Killing spinors from classical r-matrices

Swiss National Science Foundation

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SwissMAP
The Mathematics of Physics
National Centre of Competence in Research

Dualities and Generalized Geometries

Sep. 11,2018 @ Mon-Repos, Corfu

UNIVERSITÄT BERN

AEC
ALBERT EINSTEIN CENTER
FOR FUNDAMENTAL PHYSICS

Based on 1805.00948 with

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1. Introduction • Motivation

1.1. Introduction

- AdS_d/CFT_{d-1}: attractive examples of gauge/gravity duality

d=5: [Maldacena-1998]

type IIB on $AdS_5 \times S_5$ '⟺' $\mathcal{N}$=4 SU(N) SYM in 4d.

- Intriguing: integrabile structures

A comprehensive review: [Beisert et al-2010]

allows us to determine physical quantities exactly,
even at finite coupling, without relying on supersymmetries.

e.g. amplitudes, anomalous dimensions, spectrum of strings etc.

- Significant: integrable deformations

construct a variety of examples keeping the integrability

→ Want to follow a systematic approach for such deformations.

1.2. Yang-Baxter (YB) deformations

[Klimcik], [Delduc-Magro-Vicedo], [Hoare, Tseytlin], [Matsumoto-Yoshida]...

- labelled by classical r-matrix = a solution of classical YB equation (CYBE).

$$[r_{12}, r_{13}] + [r_{12}, r_{23}] + [r_{13}, r_{23}] = 0$$

- a systematic way of performing integrable deformations:

1. Put classical r-matrix into the YB deformed sigma model action: [Kyono-Yoshida] [Arutyunov-Borsato-Frolov] etc...

$$S = -\frac{\sqrt{\lambda_c}}{4}\int_{-\infty}^{\infty} d\tau \int_0^{2\pi} d\sigma (\gamma^{ab} - \epsilon^{ab})\mathrm{STr}\left[A_a\, d \;\circ\; \frac{1}{1-\lambda R_g \circ d}(A_b)\right]$$

↑ const. deformation parameter

2. Rewrite the action to read off the deformed background data:

$$S = -\frac{\sqrt{\lambda_c}}{4}\int_{-\infty}^{\infty} d\tau \int_0^{2\pi} d\sigma \left[\gamma^{ab} G_{MN}\partial_a X^M \partial_b X^N - \epsilon^{ab} B_{MN}\partial_a X^M \partial_b X^N\right]$$
$$- \frac{\sqrt{\lambda_c}}{2} i\bar{\Theta}_I \left[\gamma^{ab}\delta^{IJ} - \epsilon^{ab}\sigma_3^{IJ}\right] e_a^m \Gamma_m D_b^{JK}\Theta_K + \mathcal{O}(\theta^4)$$

with $D_a^{IJ} = \delta^{IJ}\left(\partial_a - \frac{1}{4}\omega_a^{mn}\Gamma_{mn}\right) + \frac{1}{8}\sigma_3^{IJ} e_a^m H_{mnp}\Gamma^{np}$
$$- \frac{e^{\Phi}}{8}\left[\epsilon^{IJ}\Gamma^p F_p + \frac{1}{3!}\sigma_1^{IJ}\Gamma^{pqr}F_{pqr} + \frac{1}{2\cdot 5!}\epsilon^{IJ}\Gamma^{pqrst}F_{pqrst}\right] e_a^m \Gamma_m$$

1.2. Yang-Baxter (YB) deformations

[Klimcik], [Delduc-Magro-Vicedo], [Hoare, Tseytlin], [Matsumoto-Yoshida]...

TODAY

[Borsato-Wulff]

classical r-matrix is unimodular: $r^{ij}[b_i, b_j] = 0$ **for** $r = \frac{1}{2} r^{ij} b_i \wedge b_j$

☆ **The deformed background is a solution of the standard SUGRA.**

☆ **The deformations can be related to "TsT"-transformation.**

[Matsumoto-Yoshida][Osten-van Tongeren]

↑ const. deformation parameter

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1.3. What is a TsT-transformation?

