Complex Langevin analysis

of the spontaneous symmetry breaking in dimensionally reduced super Yang-Mills models

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Acknowledgments

work with

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Outline

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1. Complex Action
   - The problem...
   - A solution...?
   - A solution perhaps...!
2. The IKKT model
3. IKKT Langevin dynamics
   - The theory...
   - The study...
   - The results...

Monte Carlo

Calculating expectation value integrals,

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{D}\phi \mathcal{O}[\phi] e^{-S[\phi]}}{\int_X \mathcal{D}\phi e^{-S[\phi]}}$$

by sampling the x configuration space via markovian chains.

Markovian chains

Driven by a selection probability best defined on the Boltzmann factor e^{-S}containing the action $S : (X \longrightarrow \mathbb{R}) \longrightarrow \mathbb{R}$

Complex Action Problem

Studying Theories with Lorentzian signature require analytical continuation to Euclidean signature $x^2 = ix^0$.

The Euclidean action $S : (X \longrightarrow \mathbb{R}) \longrightarrow \mathbb{C}$ becomes complex in some systems!

e^{-S} is thus no longer a viable probabilitistic selector!

Regular Monte Carlo techniques relying on the action S fail.

Re-weighting

Using the phase-quenched action S_0 ($S = S_0 - \imath\Gamma$)

A partial solution comes with re-weighting,

$$\langle\mathcal{O}\rangle = \frac{\dfrac{\int \mathcal{D}\phi\mathcal{O}[\phi]e^{\imath\Gamma}e^{-S_0[\phi]}}{\int \mathcal{D}\phi e^{-S_0[\phi]}}}{\dfrac{\int \mathcal{D}\phi e^{\imath\Gamma}e^{-S_0[\phi]}}{\int \mathcal{D}\phi e^{-S_0[\phi]}}} = \frac{\langle\mathcal{O}e^{\imath\Gamma}\rangle_0}{\langle e^{\imath\Gamma}\rangle_0}$$

The problem is still exponentially hard!

Outline

Stochastic Quantization

[DOI: 10.1016/0370-1573(87)90144-X] Poul Damgaard, Helmuth Huffel.

Stochastic evolution of QFT

$$\frac{\partial}{\partial \tau}\phi(\tau) = \upsilon(\phi(\tau)) + \eta(\tau), \qquad \phi(\tau_0) = \phi_0 : X \longrightarrow \mathbb{R}$$

Drift $\upsilon : (X \longrightarrow \mathbb{R}) \longrightarrow \mathbb{R}$

$$\upsilon(\phi) = -\frac{\delta S}{\delta \phi}[\phi]$$

Noise $\eta : X \longrightarrow \mathbb{R}$ obeying gaussian distribution

$$\langle \eta(\tau)\eta(\tau')\rangle_{\text{noise}} = 2\delta(\tau - \tau') \qquad \varrho_{\text{noise}}(\eta) = \exp\left(-\frac{1}{4}\eta^2\right)$$

Stochastic behaviour

[DOI: 10.1016/0370-1573(87)90144-X] Poul Damgaard, Helmuth Huffel.

Properties:

- The solution ϕ is random and depends on noise η,
- Even the initial condition can be random as in $\phi_0 = \eta_0$,
- ϕ obeys a probability distribution $\varrho : (X \to \mathbb{R}) \times \mathbb{R}^+ \to \mathbb{R}$

Fokker-Planck equation for solution probability

$$\frac{\partial}{\partial \tau} \varrho(\phi; \tau) = \mathcal{L}^*(\phi) \varrho(\phi; \tau) \qquad \varrho(\phi; \tau_0) = \delta(\phi - \phi_0)$$

EV equation

$$\frac{\partial}{\partial \tau} \langle \mathcal{O}(\tau) \rangle_{\text{noise}} = \langle \mathcal{L} \mathcal{O}(\tau) \rangle_{\text{noise}}$$

Criterion of convergence

[arXiv:1802.01876] Keitaro Nagata, Jun Nishimura and Shinji Shimasaki.

EV leading contributions

$$\langle \mathcal{O}(\tau) \rangle_{\text{noise}} \sim \int_0^\infty e^{(\tau-\tau_0)u} p(u,\tau) du$$

Drift norm probability

$$p(u,\tau) = \int \delta(u(x)-u)\varrho(x,\tau)dx \qquad u = \|v\|$$

Strong criterion for convergence

$$p(u,\tau) \lesssim e^{-\kappa u} \qquad \kappa > 0$$

Stochastic quantization assertion

$$\lim_{\tau\to\infty} \langle \mathcal{O}(\tau) \rangle_{\text{noise}} = \langle \mathcal{O} \rangle_w$$

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Complexification

[arXiv:1802.01876] Keitaro Nagata, Jun Nishimura and Shinji Shimasaki.

