Dilaton

gravity + dilaton + cosmological constants on 5D orbifold $M^4 \times S^1/\mathbb{Z}_2$

$$\mathcal{S}_5 = M_5^3 \int \mathrm{d}^5x \sqrt{-g} \left(\tfrac{1}{2}\mathcal{R}_5 - \tfrac{1}{2}\partial_\alpha S \partial^\alpha S - \Lambda_b \, e^{-2(\hat{c}/\sqrt{3})S} - e^{-(\hat{c}/\sqrt{3})S} \left[\Lambda_0 \frac{\delta(y)}{\sqrt{g_{55}}} + \Lambda_\pi \frac{\delta(y-\pi R)}{\sqrt{g_{55}}} \right] \right)$$

4D flat background solution if: $\quad -\Lambda_0 = \Lambda_\pi = \pm 6 \sqrt{\frac{2}{3}\left(\frac{\Lambda_b}{\hat{c}^2-4}\right)}$

general linear dilaton background: $\quad (\hat{c}/\sqrt{3})S = \frac{2}{3}(k-p)y$

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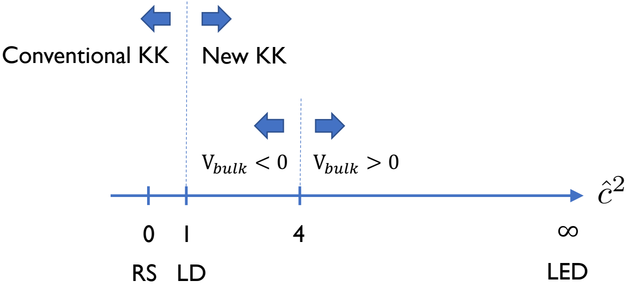
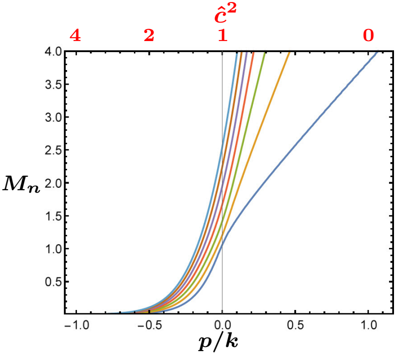
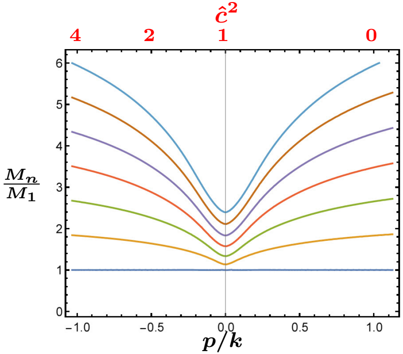


KK spectrum in GLD models

Choi, Im, Shin, 2017





- extra suppression of KK masses for $\hat{c}^2 > 1$

$$\frac{p}{k} = 2\,\frac{1-\hat{c}^2}{2+\hat{c}^2} \qquad \hat{c}^2 > 1 \;\Rightarrow\; p/k < 0$$

- RS: $\hat{c}^2 = 0 \qquad p/k = 1$
- LD: $\hat{c}^2 = 1 \qquad p/k = 0$

Hořava-Witten model

Strongly coupled $E_8 \times E_8$ heterotic M-theory $\rightarrow$ 11D SUGRA

$$\mathcal{S}_{11} = \frac{1}{2\kappa^2} \int_{\mathcal{M}^{11}} \mathrm{d}^{11}x \sqrt{-g} \left(\star\mathcal{R} - G \wedge \star G - 2\sqrt{2}\, C \wedge G \wedge G \right)$$
$$- \frac{1}{8\pi\kappa^2} \left(\frac{\kappa}{4\pi} \right)^{2/3} \sum_{i=1}^{2} \int_{\mathcal{M}^{10}_{(i)}} \mathrm{d}^{10}x \sqrt{-g}\, \mathrm{tr} F_{(i)} \wedge \star F_{(i)}$$

compactification on warped orbifold $M^4 \times X^6 \times S^1/\mathbb{Z}_2$

supersymmetry $\rightarrow$ non-zero flux: $G_{ABCD} = -\frac{\mu}{48} \epsilon_{ABCD}{}^{EF} \omega_{EF}$

$$\mu \equiv \frac{\sqrt{2}}{\pi V_0} \left(\frac{\kappa}{4\pi} \right)^{2/3} \int_{X^6} \omega \wedge \left(\mathrm{tr} F_{(1)} \wedge F_{(1)} - \frac{1}{2} \mathrm{tr} \mathcal{R} \wedge \mathcal{R} \right)$$