[Lunin-Maldacena]
[Frolov]

- Suppose that **type IIB** is compactified on the 2-torus.

$SL(2,\mathbb{Z}) \times SL(2,\mathbb{Z})$ symmetry (geometric and non-geometric)

T^2 $\tau = B_{12} + i\sqrt{g_{(12)}}$

↓ 2 isometry directions

TsT-transformation $(u,v)_\lambda$ consists of **3** steps:

deformation parameter

$$(u,v)_\lambda = \begin{cases} \text{1. T-duality} & u \to \tilde{u} \\ \text{2. Shift} & v \to v + \lambda\,\tilde{u} \\ \text{3. T-duality} & \tilde{u} \to u \end{cases}$$

Indeed, Kähler structure transforms as $\tau \to \dfrac{\tau}{1+\lambda\tau}$.

1.4. What is an M-theory TsT?

M-theory T-duality: [Ganor-Ramgoolam-Taylor] [Sen] [Aharony]

- Suppose that M-theory is compactified on the 3-torus.

$SL(3,\mathbb{Z}) \times SL(2,\mathbb{Z})$ symmetry (geometric and non-geometric)

T^3

$$\tau^M = C_{123} + i\sqrt{g_{(123)}} = \tau$$

↓ 3 isometry directions

M-theory TsT $(u,v,w)_\lambda$ consists of 5 steps:

$(u,v,w)_\lambda =$

1. reduction on w
2. T-duality $u \to \tilde{u}$
3. shift $v \to v + \lambda\,\tilde{u}$
4. T-duality $\tilde{u} \to u$
5. oxidation on w

deformation parameter (λ)

Indeed, Kähler structure transforms as $\tau^M \to \dfrac{\tau^M}{1+\lambda\tau^M}$.

1.3. What is a TsT-transformation?

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[Frolov]

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2. Shift $v \to v + \lambda\, \tilde{u}$
3. T-duality $\tilde{u} \to u$

[Hellerman-Orlando-Reffert]
[Lambert-Orlando-Reffert]
[Lambert-Orlando-Reffert-YS]

Nothing but string theory of Ω-deformation!

(deformation w/ less SUSYs)

Known: how to preserve SUSYs explicitly
(=at the level of Killing spinors)

1.4. Relation to non-commutativity (skipped)

- **Another interesting point for YB deformation**

Via field redefinition, one obtains the relations:

(open $\rightleftarrows$ closed)

$$G_{MN} = (g - Bg^{-1}B)_{MN}$$

$$G_s = g_s \left(\frac{\det(g+B)}{\det g} \right)^{1/2}$$

$$\Theta^{MN} = -\left((g+B)^{-1}B(g-B)^{-1}\right)^{MN} .$$

↑**bi-vector: measures the non-commutativity**

1.4. Relation to non-commutativity (skipped)

- **Another interesting point for YB deformation**

Via field redefinition, one obtains the relations:
(open ⇄ closed)

[Araujo-Bakhmatov-Colgain-Sakamoto-Jabbari-Yoshida]

$$G_{MN} = (g - Bg^{-1}B)_{MN}$$ → **undeformed AdS5xS5 metric**

$$G_s = g_s \left(\frac{\det(g+B)}{\det g} \right)^{1/2}$$ → **constant**

$$\Theta^{MN} = -\left((g+B)^{-1}B(g-B)^{-1}\right)^{MN}.$$ → **only relevant parameter.**

↑**bi-vector: measures the non-commutativity**

In summary,

$$\Theta^{MN} = r^{MN}$$ → **Cl. r-matrix measures the non-commutativity**

1.3. Motivations

☆ **Interplay between TsT, NC, and supersymmetry (= Killing spinor).**

- **We want to revisit the TsT deformation from the view point of Killing spinors.**