Drift term becomes complex

$$v(\phi(x)) = -\frac{\delta S[\phi]}{\delta\phi(x)}$$

ϕ is extended to complex for consistency

$$\frac{\partial\phi(\tau)}{\partial\tau} = v(\phi(\tau)) + \eta(\tau), \text{ now } \phi \text{ has to be complex}$$

Noise remains real! (compatibility condition)

$$\langle\eta(\tau)\eta(\tau')\rangle_{\text{noise}} = 2\delta(\tau-\tau') \qquad \varrho_{\text{noise}}(\eta) = \exp\left(-\frac{1}{4}\eta^2\right)$$

Discretized Langevin equation

[arXiv:1802.01876] Keitaro Nagata, Jun Nishimura and Shinji Shimasaki.

Discretized time t with n time-steps fixed or variable

$$\tau = n\bar{\epsilon}$$

Langevin process

$$\phi_{n+1} = \phi_n + \epsilon v(\phi_n) + \sqrt{\epsilon}\eta_n \qquad \phi : \mathbb{N} \longrightarrow \mathbb{C}$$

Drift $v : (\mathbb{C}) \to \mathbb{C}$

$$v(\phi_n) = -\frac{\delta S}{\delta \phi}(\phi_n)$$

Noise $\eta : \mathbb{R}$ with gaussian distribution noise : $\mathbb{R} \to \mathbb{R}_+$

$$\langle \eta_n \eta_{n'} \rangle_{\text{noise}} = 2\delta_{nn'}$$

The Type IIB string theory

Type IIB Green-Schwarz action in the Schild gauge

$$S_{\text{Schild}} = \int d^2\sigma \left(\sqrt{h} g^{-2} \left(\frac{1}{4}\{X^\mu, X^\nu\}^2 - \frac{1}{2}\imath\bar{\psi}\Gamma^\mu\{X^\mu, \psi\} \right) \right)$$

$$\{\ldots,\ldots\} \to \imath[\ldots,\ldots], \int d^2\sigma\sqrt{h}\ldots \to \mathrm{tr}\ldots \text{ and } g^{-2} = N$$

Matrix IKKT (Eucleidean) model with fixed N

$$S = S_{\text{bozon}} + S_{\text{fermion}} = -N\,\mathrm{tr}\left(\frac{1}{4}[A_\mu, A_\nu]^2 + \frac{1}{2}(\bar{\psi}_\alpha(\Gamma_\mu)_{\alpha\beta}[A_\mu, \psi_\beta]) \right)$$

Candidate for non-perturbative definition for Type IIB!

The IKKT model

Complex action

Phase produced from integrating fermions in partition function

$$Z = \int \mathcal{D}A \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp(-S) = \int \mathcal{D}A \, \mu(\mathcal{M}) \exp(-S)$$

responsible for spontaneous symmetry breaking.

Possible manifestations of the IKKT

$D = 4$, $S : \mathbb{R}$ and no SSB
$D = 6$, $S : \mathbb{C}$ with SSB ($\mu \equiv$ determinant)
$D = 10$, $S : \mathbb{C}$ with SSB ($\mu \equiv$ pfaffian)

Gaussian Expansion Method

[arXiv:1007.0883] T. Aoyama, J. Nishimura and T. Okubo.
[arXiv:1108.1293] J. Nishimura, T. Okubo and F. Sugino.

Dynamical compactification of extra dimensions

Indication of SSB of original SO(10) to SO(4) by distinction of 4 dimensional extends from the other 6.

$D = 6$

SO(6) breaks down to SO(3)... (GEM)

$D = 10$

SO(10) breaks down to... SO(3)!? (GEM)

Outline

IKKT Langevin dynamics

[arXiv:1712.07562] Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura and Stratos Kovalkov Papadoudis.

IKKT Langevin equation

$$\frac{d(A_\mu)_{ij}}{d\tau} = -\frac{\delta}{\delta(A_\mu)_{ji}}S + (\eta_\mu)_{ij} \quad A_\mu \text{ hermitian and traceless}$$

IKKT drift term

$$\frac{\delta}{\delta(A_\mu)_{ji}}S_{\text{bozon}} = N[[A_\mu A_\nu]A_\nu]_{ij}$$

$$\frac{\delta}{\delta(A_\mu)_{ji}}S_{\text{fermion}} = -\text{tr}\left(\left(\frac{\delta}{\delta(A_\mu)_{ji}}\mathcal{M}\right)\mathcal{M}^{-1}\right)$$

IKKT complexification

[arXiv:1712.07562] Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura and Stratos Kovalkov Papadoudis.