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Standard embedding (Hořava-Witten) $\mu < 0$:

- **volume V of Calabi-Yau X^6 decreases with $|x^{11}|$**
- **$\Rightarrow$ upper bound on length of 11-th dimension πR_{11}**
- $M_W \lll (\pi R_{11})^{-1} < M_{11} < M_{\mathrm{Pl}}$

Minimal heterotic M-theory

Non-standard embedding with $\mu > 0$:

- volume V of Calabi-Yau X^6 increases with $|x^{11}|$
- length of 11-th dimension πR_{11} may be quite large
- hierarchy problem of the weak vs Planck scale may be addressed

$$M_{\mathrm{Pl}}^2 \approx \mathcal{O}(10)\, M_{11}^2 (M_{11} \pi R_{11})^2$$

Relation typical for $N = 2$ flat extra dimensions

$M_{11} \sim \mathcal{O}(1)$ TeV, $\pi R_{11} \lesssim 100\,\mu$m
enough to obtain the correct value of M_{Pl}

Minimal heterotic M-theory

Compactification of 11D on Calabi-Yau X^6 with $h_{(1,1)} = 1$
(only universal hypermultiplet and gravity multiplet)

$\Rightarrow$ gravity-modulus system described by the GLD action

$$\mathcal{S}_5 = \frac{1}{\kappa_5^2} \int_{\mathcal{M}^5} \mathrm{d}^5 x \sqrt{-g} \left[\frac{1}{2}\mathcal{R}_5 - \frac{1}{2}\partial_\alpha S \partial^\alpha S - \Lambda_b \, e^{-(2\hat{c}/\sqrt{3})S} \right] - \frac{1}{\kappa_5^2} \sum_{i=1,2} \int_{\mathcal{M}^4_{(i)}} \mathrm{d}^4 x \sqrt{-g} \, \Lambda_{(i)} e^{-(\hat{c}/\sqrt{3})S}$$

with: $\hat{c}^2 = 6 \,, \quad \Lambda_b = \frac{\mu^2}{384} \,, \quad \Lambda_{(1)} = -\Lambda_{(2)} = \frac{\mu}{4\sqrt{2}}$

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$\hat{c}^2 > 1 \Rightarrow$ **non-conventional spectrum of KK states**

$$M_n^2 \approx \mathcal{O}(10^3)\, n^2\, M_{11}^2 \left(\frac{M_{11}}{M_{\mathrm{Pl}}}\right)^{5/2}$$

masses as for $N = 8/5 = 1.6$ flat extra dimensions

Heterotic M-theory with vector multiplets (in 5D)

- **compactified on CY space with the Hodge number $h_{(1,1)} > 1$**
- $h_{(1,1)}$ Kähler moduli t^i defined by $\omega = t^i \omega_i$
- intersection numbers $d_{ijk} \equiv \frac{1}{V_0} \int_{X^6} \omega_i \wedge \omega_j \wedge \omega_k$
- $h_{(1,1)}$ **flux parameters**

$$\mu_i \equiv \frac{\sqrt{2}}{\pi V_0} \left(\frac{\kappa}{4\pi} \right)^{2/3} \int_X \omega_i \wedge \left(\mathrm{tr} F_{(1)} \wedge F_{(1)} - \frac{1}{2} \mathrm{tr} \mathcal{R} \wedge \mathcal{R} \right)$$

Heterotic M-theory with vector multiplets

Simple example: $h_{(1,1)} = 2$, only $d_{112} \neq 0$

- $\mu_1 \neq 0 \qquad \mu_2 \neq 0$ (same as for $h_{(1,1)} = 1$):
  - $\hat{c}^2 = 6$
  - Planck mass as for $N = 2$ flat extra dimensions
  - $\pi R_{11} \sim 100\mu$m $\quad \Rightarrow \quad M_{11} = \mathcal{O}(1)$ TeV
  - KK spectrum similar to that of $N = 1.6$ flat extra dimensions

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  - KK spectrum similar to that of $N = 1.6$ flat extra dimensions
- $\mu_1 \neq 0 \qquad \mu_2 = 0$:
  - $\hat{c}^2 = 7$
  - Planck mass as for $N = 1.8$ flat extra dimensions
  - $\pi R_{11} \sim 100\mu$m $\quad \Rightarrow \quad M_{11} = \mathcal{O}(10)$ TeV
  - KK spectrum similar to that of $N = 1.5$ flat extra dimensions