- **We present a concise recipe for reconstructing Killing spinors for TsT-deformed backgrounds.**

- **Through the case by case study, we expect to find (empirically) some general formula of Killing spinors applicable for other YB deformations.**

The rest of my talk

1. ~~Introduction • Motivation~~

2. T-duality versus SUSY

3. Recipe for Killing spinors via TsT and examples

4. Comments

2. T-duality versus supersymmetry

Prerequisites:
Buscher rules, SUSY variations in type II SUGRA

2. A wonderful reference

PHYSICAL REVIEW D VOLUME 51, NUMBER 6 15 MARCH 1995

Duality versus supersymmetry and compactification

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(Received 4 November 1994)

We study the interplay between T duality, compactification, and supersymmetry. We prove that when the original configuration has unbroken space-time supersymmetries, the dual configuration also does if a special condition is met: the Killing spinors of the original configuration have to be independent of the coordinate which corresponds to the isometry direction of the bosonic fields used for duality. Examples of "losers" (T duals are not supersymmetric) and "winners" (T duals are supersymmetric) are given.

PACS number(s): 11.30.Pb, 04.65.+e, 04.70.Dy, 04.30.−w

I. INTRODUCTION

Target-space duality (T duality) is a powerful tool for generating new classical solutions of string theory. It can be used in the σ model context to generate new exact solutions but also in the context of the leading order in α' effective action to generate new solutions to the low-energy equations of motion. Some of these solutions have unbroken supersymmetries. The purpose of this paper is to study the generic relation between the supersymmetric properties of the original configuration and the dual one in the context of the low-energy effective action.

It has been observed that in some cases T duality preserves unbroken supersymmetry. Well-known examples

condition that guarantees the preservation of unbroken supersymmetries that it is not satisfied by these counterexamples. We will perform this analysis in the context of $N = 1$, $d = 10$ supergravity without vector fields. More general results involving Abelian and non-Abelian vector fields and higher order α' corrections will be reported elsewhere [8]. Some of the results presented in this paper were announced in [9].

The first counterexample known to us appears in a very simple case. We have found some time ago[1] that if one starts with ten-dimensional flat space (which has all supersymmetries unbroken) in polar coordinates and performs a T-duality transformation with respect to the

2. Example: loser's and winner's cases

cf: [Bergshoeff-Kallosh-Ortín]

Loser: take a flat space in type IIA

$$ds^2 = -dt^2 + d\rho^2 + \rho^2 d\varphi^2 + \sum_{I=3}^{9} dx^I dx^I \quad \rightarrow \quad \epsilon = e^{\frac{1}{2}\varphi\Gamma_{\rho\varphi}}\epsilon_0$$

T-dualize ↓

$$ds^2 = -dt^2 + d\rho^2 + \rho^{-2} d\tilde{\varphi} + \sum_{I=3}^{9} dx^I dx^I \,, \quad \Phi = -\ln\rho \quad \rightarrow \quad \Gamma_\rho \tilde{\epsilon} = 0 \rightarrow \tilde{\epsilon} = 0 \,.$$

You lose!

Winner: use two polar coodinates

$$ds^2 = -dt^2 + d\rho_1^2 + \rho_1^2 d\phi_1^2 + d\rho_2^2 + \rho_2^2 d\phi_2^2 + \sum_{I=5}^{9} dx^I dx^I$$

$$\rightarrow \quad \epsilon = e^{\frac{\phi_+}{2}(\Gamma_{\rho_1\phi_1} + \Gamma_{\rho_2\phi_2})} e^{\frac{\phi_-}{2}(\Gamma_{\rho_1\phi_1} - \Gamma_{\rho_2\phi_2})}\epsilon_0$$

$(\phi_\pm = \phi_1 \pm \phi_2)$

Consider T-duality in ϕ_+ ↓

☆ Non-zero solution is found if $\partial_{\phi_+}\epsilon = 0$.

You win!