Complexification of A_μ:

- A_μ no longer hermitian, just traceless!
  - Still, the closer it is to hermitian the better.
  - SU(N) (internal) matrix gauge symmetry becomes SL(N)
- Noise η_μ remains hermitian (and traceless!).
- Observables $\mathcal{O}$ must be holomorphic extensions of their real counter-part.

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IKKT Langevin issues

[arXiv:1712.07562] Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura and Stratos Kovalkov Papadoudis.

Imaginary part escaping

Counter with gauge cooling: exploit the gauge SU(N) gauge freedom of the action on the internal degreesof freedom to select A_μ as close to real as possible, minimizing the norm

$$\|A - A^\dagger\| = \frac{1}{DN} \mathrm{tr} \sum_\mu (A_\mu - A_\mu^\dagger)^2$$

Singular drift

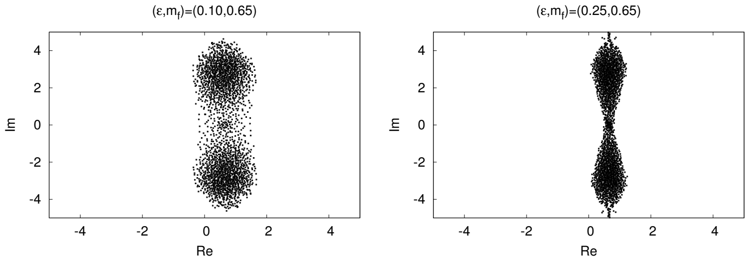
Counter with shifting fermion matrix:

$$\Delta S_{\text{fermion}} = N m_{\text{fermion}} \mathrm{tr}(\bar{\psi}_\alpha (\Gamma_6)_{\alpha\beta} \psi_\beta) \qquad \Gamma_6 = 1_2 \otimes 1_2 = 1_4$$

IKKT fermion matrix $\mathcal{M}$

[arXiv:1712.07562] Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura and Stratos Kovalkov Papadoudis.

IKKT fermion matrix eigenvalue scatter-plot



$$\Delta S_{\text{fermion}} = N m_{\text{fermion}} \text{tr}(\bar{\psi}_\alpha (\Gamma_6)_{\alpha\beta} \psi_\beta) \qquad \Gamma_6 = 1_2 \otimes 1_2 = 1_4$$

IKKT observables

[arXiv:1712.07562] Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura and Stratos Kovalkov Papadoudis.

SSB order parameter and action term

$$\lambda_\mu = N^{-1}\,\mathrm{tr}(A_\mu A_\mu) \qquad \Delta S_{\mathrm{boson}} = \frac{1}{2} N^2 \varepsilon \sum_\mu m_\mu \lambda_\mu$$

SO(6) to SO(2) granted

$$m_\mu = (0.5, 0.5, 1.0, 2.0, 4.0, 8.0)$$

$$\lambda_{12} = \frac{1}{2}(\lambda_1 + \lambda_2)$$

IKKT order parameters

[arXiv:1712.07562] Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura and Stratos Kovalkov Papadoudis.

Order parameter expectation values

$$\rho_\mu(N, \varepsilon, m_{\text{fermion}}) = R^{-1}\langle\lambda_\mu\rangle \qquad R = \sum_\nu \langle\lambda_\nu\rangle$$

Large N limit

$\lim_{N\to\infty} \rho_\mu(N, \varepsilon, m_{\text{fermion}}) = \rho_\mu(\varepsilon, m_{\text{fermion}})$

SSB limit

$\lim_{\varepsilon\to 0} \rho_\mu(\varepsilon, m_{\text{fermion}}) = \rho_\mu(m_{\text{fermion}})$

Original model limit

$\lim_{m_{\text{fermion}}\to 0} \rho_\mu(m_{\text{fermion}}) = \rho_\mu$

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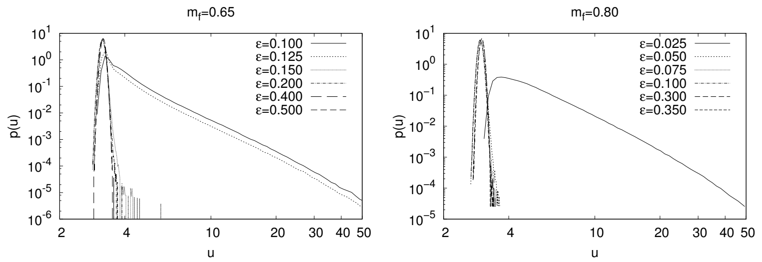
IKKT drift norm u

[arXiv:1712.07562] Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura and Stratos Kovalkov Papadoudis.