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  - KK spectrum similar to that of $N = 1.5$ flat extra dimensions
- $\mu_1 = 0 \qquad \mu_2 \neq 0$:
  - $\hat{c}^2 = 10$
  - Planck mass as for $N = 1.5$ flat extra dimensions
  - $\pi R_{11} \sim 100\mu$m $\quad \Rightarrow \quad M_{11} = \mathcal{O}(100)$ TeV
  - KK spectrum similar to that of $N = 4/3$ flat extra dimensions

Heterotic M-theory with vector multiplets

Simple example: $h_{(1,1)} = 2$, only $d_{112} \neq 0$

- $\mu_1 \neq 0 \qquad \mu_2 \neq 0 \qquad$ (same as for $h_{(1,1)} = 1$):
  - $\hat{c}^2 = 6$
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  - $\pi R_{11} \sim 100\mu$m $\quad \Rightarrow \quad M_{11} = \mathcal{O}(1)$ TeV
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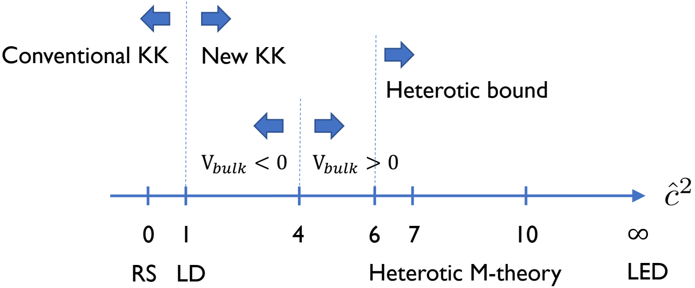
  - Planck mass as for $N = 1.5$ flat extra dimensions
  - $\pi R_{11} \sim 100\mu$m $\quad \Rightarrow \quad M_{11} = \mathcal{O}(100)$ TeV
  - KK spectrum similar to that of $N = 4/3$ flat extra dimensions

Warped product of one large (flat) and six curved extra dimensions

It is not difficult to see that in general heterotic M-theory $\hat{c}^2 \geq 6$



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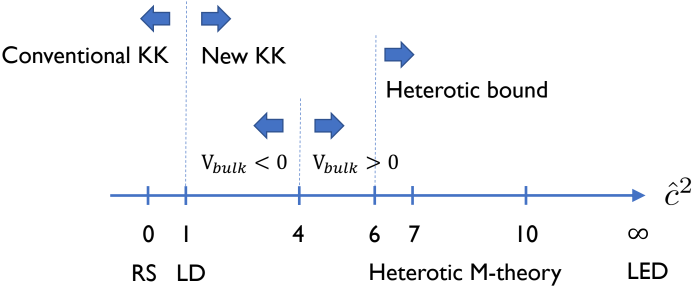

Smaller values of $\hat{c}^2 = 1,\ 4$ were obtained in 5D SUGRA
with decoupled universal hypermultiplet

Kehagias, Riotto, 2017; Antoniadis et al., 2017

Problematic from the point of view of higher dimensional string- or M-theory (LD $\hat{c}^2 = 1$ may be related to 6D "Little String Theory")

Axions – heterotic string

$$\mathcal{S}_{10} = \frac{1}{\kappa_{10}^2}\int_{\mathcal{M}^{10}}\left(\star\mathcal{R} - \frac{1}{2}H\wedge\star H - \frac{\alpha'}{4}\mathrm{tr}F\wedge\star F\right) - \int_{\mathcal{M}^{10}} B\wedge X_8 + \ldots$$

Green-Schwarz (GS) anomaly cancellation with

$X_8 = \omega^{(3,3)}\wedge qF(x) + \frac{1}{8\pi}\left(\mathrm{tr}_1 F\wedge F - \frac{1}{2}\mathrm{tr}\mathcal{R}\wedge\mathcal{R}\right)\wedge\left(\mathrm{tr}_1 F\wedge F - \mathrm{tr}_2 F\wedge F\right) + \cdots$

- modified Bianchi identity:

  $H = \mathrm{d}B + \omega_3 \qquad \mathrm{d}H = -\frac{1}{16\pi^2}\left(\mathrm{tr}F\wedge F - \mathrm{tr}\mathcal{R}\wedge\mathcal{R}\right)$

- axion fields:

  $H_{\mu\nu\rho} \to a$ (H dual in 4D to pseudoscalar) one MI axion a

  $B_{mn} \to b^i$ $(B = \frac{1}{2\pi}\omega_i^{(1,1)}b^i + \ldots)$ $h_{(1,1)}$ MD axions b^i

- couplings to $F\tilde{F}$:

  from Bianchi identity (+GS term) for MI axion a

  from GS term for MD axions b^i

- $a - b^i$ mixing due to loop corrections
- decay constants not much below the Planck scale: $f_a, f_b \sim \alpha_{\mathrm{YM}} M_{\mathrm{Pl}}$

Axions – heterotic M-theory

$$\mathcal{S}_{11} = \frac{1}{2\kappa^2}\int_{\mathcal{M}^{11}} \mathrm{d}^{11}x\sqrt{-g}\left(\star\mathcal{R} - G\wedge\star G - 2\sqrt{2}\,C\wedge G\wedge G\right)$$
$$- \frac{1}{8\pi\kappa^2}\left(\frac{\kappa}{4\pi}\right)^{2/3}\sum_{j=1}^{2}\int_{\mathcal{M}^{10}_{(i)}} \mathrm{d}^{10}x\sqrt{-g}\;\mathrm{tr}F_{(j)}\wedge\star F_{(j)}$$

- modified Bianchi identity:

  $G = \mathrm{d}C + \sum_j \omega_3^{(i)}\wedge\delta(x^{11}-x_j^{11})\mathrm{d}x^{11}$

  $\mathrm{d}G = -\frac{1}{16\pi^2}\sum_j\left(\mathrm{tr}F_{(j)}\wedge F_{(j)} - \frac{1}{2}\mathrm{tr}\mathcal{R}\wedge\mathcal{R}\right)\wedge\delta(x^{11}-x_j^{11})\mathrm{d}x^{11}$

- axion fields:

  $G_{\alpha\beta\gamma\delta}\to a$ (G dual in 5D to pseudoscalar) one MI axion a

  $C_{mn11}\to b^i$ ($C = \omega_i^{(1,1)}\wedge\mathcal{A}^i + \ldots$) $h_{(1,1)}$ vectors $\mathcal{A}^i$ (in 4d: $b^i \equiv \mathcal{A}^i_{11}$)

4D MI axion in 5D (universal) hypermultiplet

4D MD axions in 5D vector multiplets (5-th components of vectors)

Axions – heterotic M-theory

$$\mathcal{S}_5 \supset -2\pi \int_{\mathcal{M}^5} \left[\frac{1}{2V_X} \left(\mathrm{d}a + n_i \mathcal{A}^i \right) \wedge \star \left(\mathrm{d}a + n_i \mathcal{A}^i \right) + \frac{1}{2} G_{ij}\, \mathrm{d}\mathcal{A}^i \wedge \star \mathrm{d}\mathcal{A}^j \right]$$
$$+ \int_{\mathcal{M}^5} \frac{1}{4\pi}\, a \left[\left(\mathrm{tr} F_{(1)} \wedge F_{(1)} \right) \delta(x^{11}) + \left(\mathrm{tr} F_{(2)} \wedge F_{(2)} \right) \delta(x^{11} - \pi r_{11}) \right] \wedge \mathrm{d}x^{11}$$

$$G_{ij} = \int_{\mathcal{X}^6} \omega_i^{(1,1)} \wedge \star \omega_j^{(1,1)} \qquad n_i = -\frac{1}{16\pi^2} \int_{\mathcal{X}^6} \omega_i^{(1,1)} \wedge \left(\mathrm{tr}_1 F \wedge F - \frac{1}{2} \mathrm{tr} \mathcal{R} \wedge \mathcal{R} \right)$$

- couplings to $F\tilde{F}$:

  from Bianchi identity (+ Chern-Simons term) for MI axion a

  no couplings to $F\tilde{F}$! for MD axions b^i

- $a - b^i$ mixing due to quasi-kinetic mixing of a and $\mathcal{A}^i$ in 5D

- decay constants $(h_{(1,1)} = 1)$:
  due to this mixing and to warping of space-time there are 2 axion-like fields: $a_{(L,R)}$ exponentially localized at (left, right) orbifold plane

$$f_{L1} \sim 0.1 M_{11} \qquad f_{L2} \sim \frac{0.1}{\sqrt{\gamma}} \sqrt{\frac{M_{11}}{M_{\mathrm{Pl}}}}\, M_{\mathrm{Pl}}$$

$$f_{R1} \sim \frac{0.1}{\gamma} M_{\mathrm{Pl}} \qquad f_{R2} \sim 0.1 \sqrt{\gamma} \sqrt{\frac{M_{11}}{M_{\mathrm{Pl}}}}\, M_{11}$$