To be more precise,

$$\begin{cases} \tilde{\epsilon}_+ = -\dfrac{1}{\sqrt{g_{\phi_+\phi_+}}}\hat{\Gamma}_{\phi_+} \Pi^{\phi_+}\epsilon_+ \\ \tilde{\epsilon}_- = \Pi^{\phi_+}\epsilon_- \end{cases}$$

1/2 SUSY preserved!

2. Example: loser's and winner's cases

cf: [Bergshoeff-Kallosh-Ortín]

Loser: take a flat space in type IIA

$$ds^2 = -dt^2 + d\rho^2 + \rho^2 d\varphi^2 + \sum_{I=3}^{9} dx^I dx^I \quad \rightarrow \quad \epsilon = e^{\frac{1}{2}\varphi\Gamma_{\rho\varphi}}\epsilon_0$$

T-dualize ↓

$$ds^2 = -dt^2 + d\rho^2 + \rho^{-2} d\tilde{\varphi} + \sum_{I=3}^{9} dx^I dx^I \,, \qquad \Phi = -\ln\rho \quad \rightarrow \quad \Gamma_\rho \tilde{\epsilon} = 0 \rightarrow \tilde{\epsilon} = 0 \,.$$

You lose!

👎😈

Winner: use two polar coodinates

$$ds^2 = -dt^2 + d\rho_1^2 + \rho_1^2 d\phi_1^2 + d\rho_2^2 + \rho_2^2 d\phi_2^2 + \sum_{I=5}^{9} dx^I dx^I$$

$$\rightarrow \quad \epsilon = e^{\frac{\phi_+}{2}(\Gamma_{\rho_1\phi_1}+\Gamma_{\rho_2\phi_2})} e^{\frac{\phi_-}{2}(\Gamma_{\rho_1\phi_1}-\Gamma_{\rho_2\phi_2})}\epsilon_0$$

$(\phi_\pm = \phi_1 \pm \phi_2)$

Consider T-duality in ϕ_+ ↓

☆ Non-zero solution is found if $\partial_{\phi_+}\tilde{\epsilon} = 0$.

To be more precise,

$$\begin{cases} \tilde{\epsilon}_+ = -\dfrac{1}{\sqrt{g_{\phi_+\phi_+}}}\hat{\Gamma}_{\phi_+}\Pi^{\phi_+}\epsilon_+ \\ \tilde{\epsilon}_- = \Pi^{\phi_+}\epsilon_- \end{cases}$$

Π^{ϕ_+} : a projector orthogonal to $(\Gamma_{\rho_1\phi_1}+\Gamma_{\rho_2\phi_2})$

[Kosmann, 1971]
[Kelekci-Lozano -Macpherson-Colgain]
[Sfetsos-Thompson]

2. Remark on Dérivées de Lie des spineurs

Indeed, the condition to remove the dependence of isometry directions from the Killing spinor can be written using Kosmann Lie spinorial derivative:

$$\mathcal{L}_K \epsilon = K^m \nabla_m \epsilon + \frac{1}{4}(dK)_{mn}\Gamma^{mn}\epsilon \overset{!}{=} 0$$

(K is a Killing vector of the isometry direction to be T-dualized.)

In general, for type IIA ⇆ type IIB via T-duality See [Kelekci-Lozano-Macpherson-Colgain]

$$\delta\tilde{\lambda} = 0, \delta\tilde{\Psi} = 0 \quad \textbf{if} \quad \delta\lambda = 0, \delta\Psi = 0, \text{ and } \mathcal{L}_{\partial_z}\epsilon = 0$$

and

$$\tilde{\epsilon}_+ = -\Gamma^z \epsilon_+$$
$$\tilde{\epsilon}_- = \epsilon_-$$

(indep. of isometry directions)

used for TsT! →→→

3. Recipe for Killing spinors via TsT

3.1. Recipe for the Killing spinor

Take the TsT transformation denoted by $(u, v)_\lambda$.

