AdS6 T-duals and the recent classification by D'Hoker et al of AdS6 x S2 geometries in Type IIB

Yolanda Lozano (U. Oviedo)

Dualities and Generalized Geometries
Corfu, September 2018

Universidad de Oviedo

1. Introduction & motivation

The AdS_6/CFT_5 correspondence remains much less understood than its cousins in other dimensions

This is largely due to the fact that in 5d there are no maximally supersymmetric CFTs

Indeed, in 5d the superconformal algebra is unique: $F(4)$
It contains 8 supersymmetries (enhanced to a total of 16)

Accordingly, AdS_6 backgrounds are quite unique:

There is a unique AdS_6 solution to massive Type IIA (the Brandhuber-Oz solution (Brandhuber, Oz'99)), there are no solutions in M-theory, and a kind of no-go theorem existed in Type IIB (Passias'12)

AdS_6/CFT_5 duals

5d gauge theories are non-renormalizable:

$$[g^2] = M^{-1} \quad \rightarrow \quad g^2 E \quad \rightarrow \quad \text{UV completion}$$

Indeed, they can flow to strongly coupled CFTs in the UV for specific gauge groups and matter content

(Seiberg'96; Intriligator, Morrison, Seiberg'97)

String theory realisations are known in some cases that can be used to construct their AdS_6 duals

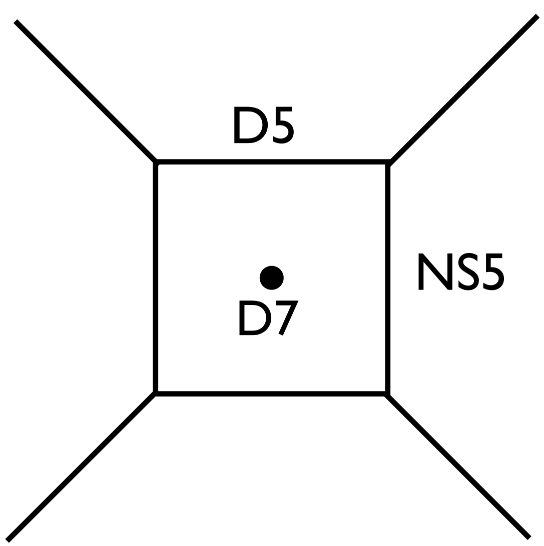
For example, $Sp(N)$ (with specific matter content) can be realised in Type I' in a D4/D8/O8 system (Seiberg'96)

The Brandhuber-Oz solution arises as its near horizon geometry

More general string theory realisations can be given in terms of (p,q) 5-brane webs in Type IIB (Aharony, Hanany'97; Aharony, Hanany, Kol'97)

The 5d field theory lives on D5-branes which are of finite extent on one dimension and are embedded in a web of semi-infinite (p,q) 5-branes as well as optional 7-branes (DeWolfe, Hanany, Iqbal, Katz'99)

These webs are 5d realisations of Hanany-Witten brane set-ups



Their AdS_6 duals remained unknown till the recent work of
D'Hoker, Gutperle, Karch, Uhlemann'16; D'Hoker, Gutperle, Uhlemann'17,18

These works provide a complete classification of AdS_6 solutions to Type IIB SUGRA, which are firm candidates for holographic duals of CFTs living in (p,q) 5-brane webs.

T-duality played a very important role in these developments:

The first AdS_6 solutions in Type IIB were constructed by acting with T-duality (Abelian and non-Abelian) on the Brandhuber-Oz solution

(Y.L., O Colgain, Rodriguez-Gomez, Sfetsos'12; Y.L., O Colgain, Rodriguez-Gomez'13)

The existence of these solutions raised the interest in the study of classifications of AdS6 solutions in Type IIB:

Apruzzi, Fazzi, Passias, Rosa, Tomasiello'14; Kim, Kim, Suh'15; Kim, Kim'16; Gutowski, Papadopoulos'17,

which culminated with the work of

D'Hoker, Gutperle, Karch, Uhlemann'16; D'Hoker, Gutperle, Uhlemann'17,18

The Abelian T-dual solution describes the same 5d $Sp(N)$ fixed point theory as the Brandhuber-Oz solution but now in terms of a D5, NS5, D7/O7 brane system in Type IIB

The CFT dual of the non-Abelian T-dual solution is not known. In principle it can be different from the previous fixed point theory, because non-Abelian T-duality has not been proved to be a string theory symmetry