Axions – heterotic M-theory

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$$+ \int_{\mathcal{M}^5} \frac{1}{4\pi}\, a \left[\left(\mathrm{tr} F_{(1)} \wedge F_{(1)} \right) \delta(x^{11}) + \left(\mathrm{tr} F_{(2)} \wedge F_{(2)} \right) \delta(x^{11} - \pi r_{11}) \right] \wedge \mathrm{d}x^{11}$$

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$$f_{L1} \sim 0.1 M_{11} \qquad f_{L2} \sim \frac{0.1}{\sqrt{\gamma}} \sqrt{\frac{M_{11}}{M_{\rm Pl}}}\, M_{\rm Pl}$$

$$f_{R1} \sim \frac{0.1}{\gamma} M_{\rm Pl} \qquad f_{R2} \sim 0.1 \sqrt{\gamma} \sqrt{\frac{M_{11}}{M_{\rm Pl}}}\, M_{11}$$

We need $M_{11} \ll M_{\rm Pl}$ in order to get (at least one) $f \ll M_{\rm Pl}$

Axions – heterotic M-theory

- **Fields a_L and a_R are both massless at the perturbative level**
- **They get masses from instanton effects**
- **Mass eigenstates a_h and a_l are mixtures of a_L and a_R**
- **In the interesting limit of $M_{11} \ll M_{\rm Pl}$ there is strong hierarchy of axion masses $m_h \gg m_l$**

$$f_{h1} \sim 0.1 M_{11} \qquad\qquad f_{h2} \sim 0.1\gamma^{-1/2}\sqrt{M_{\rm Pl} M_{11}}$$

$$f_{l1} \gg M_{\rm Pl} \qquad\qquad f_{l2} \sim 0.1\gamma^{1/2} M_{11}\sqrt{\frac{M_{11}}{M_{\rm Pl}}}$$

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- In the interesting limit of $M_{11} \ll M_{\rm Pl}$ there is strong hierarchy of axion masses $m_h \gg m_l$

$$f_{h1} \sim 0.1 M_{11} \qquad f_{h2} \sim 0.1\gamma^{-1/2}\sqrt{M_{\rm Pl}M_{11}}$$

$$f_{l1} \gg M_{\rm Pl} \qquad f_{l2} \sim 0.1\gamma^{1/2} M_{11}\sqrt{\frac{M_{11}}{M_{\rm Pl}}}$$

- only a_h may play the role of QCD axion
  if M_{11} is in the "axion window" moved up by one order of magnitude: $10^{10}\,\mathrm{GeV} \lesssim M_{11} \lesssim 10^{13}\,\mathrm{GeV}$
- a_l is ULA with negligible coupling to the observable sector

Summary

- General Linear Dilaton models (5D)
  - 2-parameter class of potential solutions to the hierarchy problem (using continuous clockwork mechanism)
  - there are consistent UV-completions of GLD models
    but probably only for a very limited discrete set of parameters
- Heterotic M-theory may be such 11D UV-completion
  - non-standard embedding (of spin connection in the gauge group) is necessary
  - minimal version (modification of Hořava-Witten model)
    - Planck-scale hierarchy as for 2 flat extra dimensions
    - KK spectrum as for 1.6 flat extra dimensions
    - M_{11} scale not very much higher than the weak scale is possible
  - heterotic bound: $\hat{c}^2 \geq 6$
  - models constructed only for $\hat{c}^2 = 6,\ 7,\ 10$
- Previously found 5D SUGRA models with $\hat{c}^2 = 1,\ 4$:
  uplift to higher dimensional string- or M-theory seems to be problematic

Summary

Axions in heterotic M-theory

- origin:
  - one (MI) a dual in 5D to tensor field strength G
  - $h_{(1,1)}$ (MD) b^i coming from harmonic (1,1) forms on CY (b^i are 5-th components of 5D vectors)
- couplings to $F\tilde{F}$:
  - a : from modified Bianchi identities (+ CS term)
  - b^i: no direct couplings in 11D (only through mixing with a)
- a and b^i mix
  (due to modified Bianchi identity and instanton effects)
- decay constants:
  may be quite small in the case of non-standard embedding
  (much smaller than $\sim 10^{16}$ GeV characteristic for heterotic string)
- Exactly one axion in the minimal version of heterotic M-theory may have properties necessary to solve the strong CP problem