In summary,

☆ **Killing spinors** $\epsilon^{(\text{fin})}$ **after TsT:**

$$\epsilon_+^{(\text{fin})} = \frac{1}{\sqrt{g_{uu}^{(\text{fin})}}}\hat{\Gamma}_u^{(\text{fin})} \frac{1}{\sqrt{g_{uu}^{(\text{in})}}}\hat{\Gamma}_u^{(\text{in})} \Pi^{\text{TsT}} \epsilon_+^{(\text{in})}$$

$$\epsilon_-^{(\text{fin})} = \Pi^{\text{TsT}} \epsilon_-^{(\text{in})}$$

← initial, undeformed

☆ **The insertion of projector(s)** Π^{TsT} **gets rid of u/v-dependence.**

☆ **The projector can be understood the vanishing Kosmann Lie deriv.** $\mathcal{L}_K \epsilon = 0$

3.2 Case by Case study (skipped)

☆ Since TsT needs two compact directions $(u, v)_\lambda$,
we can have a couple of possibilities for the action of U(1):

For flat space:

susy:

(i) U(1)'s act freely on both u and v. — completely preserved

(ii) U(1)'s act freely on u, but not on v. — Mixing needed

(iii) U(1)'s do not act freely on u or v at all. — Mixing needed

For AdS5xS5, (ii) and (iii), of course.

☆ (ii) and (iii) differ from the types of NC parameters.

3.2 Example: Lunin-Maldacena like

[Lunin-Maldacena]

Take a 10d flat space:

$$ds^2 = -(dx^0)^2 + (dx^1)^2 + \sum_{i=1}^{3}(d\rho_i^2 + \rho_i^2 d\phi_i^2) + (dx^9)^2$$

After introducing $\phi_1 = \psi - \varphi_1\,, \quad \phi_2 = \psi + \varphi_1 + \varphi_2\,, \quad \phi_3 = \psi - \varphi_2$ **,**

do the TsT $(\varphi_1, \varphi_2)_\lambda$ **(**$\rightarrow$ $r = (n_{12} + n_{34}) \wedge (n_{34} + n_{56})$ **).**

Using the recipe,

$$\begin{cases} \epsilon_+^{(\text{fin})} = \frac{1+\lambda(\rho_1\rho_2\Gamma_{\phi_1\phi_2}+\rho_2\rho_3\Gamma_{\phi_2\phi_3}+\rho_3\rho_1\Gamma_{\phi_3\phi_1})}{\sqrt{1+\lambda^2(\rho_1^2\rho_2^2+\rho_2^2\rho_3^2+\rho_3^2\rho_1^2)}}\epsilon_+^{(\text{in})} \\ \epsilon_-^{(\text{fin})} = \epsilon_-^{(\text{in})} \end{cases}$$

with $\epsilon^{(\text{in})} = e^{\frac{\psi}{2}(\Gamma_{\rho_1\phi_1}+\Gamma_{\rho_2\phi_2}+\Gamma_{\rho_3\phi_3})}\Pi^{\varphi_1}\Pi^{\varphi_2}\epsilon_0\,.$

1/4 SUSY!

Note! **The associated non-commutative parameter is given by**

$$\Theta = -\lambda(\partial_{\phi_1} \wedge \partial_{\phi_2} + \partial_{\phi_2} \wedge \partial_{\phi_3} + \partial_{\phi_3} \wedge \partial_{\phi_1})\,.$$

3.2 Example: Lunin-Maldacena like

[Lunin-Maldacena]