In fact, examples so far show that NATD may change the CFT dual to the AdS backgrounds in which it is applied

In all these examples the non-Abelian T-dual background is associated to QFT living on (Dp,NS5) Hanany-Witten brane set-ups

(Y.L, Núñez'16; Y.L, Macpherson, Montero, Núñez'16; Y.L., Núñez, Zacarías'17; Itsios, Y.L., Montero, Núñez'17)

In this talk we will describe how the Abelian T-dual and non-Abelian T-dual solutions fit in the formalism of DGU

This will require extending their global classification

We will propose a candidate CFT dual to the non-Abelian T-dual solution

Based on:

Y.L., Macpherson, Montero: 1809.xxxx

Outline:

1. Introduction and motivation
2. Brief review of the DGU global solutions
3. The Brandhuber-Oz solution of massive IIA
4. The Abelian T-dual of Brandhuber-Oz
5. The non-Abelian T-dual of Brandhuber-Oz
6. Conclusions

2. Brief review of the DGU global solutions

$AdS_6 \times S^2 \times \Sigma_2$ solutions, compatible with the $SO(2,5) \times SO(3)$ bosonic subalgebra of $F(4)$. 16 SUSYs

Σ_2 is a Riemann surface with a boundary

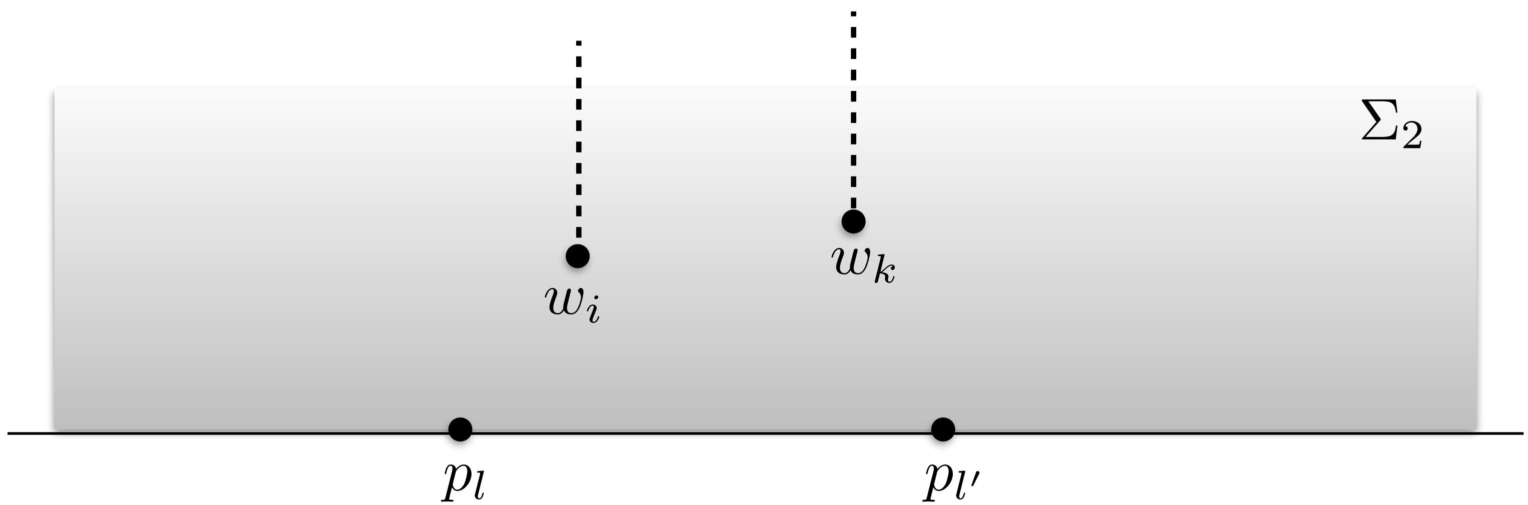
General solutions are expressed in terms of two locally holomorphic functions $\mathcal{A}_\pm$ on Σ_2

Allowing $\partial \mathcal{A}_\pm$ to have simple poles on the boundary, $\mathcal{A}_\pm$ were explicitly computed for Σ_2 the upper half plane.