IKKT drift histogram

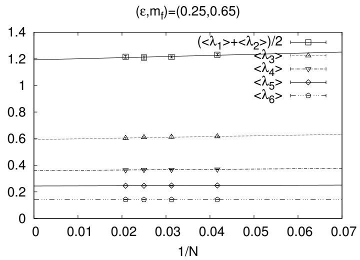


$$u^2 = \frac{1}{6N^3} \sum_{\mu} \sum_{ij} \left| \frac{\partial S}{\partial (A_\mu)_{ji}} \right|^2$$

IKKT order parameters $N \to \infty$

[arXiv:1712.07562] Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura and Stratos Kovalkov Papadoudis.

Eliminating finite-size effects at the large-N limit $N \to \infty$

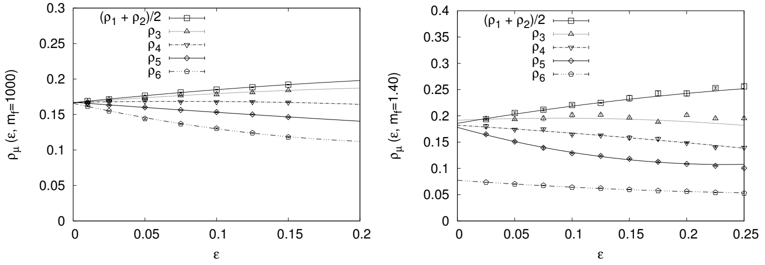


$$\langle\lambda_\mu\rangle(N,\varepsilon,m_{\text{fermion}}) \sim \langle\lambda_\mu\rangle(\varepsilon,m_{\text{fermion}}) + \alpha N^{-1}$$

IKKT order parameters $\varepsilon \to 0$

[arXiv:1712.07562] Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura and Stratos Kovalkov Papadoudis.

Quadratic extrapolation to SSB at $\varepsilon \to 0$



$$\rho_\mu(\varepsilon, m_{\text{fermion}}) \sim \rho_\mu(m_{\text{fermion}}) + \alpha\varepsilon + \beta\varepsilon^2$$

IKKT order parameters $\varepsilon \to 0$

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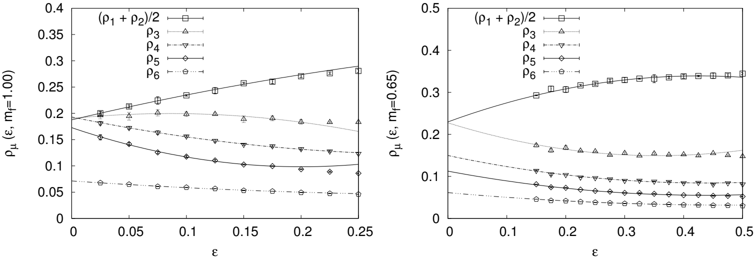
Quadratic extrapolation to SSB at $\varepsilon \to 0$

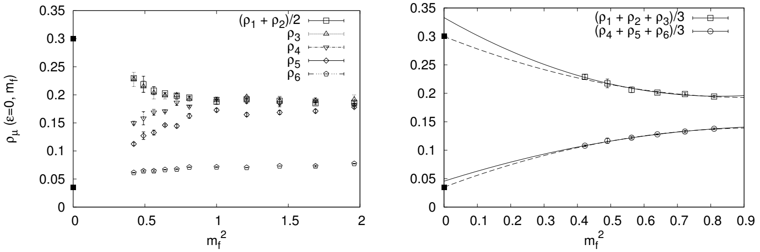


$$\rho_\mu(\varepsilon, m_{\text{fermion}}) \sim \rho_\mu(m_{\text{fermion}}) + \alpha\varepsilon + \beta\varepsilon^2$$

IKKT order parameters $m_{\text{fermion}} \to 0$

[arXiv:1712.07562] Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura and Stratos Kovalkov Papadoudis.

Quadratic extrapolation to original model at $m_{\text{fermion}} \to 0$



$$\rho_\mu(m_{\text{fermion}}) \sim \rho_\mu + \alpha m_{\text{fermion}}^2 + \beta m_{\text{fermion}}^4$$

Summary

[arXiv:1712.07562] Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura and Stratos Kovalkov Papadoudis.

Conclusions:

- Spontaneous Symmetry Breaking of the rotational symmetry of the $D = 6$ IKKT model was observed,
- At $N \to \infty$, $\varepsilon \to 0$, as m_{fermion} decreases, symmetry starts to breakdown to SO(3) as expected by the GEM result.

Outlook:

- Study of the $D = 10$ IKKT model (true reduced Type IIB superstring model)
- Looking for SO(10) $\to$ SO(4) for meaningful interpretation of dynamical compactification to 4-dimensional space-time.

~ The End ~