Using the NC parameter and open string frame,

$$\begin{cases} \epsilon_+^{(\mathrm{fin})} = \dfrac{1+\lambda(\rho_1\rho_2\Gamma_{\phi_1\phi_2}+\rho_2\rho_3\Gamma_{\phi_2\phi_3}+\rho_3\rho_1\Gamma_{\phi_3\phi_1})}{\sqrt{1+\lambda^2(\rho_1^2\rho_2^2+\rho_2^2\rho_3^2+\rho_3^2\rho_1^2)}}\,\epsilon_+^{(\mathrm{in})} \\ \epsilon_-^{(\mathrm{fin})} = \epsilon_-^{(\mathrm{in})} \end{cases}$$

with $\epsilon^{(\mathrm{in})} = e^{\frac{\psi}{2}(\Gamma_{\rho_1\phi_1}+\Gamma_{\rho_2\phi_2}+\Gamma_{\rho_3\phi_3})}\Pi^{\varphi_1}\Pi^{\varphi_2}\epsilon_0\,.$

Find some general formula

$$\epsilon_+^{(\mathrm{fin})} = \frac{1+\frac{1}{2}\Theta^{MN}\hat{\Gamma}_{MN}}{\sqrt{1+\frac{1}{2}\Theta^{MN}\Theta_{MN}}}\epsilon_+^{(\mathrm{in})}$$

$$= \exp\left(\frac{\arctan(\frac{1}{2}\Theta^{MN}\Theta_{MN})}{\sqrt{\frac{1}{2}\Theta^{MN}\Theta_{MN}}}\cdot\frac{1}{2}\Theta^{MN}\hat{\Gamma}_{MN}\right)\epsilon_+^{(\mathrm{in})}\,.$$

with $\Theta = -\lambda(\partial_{\phi_1}\wedge\partial_{\phi_2}+\partial_{\phi_2}\wedge\partial_{\phi_3}+\partial_{\phi_3}\wedge\partial_{\phi_1})\,.$

☆ **This formula applies in the examples for flat space and AdS in the paper!**

3.3 Trial: M-theory TsT

Never mind "integrability" in M-theory.

Consider the AdS7xS4 background:

$$ds^2 = dr^2 + e^r\left(-dx_0^2 + dx_1^2 + \sum_{i=1}^{2}(d\rho_i^2 + \rho_i^2 d\varphi_i^2)\right) + d\theta_2^2 + \sin^2\theta_2(d\theta_1^2 + \cos^2\theta_1 d\phi_1^2 + \sin^2\theta_1 d\phi_2^2)$$

$$C_3 = -\frac{3}{4}\cos 2\theta_1 \sin^3\theta_2 d\theta_2 \wedge d\phi_1 \wedge d\phi_2\,.$$

Introducing $\phi_+ = \frac{\phi_1+\phi_2}{2}\,,\quad \varphi_+ = \frac{\varphi_1+\varphi_2}{2}$ **, do the M-theory TsT** $(\phi_+, x^1, \varphi_+)_\lambda$

The deformed Killing spinor preserves 4 supercharges and takes the form

$$\epsilon^{(\text{fin})} = \Delta^{1/6}\left[\Gamma_- + \frac{1-\lambda(e^{r/2})^2\sin\theta_2\sqrt{\rho_1^2+\rho_2^2}(\cos\theta_1\Gamma_{\phi_1 x^1} + \sin\theta_1\Gamma_{\phi_2 x^1})}{\Delta}\Gamma_+\right]\epsilon^{(\text{in})}$$

with $\Delta = \sqrt{1+\lambda^2 e^{2r}\sin^2\theta_2(\rho_1^2+\rho_2^2)}$

☆ **This suggests a notion for non-commutativity in M-theory?**

4. Comments

4. Conclusions and Outlook

☆ We reconstructed the Killing spinor in the TsT deformed background using the open string metric and NC parameter Θ .

☆ The exponential factor written by Θ is expected to apply in more complicated YB deformations (higher-rank, non-Abelian…). [Orlando-Reffert-Sakamoto-Yoshida], [Borsato-Wulff]

☆ Kosmann Lie derivative condition in generalized supergravity?

not changed to my best knowledge.

☆ Ω-deformation has been linked to the YB deformation. Its gravity dual?

☆ We tried the M-theory TsT.
Is the result related to non-commutativity in M-theory? 3-Lie bracket?

Thank you!

Ευχαριστώ!