The solutions were shown to behave near the poles as the near brane limit of (p,q) 5-branes

The poles were interpreted as the remnants of external (p,q) 5-branes in a 5-brane web

7-branes were included in a later paper, by allowing the supergravity fields to have non-trivial $SL(2,\mathbb{R})$ monodromy at isolated punctures in the interior of Σ_2



This is consistent with a supersymmetric D5,NS5,D7 intersection:

$$\begin{array}{rc|ccccccccc}
D5: & \times & \times & \times & \times & \times & \times & - & - & - & - \\
\mathrm{NS5}: & \times & \times & \times & \times & \times & - & - & - & - & \times \\
D7: & \times & \times & \times & \times & \times & - & \times & \times & \times & -
\end{array}$$

Specific examples of 5d CFTs and their AdS duals have been discussed recently, and holographic quantities have been tested directly against their field theory counterparts

(Bergman, Rodriguez-Gomez, Uhlemann'18)

3. The Brandhuber-Oz solution of massive IIA

The near horizon geometry of the D4, D8/O8 system is a fibration of AdS_6 over half- S^4 with an S^3 boundary at the position of the O8-plane, preserving 16 SUSYs

$$ds^2 = \frac{W^2\,L^2}{4}\left[9\,ds^2(AdS_6) + 4\,ds^2(S^4)\right] \qquad \theta \in [0, \frac{\pi}{2}]$$

$$F_4 = 5\,L^4\,W^{-2}\,\sin^3\theta\,d\theta \wedge \mathrm{Vol}(S^3)$$

$$e^{-\phi} = \frac{3\,L}{2\,W^5}\,, \qquad W = (m\,\cos\theta)^{-\frac{1}{6}} \qquad m = \frac{8-N_f}{2\pi l_s}$$

- $SO(5)$ symmetry broken to $SO(4) \sim SU(2) \times SU(2)$:

  $SU(2) \leftrightarrow SU(2)_R$ R-symmetry of the field theory

  $SU(2) \leftrightarrow$ global symmetry massless antisym. hyper

4. The Abelian T-dual of Brandhuber-Oz

Take the $AdS_6 \times S^4$ background

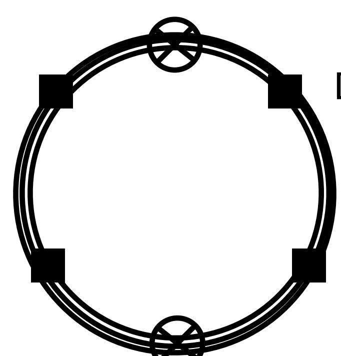
$$ds^2 = \frac{W^2 L^2}{4}\Big[9ds^2(AdS_6) + 4\Big(d\theta^2 + \sin^2\theta ds^2(S^3)\Big)\Big]$$

$$F_4 = 5L^4 W^{-2} \sin^3\theta \, d\theta \wedge \mathrm{Vol}(S^3)$$

and dualise w.r.t. the U(1) fibre

The resulting background preserves all SUSY

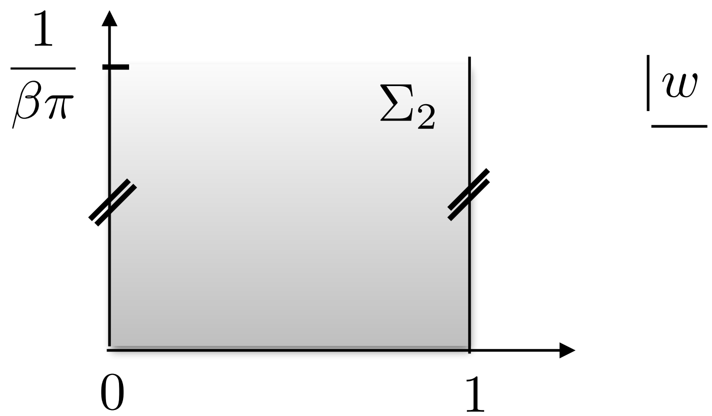
It realises the $Sp(N)$ fixed point theory in a D5,NS5,D7/O7 brane system:



The Abelian T-dual as a DGU global solution

The Riemann surface is an annulus:

$$w = \frac{\psi}{\pi} + \frac{i}{\beta\pi}(1 - (\cos\theta)^{2/3})$$



DGU discussed (p,q) 5-branes on the annulus. We added the D7 branes. The resulting holomorphic functions are given by:

$$\mathcal{A}_\pm = \mathcal{A}^0_\pm + \sum_{l=1}^{L} Y^l_\pm \log(\theta_1(w - p_l|\tau)) + (r \mp is)\int_1^w dz f(z) \sum_{l=1}^{L} Y^l \partial_z \log(\theta_1(z - p_l|\tau))$$

$\theta_1(w|\tau)$ are Jacobi theta-functions, that ensure the required periodicities under $w \to w+1,\ w \to w+\tau$

$$f(w) = \sum_{j=i}^{I} \frac{n_i^2}{4\pi}\left(\log\left(\gamma_i \frac{\theta_1(w - w_i|\tau)}{\theta_1(w - \overline{w_i}|\tau)}\right) - \frac{2\pi i}{\tau}(w_i - \overline{w_i})w\right)$$

The $\mathcal{A}_\pm$ of the ATD solution are reproduced from this expression

First explicit solution for the annulus (with D7/O7 branes)

5. The non-Abelian T-dual of Brandhuber-Oz

Take the $AdS_6 \times S^4$ background

$$ds^2 = \frac{W^2 L^2}{4}\left[9ds^2(AdS_6) + 4\Big(d\theta^2 + \sin^2\theta\, ds^2(S^3)\Big)\right]$$

$$F_4 = 5L^4 W^{-2} \sin^3\theta\, d\theta \wedge \mathrm{Vol}(S^3)$$

Dualise it w.r.t. one of the $SU(2)$ symmetries

In spherical coordinates adapted to the remaining $SU(2)$:

$$ds^2 = \frac{W^2\, L^2}{4}\left[9\, ds^2(AdS_6) + 4\, d\theta^2\right] + e^{-2\,A}\, dr^2 + \frac{r^2\, e^{2\,A}}{r^2 + e^{4\,A}}\, ds^2(S^2)$$

$$B_2 = \frac{r^3}{r^2 + e^{4\,A}}\, \mathrm{Vol}(S^2) \qquad e^{-\phi} = \frac{3\,L}{2\,W^5}\, e^{A}\sqrt{r^2 + e^{4\,A}}$$

$$F_1 = -G_1 - m\, r\, dr \qquad F_3 = \frac{r^2}{r^2 + e^{4\,A}}\left[-r\, G_1 + m\, e^{4A}\, dr\right] \wedge \mathrm{Vol}(S^2)$$

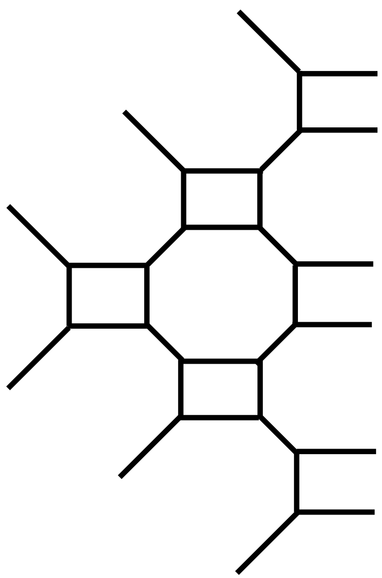
- It solves the IIB equations of motion, preserving all SUSY
- What about r ?
  - Background perfectly smooth for all $r \in \mathbb{R}^+$
  - No global properties inferred from the NATD
  - How do we interpret the running of r to infinity in the CFT?
- Boundary at $\theta = \pi/2$ inherited
- Singular at $\theta = 0$ where the original S^3 shrinks (due to the presence of NS5-branes)

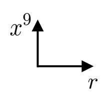
  This is the tip of a cone with S^2 boundary $\rightarrow$ We have to care about large gauge transformations

- Large gauge transformations modify the quantised charges such that $N_{D5} = nN_{D7}$ in each $[n\pi, (n+1)\pi]$ interval

We have also NS5 charge, such that every time we cross a π interval one unit of NS5 charge is created

This is compatible with a D5/NS5 brane set-up:





The NS5 are bent due to the different number of D5 ending on each side

There is a symmetry under $x^9 \to -x^9$ due to the orientifold action

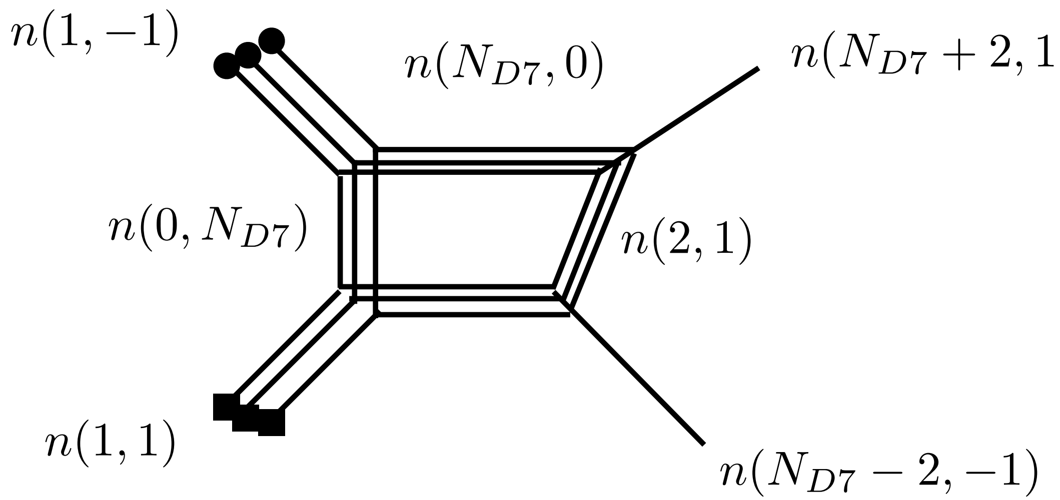
Add the orientifold:

A worldsheet analysis shows that under NATD: $\Omega \to I_\chi \Omega$

$\Rightarrow$ The O8 plane is mapped onto a O5 plane, located at $r = 0$

This O5 is in fact a O7 wrapped on the S^2

Then:



Here:

We have completed the infinite brane set-up with semi-infinite (p,q) 5-branes

$Sp(N_{D7}) \times Sp(2N_{D7}) \times \cdots \times Sp(nN_{D7})$ fixed point theory

At each interval the condition $N_F \leq 2N+4$ for the existence of a $Sp(N)$ UV fixed point theory is satisfied, according to the classification in Intriligator, Morrison, Seiberg'97

This is supported by the central charge and EE, which scale with $N^{5/2}$

The non-Abelian T-dual as a DGU global solution

6. Conclusions

The Abelian and non-Abelian T-duals of Brandhuber-Oz, the first AdS6 solutions known in Type IIB SUGRA, fit within an extension of the $AdS_6 \times S^2$ global solutions with 7-branes recently classified by D'Hoker, Gutperle and Uhlemann

Showing this required extending the formalism of DGU to the strip and including D7-branes in the annulus

Non-trivial examples involving smeared NS5 and D7 branes, and O7

The Abelian T-dual solution provides the first known example for the annulus

We have given a candidate CFT dual to the non-Abelian T-dual solution, consistent with the classification of 5d fixed point theories by Intriligator, Morrison and Seiberg

Would be interesting:

To find explicit solutions for the strip other than the NATD

Clarify further the dual CFT of the NATD:

- Computation of observables in the dual CFT
- Careful study of global symmetries, enhancement..

Compute the backreaction in the geometry of the external 5-branes added to complete the infinite quiver $\longrightarrow$

Complete the non-Abelian T-dual solution

Would be interesting:

To find explicit solutions for the strip other than the NATD

Clarify further the dual CFT of the NATD:

- Computation of observables in the dual CFT
- Careful study of global symmetries, enhancement..

Compute the backreaction in the geometry of the external 5-branes added to complete the infinite quiver $\longrightarrow$

Complete the non-Abelian T-dual solution

THANKS!

2. Basics of NATD

Using the string sigma-model Rocek and Verlinde proved that Abelian T-duality is a symmetry to all orders in g_s and α'

(Buscher'88; Rocek, Verlinde'92)

The extension to arbitrary wordsheets determines the global properties of the dual variable:

$$\theta \in [0, 2\pi] \quad \xrightarrow{\mathsf{T}} \quad \tilde{\theta} \in [0, 2\pi]$$

In the non-Abelian case neither proof works

Variables living in a group manifold are substituted by variables living in its Lie algebra

$$g \in SU(2) \quad \xrightarrow{\mathsf{NAT}} \quad \chi \in \mathbb{R}^3$$

In the absence of global information the new variables remain non-compact

In more detail:

Rocek and Verlinde's formulation of Abelian T-duality for ST in a curved background (Buscher'88) :

$$\boxed{S = \frac{1}{4\pi\alpha'}\int\Big(g_{\mu\nu}\, dX^\mu \wedge *dX^\nu + B_{\mu\nu}\, dX^\mu \wedge dX^\nu\Big) + \frac{1}{4\pi}\int R^{(2)}\phi}$$

i) Identify an Abelian isometry: $X^\mu = \{\theta, X^\alpha\}$ such that

$\theta \to \theta + \epsilon$ and $\partial_\theta(\text{backgrounds}) = 0$

ii) Gauge the isometry: $d\theta \to D\theta = d\theta + A$

A non-dynamical gauge field / $\delta A = -d\epsilon$

iii) Add a Lagrange multiplier term: $\tilde{\theta}\, dA$, such that

$$\int \mathcal{D}\tilde{\theta} \;\rightarrow\; dA = 0 \;\Rightarrow\; A \text{ exact}$$

(in a topologically trivial worldsheet)

+ fix the gauge: $A = 0 \rightarrow$ Original theory

iv) Integrate the gauge field

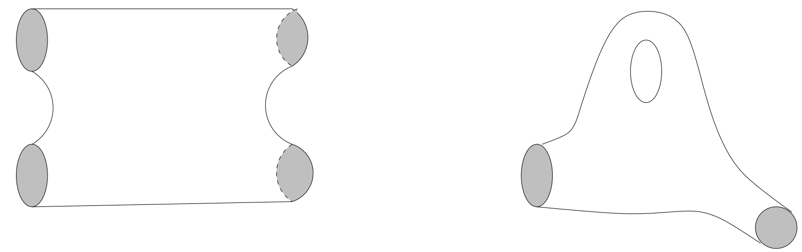
+ fix the gauge: $\theta = 0 \quad \rightarrow$ Dual sigma model:

$\{\theta, X^\alpha\} \rightarrow \{\tilde{\theta}, X^\alpha\}$ and

$(\tilde{g}, \tilde{B}_2, \tilde{\phi})$ given by Buscher's formulae

- Conformal invariance? Original and dual theories can be obtained from the gauged Lagrangian either gauging a vectorial or an axial combination of chiral currents

- Arbitrary worldsheets? (symmetry of string perturbation theory):



(a) (b)

Large gauge transformations: $$\oint_\gamma d\epsilon = 2\pi n\,;\;\; n \in \mathbb{Z}$$

To fix them:

Multivalued Lagrange multiplier: $$\oint_\gamma d\tilde{\theta} = 2\pi m\,;\;\; m \in \mathbb{Z}$$

such that

$$\int [\text{exact}] \;\rightarrow\; dA = 0 \qquad + \qquad \int [\text{harmonic}] \Rightarrow A \text{ exact}$$

This fixes the periodicity of the dual variable

Non-Abelian T-duality

(De la Ossa, Quevedo'93)

Non-Abelian continuous isometry: $X^m \to g^m_{\ n} X^n \,, g \in G$

i) Gauge it: $dX^m \to DX^m = dX^m + A^m_{\ n} X^n$

$A \in$ Lie algebra of G $\qquad A \to g(A+d)g^{-1}$

ii) Add a Lagrange multiplier term: $\mathrm{Tr}(\chi F)$

$F = dA - A \wedge A$

$\chi \in$ Lie Algebra of G, $\quad \chi \to g\chi g^{-1}$, such that

$\int \mathcal{D}\chi \;\to\; F = 0 \;\Rightarrow\; A$ exact
(in a topologically trivial worldsheet)

+ fix the gauge: $A = 0 \;\Rightarrow$ Original theory

iii) Integrate the gauge field + fix the gauge $\rightarrow$ Dual theory

However:

- Non-involutive
- Higher genus generalization? Set to zero $W_\gamma = P\, e^{\oint_\gamma A}$
- Global properties?

  For SU(2): $\chi \in \mathbb{R}^3$: Global completion of $\mathbb{R}^3$?

- Conformal invariance not proved in general (only vectorial gauging)

True symmetry in String Theory?

Still useful as a solution generating technique

Still, NATD has been proved to be a very useful solution generating technique (Sfetsos and Thompson (2010))

But we will have to see how we interpret the non-compact directions in the context of AdS/CFT